

## Cancellation of neutral $\Delta S \neq 0$ hadronic processes without charmed particles\*

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The  $SU_2 \otimes U_1$  gauge model of weak and electromagnetic interactions introduced by Weinberg and Salam is extended to the group  $SU_2 \otimes U_1 \otimes U_1$ . A natural cancellation of the effective Lagrangian for neutral strangeness-changing semileptonic processes is achieved without using charmed quarks. The additional neutral vector boson may have an arbitrarily small mass.

### I. INTRODUCTION

In the renormalizable unified theory of weak and electromagnetic interactions developed by Weinberg and Salam,<sup>1</sup> the incorporation of hadrons runs into the problem of suppressing the  $\Delta S = 1$  neutral hadronic currents. A popular way of circumventing this difficulty is by the addition of at least one more "charmed" quark,  $\mathcal{P}'$ , to the conventional  $\mathcal{P}, \mathcal{X}, \lambda$  triplet.<sup>2</sup> This postulate has received much theoretical interest,<sup>3</sup> but it is still necessary to explore alternative mechanisms.

The possibility we investigate is the extension of the model based on the group  $SU_2 \otimes U_1$  to one based on the group  $SU_2 \otimes U_1 \otimes U_1$ , where the additional vector boson is responsible for the cancellation of  $\Delta S = 1$  neutral currents. This scheme was investigated previously by Schechter and Ueda<sup>4</sup> but their analysis was not sufficiently general. For example, they did not explore all possible Higgs-Kibble systems. It will be shown that we can eliminate the  $\Delta S = 1$  semileptonic effective Lagrangian in a natural manner.<sup>5</sup> The leptonic sector in this model is compatible with present data on  $\nu_e$  and  $\nu_\mu$  scattering,<sup>6</sup> whereas that of Schechter and Ueda is not. The leptonic sector, furthermore, is almost identical to the  $SU_2 \otimes U_1 \otimes U_1$  model of Dolgov et al.<sup>7</sup> However, their motivation for studying this model is different from ours and their Higgs system is not as general. In particular, they made no attempt to include hadrons. We do not consider higher-order induced effects in this paper.

Since this work was commenced, the discovery of  $J$  and  $\psi$  resonances with masses between 3 GeV and 4 GeV,<sup>8</sup> and the suspicion that they may have something to do with weak interactions makes it interesting to investigate the possibility of a unified gauge theory of weak and electromagnetic interactions with a neutral vector boson much

lighter than the  $Z$  of the Weinberg-Salam model. The group  $SU_2 \otimes U_1 \otimes U_1$  is a possible group for a model which exhibits this feature. It is to be emphasized that the latter reason for the extension of the  $SU_2 \otimes U_1$  model to  $SU_2 \otimes U_1 \otimes U_1$  is logically independent of the former, and that the possibility of a neutral vector boson lighter than the  $Z$  of the Weinberg-Salam model is interesting<sup>9</sup> independently of any identification with the new resonances.

The group  $SU_2 \otimes U_1 \otimes U_1$  is a possible subgroup, for the weak and electromagnetic interactions, of the gauge group of the world,<sup>10</sup> rather than the group  $SU_2 \otimes U_1$ . A model based upon the group  $SU_2 \otimes U_1 \otimes U_1$  has been discussed elsewhere within a different context.<sup>11</sup>

### II. THE $SU_2 \otimes U_1 \otimes U_1$ REPRESENTATIONS

In complete correspondence with the Weinberg-Salam model<sup>1</sup> for the leptons we consider

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L = \frac{1}{2}(1 + \gamma_5) \begin{pmatrix} \nu_e \\ e \end{pmatrix}$$

as a left-handed doublet, and  $e_R = \frac{1}{2}(1 - \gamma_5)e$  as a right-handed singlet, under  $SU_2$ . The muon is treated analogously. We group the conventional quark states,  $\mathcal{P}, \mathcal{X}, \lambda$ , into a left-handed doublet,

$$\psi_L = \begin{pmatrix} \mathcal{P} \\ \mathcal{X} \end{pmatrix}_L,$$

a left-handed singlet,  $\lambda_{C_L}$ , and three right-handed singlets,  $\mathcal{P}_R, \mathcal{X}_R, \lambda_R$ , where

$$\mathcal{X}_C = \mathcal{X} \cos \theta_C + \lambda \sin \theta_C,$$

$$\lambda_C = \lambda \cos \theta_C - \mathcal{X} \sin \theta_C$$

are the Cabibbo rotated states. ( $\theta_C$  is the Cabibbo angle.) We assume an approximate  $SU_3$ -symmetric

strong-interaction Lagrangian.

The choice of Higgs-Kibble scalar fields,<sup>12</sup> whose vacuum expectation values break the symmetry down to the electromagnetic  $U_1$  group and simultaneously give mass terms to the leptons, quarks, and vector bosons in the Lagrangian, is not unambiguous. There must be several scalars to provide the four vector bosons with masses, and there must be at least one scalar  $SU_2$  doublet so that the electron (and muon) and the  $\mathcal{P}$  quark can obtain masses through the Higgs-Kibble mechanism.

The following simple possibilities for the scalar fields exist:

(a) one complex doublet coupled to the leptons and the quarks, together with one singlet coupled to the quarks;

(b) two complex doublets, one coupled to the leptons and the other coupled to the quarks;

(c) two complex doublets, one coupled to the leptons and the other coupled to the quarks, together with a singlet coupled to the quarks.

We shall discuss the situation in terms of possibility (b) since only in this case does the *natural* cancellation of the effective Lagrangian occur. Other features discussed occur with all three possibilities, and case (b) is therefore representative; only trivial modifications are required to transfer to cases (a) or (c).

### III. THE TWO-DOUBLET REPRESENTATION

Denoting the gauge coupling constant of  $SU_2$  by  $g$ , and absorbing the coupling constants of the  $U_1$  groups into the hypercharges ( $q_i, p_i$ ), the general renormalizable Lagrangian, for weak and electromagnetic interactions, invariant under the group  $SU_2 \otimes U_1 \otimes U_1$ , is  $\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{scalars}} + \mathcal{L}_{\text{leptons}} + \mathcal{L}_{\text{hadrons}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{pot}}$ , where

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}\vec{F}_{\mu\nu} \cdot \vec{F}^{\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} - \frac{1}{4}C_{\mu\nu} C^{\mu\nu}, \quad (3.1)$$

$$\begin{aligned} \mathcal{L}_{\text{scalars}} = & -\frac{1}{2}[\partial_\mu \phi_1 - \frac{1}{2}ig\vec{W}_\mu \cdot \vec{\tau}\phi_1 + i(q_1 B_\mu + p_1 C_\mu)\phi_1]^2 \\ & - \frac{1}{2}[\partial_\mu \phi_2 - \frac{1}{2}ig\vec{W}_\mu \cdot \vec{\tau}\phi_2 + i(q_2 B_\mu + p_2 C_\mu)\phi_2]^2, \end{aligned} \quad (3.2)$$

$$\begin{aligned} \mathcal{L}_{\text{leptons}} = & -\bar{e}_R \gamma^\mu [\partial_\mu - i(q_3 B_\mu + p_3 C_\mu)] e_R + (e - \mu) \\ & - \bar{L} \gamma^\mu [\partial_\mu - \frac{1}{2}ig\vec{W}_\mu \cdot \vec{\tau} - i(q_4 B_\mu + p_4 C_\mu)] L + (e - \mu), \end{aligned} \quad (3.3)$$

$$\begin{aligned} \mathcal{L}_{\text{hadrons}} = & -\bar{\psi}_L \gamma^\mu [\partial_\mu - \frac{1}{2}ig\vec{W}_\mu \cdot \vec{\tau} - i(q_5 B_\mu + p_5 C_\mu)] \psi_L \\ & - \bar{\chi}_{CL} \gamma^\mu [\partial_\mu - i(q_6 B_\mu + p_6 C_\mu)] \chi_{CL} \\ & - \bar{\mathcal{P}}_R \gamma^\mu [\partial_\mu - i(q_7 B_\mu + p_7 C_\mu)] \mathcal{P}_R \\ & - \bar{\mathcal{X}}_R \gamma^\mu [\partial_\mu - i(q_8 B_\mu + p_8 C_\mu)] \mathcal{X}_R \\ & - \bar{\lambda}_R \gamma^\mu [\partial_\mu - i(q_9 B_\mu + p_9 C_\mu)] \lambda_R, \end{aligned} \quad (3.4)$$

$$\begin{aligned} \mathcal{L}_{\text{int}} = & G_e (\bar{L} \phi_1 e_R + \text{H.c.}) + G_\mu (e - \mu) \\ & + G_1 (\bar{\psi}_L i\sigma_2 \phi_2^* \mathcal{P}_R + \text{H.c.}) + G_2 (\bar{\psi}_L \phi_2 \mathcal{X}_R + \text{H.c.}) \\ & + G_3 (\bar{\psi}_L \phi_2 \lambda_R + \text{H.c.}) + G_4 (\bar{\chi}_{CL} \mathcal{X}_R + \text{H.c.}) \\ & + G_5 (\bar{\lambda}_{CL} \lambda_R + \text{H.c.}), \end{aligned} \quad (3.5)$$

$$\mathcal{L}_{\text{pot}} = V(\phi_1, \phi_2). \quad (3.6)$$

The definitions of the field tensors  $\vec{F}_{\mu\nu}, B_{\mu\nu}, C_{\mu\nu}$  in (3.1) are

$$\vec{F}_{\mu\nu} = \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu + g\vec{W}_\mu \times \vec{W}_\nu,$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,$$

$$C_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu.$$

We note that the gauge fields  $B_\mu, C_\mu$  may be redefined without loss of generality so that

$$q_1 = q_2 = q.$$

Spontaneous symmetry breaking occurs when the scalar fields

$$\phi_i = \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix} \quad (i = 1, 2)$$

are allowed to develop the vacuum expectation values

$$\langle \phi_i \rangle_0 = \begin{pmatrix} 0 \\ F_i \end{pmatrix} \quad (i = 1, 2). \quad (3.7)$$

In the tree approximation, the charged vector mesons  $W_\mu^\pm = (1/\sqrt{2})(W_\mu^1 \mp iW_\mu^2)$  now, by the Higgs-Kibble mechanism,<sup>12</sup> acquire the mass

$$M_W^2 = \frac{1}{4}g^2(F_1^2 + F_2^2).$$

The neutral vector-meson mass matrix is given, from (3.2), by the expression  $(\frac{1}{2}gW_\mu^3 + qB_\mu + p_1 C_\mu)^2 F_1^2 + (\frac{1}{2}gW_\mu^3 + qB_\mu + p_2 C_\mu)^2 F_2^2$ , and we define the photon field

$$A_\mu = e \left( \frac{1}{g} W_\mu^3 - \frac{1}{2q} B_\mu \right),$$

where  $e^2 = 4q^2 g^2 / (g^2 + 4q^2)$  is the square of the electric charge. We let the mass matrix of the vector mesons be diagonal on the basis  $A_\mu, X_\mu, Z_\mu$ , where

$$\begin{aligned} X_\mu = & (gW_\mu^3 + 2qB_\mu + \lambda_- C_\mu)(g^2 + 4q^2 + \lambda_-^2)^{-1/2}, \\ Z_\mu = & (gW_\mu^3 + 2qB_\mu + \lambda_+ C_\mu)(g^2 + 4q^2 + \lambda_+^2)^{-1/2}, \end{aligned} \quad (3.8)$$

where  $\lambda_+$  and  $\lambda_-$  are real for  $CP$  conservation, and

$$g^2 + 4q^2 + \lambda_+ \lambda_- = 0 \quad (3.9)$$

for orthogonality. The eigenvalues  $\Lambda_- = M_X^2$  and  $\Lambda_+ = M_Z^2$  of the mass matrix are given by

$$\begin{aligned} \Lambda_{\pm} = & \frac{1}{2} [p_1^2 F_1^2 + p_2^2 F_2^2 + \frac{1}{4}(g^2 + 4q^2)(F_1^2 + F_2^2)] \\ & \pm \frac{1}{2} [(p_1^2 F_1^2 + p_2^2 F_2^2 + \frac{1}{4}(g^2 + 4q^2)(F_1^2 + F_2^2))^2 \\ & - (p_1 - p_2)^2 (g^2 + 4q^2) F_1^2 F_2^2]^{1/2}. \end{aligned} \quad (3.10)$$

A useful identity is

$$\frac{2(p_1 F_1^2 + p_2 F_2^2)}{F_1^2 + F_2^2} \lambda_{\pm} = g^2 \frac{\Lambda_{\pm}}{M_W^2} - (g^2 + 4q^2), \quad (3.11)$$

and it is obvious by inspection of the mass matrix that

$$M_X^2 + M_Z^2 = M_W^2 + p_1^2 F_1^2 + p_2^2 F_2^2 + q^2 (F_1^2 + F_2^2),$$

so that there is no lower bound on  $M_X^2$  when  $M_Z^2$  is sufficiently large. This may be seen by considering  $(p_1 - p_2)^2$  sufficiently small in (3.10). This is in contrast to the model discussed in Ref. (9).

We now investigate the constraints on the hypercharges. The correct coupling of the leptons and the quarks to the photon implies that

$$\begin{aligned} q_3 &= 2q, \\ q_4 &= q, \\ q_5 &= b - q, \\ q_7 &= b - 2q, \end{aligned} \quad (3.12)$$

where

$$q_6 = q_8 = q_9 = b.$$

Also, gauge invariance of the terms in  $\mathcal{L}_{\text{int}}$ , (3.5),

which are necessary and sufficient to yield a diagonal lepton and quark mass matrix upon symmetry breaking,<sup>12</sup> implies that

$$\begin{aligned} p_4 &= p_3 - p_1, \\ p_5 &= c - p_2, \\ p_7 &= c - 2p_2, \end{aligned} \quad (3.13)$$

where

$$p_6 = p_8 = p_9 = c.$$

Hence, only 6, viz.,  $q, p_1, p_2, p_3, b, c$ , of the 18 hypercharges introduced are arbitrary.

$\mathcal{L}_{\text{pot}} = V(\phi_1, \phi_2)$  is the gauge-invariant potential which is a polynomial in  $\phi_1$  and  $\phi_2$  of maximum dimension 4 for renormalizability. The general polynomial satisfying these requirements can indeed have a minimum, in the tree approximation, at the vacuum expectation values  $\langle \phi_i \rangle_0$  given in (3.7). With this potential and the terms in  $\mathcal{L}_{\text{int}}$ , (3.5), all particles in the model, except the photon and the neutrino, have a mass upon spontaneous symmetry breaking. The couplings  $G_e, G_\mu, G_i$  ( $i = 1, 5$ ) are determined by the fermion masses and the Cabibbo angle, which are free in the model.

The Fermi coupling constant for  $\mu$  decay is recovered in the local limit by the identification

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}. \quad (3.14)$$

#### IV. THE NEUTRAL SEMILEPTONIC LAGRANGIAN

The coupling of the leptons and the quarks to the bosons  $X$  and  $Z$  is given by the following Lagrangian, using (3.3), (3.4), (3.8), (3.12), and (3.13):

$$\begin{aligned} \mathcal{L}^{\text{neutral}} = & \frac{1}{2} i X_\mu \{ [g^2 + 4q^2 + 2(p_3 - p_1)\lambda_-] \bar{\nu}_L \gamma_\mu \nu_L - [g^2 - 4q^2 - 2(p_3 - p_1)\lambda_-] \bar{e}_L \gamma_\mu e_L \\ & + 2(4q^2 + p_3 \lambda_-) \bar{e}_R \gamma_\mu e_R \\ & - (4qb + 2c\lambda_-) (\bar{\mathcal{P}} \gamma_\mu \mathcal{P} + \bar{\mathcal{N}} \gamma_\mu \mathcal{N} + \bar{\lambda} \gamma_\mu \lambda) + 4[2q^2 + p_2 \lambda_-] \bar{\mathcal{P}} \gamma_\mu \mathcal{P} \\ & - (g^2 + 4q^2 + 2p_2 \lambda_-) (\bar{\mathcal{P}}_L \gamma_\mu \mathcal{P}_L - \bar{\mathcal{N}}_{CL} \gamma_\mu \mathcal{N}_{CL}) \} \\ & \times (g^2 + 4q^2 + \lambda_-^2)^{-1/2} \\ & + \frac{1}{2} i Z_\mu (\lambda_- - \lambda_+). \end{aligned} \quad (4.1)$$

A necessary, but of course not sufficient, condition for a conspiracy between  $X_\mu$  and  $Z_\mu$  to cancel semileptonic  $\Delta S = 1$  neutral processes is that the leptonic currents coupled to  $X_\mu$  and  $Z_\mu$  should be proportional. This follows if we do not allow the leptons to transform under the additional  $U_1$  group, and implies that

$$p_4 = p_3 = p_1 = 0. \quad (4.2)$$

We now evaluate the local limit of Eq. (4.1) to find the effective Lagrangian for  $\Delta S = 1$  neutral semileptonic (sl) processes:

$$\mathcal{L}_{\text{sl}}^{\Delta S=1} = - \left( \frac{g^2 + 4q^2 + 2p_2\lambda_-}{(g^2 + 4q^2 + \lambda_-^2)M_X^2} + \frac{g^2 + 4q^2 + 2p_2\lambda_+}{(g^2 + 4q^2 + \lambda_+^2)M_Z^2} \right) l^\mu j_\mu^{\Delta S},$$

where

$$l_\mu = \frac{1}{2}[(g^2 + 4q^2)\bar{\nu}_L\gamma_\mu\nu_L - (g^2 - 4q^2)\bar{e}_L\gamma_\mu e_L + 8q^2\bar{e}_R\gamma_\mu e_R],$$

$$j_\mu^{\Delta S} = \frac{1}{2}\sin\theta_C\cos\theta_C(\bar{\mathcal{N}}_L\gamma_\mu\lambda_L + \bar{\lambda}_L\gamma_\mu\mathcal{N}_L).$$

It is trivial to show that  $\mathcal{L}_{\text{sl}}^{\Delta S=1} \equiv 0$  using Eqs. (3.9), (3.10) and (3.11). This is surprising since one would expect, as in Ref. 4, that this could only be achieved by some prudent choice of the otherwise arbitrary parameters  $q$  and  $p_2$ . The cancellation is therefore in some sense natural.<sup>5</sup> All we need is the condition that the leptons do not transform under the additional  $U_1$  group.

The problem of suppressing  $\Delta S=1$  effects when the local limit is not valid remains (e.g. in  $K$  decay where  $q^2 = m_K^2$ ), especially if we wish  $M_X$  to be small enough to associate  $X_\mu$  with one of the new resonances<sup>8</sup> below 4 GeV. An investigation of higher-order processes is necessary to see if the induced  $\Delta S=1$  semileptonic decays can be sufficiently suppressed.

## V. THE ELIMINATION OF THE $\Delta S \neq 0$ CURRENT

Even with the remarkable cancellation discussed in the last section,  $\Delta S=2$  neutral current processes still exist, as can be seen by inspection of Eq. (4.1). However, the question arises of eliminating the strangeness-changing neutral current entirely by choosing  $p_2$  and  $q$  such that the following relations are valid:

$$g^2 + 4q^2 + 2p_2\lambda_- = 0, \quad (5.1)$$

$$g^2 + 4q^2 + 2p_2\lambda_+ = 0.$$

This implies that  $\lambda_+ = \lambda_-$  so  $X_\mu$  and  $Z_\mu$  become degenerate and the orthogonality condition (3.9) is violated. Nor is it possible to make the equalities of (5.1) only approximate and obtain suppression of  $\Delta S=2$  processes. In fact such processes in this model will be of order  $G_F$ . This indicates that further studies are required to construct models where vector bosons, rather than charmed quarks, are responsible for suppression of strangeness-changing processes.

## VI. BOUNDS FROM LEPTONIC SCATTERING

One of the problems encountered in the early attempt at utilizing the  $SU_2 \otimes U_1 \otimes U_1$  model<sup>4</sup> was compatibility with neutrino-electron scattering experimental data.<sup>6</sup> The theory presented here is

quite compatible with data on elastic neutrino interactions. To see this we examine the effective Lagrangian for the four-fermion electron neutrino interaction which may be written as [using Eqs. (3.14), (4.1)],

$$L_{\text{eff}}(e\nu_e) = - \frac{G_F}{\sqrt{2}} \bar{\nu}_e\gamma_\mu(1+\gamma_5)\nu_e\bar{e}\gamma_\mu(C_V + C_A\gamma_5)e,$$

where

$$C_V = 1 - \frac{4 - 3\beta^2}{2\beta^2} \left( \frac{F_1^2 + F_2^2}{F_1^2} \right) - \frac{F_2^2}{4F_1^2} \left[ \rho_2(\rho_1 - \rho_2) + 5\rho_2 - 3\rho_1 + \frac{2}{\beta^2}(2\rho_1 - \rho_2) \right], \quad (6.1)$$

and

$$C_A = 1 - \frac{1}{2} \left( \frac{F_1^2 + F_2^2}{F_1^2} \right) - \frac{F_2^2}{4F_1^2} [\rho_1(\rho_2 - \rho_1) - \rho_2 - \rho_1]. \quad (6.2)$$

We have used the following definitions:

$$\beta^2 = 1 + 4q^2/g^2 = \frac{1}{\cos^2\theta_w} > 1, \quad (6.3)$$

$$\rho_1 = \frac{p_1}{g\rho}, \quad \rho_2 = \frac{2p_3 - p_1}{g\rho},$$

and

$$\rho = \frac{p_1 F_1^2 + p_2 F_2^2}{g(F_1^2 + F_2^2)}.$$

In the case discussed in Sec. IV, Eqs. (4.2) and (6.3) imply that  $\rho_1 = \rho_2 = 0$ , so that

$$C_V = 1 - \frac{4 - 3\beta^2}{2\beta^2} \left( \frac{F_1^2 + F_2^2}{F_1^2} \right),$$

$$C_A = 1 - \frac{1}{2} \left( \frac{F_1^2 + F_2^2}{F_1^2} \right).$$

The condition  $\beta^2 > 1$  may be used to calculate the allowed region for the parameters  $C_A$  and  $C_V$  which, on a  $(C_A, C_V)$  plot, is the interior of the trapezoid bounded by

$$C_A = 0, \quad C_A = \frac{1}{2}, \quad C_A = C_V, \quad C_A = -\frac{1}{3}C_V + \frac{4}{3}.$$

The intersection of this region with the experimentally allowed region<sup>6</sup> is finite.

If Eq. (4.2) is not true, [i.e., leptons transform under the extra  $U_1$  group,] it becomes more difficult to find the allowed range for the parameters  $C_A$  and  $C_V$ , but other simple cases do exist; e.g., in the singlet-doublet model [case (a) of Sec. II], Eqs. (6.1), (6.2), and (6.3) are correct providing

$$\rho = p_1/g.$$

This implies that  $\rho_1 = 1$  so that  $C_A = \frac{1}{2}$ . Clearly, solutions exist for which  $C_V$  has reasonable values.

The recently discovered resonances<sup>8</sup> are usually assumed to be  $1^-$  states. However, the angular distribution of the decay products, especially in the leptonic channels, is uncertain. If the  $X_\mu$  of this theory were to be identified as one of the recent resonances, the decay angular distribution is determined by the ratio of the couplings to the vector and axial-vector currents  $\bar{e}\gamma_\mu e$  and  $\bar{e}\gamma_\mu\gamma_5 e$ . These couplings come from the term  $X_\mu(g_V\bar{e}\gamma_\mu e + g_A\bar{e}\gamma_\mu\gamma_5 e)$  in the Lagrangian (4.1), so that the coupling constants  $g_V$  and  $g_A$  are given by, using (3.9), (3.10), (3.11), and (6.3),

$$\begin{aligned} \frac{\alpha_V}{\alpha} &= \frac{g_V^2}{e^2} = \frac{1}{16(\beta^2 - 1)} \frac{M_Z^2 - \beta^2 M_W^2}{M_Z^2 - M_X^2} \\ &\quad \times \left[ 4 - 3\beta^2 + \rho_2 \left( \beta^2 - \frac{M_X^2}{M_W^2} \right) \right]^2, \\ \frac{\alpha_A}{\alpha} &= \frac{g_A^2}{e^2} = \frac{1}{16(\beta^2 - 1)} \frac{M_Z^2 - \beta^2 M_W^2}{M_Z^2 - M_X^2} \\ &\quad \times \left[ \beta^2 - \rho_1 \left( \beta^2 - \frac{M_X^2}{M_W^2} \right) \right]^2. \end{aligned}$$

In the case discussed in Sec. IV,  $\rho_1 = \rho_2 = 0$ ; it is interesting to observe that the simple choice  $\beta^2 \approx \frac{4}{3}$  implies  $\alpha_V/\alpha_A \approx 0$ , which would give a  $(1 + \cos^2\phi)$  distribution for the electrons in the decay  $X_\mu \rightarrow e^+e^-$ . The identification is, of course, purely speculative, but it is nevertheless interesting that such a possibility exists.

## VII. OTHER REPRESENTATIONS

With case (a), the doublet-singlet representation, and case (c), the two-doublet-one-singlet representation, the cancellation of the effective local interaction for  $\Delta S = 1$  neutral semileptonic processes cannot be achieved naturally.

While we are making an Abelian extension of the  $SU_2 \otimes U_1$  model, the question arises about yet another model based on the Abelian extension to  $SU_2 \otimes U_1 \otimes U_1 \otimes U_1$ . In this situation the proportionality of neutral leptonic currents is no longer necessary for the cancellation of the effective neutral  $\Delta S = 1$  semileptonic interaction. However, it is necessary, for the elimination of the entire strangeness changing neutral current, that at least two of the three neutral vector bosons are degenerate, as in Sec. V. One cannot then avoid the degeneracy in this manner.

## VIII. CONCLUSION

We have shown it is possible to extend the Weinberg-Salam  $SU_2 \otimes U_1$  model to have an additional neutral vector boson, and still retain compatibility with neutrino data. It is possible to choose representations for the Higgs-Kibble scalar fields such that the  $\Delta S = 1$  neutral semileptonic processes are canceled in the local limit in a manner more natural than would be expected. In this case it is possible to have a neutral vector boson of *arbitrarily* low mass. This is an important departure from the Weinberg-Salam model, and the model of Bég and Zee.<sup>9</sup> A charmed  $\mathcal{C}'$  quark is not required and we can safely assume an underlying  $SU(3)$  symmetry for the hadrons.

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