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"Natural" left-right symmetry

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It is remarked that left-right symmetry of the starting gauge interactions is a "natural" symmetry if it is broken in no way except possibly by mass terms in the Lagrangian. The implications of this result for the unification of coupling constants and for parity nonconservation at low and high energies are stressed.

In a recent note[1] it has been pointed out that a left $\leftrightarrow$ right discrete symmetry[2] in the starting gauge interactions provides a desirable basis for introducing CP violation into such a scheme, in that such a gauge symmetry, subjected to spontaneous symmetry breaking, automatically satisfies the relation $\eta_{+-}=\eta_{00}$ and links the suppression of CP violation to the *known* suppression of $(V+A)$ weak interactions. The purpose of this note is to point out that such a discrete symmetry has a second desirable feature in that it can indeed be preserved as a "natural" symmetry, provided we ensure that the symmetry in question is broken, if at all, *only in a "soft" manner*, i.e., via mass terms of the Higgs scalars.

We point out that such a soft symmetry breaking is, on the one hand, sufficient to guarantee the desired mass splittings between the left- and the right-handed gauge bosons (with $m_{W_R}\gg m_{W_L}$), so as to account for parity nonconservation and dominant $(V-A)$ character of the known weak interactions. On the other hand, it does not lead to infinite corrections in higher orders to the difference between the left- and the right-handed gauge coupling constants g_L and g_R, so that (g_L-g_R) is *calculable*. This in turn has important implications: (1) A unifying gauge model of the type $\mathrm{SU}(n)_L\times\mathrm{SU}(n)_R$ based[3] on $2n$ four-component fermions (which may comprise gauge groups of Ref. 2 as subgroups), subjected to a discrete left $\leftrightarrow$ right symmetry as above, can be described by *a single coupling constant*,[4] a feature which is desirable from the point of view of unification of all forces and all matter. (2) Parity nonconservation (like CP nonconservation) can be interpreted as a low-energy phenomenon to disappear at high energies, as conjectured in Refs. 1 and 2. We also briefly discuss the implications of such a model for neutral-current phenomenology.

To make our discussions specific, let us assume, for illustration only, that the gauge group is of the form

$$\mathcal{G}_0=\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R\otimes G, \tag{1}$$

where $\mathrm{SU}(2)_L$ generates $(V-A)$ known weak interactions, $\mathrm{SU}(2)_R$ generates *parallel* $(V+A)$ interactions in the manner suggested in Ref. 2, while G denotes any gauge group commuting with $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ and generating gauge interactions with $L\leftrightarrow R$ symmetry. [Examples[2] of G are $\mathrm{SU}(4)^{\mathrm{color}}_{L+R}$ or $\mathrm{SU}(3)^{\mathrm{color}}_{L+R}\times\mathrm{U}(1)_{L+R}$.] We will assume that the bare coupling constants $g_L^{(0)}$ and $g_R^{(0)}$ for the gauge groups $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ are *equal;* in this case the gauge interactions generated by the entire group $\mathcal{G}_0$ are $L\leftrightarrow R$ symmetric. We then show that the renormalized coupling constants g_L and g_R are equal up to finite radiative corrections as long as the $L\leftrightarrow R$ symmetry is not broken by the Higgs potential in a hard manner. [It will be clear from our discussions that the results obtained are not limited to the special choice of the gauge group and are more general; in particular they apply to a gauge group of the form $\mathrm{SU}(n)_L\times\mathrm{SU}(n)_R$ as long as one satisfies the condition of soft breaking.]

Once again for illustration only, let us choose the following sets of Higgs scalars:

$$\begin{array}{ll}\phi_{1,2} & (2,2,1)\\ \chi_L & (2,1,1)\\ \chi_R & (1,2,1)\,.\end{array}$$

[Additional Higgs scalars needed to generate masses for the gauge mesons of the group G should be chosen to have either the representation content $(1,1,m)$ or should consist of parallel multiplets, i.e., $\xi_L=(l,1,m)$ and $\xi_R=(1,l,m)$, etc., so that their contributions to the Higgs potential may also be written in a $L\leftrightarrow R$ symmetric manner. We do not exhibit such multiplets for ease of writing.]

The choice of the potential consistent with renormalizability and the $L\leftrightarrow R$ symmetry as mentioned above is given by

$$V(\phi_{1,2},\chi_L,\chi_R)=\sum_{i,j} a_{ij}\,\mathrm{Tr}(\phi_i^\dagger\phi_j)+\sum_{i,j,k,l=1,2,3,4} b_{ijkl}\,\mathrm{Tr}(\phi_i^\dagger\phi_j\phi_k^\dagger\phi_l)+\sum_{i,j,k,l=1,2,3,4} c_{ijkl}\,\mathrm{Tr}(\phi_i^\dagger\phi_j)\mathrm{Tr}(\phi_k^\dagger\phi_l)$$
$$+\sum_{i,j=1,2,3,4} d_{ij}\,\mathrm{Tr}(\phi_i^\dagger\phi_j)(\chi_L^\dagger\chi_L+\chi_R^\dagger\chi_R)+e[(\chi_L^\dagger\chi_L)^2+(\chi_R^\dagger\chi_R)^2]+f(\chi_L^\dagger\chi_L)(\chi_R^\dagger\chi_R)+\mu_L^2\chi_L^\dagger\chi_L+\mu_R^2\chi_R^\dagger\chi_R, \tag{2}$$

where[5]

$$\phi_3\equiv\tau_2\phi_1^*\tau_2,\quad \phi_4\equiv\tau_2\phi_2^*\tau_2. \tag{3}$$

Note that cubic coupling of the type $\chi_L^\dagger\phi\chi_R$ can be forbidden by imposing the discrete symmetry $\chi_L\to-\chi_L$.

Note the important point that except for the mass terms for χ_L and χ_R, the rest of the potential respects the $L\leftrightarrow R$ symmetry.

First we remark that it is easy to arrange (with appropriate choice of signs[6]) that the vacuum expectation values $\langle\chi_L\rangle$ and $\langle\chi_R\rangle$ are each nonzero, and secondly (if $\mu_R\gg\mu_L$), one would obtain $\langle\chi_R\rangle\gg\langle\chi_L\rangle$, which in turn would lead to $m_{W_R}\gg m_{W_L}$. This is sufficient to account for the phenomenology of the known weak interactions.

On the other hand, we now argue that a breaking of the $L\leftrightarrow R$ symmetry due solely to unequal mass terms for χ_R and χ_L cannot lead to infinite correction to the relation $g_L=g_R$. The reasons for this are the following:

(1) The corrections in question due to mass terms are always more convergent by at least two powers of momenta than those due to interactions.

(2) The vertex coupling ($W\bar\psi\psi$ or $W_\mu\phi\partial_\mu\phi$, etc.), all the quartic scalar field couplings, and the kinetic-energy terms[7] being of dimension four, the most divergent corrections to them (in a renormalizable field theory) are necessarily logarithmic, arising from the most singular terms in the propagator.[8] (We work in the R gauge so that the vector as well as scalar propagators behave as k^{-2} for large k^2). This makes it clear that the infinite corrections to the vertex as well as to the quartic scalar-field couplings must be *independent* of the masses of the Higgs bosons. Hence the result $g_L=g_R+$(finite corrections).

The working of this general argument may be verified by examining explicitly the Feynman dia-

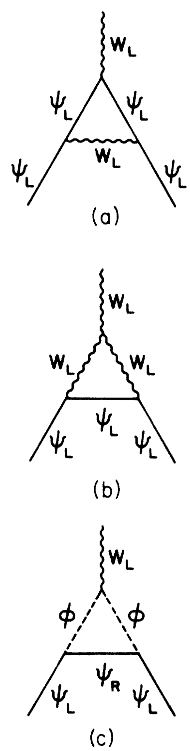

FIG. 1. One-loop corrections to g_L.

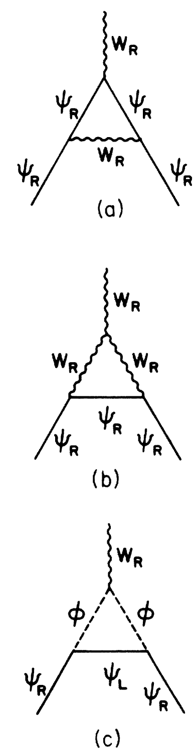

FIG. 2. One-loop corrections to g_R.

grams for radiative correction to g_L and g_R. Below, we exhibit these for vertices with $(W_L\psi_L\psi_L)$ and $(W_R\psi_R\psi_R)$ external lines with one and two loops only, although the result holds to all orders. In drawing these diagrams, we have allowed for the possibility that the ϕ fields may, in general, possess Yukawa couplings of the form $(\bar{\psi}_L\phi\psi_R + \text{H.c.})$, which are needed to ultimately provide masses to the fermions.

The one-loop corrections to g_L and g_R with fermion–gauge-boson external lines are exhibited in Figs. 1 and 2 respectively. It is clear that if the bare coupling constants $g_L^{(0)}$ and $g_R^{(0)}$ are equal, the infinite corrections due to Fig. 1 are exactly equal to those of Fig. 2. Finite corrections due to Figs. 1 and 2 will of course differ since $m_{W_L} \neq m_{W_R}$. [Note, we have not exhibited diagrams of classes 1(c) and 2(c) leading to $(W_L\psi_R\psi_R)$ and $(W_R\psi_L\psi_L)$ external lines, respectively. These are new induced vertices, but these diagrams are convergent anyhow.]

The two-loop corrections to g_L with $(W_L\psi_L\psi_L)$ external lines are exhibited in Fig. 3. Counterpart diagrams showing analogous corrections to g_R with $(W_R\psi_R\psi_R)$ external lines are not drawn, but may be obtained from Fig. 3 with the substitution $L \leftrightarrow R$ for every line. Once again, with $g_L^{(0)} = g_R^{(0)}$, the divergent parts of Fig. 3 match those of the counterpart diagrams mentioned above.

It is worthwhile pointing out the graphs which, in the absence of left-right symmetry of the quartic couplings, would make different divergent contribution to $(g_L - g_R)$. This happens starting at the two-loop level for the graphs involving $\phi\phi W$ vertices. (See diagrams in Fig. 4; the right-handed counterpart diagrams are obtained by the substitution $L \leftrightarrow R$). Note that the divergent contributions from these diagrams are $L \leftrightarrow R$ symmetric provided we choose the quartic couplings $\mathrm{Tr}(\phi^\dagger\phi)(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R)$ in the $L \leftrightarrow R$ symmetric[9] form in the first place.

It is clear from the line of arguments presented above, in particular the general arguments, that they are not limited to the special choice of the gauge group, and we conclude that the class of renormalizable gauge models, which respect the $L \leftrightarrow R$ discrete symmetry everywhere in the Lagrangian except possibly in the scalar mass terms, preserve the "naturalness" of the $L \leftrightarrow R$ symmetry in spite of radiative corrections. This in turn has the following implications: (a) Local gauge symmetries of the form $\mathrm{SU}(n)_L \times \mathrm{SU}(n)_R$, subjected to the discrete symmetry as mentioned above, involve a single fundamental coupling constant.
(b) Since all masses and mass differences [like $(\mu_R - \mu_L)$ and $(m_{W_R} - m_{W_L})$] can be neglected in a renormalizable theory at sufficiently high energy, parity nonconservation should disappear in theories of the type discussed here at high energies ($\gg m_{W_R}$), parity nonconservation at low energies being primarily[10] a consequence of the large mass difference between W_R and W_L. Lastly, we observe that in the case of the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times G$ gauge group, the weak mixing angle θ_W is predicted[2,11] to be $\sin^2\theta_W = g_R^2/(g_R^2 + g_L^2) \simeq 0.5$.

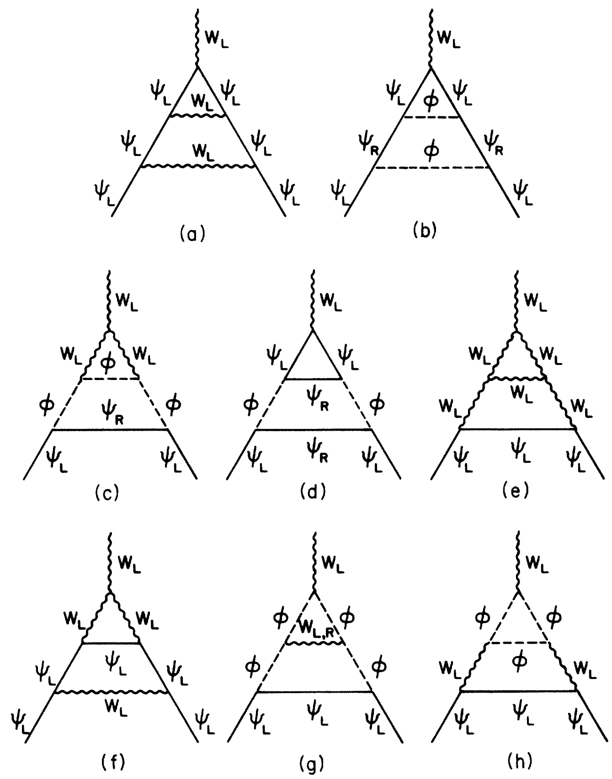

FIG. 3. Two-loop corrections to g_L. Counterpart diagrams for corrections to g_R are obtained by the substitution $L \longleftrightarrow R$ for every line.

We thank Professor Abdus Salam for several illuminating discussions on the notion of left-right symmetry.

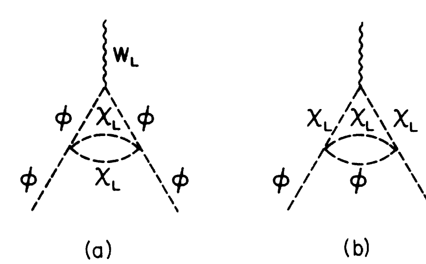

FIG. 4. Corrections to $\phi\phi W$ vertex sensitive to the left-right symmetry of the quartic couplings.

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[1]R. N. Mohapatra and J. C. Pati, Phys. Rev. D 11, 566 (1975).

[2]Such a discrete symmetry was first suggested by J. C. Pati and Abdus Salam, Phys. Rev. Lett. 31, 661 (1973); Phys. Rev. D 10, 275 (1974).

[3]One must choose $2n$ rather than n (if $n>2$) to avoid Adler-Bell-Jackiw anomalies. For the basic model of Ref. 2, the group in question will be $SU(16)_L \times SU(16)_R$, which if gauged will require a new 16-plet of four-component fermions to avoid anomalies.

[4]The desirability of a single coupling constant has been advanced by J. C. Pati and Abdus Salam, Phys. Rev. D 8, 1240 (1973); and H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974). Note that the manner of obtaining a single coupling constant for $SU(n)_L \times SU(n)_R$ local symmetry with the $L \longleftrightarrow R$ reflection symmetry, as suggested here, is distinct from that obtained under a single $SU(n)$ local symmetry suggested by Georgi and Glashow.

[5]The appearance of ϕ_3 and ϕ_4 related to ϕ_1 and ϕ_2 is typical of the SU(2) group.

[6]The appropriate choice of couplings which gives the desired pattern of vacuum expectation values mentioned is $\mu_L^2 < 0$ and $\mu_R^2 < 0$, $\mu_R \gg \mu_L$, and $f \ll \langle\chi_2\rangle^2/\langle\chi_R\rangle^2$; then we get

$$\langle\phi_{1,2}\rangle = \begin{pmatrix} \kappa_{1,2}e^{i\xi_{1,2}} & 0 \\ 0 & \kappa'_{1,2}e^{i\xi'_{1,2}} \end{pmatrix},$$

$$\langle\chi_L\rangle = \frac{-\mu_L^2 - f\langle\chi_R\rangle^2 - D}{e},$$

$$\langle\chi_R\rangle = \frac{-\mu_R^2 - f\langle\chi_L\rangle^2 - D}{e},$$

where $D = \sum_{ij} d_{ij}\,\mathrm{Tr}(\langle\phi_i^\dagger\rangle\langle\phi_j\rangle)$. One gets for the masses $m_{W_L^+}$ and $M_{W_R^+}$ the following values:

$$m_{W_L^+}{}^2 \simeq g_L^2(\kappa_1^2 + \kappa_2^2 + \langle\chi_L\rangle^2)$$

and

$$m_{W_R^+}{}^2 \simeq g_R^2(\kappa_1^2 + \kappa_2^2 + \langle\chi_R\rangle^2).$$

Therefore, if $\kappa_i \approx \langle\chi_L\rangle$, we get $m_{W_R^+}{}^2 \gg m_{W_L^+}{}^2$. Note that the complex $\langle\phi_{1,2}\rangle$ given above generate CP violation as suggested in Ref. 1.

[7]This implies that the wave-function renormalizations, which enter into the definitions of the renormalized coupling constants, are at most logarithmically divergent, and therefore their divergent parts are independent of mass terms and hence $L \longleftrightarrow R$ symmetric (following the same arguments as discussed in the text).

[8]This result actually follows from a theorem due to K. Symanzik [in *Fundamental Interactions at High Energies*, edited by A. Perlmutter, *et al.* (Gordon and Breach, New York, 1970)] quoted by T. Hagiwara and B. W. Lee, Phys. Rev. D 7, 459 (1973). The theorem states that when a total symmetry of the Lagrangian is "softly" broken, vertices of higher dimension (than those of the symmetry-breaking terms) suffer only finite renormalization due to the symmetry-breaking terms. Here the global symmetry in question is the reflection symmetry which takes $L \longleftrightarrow R$.

[9]In a manner similar to that discussed for $W\psi\psi$ and $W\phi\phi$ couplings, it is, of course, easy to verify diagrammatically that divergent parts of radiative corrections to quartic couplings also remain $L \longleftrightarrow R$ symmetric, once the starting Lagrangian is $L \longleftrightarrow R$ symmetric except for $\mu_L \neq \mu_R$.

[10]Note that parity-nonconservation at low energies has two sources: (1) $m_{W_R} \gg m_{W_L}$ and (2) $(g_L - g_R) \neq 0$. The former source leads to parity violation of order $(g_L^2/m_{W_L}{}^2)$, while the latter, by itself, would lead to parity violation of order $g_L(g_L - g_R)/m_{W_L}{}^2 \approx \alpha G_F$, which is small compared to G_F. Thus, observed parity violation to order G_{Fermi} is entirely a consequence of $m_{W_R} \gg m_{W_L}$, which, in turn, can be attributed to a *joint consequence* of spontaneous symmetry breaking and the choice $\mu_R \neq \mu_L$. If one can dispense with the Higgs-Kibble mechanism in favor of dynamical symmetry breaking, one could attribute parity nonconservation in theories of the type discussed here entirely to spontaneous symmetry breaking.

[11]If one assumes that left- and right-handed Cabibbo angles are comparable and that the CP-violating phase is maximal, from Ref. 1. We may deduce that $(m_{W_L}/m_{W_R})^2 \simeq |\eta_{+-}| \simeq 10^{-3}$. Since in the scheme presented in this paper $|(g_L - g_R)/g_L|$ is of order $\alpha \ln(m_{W_R}/m_{W_L})^2$, we expect g_L to differ from g_R by at most a few percent, and therefore $\sin^2\theta_W \simeq 0.5$.