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SU(4) symmetry and the new resonances

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Using as a basis the idea that the new resonances discovered recently at the Stanford Linear Accelerator Center and at Brookhaven National Laboratory can be fitted into an SU(4)-symmetric scheme, we examine in detail the production and decay mechanisms of these resonances. A generalization of the Weinberg spectral-function sum rule is proposed which leads to quantitative predictions for the production cross sections of these resonances in $e\bar{e}$ annihilation. Several decay mechanisms are discussed, and arguments are presented, albeit somewhat qualitatively, to suggest that the dominant decay modes are $\psi \to \eta'\gamma$ and $\psi' \to \psi + 2\pi$. As an aid to the experimentalist searching for charmed particles, we have also tabulated the predicted masses of various mesons and baryons, using both the linear and the quadratic mass formulas.

I. INTRODUCTION

Recent experiments[1] have discovered the existence of a new resonance (hereafter called ψ) with a mass and width given by

$$M_\psi = 3.105 \pm 0.003 \text{ GeV}, \quad \Gamma_\psi \le 1.9 \text{ MeV}. \tag{1.1}$$

More recently, SLAC has reported[2] the observation of another resonance (hereafter called ψ') with the parameters

$$M_{\psi'} = 3.695 \pm 0.004 \text{ GeV}, \quad \Gamma_{\psi'} \le 2.7 \text{ MeV}. \tag{1.2}$$

A special feature of these resonances is their rather small width, so that if ψ and ψ' have strong interactions with other hadrons, the strong suppression of the decay to ordinary hadrons would seem to require the existence of a new quantum number. Such an interpretation has in fact been advanced recently by us[3] and by many others,[4-6] using as a basis the idea that ψ and ψ' may be bound states of the type $q_4\bar{q}_4$ where q_4 is a fourth quark, usually referred to as the charmed quark. With this interpretation, we identify ψ with the ground state of the $q_4\bar{q}_4$ complex and ψ' with a higher radial or rotational excitation. We may mention here that ψ and/or ψ' may not have any strong interactions with other hadrons (or if they do, these may be of some special type). One may in this case interpret ψ or ψ' to be an intermediate vector boson, as proposed by some authors.[7] In the present paper, however, we shall not pursue this type of interpretation.

We shall assume that both ψ and ψ' have $J^{PG} = 1^{--}$ and transform as isotopic spin singlets. Our hypothesis, stated more precisely, is that ψ is a member of the $\underline{15} \oplus \underline{1}$ representation of the SU(4) group. This group has been studied earlier by several authors.[8] More recently, as is well known,[9] the SU(4) group has been of great interest in the unified theory of weak and electromagnetic interactions of Weinberg and Salam. We shall denote the $\underline{15}$ representation of vector mesons as V_i ($i = 1, 2, \ldots, 15$) and the SU(4) singlet as V_0, and shall write them together as V_α ($\alpha = 0, 1, 2, \ldots, 15$). Note that $V_{1,\ldots,8}$ and V_{15} transform as the $\underline{8} \oplus \underline{1}$ representation (nonet) of SU(3). Besides the nonet, the $\underline{15} \oplus \underline{1}$ representation of the SU(4) contains an $\mathrm{SU}(\overline{3})$ charm-carrying triplet (C_u, C_d, C_s) consisting of the $I = \frac{1}{2}$ vector mesons C_u, C_d and an $I = 0$ meson C_s, a corresponding charge-conjugate SU(3) triplet $(\overline{C}_u, \overline{C}_d, \overline{C}_s)$, and the SU(4) singlet V_0. Note that if SU(4) is broken down to the level of SU(3), V_0 and V_{15} will mix, and if it is further broken down to the level of SU(2), all the three states V_0, V_8, and V_{15} will mix. Diagonalization of the 3×3 mass matrix in this sector would then generate the physical states ω, ϕ, and ψ.

In a similar manner, we have proposed[10] that ψ' is a member of another $\underline{15} \oplus \underline{1}$ representation V_α ($\alpha = 0, 1, \ldots, 15$) of $\mathrm{SU}(\overline{4})$. The analogs of the members of the SU(3) nonet will be designated by ρ', $K^{*\prime}$, ω', and ϕ', which are likely to be the first rotationally excited states with the orbital angular momentum $l = 2$. Although there is no conclusive evidence for the existence of these resonances, some of them have in fact been reported in recent experiments.[11] Once again, here the members V_0', V_8', and V_{15}' would mix to yield the physical states ω', ϕ', and ψ'.

We show in Sec. II that the mass matrix diagonalization indeed leads to the result that ψ and ψ' are almost pure $q_4\bar{q}_4$ states to a high degree. This fact is closely related to the approximate validity of both the Schwinger and the nonet mass formulas for the 1^- SU(3) nonets.

In Sec. III we discuss the production cross section of ψ and ψ' from $e\bar{e}$ annihilation, assuming that they are coupled to the $e\bar{e}$ system through one photon exchange. We compare the coupling of ψ and ψ' to the photon by using the Weinberg spectral-function sum rule. We also discuss suitable generalizations of the sum rule and use these to estimate the photon couplings individually. At the moment, an integrally charged fourth quark appears to be favored over the usually assumed fractionally charged one, although the conclusion is by no means definite. Finally, various decay modes of ψ and ψ' are discussed in Sec. IV. It appears possible that the dominant decay mode of ψ could be the radiative process $\psi \to \psi_P + \gamma$ or $\psi \to \eta' + \gamma$, where ψ_P is the pseudoscalar analog of ψ with expected mass of 2.75 GeV and η' is the $I = Y = 0$ member of the usual 0^- SU(3) nonet with mass 960 MeV.

II. MASS FORMULAS

We assume in direct analogy to the SU(3) theory[12] that the mass splitting arises from an interaction

$$H_{\text{int}} = T^8 + \beta T^{15}, \tag{2.1}$$

where T^8 and T^{15} belong to the same 15-plet of SU(4). Note that T^{15} breaks SU(4) down to the level of SU(3), and T^8 breaks SU(3) down to SU(2) in the usual manner. Since SU(4) will almost certainly be a worse symmetry than SU(3), we expect $\beta \gg 1$. In terms of the corresponding SU(4) tensor notation T^{μ}_{ν} ($\mu, \nu = 1, 2, 3, 4$), (2.1) can be rewritten as (apart from a common multiplicative constant)

$$H_{\text{int}} = T^3_3 + yT^4_4, \tag{2.2}$$

$$y = \tfrac{1}{3}(1 + 2\sqrt{2}\,\beta). \tag{2.3}$$

Note that if the quark mass terms are solely responsible for the breaking of both SU(3) and SU(4) symmetries, then the parameter y in (2.2) is given by

$$y = \frac{m_4 - m_1}{m_3 - m_1}, \tag{2.4}$$

where m_μ ($\mu = 1, 2, 3, 4$) is the bare mass of the μth quark.

The matrix elements of the mass matrix for a $\underline{15} \oplus \underline{1}$ representation can then be written as

$$\begin{aligned}(M)_{ij} &= \overline{M}\delta_{ij} + D(d_{i8j} + \beta d_{i15j}),\\ (M)_{0j} &= A(\delta_{8j} + \beta\delta_{15j}),\\ (M)_{00} &= M_0,\end{aligned} \tag{2.5}$$

where $i, j = 1, 2, \ldots, 15$. The symbol M stands for either squared masses or linear masses. $\overline{M}$ and M_0 are the SU(4) invariant masses (square or linear as the case may be) of the 15-plet and the singlet, respectively. Finally, D and A are the appropriate reduced matrix elements. The matrix (2.5) contains the off-diagonal terms $(M)_{8,15}$, $(M)_{0,8}$, and $(M)_{0,15}$. For the $\underline{15} \oplus \underline{1}$ representation V_α, the five unknown parameters in (2.5) can be determined[3,13] by using the known masses of ρ, K^*, ω, ϕ, and ψ. Expressing the quark content by

$$|\psi\rangle = a\,|q_4\bar{q}_4\rangle + b\,|q_3\bar{q}_3\rangle + c\,|q_1\bar{q}_1 + q_2\bar{q}_2 + q_3\bar{q}_3\rangle, \tag{2.6}$$

we state our numerical results for the diagonalization of the squared or linear mass matrix for the multiplet V_α as follows:

(i) Squared mass matrix. Numerically we obtain

$$\begin{aligned}&\beta = 21.60, \quad y = 20.70,\\ &a = 0.998, \quad b = 3.46 \times 10^{-3}, \quad c = 3.51 \times 10^{-2},\end{aligned} \tag{2.7}$$

using as input the squared central mass values of ρ, K^*, ω, and ϕ as given in the particle data tables,[11] and for ψ the squared mass value from the experimental result (1.1). Hereafter, we shall call the admixtures b and c of the usual quarks in (2.6) quark-leakage coefficients. We would like to remark that these leakage coefficients b and c are extremely sensitive functions of the input mass values. In particular, lowering the ρ mass, for example, by one standard deviation (the ρ mass of all the input masses has the largest quoted errors in the particle data table) changes the values of b and c into

$$b = 5.70 \times 10^{-4}, \quad c = 6.64 \times 10^{-3}. \tag{2.8}$$

(ii) Linear mass matrix. Diagonalization of the linear mass matrix gives

$$\begin{aligned}&\beta = 9.80, \quad y = 9.57,\\ &a = 1.00, \quad b = -2.16 \times 10^{-4}, \quad c = 4.43 \times 10^{-4},\end{aligned} \tag{2.9}$$

and as before the solution for b and c has a rather sensitive dependence on the input masses. Nevertheless, the results (2.7)–(2.9) do confirm that to a good approximation ψ is a $q_4\bar{q}_4$ quark state. The same remark also applies for the 2^+ multiplet of mesons. However, as we shall see shortly, this is not the case for the corresponding 0^- (and 0^+) state ψ_P (and ψ_S) where the quark-leakage coefficients are nearly 15 ~ 20%. These facts have an interesting consequence for decays of ψ, ψ', and ψ_P, as we will show in Sec. IV.

In order to understand in some detail the reason for the smallness and the sensitivity of the quark-leakage coefficients for the 1^- and 2^+ multiplets but not for the 0^+ case, we have carried out the diagonalization of the mass matrix analytically in the approximation of large y. Neglecting terms of orders y^{-2} and y^{-3} respectively for expressions of c and b, we find[14]

$$c=\frac{1}{2y(K^*-\rho)}\left(\frac{\omega+\phi-2K^*}{3}+f\right), \tag{2.10}$$

$$b=\frac{1}{y^2(K^*-\rho)}\left(\frac{\omega+\phi-2K^*}{6}+f\right), \tag{2.11}$$

where f is defined by

$$\tfrac{8}{3}(K^*-\rho)\frac{f}{y}=\left(\phi-\frac{4K^*-\rho}{3}\right)\left(\frac{4K^*-\rho}{3}-\omega\right)-\tfrac{8}{9}(K^*-\rho)^2 \tag{2.12}$$

and the particle symbols in the above equations refer to the quadratic or linear masses of the corresponding particles. For the large-y limit, the mass of ψ is also approximately given by

$$\psi-\rho=2y(K^*-\rho). \tag{2.13}$$

Now, numerically the right-hand side of (2.12) is almost zero. In fact,

$$\left(\phi-\frac{4K^*-\rho}{3}\right)\left(\frac{4K^*-\rho}{3}-\omega\right)=\tfrac{8}{9}(K^*-\rho)^2 \tag{2.14}$$

is the well-known Schwinger mass formula[15] which is very well satisfied by both the quadratic and the linear masses. Thus, slight changes in the input masses in (2.12) cause sizable changes in f, even reversing its sign. Furthermore,

$$\omega+\phi-2K^*=0 \tag{2.15}$$

is the usual nonet mass formula[16] which is also very well satisfied by the quadratic masses as well as the linear masses. Thus, the smallness of the coefficients b and c is due not only to the largeness of y, but also to the fact that the experimental masses for the 1^- multiplets satisfy the Schwinger and the nonet formulas very well. Also it is for the latter reason that the leakage coefficients are rather sensitive to the input masses. This remark also applies to the 2^+ multiplet. However, for 0^- and 0^+ cases this is no longer true, and indeed we find large quark-leakage coefficients for ψ_P and ψ_S.

The mixing between V_0', V_8', and V_{15}' can be handled in a similar fashion. In this case, since the parameter β (or y) appearing in the Hamiltonian (2.1) or (2.2) is known from (2.7) or (2.9), we need only four input masses to determine the four unknown parameters $\overline{M}'$, D', A', and M_0', analogous to the corresponding quantities in the previous case. We choose as input the masses of ψ', ω', $K^{*\prime}$, and ρ'. We identify ω' with $\omega(1675)$ and ρ' with $\rho'(1600)$ from the particle data, but there are several probable candidates for $K^{*\prime}$, and we choose somewhat arbitrarily the state $K_N(1660)$, the lowest-mass state that could have the required quantum numbers of $K^{*\prime}$. The evidence for such a state is not compelling, and our present assignment should be considered tentative. However, it is interesting to observe that another nonet mass formula[16] $M_{\rho'}=M_{\omega'}$ is reasonably well satisfied for the present assignment of ρ' and ω'. Using these masses of ψ', ω', and $K^{*\prime}$, we have checked that the state ψ' indeed comes out to be predominantly a $q_4\overline{q}_4$ state, if we choose $M_{\rho'}=1.57$ GeV, for the quadratic mass matrix and if $M_{\rho'}=1.55$ GeV, in the case of the linear mass matrix. In either case, $M_{\rho'}$ is reasonably close to the mass of $\rho'(1600)$, especially if one keeps in mind that ρ' is a very broad structure with a width of about 400 MeV. Furthermore, we predict the mass value of ϕ':

$$\begin{aligned} M_{\phi'}&=1.9\text{ GeV}\quad\text{(quadratic formula)},\\ M_{\phi'}&=2.0\text{ GeV}\quad\text{(linear formula)}.\end{aligned} \tag{2.16}$$

Obviously the specific values (2.16) may change somewhat with a better knowledge of the input parameters. From the known experimental decay width $\Gamma(\omega'\to\rho\pi)\approx 100$ MeV, we could compute the decay width $\Gamma(\phi'\to K\overline{K}^*)$ using the SU(3) nonet symmetry to obtain

$$\Gamma(\phi'\to K\overline{K}^*)+\Gamma(\phi'\to K^*\overline{K})\approx 60\text{ MeV}. \tag{2.17}$$

It might be tempting to identify ϕ' with the structure observed[17] in the $p\overline{p}$ reaction at 1.9 GeV, but our estimated width (2.17) seems to be too large.

We have seen that both ψ and ψ' can indeed be regarded as almost pure bound states of $q_4\overline{q}_4$. The mass values of the charmed particles in both the V_α and V_α' multiplets can now be predicted since all relevant parameters in the mass matrix are known. We have listed these in Table I for the cases of the quadratic and the linear mass matrices.

Since the parameter β (or y) is known, we can also compute masses of the unknown members of the $\underline{15}\oplus\underline{1}$ multiplet of the pseudoscalar mesons, using masses of π, K, η, and η' as input. Using both the quadratic and the linear formulas, we have listed these masses in Table I. As we emphasized earlier, a special feature of the pseudoscalar mesons to be noted is the well-known fact that the masses of π, K, η, and η' do not satisfy the Schwinger and nonet formulas with either the quadratic or the linear masses. As a result, the quark-leakage coefficients for ψ_P are quite large, as already mentioned. Indeed, the exact numerical diagonalization confirms this feature. This fact provides a useful guide in the

TABLE I. Predicted masses of the new hadrons based on the SU(4) mass formulas. In using the masses of the usual hadrons as input, we have, wherever required, averaged over the masses of the particles in an isomultiplet. For the quark content of the baryons, we have not exhibited the normalized wave functions with appropriate symmetry and antisymmetry properties. For the charmed quark q_4, we have adopted $Q=\frac{2}{3}$ and $I=0$.

SU(3) representation	Quark content	Predicted masses (GeV) Linear formula	Predicted masses (GeV) Quadratic formula
$J^P=1^-$ mesons			
$\underline{\bar{3}}$ $(C_u^+(V),\ C_d^0(V))$	$(\bar{q}_2 q_4,\ \bar{q}_1 q_4)$	1.940	2.190
$C_s^+(V)$	$\bar{q}_3 q_4$	2.062	2.236
$J^P=1^-$ meson recurrences			
$\underline{\bar{3}}$ $(C_u^+(V)',\ C_d^0(V)')$	$(\bar{q}_2 q_4,\ \bar{q}_1 q_4)$	2.603	2.895
$C_s^+(V)'$	$\bar{q}_3 q_4$	2.713	2.944
$J^P=0^-$ mesons			
$\underline{\bar{3}}$ $(C_u^+(P),\ C_d^0(P))$	$(\bar{q}_2 q_4,\ \bar{q}_1 q_4)$	3.561	2.171
$C_s^+(P)$	$\bar{q}_3 q_4$	3.919	2.222
$\underline{1}$ [a] $\psi(P)$	$\sim\bar{q}_4 q_4$ [b]	5.599	2.755
$J^P=2^+$ mesons			
$\underline{\bar{3}}$ $(C_u^+(T),\ C_d^0(T))$	$(\bar{q}_2 q_4,\ \bar{q}_1 q_4)$	2.372	2.816
$C_s^+(T)$	$\bar{q}_3 q_4$	2.483	2.869
$\underline{1}$ [a] $\psi(T)$	$\sim\bar{q}_4 q_4$	3.414	3.800

		Linear 1^- masses	Quadratic 1^- masses	
		Linear formula	Linear formula	Quadratic formula
$J^P=\frac{1}{2}^+$ baryons				
$\underline{6}$ $(B_c^{++},\ B_c^+,\ B_c^0)$	$(q_1 q_1 q_4,\ q_1 q_2 q_4,\ q_2 q_2 q_4)$	3.371	6.200	3.478
$(B_c^+,\ B_c^0)$	$(q_1 q_3 q_4,\ q_2 q_3 q_4)$	3.569	6.397	3.537
B_c^0	$q_3 q_3 q_4$	3.751	6.579	3.599
$\underline{\bar{3}}$ $(B_c^+,\ B_c^0)$	$(q_1 q_3 q_4,\ q_2 q_3 q_4)$	2.863	4.830	2.982
B_c^+	$q_1 q_2 q_4$	2.630	4.596	2.898
$\underline{3}$ $(B_c^{++},\ B_c^+)$	$(q_1 q_4 q_4,\ q_2 q_4 q_4)$	4.568	8.788	4.312
B_c^+	$q_3 q_4 q_4$	4.822	9.042	4.375
$J^P=\frac{3}{2}^+$ baryons				
$\underline{6}$ $(\Delta_c^{++},\ \Delta_c^+,\ \Delta_c^0)$	$(q_1 q_1 q_4,\ q_1 q_2 q_4,\ q_2 q_2 q_4)$	2.634	4.264	3.213
$(\Delta_c^+,\ \Delta_c^0)$	$(q_1 q_3 q_4,\ q_2 q_3 q_4)$	2.786	4.416	3.275
Δ_c^0	$q_3 q_3 q_4$	2.935	4.564	3.340
$\underline{3}$ $(\Delta_c^{++},\ \Delta_c^+)$	$(q_1 q_4 q_4,\ q_2 q_4 q_4)$	4.035	7.294	4.374
Δ_c^+	$q_3 q_4 q_4$	4.187	7.446	4.419
$\underline{1}$ Δ_c^{++}	$q_4 q_4 q_4$	5.437	10.325	5.285

[a] Mostly an SU(3) singlet but with some admixture from the 8th component of the octet.

[b] There is a 15-20% leakage to $\bar{q}_1 q_1 + \bar{q}_2 q_2$ and $\bar{q}_3 q_3$ states.

search for the dominant decay mode of ψ, which will be discussed in Sec. IV

As an aid to the experimental search for the new hadrons that the SU(4) theory implies, we have also listed in Table I the predicted masses of the unknown members of the $\underline{15}\oplus\underline{1}$ representation of 2^+ mesons as well as of the $\underline{20}$-plets of $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons. For the baryons we have computed three sets of predictions. We may use for β the value in Eq. (2.7) determined by using a quadratic mass matrix for the 1^- mesons, and use this value as an input in either the linear mass formula or the quadratic mass formula for the baryons.[10,18] Alternatively, we may use β from Eq. (2.9) obtained by using a linear mass matrix for the 1^- mesons, and use this in the linear mass formula for the baryons. Since the mass formulas for the baryons have been derived elsewhere[10,18] we do not repeat this discussion here.

Although we have computed mass formulas in both linear and quadratic forms, we believe that the use of the quadratic mass formula for both the mesons and the baryons is a more plausible choice for the following reasons. First of all, the experimental straight line of the Chew-Frautschi Regge-pole plot indicates that the use of M^2 rather than M itself is more natural, at least for the leading trajectories. Second,[19] M^2 is the Casimir invariant of the Poincaré group, independent of any particular frame. Third, any calculation of energy eigenvalues in the infinite-momentum frame automatically leads to the quadratic formula.[20] Finally, calculations[21] on the basis of asymptotic chiral symmetry indeed give quadratic mass formulas for both the baryons and the mesons. We should also remark that the use of the quadratic SU(3) mass formula for the baryons is slightly better[19,21] than the corresponding linear one, although both formulas are rather well satisfied. However, if the SU(4) multiplets are discovered, since there is a sizable difference in the predicted mass values in the various cases (see Table I), it is interesting to note that the long-standing question of choosing between the quadratic and the linear mass formulas may actually be settled numerically.

In ending this section, we briefly discuss the $\underline{15}\oplus\underline{1}$ multiplets of 0^+ and 1^+ mesons. Unfortunately, even at the SU(3) level, the status of these multiplets is not well established. If we choose the B meson as well as one of the K_A states listed in the particle data to belong to the multiplet $\underline{15}\oplus\underline{1}$ of SU(4), then we roughly estimate the mass of $\psi\,(1^+)$ to be 3.0–4.0 GeV. With respect to the 0^+ meson, the situation depends sensitively upon the exact mass of the so-called κ meson, and the mass value of ψ_S could be as high as 5 GeV.

III. GENERALIZED WEINBERG SUM RULE

We assume that ψ and ψ' are produced in $e\bar{e}$ annihilation due to one photon exchange. If G_ψ (G_ψ') denotes the coupling of ψ (ψ') to the photon, the total production cross section for $e\bar{e}\to\psi$ integrated over the width of the resonance is given by[3]

$$A_\psi = \int ds\, \sigma_\psi(s) = \pi(4\pi\alpha)^2 G_\psi^{\,2}/M_\psi^{\,4}, \qquad (3.1)$$

where α is the fine-structure constant. The same formula also applies for the $e\bar{e}\to\psi'$ cross section. We may remark that (3.1) is derived under the assumption of unpolarized e and $\bar{e}$ beams. However, even if e and $\bar{e}$ are polarized perpendicular to the beam direction, with opposite polarizations for e and $\bar{e}$, the formula (3.1) is still valid, irrespective of the degree of polarization. Experimentally, from the figures of the SLAC data,[1,2] we estimate (ignoring radiative corrections)[22]

$$\begin{aligned} A_\psi &\simeq 1.0\times10^{-4}(1+R), \\ A_{\psi'} &\simeq 0.5\times10^{-4}(1+R'), \end{aligned} \qquad (3.2)$$

where R and R' are fractions of purely neutral decay modes for ψ and ψ', i.e.,

$$R = \frac{\Gamma(\psi\to\text{neutral})}{\Gamma(\psi\to\text{charged})}\,. \qquad (3.3)$$

The expression for R' is obtained by replacing $\psi\to\psi'$ in (3.3). We expect that R and R' are perhaps small.

In order to compare the formula (3.1) with (3.2), we now proceed to estimate the coupling constants G_ψ and $G_{\psi'}$ by means of the Weinberg sum rule.[23] Several simple calculations based on this idea have been independently proposed by various authors,[3,5] including us. Here, we shall investigate the problem in greater detail. To start with, we assume the validity of the asymptotic U(4) symmetry.[24] Then, the first spectral sum rule is expressed as

$$\int_0^\infty \frac{dm^2}{m^2}\,\rho(V^\alpha, V^\beta; m^2) = A\,\delta_{\alpha\beta}, \qquad (3.4)$$

where $\alpha, \beta = 0, 1, \ldots, 15$ and $\rho^{\alpha\beta}=\rho(V^\alpha, V^\beta; m^2)$ is the usual spectral function[24] for vector currents. In particular (3.4) gives

$$\int_0^\infty \frac{dm^2}{m^2}\,[2\rho(V^3, V^3; m^2) - \rho(V^c, V^c, m^2)] = 0, \qquad (3.5)$$

where the charmed current V^c is defined by

$$V_\mu^c = \frac{1}{\sqrt{2}}\,(V_\mu^0 - \sqrt{3}\;V_\mu^{15})\,. \qquad (3.6)$$

We should emphasize the fact that the relation

(3.4) is formal and that the integral in Eq. (3.4) may actually diverge. However, the difference in Eq. (3.5) may still be convergent and meaningful.

In the quark model we note that

$$V_\mu^c = i\bar{q}_4\gamma_\mu q_4 \tag{3.7}$$

is purely the q_4 current. Now in Sec. II we showed that both ψ and ψ' have predominantly a $q_4\bar{q}_4$ quark content. So if we use pole dominance in Eq. (3.5), the contribution to the $\rho(V^c, V^c; m^2)$ term will come from ψ and ψ' states. On the other hand, both ρ and ρ' will contribute to the $\rho(V^3, V^3; m^2)$ term. Therefore, we find

$$2\left(\frac{G_\rho^2}{M_\rho^2} + \frac{G_{\rho'}^2}{M_{\rho'}^2}\right) = \frac{g_\psi^2}{M_\psi^2} + \frac{g_{\psi'}^2}{M_{\psi'}^2} . \tag{3.8}$$

The coupling constants in Eq. (3.8) are defined as

$$\begin{aligned} \langle 0 | V_\mu^3 | \rho^0(p)\rangle &= \frac{1}{(2p_0V)^{1/2}}\,\epsilon_\mu(p) G_\rho , \\ \langle 0 | V_\mu^c \,| \psi(p)\rangle &= \frac{1}{(2p_0V)^{1/2}}\,\epsilon_\mu(p) g_\psi , \end{aligned} \tag{3.9}$$

with similar definitions of G_ρ' and g_ψ'.

In order to relate g_ψ and g_ψ' with G_ψ and $G_{\psi'}$, we have to know the structure of the electromagnetic current j^{em}. In the SU(4) theory, we expect a form

$$j_\mu^{\text{em}} = V_\mu^3 + \frac{1}{\sqrt{3}}\, V_\mu^8 + Z V_\mu^c , \tag{3.10}$$

where Z is the electric charge of the charmed quark q_4 in units of e. In the fractionally charged quark model discussed by Glashow[9] *et al.*, $Z=\frac{2}{3}$. Using (3.10), the electromagnetic couplings of ψ and ψ' to the photon are given by

$$G_\psi = Z g_\psi , \quad G_{\psi'} = Z g_{\psi'} . \tag{3.11}$$

Also, owing to the V_μ^3 term in Eq. (3.10), the quantities G_ρ and G_ρ' are just the couplings of ρ and ρ' to the electromagnetic current. Then, the sum rule (3.8) gives

$$\frac{G_\psi^2}{M_\psi^2} + \frac{G_{\psi'}^2}{M_{\psi'}^2} = 2Z^2\left(\frac{G_\rho^2}{M_\rho^2} + \frac{G_{\rho'}^2}{M_{\rho'}^2}\right) , \tag{3.12}$$

or using (3.1)

$$A_\psi M_\psi^2 + A_{\psi'} M_{\psi'}^2 = 2\pi Z^2 (4\pi\alpha)^2 \left(\frac{G_\rho^2}{M_\rho^2} + \frac{G_{\rho'}^2}{M_{\rho'}^2}\right) . \tag{3.13}$$

Now, the electromagnetic coupling of ρ is well known from experiments, and the experimental result is consistent with the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation $G_\rho^2/M_\rho^2 = f_\pi^2$, where f_π is the $\pi\to\mu\nu$ decay constant. Also, a 4π enhancement at 1.6 GeV has been observed in $e\bar{e}$ annihilation, and if one assumes this is due to ρ', the experiments yield[25] the following estimate for G_ρ':

$$\frac{G_{\rho'}^2}{M_{\rho'}^2} \approx \frac{G_\rho^2}{M_\rho^2} \simeq f_\pi^2 . \tag{3.14}$$

If we use the standard fractionally charged quark model, we may take $Z=\frac{2}{3}$. Then, the experimental estimate (3.2) with $R=R'=0$ gives for the left-hand side of Eq. (3.13) the numerical result 16×10^{-4} (GeV)^2, which is to be compared with the estimate 9×10^{-4} (GeV)^2 for the right-hand side. Since we expect $R>0$ and $R'>0$, the real discrepancy will become worse, perhaps by a factor of 2. However, the integrally charged case $|Z| = 1$ would satisfy (3.13) quite well, provided that R and R' are small. Quark models with such an integrally charged assignment for the q_4 quark can be constructed, although they are somewhat artificial and lack the appeal of the fractionally charged model.[26] At present, however, we do not take the discrepancy seriously since our input estimates, particularly for the parameters of ρ', may be in substantial error. Furthermore, we may draw attention to the possibility that the pole-dominated form (3.12) of the spectral sum rule may not be reliable. In the particle spectrum, there could well be higher excited states, hitherto undiscovered, which could upset the saturation of the sum rule. This may be particularly serious since, for example, the estimate (3.14) of the ρ' parameter shows that both ρ and ρ' make comparable contributions to the sum rule (3.13). These doubts notwithstanding, there is a well-known sum rule[27] relating the leptonic decay widths of ρ, ω, and ϕ which is in good agreement with the experiments. The validity of this sum rule [see Eq. (3.20)] would be accidental if the higher excited states of ρ, ω, and ϕ were to make sizable contributions. A possible resolution of this paradoxical situation may be that the first spectral sum rule is actually valid individually for each multiplet, analogous to the local duality sum rule.

We propose accordingly the validity of such local (or individual) sum rules,

$$\frac{G_\psi^2}{M_\psi^2} = 2Z^2\,\frac{G_\rho^2}{M_\rho^2} , \tag{3.15}$$

$$\frac{G_{\psi'}^2}{M_{\psi'}^2} = 2Z^2\,\frac{G_{\rho'}^2}{M_{\rho'}^2} . \tag{3.16}$$

A possible way in which such local sum rules [(3.15) and (3.16)] can be obtained is as follows. If we apply the asymptotic symmetry requirement on the two-point propagator function of the unrenormalized vector field operator $\phi_\mu^\alpha(x)$ belonging to a given U(4) representation ($\alpha = 0, 1, 2, \ldots, 15$), we obtain an analog of the first spectral sum rule,

$$Z_\alpha / M_\alpha{}^2 = \text{constant}, \tag{3.17}$$

where the renormalization constants Z_α are defined by

$$\langle 0 | \phi_\mu^\alpha(0) | V^\alpha(p)\rangle = \frac{1}{(2p_0 V)^{1/2}} (Z_\alpha)^{1/2} \epsilon_\mu(p). \tag{3.18}$$

If we now were to use phenomenologically a field-current identity in the form

$$V_\mu^\alpha = c\phi_\mu^\alpha + c'\phi_\mu'^\alpha + \cdots, \tag{3.19}$$

where the primed fields belong to the V_α' representation, the sum rule (3.17) would lead to (3.15), and an analog of (3.17) for the 16-plet $\phi_\mu'^\alpha$ would yield (3.16). Furthermore, since ψ is predominantly a $q_4\bar{q}_4$ quark structure with negligible admixture from $q_1\bar{q}_1 + q_2\bar{q}_2$ and $q_3\bar{q}_3$ states, the sum rule[27]

$$\tfrac{1}{3} M_\rho \Gamma(\rho \to l\bar{l}) = M_\omega \Gamma(\omega \to l\bar{l}) + M_\phi \Gamma(\phi \to l\bar{l}) \tag{3.20}$$

would virtually remain unaltered. A sum rule similar to (3.20) is then also expected to hold for the leptonic decays of ρ', ω', and ϕ'. Another way to understand the validity of the local sum rules (3.15) and (3.16) is to use the current-mixing theory based upon the field algebra[28] separately for ϕ_α and ϕ_α' fields with the ansatz (3.19).

A remarkable feature of the local sum rules is that they are consistent with the broken SU(4) formula,

$$(g_{jl})^2 = g_0{}^2 \delta_{jl} + c[d_{j8l} + \beta d_{j15l}], \tag{3.21}$$

where g_{jl} ($j, l = 0, 1, 2, \ldots, 15$) is defined by

$$\langle 0 | V_\mu^j | V^l(p)\rangle = \frac{1}{(2p_0 V)^{1/2}} g_{jl} \epsilon_\mu(p). \tag{3.22}$$

Indeed, together with the quadratic mass formula for the 16-plet, Eq. (3.21) can reproduce the sum rule (3.15). Note that the parameter β appearing in Eq. (3.21) is the same SU(4)-breaking constant as in Eq. (2.1). Although the validity of Eq. (3.21) appears to be reasonable from an orthodox broken-SU(4)-symmetry viewpoint, its compatibility with (3.15) may be purely accidental. We should remark that Eq. (3.21) is reminiscent of the questionable broken second Weinberg sum rule.[29] Therefore, a more careful investigation is perhaps called for.

Now returning to the original discussion we shall consider the experimental verifications of (3.15) and (3.16). First, the sum rule (3.15) is quite well satisfied with the choice of the integral charge $Z = 1$ and with a small R (≈ 0.10). With respect to (3.16), it is more convenient to rewrite it as

$$\frac{M_{\psi'}{}^2 A_{\psi'}}{M_\psi{}^2 A_\psi} = \frac{G_{\rho'}{}^2}{M_{\rho'}{}^2} \frac{M_\rho{}^2}{G_\rho{}^2} \tag{3.23}$$

in view of Eq. (3.1). Using the estimate (3.14), we obtain

$$A_{\psi'} \approx 0.7 A_\psi, \tag{3.24}$$

which can be consistent with the experimental value (3.2) if we have $R' \approx 0.4$. Such a rather large neutral decay rate of ψ' is perhaps unrealistic. However, the estimate (3.25) depends sensitively on the value[25] of G_ρ', so that it would be premature to reject the validity of (3.16) at the present time.

In closing this section, we may comment on the curious empirical equality (3.14) between $G_{\rho'}{}^2/M_{\rho'}{}^2$ and $G_\rho{}^2/M_\rho{}^2$. If verified by future experiments this may perhaps indicate some hidden higher symmetry group such as SU(8)⊗O(3) which is an extension of the usual SU(6)⊗O(3) group of the three-quark model, where O(3) is the space-rotation group.

IV. DECAY MODES OF ψ AND ψ

The purely leptonic decay modes $\psi \to l\bar{l}$ and $\psi' \to l\bar{l}$ would be mediated by a virtual photon, and one would obtain[3]

$$\Gamma(\psi \to l\bar{l}) = \frac{4\pi\alpha^2}{3} \frac{G_\psi{}^2}{M_\psi{}^3} = \frac{A_\psi M_\psi}{12\pi^2}, \tag{4.1}$$

with a similar formula for $\psi' \to l\bar{l}$. Using the estimate (3.2), we calculate

$$\begin{aligned} \Gamma(\psi \to l\bar{l}) &\simeq 2.6(1+R)\ \text{keV}, \\ \Gamma(\psi' \to l\bar{l}) &\simeq 1.6(1+R')\ \text{keV}. \end{aligned} \tag{4.2}$$

Experimentally, we know[30]

$$\frac{\Gamma(\psi \to \text{charged})}{\Gamma(\psi \to \mu\bar{\mu})} \approx 16, \tag{4.3}$$

so that (4.2) and (4.3) give

$$\Gamma(\psi \to \text{all}) \approx 42(1+R)^2\ \text{keV}. \tag{4.4}$$

With respect to $\psi' \to \mu\bar{\mu}$, it appears experimentally that it represents only a very tiny fraction of less than 1% of the total decay rate. Hence, the total decay width of $\psi' \to$ all could be 200 keV–1 MeV.

Next let us consider the hadronic decays of ψ. First of all, this could go through a virtual-photon exchange mechanism $\psi \to \gamma \to$ hadrons. In this case, an argument based upon the usual quark-parton model would give

$$\frac{\Gamma_\gamma(\psi \to \text{hadrons})}{\Gamma_\gamma(\psi \to l\bar{l})} = \sum_j Q_j{}^2, \tag{4.5}$$

where Q_j is the charge of the jth quark. For the 3-color fractionally charged quartet model, we have $\sum_j Q_j^2 = \frac{10}{3}$, and we expect $\Gamma_\gamma(\psi \to \text{hadrons}) \approx 10$ keV. However, this is too small a value when compared with the experimental result (4.3), and we have to look for a different mechanism for the dominant hadronic decays of ψ and ψ'.

An important question is why ψ and ψ' do not decay strongly into ordinary hadrons consisting of the SU(3) quarks q_1, q_2, q_3 and the corresponding antiquarks, leading to large widths typical of strong decays. In terms of the quark content of ψ and ψ', such decays could go through either or both of the following possibilities: (a) The $q_4\bar{q}_4$ state which predominantly makes up the ψ and ψ' structures may annihilate into vacuumlike (or gluon) states, subsequently coupling to the ordinary quark structure. (b) ψ and ψ' may decay into ordinary hadrons through the small leakage coefficients of the ordinary quarks. The situation here is similar to the one encountered in the more familiar decay $\phi \to \rho\pi$, where the final hadrons are made up of the quarks q_1, q_2 and their antiparticles, and (through diagonalization of the mass matrix) ϕ is known to be predominantly a $q_3\bar{q}_3$ bound state. It is well known that the small decay rate for $\phi \to \rho\pi$ can in fact be accounted for by the presence of the small quark-leakage coefficient for the structure $q_1\bar{q}_1 + q_2\bar{q}_2$ contained in the physical ϕ state. This follows from the nonet hypothesis,[16] which works well for both the 1^- and the 2^+ multiplets. In terms of the quark lines, this hypothesis equivalently forbids the pair annihilation diagrams of quark-antiquark lines into the vacuumlike (or gluon) states, a feature first recognized explicitly by Zweig and Iizuka,[31] and we shall refer to it hereafter as the OZI rule. Subsequently, a close relationship of this rule with the duality diagrams was also recognized.[32] In this paper we shall adopt this rule as a working hypothesis, and consequently discard the possibility (a).

We believe that this philosophy is perhaps consistent with the quark-parton model if we take a view that rearrangement of a quark pair into ordinary hadrons is possible only when the total energy of the system is much larger than the quark masses. Then the hadronic decays of ψ and ψ' by strong interactions are possible only through the mechanism (b). Since the leakage coefficients of the ordinary quarks are small for the 1^- multiplets, we can readily understand, at least qualitatively, the narrow widths of ψ and ψ'. In this connection, we should mention that an alternative and opposite viewpoint has been discussed by De Rújula and Glashow.[33] Ultimately, the future complete theory must take into account all these different mechanisms, but hereafter we follow the OZI quark rule supplemented by our philosophy of the quark leakage.

Since the hadronic decay of ψ can go through the quark leakage, and since the SU(3)-singlet coefficient c in (2.6) is larger than the other coefficient b, the total hadronic decay rate via this mechanism would be given crudely by

$$\Gamma_{\text{q.l.}}(\psi \to \text{hadrons}) \approx 3|c|^2 \frac{M_\psi}{12\pi}, \tag{4.6}$$

where the dominant leakage coefficient c is given by (2.6) or (2.10). Depending upon whether we use the quadratic case (2.7) or the linear case (2.9), we estimate

$$\Gamma_{\text{q.l.}}(\psi \to \text{hadrons}) \approx 300 \text{ keV} \quad \text{(quadratic)},$$

or (4.7)

$$\Gamma_{\text{q.l.}}(\psi \to \text{hadrons}) \approx 0.04 \text{ keV} \quad \text{(linear)}.$$

To exhibit the sensitive dependence of the coefficient c on input masses, if we lower m_ρ by one standard deviation, we get

$$\begin{aligned}\Gamma_{\text{q.l.}}(\psi \to \text{hadrons}) &\approx 11 \text{ keV} \quad \text{(quadratic)},\\ \Gamma_{\text{q.l.}}(\psi \to \text{hadrons}) &\approx 200 \text{ keV} \quad \text{(linear)}.\end{aligned} \tag{4.8}$$

It is clear that because of the enormous sensitivity of the result to small changes in input parameters, the quantitative results in (4.7) and (4.8) cannot be taken too seriously. In fact it may be important even to consider the electromagnetic mass differences. Note in particular that it would be premature to conclude from the large width in (4.7) for the quadratic case that the mass formula for 1^- mesons should be linear. This mechanism in fact may even be quite unimportant for the decay of ψ.

For the decay of ψ', we similarly estimate

$$\begin{aligned}\Gamma_{\text{q.l.}}(\psi' \to \text{hadrons}) &\approx 0.1 \text{ MeV} \quad \text{(quadratic)},\\ \Gamma_{\text{q.l.}}(\psi' \to \text{hadrons}) &\approx 0.5 \text{ MeV} \quad \text{(linear)}.\end{aligned} \tag{4.9}$$

In this case the input masses in the mass-matrix diagonalization have much larger uncertainties, so that the estimates (4.9) are probably much less reliable. We may also note here that for ψ_P, the 0^- analog of ψ, the quark-leakage coefficients are much larger, and relatively insensitive, as emphasized in Sec. II. The quadratic mass formula predicts $M(\psi_P) = 2.75$ GeV, whereas in the linear case $M(\psi_P) \simeq 5.60$ MeV, so that for ψ_P we expect a sizable width due to quark leakage;

$$\begin{aligned}\Gamma_{\text{q.l.}}(\psi_P \to \text{hadrons}) &\approx 4.4 \text{ MeV} \quad \text{(quadratic)},\\ \Gamma_{\text{q.l.}}(\psi_P \to \text{hadrons}) &\approx 12 \text{ MeV} \quad \text{(linear)}.\end{aligned} \tag{4.10}$$

Another important decay mechanism for ψ is the radiative decay

$$\psi \to \psi_P + \gamma \to \gamma + \text{hadrons}, \tag{4.11}$$

$$\psi \to \eta' + \gamma \to \gamma + \text{hadrons}, \tag{4.12}$$

$$\psi \to \eta + \gamma \to \gamma + \text{hadrons}. \tag{4.13}$$

Let us consider[22] first the decay (4.11). Note that $\psi \to \psi_P + \gamma$ is energetically allowed only if we use the mass value $M(\psi_P) \simeq 2.75$ GeV determined from the quadratic mass formula. In this case, the decay is allowed by the OZI rule. Using the exact U(4) symmetry as a guide, we estimate

$$\frac{\Gamma(\psi \to \psi_P\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \left[\frac{\mu(\psi \to \psi_P)}{\mu(\omega \to \pi^0)}\right]^2 \left(\frac{\omega + \pi_0}{\psi + \psi_P}\right)^2 \left(\frac{k_\psi}{k_\omega}\right)^3, \tag{4.14}$$

where μ denotes the transition magnetic moment in units of the Bohr magneton and k is the photon energy. Since we are dealing with a magnetic dipole transition, we have extracted the mass ratio $(\omega + \pi_0)^2/(\psi + \psi_P)^2$ explicitly in (4.14) in conformity with the usual practice. In the exact U(4) symmetry, we expect the ratio of the transition magnetic moments to be $\sqrt{2}$, so that we obtain

$$\Gamma(\psi \to \psi_P\gamma) \approx 40 \text{ keV} \tag{4.15}$$

if we use the known experimental value $\Gamma(\omega \to \pi^0\gamma) \simeq 0.8$ MeV. If the estimate (4.15) is seriously believed then the decay $\psi \to \psi_P\gamma$ may well be a dominant decay mode of ψ, especially if in the present case of the quadratic mass formulas we accept the estimate (4.8) as typical for $\psi \to$ hadrons. An important signature for the decay $\psi \to \psi_P + \gamma$ would be the emission of a monochromatic photon with energy $E_\gamma \simeq 330$ MeV. One should keep in mind that in our numerical estimate (4.15) use has been made of the U(4) symmetry, which may be quite inaccurate. Furthermore, if the linear mass formula is used, this decay is energetically forbidden.

The decay mode $\psi \to \eta' + \gamma$ may actually be quite important too. The reason is the following. The diagonalization of the (quadratic) mass matrix leads to a large quark-leakage component of $q_4\bar{q}_4$ in the physical η' state (but not in η), which is of the order of 20%. Hence, the OZI rule requires

$$\frac{\Gamma(\psi \to \eta'\gamma)}{\Gamma(\psi \to \psi_P\gamma)} \approx |c'|^2 \left(\frac{k'}{k}\right)^3, \tag{4.16}$$

where c' is the $q_4\bar{q}_4$ quark-leakage coefficient of η' and k' and k are the photon energies in the two processes. The suppression due to quark leakage in (4.16) is roughly compensated by the gain in the phase volume, so crudely we expect

$$\Gamma(\psi \to \eta'\gamma) \approx \Gamma(\psi \to \psi_P\gamma). \tag{4.17}$$

We would like to point out that one of the final observable decay products of $\psi \to \eta'\gamma$ is $\pi^+\pi^-\pi^+\pi^-\pi^0\gamma$ due to $\eta' \to \eta\pi^+\pi^-$ followed by $\eta \to \pi^+\pi^-\pi^0$. Hence we expect that this process would constitute a sizable background (with a monochromatic photon of energy $\simeq 1.4$ GeV) to the reported decay $\psi \to \omega\pi^+\pi^-$. If a linear mass matrix is employed, c' is again of order 20%, and although $\psi \to \psi_P\gamma$ is not allowed, we may expect the width of $\psi \to \eta'\gamma$ in this case to be comparable to the corresponding one in the quadratic case. In this case if we were to take the estimate (4.7) seriously, $\psi \to \eta'\gamma$ would presumably be the dominant decay mode of ψ. Also note that the decay rate for $\psi \to \eta\gamma$ can be estimated similarly in terms of $\psi \to \eta'\gamma$, and we find that $\Gamma(\psi \to \eta\gamma)$ is about 10% of $\Gamma(\psi \to \eta'\gamma)$ in the linear case but about 2% of $\Gamma(\psi \to \eta'\gamma)$ in the quadratic case.

We may remark here that for all the three types of decay mechanisms we have considered here for ψ, i.e., $\psi \to \gamma \to$ hadrons, $\psi_{\text{q.l.}} \to$ hadrons, and the radiative decays of ψ, the effective decay interaction is a U-spin scalar. Then for all these decay mechanisms, as well as for the gluon annihilation mechanism of De Rújula and Glashow[33] (if it is important), we predict

$$\begin{gathered}\frac{d^3}{dp^3}\Gamma(\psi \to K^+(p) + \text{anything}) = \frac{d^3}{dp^3}\Gamma(\psi \to \pi^+(p) + \text{anything}), \\ \frac{d^3}{dp^3}\Gamma(\psi \to \pi^0(p) + \text{anything}) = 3\frac{d^3}{dp^3}\Gamma(\psi \to \eta(p) + \text{anything})\end{gathered} \tag{4.18}$$

for the semi-inclusive decay rate with a given momentum p. For the integrated rates, these are rewritten as

$$\begin{gathered}\langle n(K^+)\rangle\,\Gamma(\psi \to K^+ + \text{anything}) = \langle n(\pi^+)\rangle\,\Gamma(\psi \to \pi^+ + \text{anything}), \\ \langle n(\pi^0)\rangle\,\Gamma(\psi \to \pi^0 + \text{anything}) = \frac{\langle n(\eta)\rangle}{3}\,\Gamma(\psi \to \eta + \text{anything}),\end{gathered} \tag{4.19}$$

where $\langle n(P)\rangle$ is the average multiplicity of the meson P. In the above derivation, we have neglected the mass differences among members of the 0^- meson multiplet.[34] Unfortunately, this is not possible for the rather important radiative decay $\psi\to\eta'+\gamma$. Note that since η' may decay into $\eta'\to\eta\pi^+\pi^-\to(\pi^+\pi^-\pi^0)\pi^+\pi^-$, and there is no corresponding mechanism for $K\bar{K}$ decays, we have to subtract the mechanism $\psi\to\eta'\gamma$ when we use (4.18) or (4.19).

Finally, we would like to explain qualitatively the reported dominant decay mode of ψ', i.e., $\psi'\to\psi\pi^+\pi^-$. The OZI quark rule allows a strong virtual decay

$$\psi'\to\psi+\psi_S\,, \tag{4.20}$$

where ψ_S is the scalar analog of ψ_P with $J^P=0^+$. As mentioned in Sec. II, the status of the 0^+ nonet itself is not clear at present; however, we expect that the above decay is energetically forbidden. At the same time, as for ψ_P, the quark-leakage coefficients in ψ_S are expected to be quite large. We draw this inference from the fact that the various listed masses or mass ranges for the 0^+ nonet do not satisfy either the Schwinger or the nonet formula. Therefore, the virtual ψ_S could decay into a 2π state via the quark leakage with a probability which is not too small. Although reliable quantitative calculations are difficult to make out at present, we believe on the basis of the foregoing qualitative arguments that the mechanism (4.20) may well account for the decay $\psi'\to\psi+\pi^+\pi^-$. Another possible mechanism is through the virtual decay $\psi'\to\psi+\epsilon$, since the 0^+ meson ϵ corresponding to the 2π resonance with $I=Y=0$ is expected to contain a large $q_4\bar{q}_4$ leakage coefficient.

To summarize, we have examined in this paper in some detail the production and decay mechanisms of ψ and ψ', based on the hypothesis that they belong to the $\underline{15}\oplus\underline{1}$ representations V_α and V'_α, respectively, of the SU(4) group. Although our estimates at times have been rather qualitative, we believe that the various known experimental features of the new resonances at the present time fit in quite well with the SU(4) hypothesis. The most direct confirmation of the SU(4) theory would be the discovery of charmed mesons and baryons. As an aid to the experimentalist, we have tabulated the various masses of the yet to be discovered particles.

In conclusion, we may point out an amusing parallel between the 20-plet of baryons on the one hand and the 20 amino acids in the DNA molecule on the other. Both schemes are based on the existence of four fundamental entities, with three units representing each of 20 baryons or amino acids. Is nature trying to tell us something important here?

Note added in proof. For the $\underline{15}\oplus\underline{1}$ multiplet of pseudoscalar mesons, one may alternatively use the $E(1416)$ meson instead of $\eta'(958)$. Diagonalization of the mass matrix with masses of π, K, η, and E as input then yields the following results: (i) $M(\psi_P)=3.026$ GeV for the quadratic mass matrix and $M(\psi_P)=6.077$ GeV in the linear case. In the quadratic case the mass of ψ_P is close to the estimates made for the paracharmonium mass. (ii) The quark content of ψ_P now has much smaller leakage coefficients in contrast to the case discussed in the test. We also note that the Schwinger mass formula is better satisfied in this case. In the present case the discussion of the radiative decays of ψ, Eqs. (4.11)–(4.13), will be considerably changed: (a) $\psi\to\psi_P\gamma$, as before, is allowed energetically if we use the quadratic mass formula. However, the phase space is now very small and Eq. (4.14) leads to $\Gamma(\psi\to\psi_P\gamma)\simeq 0.5$ keV. (b) The decay rate for $\psi\to E\gamma$ will also be much smaller, since the $q_4\bar{q}_4$ quark leakage coefficient of E is much smaller now [see Eq. (4.16)]. The decay rate for $\psi\to\eta\gamma$ is negligible as before.

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