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Nonleptonic decays of hadrons and the $\Delta I = \frac{1}{2}$ rule

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The recent data for $K \to 2\pi$, $\Xi \to \Lambda\pi$, and $\Lambda \to N\pi$ decays are used to extract solutions for the parameters of these decays. The available $K \to 3\pi$, $\Sigma \to N\pi$, and $\Omega \to \Xi\pi$ data are not sufficient to determine the parameters of these decays. No preference is found for a $\Delta I = \frac{1}{2}$ solution in any of the three analyzed decays. It may be possible to distinguish between $\Delta I = \frac{1}{2}$ and $\Delta I \neq \frac{1}{2}$ solutions for $\Lambda \to N\pi$ by means of a future measurement of the transverse polarization of the neutron in the $\Lambda \to n\pi^0$ decay.

I. INTRODUCTION

In virtually every elementary particle physics text and in many journal articles one reads that the nonleptonic decays of hadrons satisfy the $\Delta I = \frac{1}{2}$ rule. However, in 1968 and 1971 we showed[1] that the $K \to 2\pi$ data available then did not favor a $\Delta I = \frac{1}{2}$ solution over a $\Delta I \neq \frac{1}{2}$ solution for the decay parameters. Herein we show that this is still the case for $K \to 2\pi$ and is also true for $\Xi \to \Lambda\pi$ and $\Lambda \to N\pi$. The data for $K \to 3\pi$, $\Sigma \to N\pi$, and $\Omega \to \Xi\pi$ are insufficient for us to make any such judgments for these decays.

So, as the data presently stand, there is no reason to believe that the $\Delta I = \frac{1}{2}$ rule is valid. We show herein, however, that there is some hope that a measurement of the transverse polarization of the neutron in $\Lambda \to n\pi^0$ decay would distinguish between $\Delta I = \frac{1}{2}$ and $\Delta I \neq \frac{1}{2}$ solutions.

In the next section we analyze the most recent $K \to 2\pi$ data using the formalism presented in Refs. 1. Section III contains an analysis of the $\Xi \to \Lambda\pi$ decays and Sec. IV contains an analysis of the $\Lambda \to N\pi$ decays. In Sec. V we explain the situation for the $\Sigma \to N\pi$, $\Omega \to \Xi\pi$, and $K \to 3\pi$ decays for which there are not sufficient data for analyses. Finally, in Sec. VI we summarize the noncommittal verdict of the nonleptonic decay data of hadrons regarding the validity of the $\Delta I = \frac{1}{2}$ rule.

II. $K \to 2\pi$ DECAYS

The details of these decays are given in Refs. 1, so we only summarize the situation here. The decays are

$$K^{\pm} \to \pi^{\pm} + \pi^{0}(A_{\pm 0}),$$

$$K_S \begin{array}{l} \nearrow \pi^+ + \pi^-(A^S_{+-}) \\ \searrow \pi^0 + \pi^0(A^S_{00}) \end{array},$$

and

$$K_L \begin{array}{l} \nearrow \pi^+ + \pi^-(A^L_{+-}) \\ \searrow \pi^0 + \pi^0(A^L_{00}) \end{array}, \tag{1}$$

where the symbols for the decay amplitudes are indicated inside the parentheses. For CPT invariance the ratio $|A_{+0}|^2/|A_{-0}|^2$ should be unity, which it is[2] within an error of $\pm 0.11\%$. So in this and all other decays considered herein we assume CPT invariance. (The analyses are impossible to perform for CPT noninvariance because there are then many more parameters than there are available data.)

Typically we want to calculate a decay amplitude such as $\langle K_S \to \pi^+ + \pi^-\rangle = \langle \pi^+\pi^- | H | K_S\rangle$. To calculate this and the other decay amplitudes we need the initial states

$$|K^{\pm}\rangle = |\tfrac{1}{2}, \pm\tfrac{1}{2}\rangle_{\pm},$$

$$|K_S\rangle = \alpha(\epsilon)\left[(1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle\right],$$

and

$$|K_L\rangle = \alpha(\epsilon)\left[(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle\right], \tag{2}$$

where

$$|K^0\rangle = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle_+,$$
$$|\bar{K}^0\rangle = |\tfrac{1}{2}, +\tfrac{1}{2}\rangle_-, \tag{3}$$
$$\alpha(\epsilon) \equiv 1/[2(1+|\epsilon|^2)]^{1/2},$$

and we use the isospin (I) and strangeness (S) notation $|I, I_z\rangle_S$. The complex parameter ϵ is the usual CP-violation parameter of order 10^{-3} in magnitude. We also need the final states

$$\langle\pi^+\pi^0| = \langle 2, \pm 1|e^{i\delta_2},$$
$$\langle\pi^+\pi^-| = (\tfrac{1}{3})^{1/2}\langle 2,0|e^{i\delta_2} + (\tfrac{2}{3})^{1/2}\langle 0,0|e^{i\delta_0},$$
and
$$\langle\pi^0\pi^0| = (\tfrac{2}{3})^{1/2}\langle 2,0|e^{i\delta_2} - (\tfrac{1}{3})^{1/2}\langle 0,0|e^{i\delta_0}, \tag{4}$$

where δ_I is the s-wave $\pi\pi$ scattering phase shift for isospin I at the $\pi\pi$ c.m. energy equal to the kaon mass and we use the isospin notation $\langle I, I_z|$. The final state $\pi\pi$ interaction is accounted for in the usual way by the inclusion of the $\pi\pi$ phase shifts in the manner shown.

The basic isospin amplitudes are

$$\langle 0,0|H|K^0\rangle \equiv B_0 e^{i\delta_0}, \quad \langle 0,0|H|\bar{K}^0\rangle \equiv B_0 e^{i\delta_0},$$
$$\langle 2,0|H|K^0\rangle \equiv B_2 e^{i\delta_2}, \quad \langle 2,0|H|\bar{K}^0\rangle \equiv B_2^* e^{i\delta_2}, \tag{5}$$
$$\langle 2,1|H|K^+\rangle \equiv B_+ e^{i\delta_2}, \text{ and } \langle 2,-1|H|K^-\rangle \equiv B_+^* e^{i\delta_2},$$

where the forms given are determined by CPT invariance (with the final state interaction, i.e., the δ's, set equal to zero) and B_0 is set to be real because there is an arbitrary overall phase factor. For CP conservation B_+ and B_2 would also be real. Since CP violation is a small effect, $\mathrm{Im}B_+$ and $\mathrm{Im}B_2$ are of the order 10^{-3} compared to B_0. The decay amplitudes in terms of these basic amplitudes are

$$A_{+0} = \langle\pi^+\pi^0|H|K^+\rangle = B_+ e^{i\delta_2}, \quad A_{-0} = \langle\pi^-\pi^0|H|K^-\rangle = B_+^* e^{i\delta_2}$$
$$A_{+-}^S = \alpha(\epsilon)[(1+\epsilon)\langle\pi^+\pi^-|H|K^0\rangle + (1-\epsilon)\langle\pi^+\pi^-|H|\bar{K}^0\rangle],$$
$$A_{00}^S = \alpha(\epsilon)[(1+\epsilon)\langle\pi^0\pi^0|H|K^0\rangle + (1-\epsilon)\langle\pi^0\pi^0|H|\bar{K}^0\rangle],$$
$$A_{+-}^L = \alpha(\epsilon)[(1+\epsilon)\langle\pi^+\pi^-|H|K^0\rangle - (1-\epsilon)\langle\pi^+\pi^-|H|\bar{K}^0\rangle],$$
and
$$A_{00}^L = \alpha(\epsilon)[(1+\epsilon)\langle\pi^0\pi^0|H|K^0\rangle - (1-\epsilon)\langle\pi^0\pi^0|H|\bar{K}^0\rangle], \tag{6}$$

where

$$\langle\pi^+\pi^-|H|K^0\rangle = (\tfrac{1}{3})^{1/2}B_2 e^{i\delta_2} + (\tfrac{2}{3})^{1/2}B_0 e^{i\delta_0},$$
$$\langle\pi^+\pi^-|H|\bar{K}^0\rangle = (\tfrac{1}{3})^{1/2}B_2^* e^{i\delta_2} + (\tfrac{2}{3})^{1/2}B_0 e^{i\delta_0},$$
$$\langle\pi^0\pi^0|H|K^0\rangle = (\tfrac{2}{3})^{1/2}B_2 e^{i\delta_2} - (\tfrac{1}{3})^{1/2}B_0 e^{i\delta_0}, \tag{7}$$
and
$$\langle\pi^0\pi^0|H|\bar{K}^0\rangle = (\tfrac{2}{3})^{1/2}B_2^* e^{i\delta_2} - (\tfrac{1}{3})^{1/2}B_0 e^{i\delta_0}.$$

By a short study of the initial and final states one observes that the decay Hamiltonian H can change the isospin by either $\frac{1}{2}$, $\frac{3}{2}$, or $\frac{5}{2}$. Therefore, the most general decay Hamiltonian can be written as

$$H = H_{1/2}{}^{\pm 1/2} + H_{3/2}{}^{\pm 1/2} + H_{5/2}{}^{\pm 1/2}, \tag{8}$$

where the subscript is ΔI and the superscript is ΔI_z. Upon sandwiching this Hamiltonian between the initial and final states in the basic amplitudes and using the Wigner-Eckart theorem we obtain

$$B_0 = A_1/\sqrt{2},$$
$$B_2 = (A_3 + A_5)/\sqrt{2}, \tag{9}$$
and
$$B_+ = (\tfrac{3}{4})^{1/2}(A_3 - \tfrac{2}{3}A_5),$$

where $A_n \equiv \langle I \| H_{n/2} \| \tfrac{1}{2}\rangle$ is the reduced matrix element for $\Delta I = n/2$ connecting the initial state with isospin $\frac{1}{2}$ to the final state with isospin I ($I=0$ for $n=1$ and $I=2$ for $n=3$ or 5). The $\Delta I = \frac{1}{2}$ rule states that $|A_3| \ll A_1$ and $|A_5| \ll A_1$ or $|B_2| \ll B_0$ and $|B_+| \ll B_0$.

So we have the following kinds of parameters:

(1) CP-violation parameters ($\mathrm{Re}\epsilon$, $\mathrm{Im}\epsilon$, $\mathrm{Im}B_2$, and $\mathrm{Im}B_+$);
(2) $\Delta I \neq \frac{1}{2}$ parameters ($\mathrm{Re}B_2$ and $\mathrm{Re}B_+$);
(3) $\Delta I = \frac{1}{2}$ parameter (B_0).

If, in any good fit to the data, one of the parameters $\mathrm{Re}B_2$ or $\mathrm{Re}B_+$ is not small compared to B_0, then the $\Delta I = \frac{1}{2}$ rule is not proved to be valid.

The experimentally determined quantities[2] are

$$R_S \equiv \frac{\rho_{+-}^S}{\rho_{00}^S}\left|\frac{A_{+-}^S}{A_{00}^S}\right|^2 \equiv \frac{\rho_{+-}^S}{\rho_{00}^S}\bar{R}_S = 2.205 \pm 0.030,$$
$$R_{S+} \equiv \frac{\rho_{+-}^S}{\rho_{+0}^+}\left|\frac{A_{+-}^S}{A_{+0}}\right|^2 \equiv \frac{\rho_{+-}^S}{\rho_{+0}^+}\bar{R}_{S+} = 454.6 \pm 5.3, \tag{10}$$

$$\eta_{+-} \equiv |\eta_{+-}|\exp(i\phi_{+-}) \equiv \frac{A_{+-}^L}{A_{+-}^S} = (2.17 \pm 0.07)\times 10^{-3} \times \exp[i(46.6° \pm 2.5°)],$$
and
$$\eta_{00} \equiv |\eta_{00}|\exp(i\phi_{00}) \equiv \frac{A_{00}^L}{A_{00}^S} = (2.25 \pm 0.09)\times 10^{-3} \times \exp[i(49° \pm 13°)], \tag{11}$$

where the two-body phase space factors for $a \to b + c$ are[3]

$$\rho_{bc}^a = \{[M_a{}^2 - (M_b + M_c)^2][M_a{}^2 - (M_b - M_c)^2]\}^{1/2}/M_a{}^2 \tag{12}$$

in terms of the masses of a, b, and c. Note that the experimental value, $49°\pm13°$, given for ϕ_{00} is obtained[2] by combining two experiments.[4] The earlier experiment obtained a value of $51°\pm30°$ and the later experiment yielded $38°\pm25°$. The same group did both experiments and claimed that the later experiment is the more reliable of the two. We shall use both the combined value and the later value in our analysis.

The equations for the experimentally determined quantities in terms of the parameters given above can be found in Refs. 1. Here we give approximate expressions for $|\epsilon|\ll \mathrm{Re}B_n$ and $\mathrm{Im}B_n \ll \mathrm{Re}B_n$:

$$
\begin{aligned}
\bar{R}_S &\cong \frac{2+(\mathrm{Re}\bar{B}_2)^2+2\sqrt{2}f\,\mathrm{Re}\bar{B}_2}{1+(\mathrm{Re}\bar{B}_2)^2-2\sqrt{2}f\,\mathrm{Re}\bar{B}_2}\,,\\
\bar{R}_{S+} &\cong \frac{4}{9}\,\frac{2+(\mathrm{Re}\bar{B}_2)^2+2\sqrt{2}f\,\mathrm{Re}\bar{B}_2}{|B_+|^2}\,,\\
\eta_{+-} &\cong \epsilon+\frac{ih\,\mathrm{Im}\bar{B}_2}{\sqrt{2}+h\,\mathrm{Re}\bar{B}_2}\,,\\
\eta_{00} &\cong \epsilon-\frac{i\sqrt{2}h\,\mathrm{Im}\bar{B}_2}{1-\sqrt{2}h\,\mathrm{Im}\bar{B}_2}\,,
\end{aligned}
\tag{13}
$$

where $B_n \equiv B_n/B_0$, $h\equiv \exp[i(\delta_2-\delta_0)]$, and $f\equiv\cos(\delta_2-\delta_0)$.

Note that the $\pi\pi$ s-wave scattering phase shifts enter in only in the combination $\delta_2-\delta_0$. These $\pi\pi$ phase shifts are experimentally determined in $\pi p\to\pi p\pi$ reactions. Therefore, we can use $\delta_2-\delta_0$ both as a parameter to be varied in the expressions above and as an experimental quantity. We use the $\delta_2-\delta_0$ value determined by Estabrooks *et al.*[5] except that we approximately double the error to include other recent determinations of its value; the value we use is $-49°\pm5°$.

Since the experimental values of $\bar{R}_S$ and $\bar{R}_{S+}$ are approximately 2 and 450, respectively, it appears probable from the equation above that $\mathrm{Re}\bar{B}_2\ll1$ and $\mathrm{Re}\bar{B}_+\ll1$, i.e., that $\Delta I=\frac{1}{2}$ is dominant. However, it is possible that $\mathrm{Re}\bar{B}_+\ll1$ but $\mathrm{Re}\bar{B}_2\approx1$: We can set $\bar{R}_S=2$ and solve to get $\mathrm{Re}\bar{B}_2(3\,\mathrm{Re}\bar{B}_2-6\sqrt{2}f)=0$. One solution is $\mathrm{Re}\bar{B}_2=0$, the $\Delta I=\frac{1}{2}$ dominance solution; the other solution is

$$\mathrm{Re}B_2=2\sqrt{2}f=2\sqrt{2}\cos(\delta_2-\delta_0)=1.85,\ \text{i.e.,}\ \Delta I\neq\tfrac{1}{2}\,.$$

We obtain fits to the most recent data[2] starting from the solutions found in Refs. 1. There are seven experimental numbers to be fitted ($\bar{R}_S$, $\bar{R}_{S+}$, $|\eta_{+-}|$, $|\eta_{00}|$, ϕ_{+-}, ϕ_{00}, and $\delta_2-\delta_0$) and six variable parameters ($\mathrm{Re}\epsilon$, $\mathrm{Im}\epsilon$, $|\bar{B}_+|$, $\mathrm{Re}\bar{B}_2$, $\mathrm{Im}\bar{B}_2$, and $\delta_2-\delta_0$). The results of the fits are shown in Table I. We see that the $\Delta I=\frac{1}{2}$ solution is favored when we use $\phi_{00}=49°\pm13°$ and the $\Delta I\neq\frac{1}{2}$ solution is favored when we use $\phi_{00}=38°\pm25°$.

Note in Table I that $\mathrm{Im}\bar{B}_2$ is zero within errors.

TABLE I. $K\to2\pi$ fits.

$\phi_{00}=49°\pm13°$	$\Delta I=\frac{1}{2}$ solution	$\Delta I\neq\frac{1}{2}$ solution
$\lvert\bar{B}_+\rvert$	$0.044\,97\pm0.000\,29$	0.0880 ± 0.0072
$\mathrm{Re}\bar{B}_2$	0.0406 ± 0.0063	1.68 ± 0.18
$\mathrm{Im}\bar{B}_2$	$(-0.037\pm0.053)\times10^{-3}$	$(0.06\pm0.94)\times10^{-3}$
$\mathrm{Re}\epsilon$	$(1.508\pm0.080)\times10^{-3}$	$(1.50\pm0.14)\times10^{-3}$
$\mathrm{Im}\epsilon$	$(1.597\pm0.078)\times10^{-3}$	$(1.58\pm0.33)\times10^{-3}$
$\delta_2-\delta_0$	$-49.0°\pm5.0°$	$-49.0°\pm5.0°$
χ^2	0.0401	0.5334
Probability	0.841	0.465
$\phi_{00}=38°\pm25°$		
$\lvert\bar{B}_+\rvert$	$0.044\,97\pm0.000\,29$	0.0880 ± 0.0071
$\mathrm{Re}\bar{B}_2$	0.0406 ± 0.0062	1.68 ± 0.18
$\mathrm{Im}\bar{B}_2$	$(-0.038\pm0.053)\times10^{-3}$	$(-0.7\pm1.0)\times10^{-3}$
$\mathrm{Re}\epsilon$	$(1.512\pm0.080)\times10^{-3}$	$(1.60\pm0.12)\times10^{-3}$
$\mathrm{Im}\epsilon$	$(1.592\pm0.079)\times10^{-3}$	$(1.84\pm0.32)\times10^{-3}$
$\delta_2-\delta_0$	$-49.0°\pm5.0°$	$-49.0°\pm5.0°$
χ^2	0.1105	0.0156
Probability	0.739	0.900

Table II shows the results of fits to the data with $\mathrm{Im}\bar{B}_2=0$. Rather good fits to the data are obtained in all cases.

We can extend the analysis to include the semileptonic decays of kaons, as we did in our last paper.[1] Then the latest data yield fitted probabilities of $P(\Delta I=\frac{1}{2})=0.251$ and $P(\Delta I\neq\frac{1}{2})=0.367$ for $\phi_{00}=49°\pm13°$ and of $P(\Delta I=\frac{1}{2})=0.261$ and $P(\Delta I\neq\frac{1}{2})=0.714$ for $\phi_{00}=38°\pm25°$. The fitted parameters are given in Table III. Thus this combined analysis of nonleptonic and semileptonic decays of kaons favors the $\Delta I\neq\frac{1}{2}$ solution for both values of ϕ_{00}.

We conclude that the $\Delta I=\frac{1}{2}$ rule is not supported by the current $K\to2\pi$ data. A more precise measurement of ϕ_{00} is needed to settle the issue. It is interesting to note that for the $I\neq\frac{1}{2}$ solution

TABLE II. $K\to2\pi$ fits with $\mathrm{Im}\bar{B}_2=0$.

$\phi_{00}=49°\pm13°$	$\Delta I=\frac{1}{2}$ solution	$\Delta I\neq\frac{1}{2}$ solution
$\lvert\bar{B}_+\rvert$	$0.044\,97\pm0.000\,29$	0.0880 ± 0.0072
$\mathrm{Re}\bar{B}_2$	0.0406 ± 0.0063	1.69 ± 0.18
$\mathrm{Re}\epsilon$	$(1.509\pm0.081)\times10^{-3}$	$(1.509\pm0.081)\times10^{-3}$
$\mathrm{Im}\epsilon$	$(1.601\pm0.078)\times10^{-3}$	$(1.601\pm0.078)\times10^{-3}$
$\delta_2-\delta_0$	$-49.0°\pm5.0°$	$-49.0°\pm5.0°$
χ^2	0.5252	0.5252
Probability	0.769	0.769
$\phi_{00}=38°\pm25°$		
$\lvert\bar{B}_+\rvert$	$0.044\,97\pm0.000\,29$	0.0882 ± 0.0072
$\mathrm{Re}\bar{B}_2$	0.0406 ± 0.0063	1.69 ± 0.18
$\mathrm{Re}\epsilon$	$(1.514\pm0.081)\times10^{-3}$	$(1.514\pm0.081)\times10^{-3}$
$\mathrm{Im}\epsilon$	$(1.596\pm0.079)\times10^{-3}$	$(1.596\pm0.079)\times10^{-3}$
$\delta_2-\delta_0$	$-49.0°\pm5.0°$	$-49.0°\pm5.0°$
χ^2	0.6095	0.6095
Probability	0.737	0.737

TABLE III. Combined nonleptonic and semileptonic kaon decay fits.

$\phi_{00}=49°\pm 13°$	$\Delta I=\frac{1}{2}$ solution	$\Delta I\neq\frac{1}{2}$ solution
$\lvert\overline{B}_+\rvert$	$0.044\,97\pm 0.000\,28$	0.0879 ± 0.0069
$\mathrm{Re}\overline{B}_2$	0.0406 ± 0.0065	1.68 ± 0.18
$\mathrm{Im}\overline{B}_2$	$(-0.034\pm 0.053)\times 10^{-3}$	$(-0.6\pm 0.68)\times 10^{-3}$
$\mathrm{Re}\epsilon$	$(1.593\pm 0.057)\times 10^{-3}$	$(1.629\pm 0.066)\times 10^{-3}$
$\mathrm{Im}\epsilon$	$(1.557\pm 0.074)\times 10^{-3}$	$(1.786\pm 0.24)\times 10^{-3}$
$\delta_2-\delta_0$	$-49.1°\pm 5.0°$	$-49.2°\pm 4.8°$
$\mathrm{Re}x$ [a]	0.022 ± 0.017	0.015 ± 0.018
$\mathrm{Im}\,x$	0.012 ± 0.030	0.011 ± 0.030
χ^2	2.768	2.007
Probability	0.251	0.367
ϕ_{00} $38°\pm 25°$		
$\lvert\overline{B}_+\rvert$	$0.044\,97\pm 0.000\,29$	0.0895 ± 0.0067
$\mathrm{Re}\overline{B}_2$	0.0407 ± 0.0063	1.72 ± 0.17
$\mathrm{Im}\overline{B}_2$	$(-0.035\pm 0.053)\times 10^{-3}$	$(-1.10\pm 0.82)\times 10^{-3}$
$\mathrm{Re}\epsilon$	$(1.597\pm 0.059)\times 10^{-3}$	$(1.651\pm 0.067)\times 10^{-3}$
$\mathrm{Im}\epsilon$	$(1.551\pm 0.075)\times 10^{-3}$	$(1.91\pm 0.27)\times 10^{-3}$
$\delta_2-\delta_0$	$-49.1°\pm 5.0°$	$-48.0°\pm 4.7°$
$\mathrm{Re}x$	0.021 ± 0.017	0.010 ± 0.018
$\mathrm{Im}x$	0.010 ± 0.030	0.011 ± 0.030
χ^2	2.688	0.674
Probability	0.261	0.714

[a] See Refs. 1 for notation and equations; x is the ratio of the $\Delta S=-\Delta Q$ amplitude to the $\Delta S=\Delta Q$ amplitude for kaon semileptonic decays.

$\overline{B}_+\cong 0$ and $\overline{B}_2\cong\frac{5}{3}$ or $A_3\cong\frac{2}{3}A_1$ and $A_5\cong A_1$.

A further interesting question for $K\to 2\pi$ decays is: Can one fit the data with the $\Delta I=\frac{1}{2}$ dominance solution without any $\Delta I=\frac{5}{2}$ amplitude? The answer is "yes." We get fitted probabilities between 0.35 and 0.8 for all fitted attempts for both values of ϕ_{00}, with and without semileptonic data included.

III. $\Xi\to\Lambda\pi$ DECAYS

The decays are

$$\Xi^0\to\Lambda+\pi^0(\Xi_l^0)\text{ and }\Xi^-\to\Lambda+\pi^-(\Xi_l^-)\,, \tag{14}$$

where the symbols for the decay amplitudes are indicated inside the parentheses. Since parity is not conserved, the $\Lambda\pi$ final state can have angular momentum l equal to zero as well as one. We distinguish between the amplitudes for the two angular momenta by a subscript l in the symbols for the decay amplitudes.

Typically we want to calculate a decay amplitude such as $\langle\Xi^0\to\Lambda+\pi^0\rangle_l={}_l\langle\pi^0\Lambda|H|\Xi^0\rangle$. To calculate this and the other decay amplitudes we need the initial states

$$|\Xi^0\rangle=|\tfrac{1}{2},+\tfrac{1}{2}\rangle\text{ and }|\Xi^-\rangle=|\tfrac{1}{2},-\tfrac{1}{2}\rangle \tag{15}$$

in terms of isospin states $|I,I_z\rangle$, and the final states

$$_l\langle\pi^0\Lambda|={}_l\langle 1,0|e^{i\delta_2}\text{ and }{}_l\langle\pi^-\Lambda|={}_l\langle 1,-1|e^{i\delta_l} \tag{16}$$

in terms of isospin states $\langle I,I_z|$, and the $\pi\Lambda$ scattering phase shift δ_l for angular momentum l at the $\pi\Lambda$ c.m. energy equal to the Ξ mass. The final state interaction is accounted for in the usual way by the inclusion of the $\pi\Lambda$ phase shifts in the manner shown. The $\pi\Lambda$ phase shifts are not available from other experiments, so we must consider them as variable parameters in our fits to the decay data. Actually, as we shall see below, only the combination $\delta_1-\delta_0$ occurs in the equations for the decay amplitudes.

The basic isospin amplitudes are, in this case, the same as the decay amplitudes

$$\Xi_l^0={}_l\langle 1,0|H|\tfrac{1}{2},+\tfrac{1}{2}\rangle e^{i\delta_l}\equiv B_{0l}e^{i\delta_l}$$

and (17)

$$\Xi_l^-={}_l\langle 1,0|H|\tfrac{1}{2},-\tfrac{1}{2}\rangle e^{i\delta_l}\equiv B_{-l}e^{i\delta_l}\;.$$

We know that in $K\to 2\pi$ decays CP violation is a small effect. So we assume here that CP is conserved, which implies that B_{0l} and B_{-l} are real. We shall see below that the Ξ decay data are somewhat inconsistent with this assumption, but that the introduction of imaginary parts to the amplitudes would yield more parameters than there are number of data to be fitted. In the next section we shall see that the Λ decay data can be well fitted with real amplitudes. So it appears probable that the inconsistency of the Ξ data with CP conservation is due to experimental difficulties.

By a short study of the initial and final states one observes that the decay Hamiltonian H can change the isospin by either $\frac{1}{2}$ or $\frac{3}{2}$. Therefore, the most general decay Hamiltonian can be written as

$$H=H_{1/2}{}^{-1/2}+H_{3/2}{}^{-1/2}\,, \tag{18}$$

where the subscript is ΔI and the superscript is ΔI_z. Upon sandwiching this Hamiltonian between the initial and final states in the decay amplitudes and using the Wigner-Eckart theorem we obtain

$$B_{0l}=(\tfrac{1}{2})^{1/2}(A_{1l}+A_{3l})\text{ and }B_{-l}=A_{1l}+\tfrac{1}{2}A_{3l}\;, \tag{19}$$

where $A_{nl}={}_l\langle 1\|H_{n/2}\|\frac{1}{2}\rangle$ is the reduced matrix element for $\Delta I=n/2$. The $\Delta I=\frac{1}{2}$ rule states that either A_{10} or A_{11} or both are much greater than A_{30} and A_{31}.

So we have the following kinds of parameters:

(1) $\Delta I\neq\frac{1}{2}$ parameters (A_{30} and A_{31});
(2) $\Delta I=\frac{1}{2}$ parameters (A_{10} and A_{11}).

If, in any good fit to the data, one of the parameters A_{30} or A_{31} is not small compared to the larger of the parameters A_{10} and A_{11}, then the $\Delta I=\frac{1}{2}$ rule is not proved to be valid.

The experimentally determined quantities[2] are

$$R\equiv\frac{\rho_0}{\rho_-}\frac{|\Xi_0^0|^2+|\Xi_1^0|^2}{|\Xi_0^-|^2+|\Xi_1^-|^2}\equiv\frac{\rho_0}{\rho_-}\frac{R_0}{R_-}\equiv\frac{\rho_0}{\rho_-}\bar{R}=0.558\pm0.024\ ,$$

$$\alpha_0\equiv2\,\mathrm{Re}(\Xi_0^{0*}\,\Xi_1^0)/R_0=-0.44\pm0.08\ ,\qquad(20)$$

$$\alpha_-\equiv2\,\mathrm{Re}(\Xi_0^{-*}\,\Xi_1^-)/R_-=-0.393\pm0.023\ ,$$

$$\phi_0\equiv\sin^{-1}[\beta_0/(1-\alpha_0^2)^{1/2}]=21^\circ\pm12^\circ\ ,$$

and

$$\phi_-\equiv\sin^{-1}[\beta_-/(1-\alpha_-^2)^{1/2}]=2^\circ\pm7^\circ\ ,$$

where the phase space factors ρ_n are defined as in the previous section and $\beta_n\equiv2\,\mathrm{Im}(\Xi_0^{n*}\,\Xi_1^n)/R_n$. There is another quantity $\gamma_n\equiv(|\Xi_0^n|^2-|\Xi_1^n|^2)/R_n$, but one can show that $\alpha_n^2+\beta_n^2+\gamma_n^2=1$. Therefore, if α_n and β_n are known, then γ_n is fixed except for sign. So, all solutions obtained must be checked to ensure that γ_0 and γ_- have the correct signs as determined by experiment. If $\gamma_n\geq0$ then $-\pi/2\leq\phi_n\leq\pi/2$ and if $\gamma_n\leq0$ then $\pi/2\leq\phi_n\leq3\pi/2$.

We see from the values of ϕ_0 and ϕ_- given above that experimentally $\gamma_0>0$ and $\gamma_1>0$. The α_n are related to the decay symmetry of a polarized Ξ and the longitudinal polarization of the Λ. The β_n and γ_n are related to the transverse polarization of the Λ. See Ref. 6 for the details.

The equations for the experimentally determined quantities in terms of the parameters given above are

$$R_0=B_{00}^2+B_{01}^2,\quad R_-=B_{-0}^2+B_{-1}^2\ ,$$

$$\alpha_n=2B_{n0}B_{n1}f/R_n\ ,\quad\beta_n=2B_{n0}B_{n2}g/R_n\ ,\qquad(21)$$

and

$$\gamma_n=(B_{n0}^2-B_{n1}^2)/R_n\ ;$$

where $f\equiv\cos(\delta_1-\delta_0)$ and $g\equiv\sin(\delta_1-\delta_0)$. Since these equations contain only bilinear combinations of the decay amplitudes, we can only determine the amplitudes relative to some specified amplitude. We choose $A_{10}=1$ and then determine all other amplitudes relative to it.

One observes immediately from these equations that

$$\beta_0/\alpha_0=(1-\alpha_0^2)^{1/2}\sin\phi_0/\alpha_0=\frac{g}{f}=(1-\alpha^2)^{1/2}\sin\phi_-/\alpha_-=\beta_-/\alpha_-\ .\qquad(22)$$

Upon putting in the experimental numbers on both sides of this equation we got $-0.73\pm0.41=-0.08\pm0.28$. The errors on these two numbers just barely overlap; so, as mentioned previously, the formalism with CP conservation is not strongly supported by the data. However, if we allowed all but one of the reduced matrix elements to be complex (one can be taken to be real because of an arbitrary overall phase factor), we would have six amplitude ratios to be determined by fitting five experimental numbers. So, because of this and arguments previously given, we shall maintain CP conservation, but we cannot expect to achieve low χ^2's in our fits.

Note that the $\pi\Lambda$ s- and p-wave scattering phase shifts enter in Eqs. (21) only in the combination $\delta_1-\delta_0$. This is a parameter to be varied in our fits.

For $\Delta I=\frac{1}{2}$ Eqs. (21), and also more general equations without CP conservation, yield

$$\bar{R}=R_0/R_-=\tfrac{1}{2},\quad\alpha_0=\alpha_-,\ \text{and}\ \phi_0=\phi_-\ .\qquad(23)$$

(One can show that some $\Delta I\neq\frac{1}{2}$ solutions also satisfy these equations.) We see that $\phi_0=\phi_-$ is not well satisfied by the data, but this is essentially the same as the CP conservation discrepancy discussed above (because $\alpha_0\cong\alpha_-$). So, if CP is not conserved, it appears that $\Delta I\neq\frac{1}{2}$ also.

Now we can obtain fits to the most recent data[2] starting from many random inputs and from various algebraic solutions to the equations $\bar{R}=\frac{1}{2}$, $\alpha_0=\alpha_-$, and $\phi_0=\phi_-$. There are five experimental numbers ($\bar{R}$, α_0, α_-, ϕ_0, and ϕ_-. Also, one must check the signs of γ_0 and γ_- as explained above) and four variable parameters ($\bar{A}_{11}$, $\bar{A}_{30}$, $\bar{A}_{31}$, and $\delta_1-\delta_0$ where $\bar{A}_{nl}\equiv A_{nl}/A_{10}$). The results of the fits are shown in Table IV. We see that the $\Delta I\cong\frac{1}{2}$ solutions and the $\Delta I\neq\frac{1}{2}$ solutions are equally good, both types having a rather high χ^2, the reason for which is the data discrepancy described above.

We conclude from Table IV that the $\Delta I=\frac{1}{2}$ rule is not supported by the $\Xi\to\Lambda\pi$ data. Future experimental work should be directed toward more precise measurements of ϕ_0 and ϕ_-. Also, an independent determination of the $\pi\Lambda$ scattering phase shifts δ_1 and δ_0 would be helpful. It is interesting to note that for the $\Delta I\neq\frac{1}{2}$ solution $A_{30}/A_{10}\cong-\frac{4}{3}$, $A_{31}/A_{11}\cong-\frac{4}{3}$, $A_{n0}/A_{n1}\cong\pm4$.

IV. $\Lambda\to N\pi$ DECAYS

The decays are

$$\Lambda\to n+\pi^0(\Lambda_{0l})\ \text{and}\ \Lambda\to p+\pi^-(\Lambda_{-l})\ ,\qquad(24)$$

where the symbols for the decay amplitudes are indicated inside the parentheses. The same angular momentum situation and notation apply as in the last section.

TABLE IV. $\Xi \to \Lambda + \pi$ fits.

	$\Delta I \cong \frac{1}{2}$		$\Delta I \neq \frac{1}{2}$	
A_{10}	1	1	1	1
A_{11}	0.170 ± 0.065	-0.172 ± 0.060	0.226 ± 0.019	-0.226 ± 0.021
A_{30}	0.127 ± 0.064	0.128 ± 0.060	-1.3464 ± 0.0057	-1.3464 ± 0.0061
A_{31}	0.114 ± 0.11	-0.110 ± 0.11	-0.313 ± 0.034	0.313 ± 0.037
$\delta_1 - \delta_0$	164 ± 13	-16 ± 13	164 ± 13	-16 ± 13

There are two solutions of each type because the observable equations are invariant under the following simultaneous transformations:

$\delta_1 - \delta_0 \to 180° + (\delta_1 - \delta_0)$ [i.e., $\cos(\delta_1 - \delta_0) \to -\cos(\delta_1 - \delta_0)$ and $\sin(\delta_1 - \delta_0) \to -\sin(\delta_1 - \delta_0)$] and $A_{n1} \to -A_{n1}$.

All solutions have $\chi^2 \cong 1.648$ (probability = 0.20).

All solutions predict the same values of ϕ_0, ϕ_-, γ_0, and γ_-:

	Predicted values	Experimental values [a]
ϕ_0	$8.5° \pm 6.8°$	$21° \pm 12°$
ϕ_-	$7.1° \pm 5.6°$	$2° \pm 7°$
γ_0	0.882 ± 0.048	0.84
γ_-	0.913 ± 0.012	0.92

[a] See Ref. 2.

Typically we want to calculate a decay amplitude such as $\langle \Lambda \to n + \pi^0 \rangle_l = {}_l\langle \pi^0 n | H | \Lambda \rangle$. To calculate this and the other decay amplitude we need the initial state $|\Lambda\rangle = |0, 0\rangle$ in terms of the $I = 0$ isospin state and the final states

$$ {}_l\langle \pi^0 n | = (\tfrac{2}{3})^{1/2}\, {}_l\langle \tfrac{3}{2}, -\tfrac{1}{2} | e^{i\delta_{3l}} + (\tfrac{1}{3})^{1/2}\, {}_l\langle \tfrac{1}{2}, -\tfrac{1}{2} | e^{i\delta_{1l}} $$

and (25)

$$ {}_l\langle \pi^- p | = (\tfrac{1}{3})^{1/2} \langle \tfrac{3}{2}, -\tfrac{1}{2} | e^{i\delta_{3l}} - (\tfrac{2}{3})^{1/2}\, {}_l\langle \tfrac{1}{2}, -\tfrac{1}{2} | e^{i\delta_{1l}} $$

in terms of isospin states $\langle I, I_z|$ and the πN scattering phase shifts $\delta_{2I,l}$ for isospin I and angular momentum l at the πN c.m. energy equal to the Λ mass. The final state πN interaction is accounted for in the usual way by the inclusion of the πN phase shifts in the manner shown.

The basic isospin amplitudes are

$$ {}_l\langle \tfrac{3}{2}, -\tfrac{1}{2} | H | \Lambda \rangle \equiv A_{3l} e^{i\delta_{3l}} $$

and (26)

$$ {}_l\langle \tfrac{1}{2}, -\tfrac{1}{2} | H | \Lambda \rangle \equiv A_{1l} e^{i\delta_{1l}} \ . $$

We assume that CP is conserved, which implies that the A_{nl} are real. Making the A_{nl} complex would yield more parameters than there are data to be fitted. The decay amplitudes in terms of these basic amplitudes are

$$ \Lambda_{0l} = (\tfrac{2}{3})^{1/2} A_{3l} e^{i\delta_{3l}} + (\tfrac{1}{3})^{1/2} A_{1l} e^{i\delta_{1l}} $$

and (27)

$$ \Lambda_{-l} = (\tfrac{1}{3})^{1/2} A_{3l} e^{i\delta_{3l}} - (\tfrac{2}{3})^{1/2} A_{1l} e^{i\delta_{1l}} \ . $$

By a short study of the initial and final states one observes that the decay Hamiltonian H can change the isospin by either $\frac{1}{2}$ or $\frac{3}{2}$. Therefore, the most general decay Hamiltonian can be written as

$$ H = H_{1/2}{}^{-1/2} + H_{3/2}{}^{-1/2} \ , \tag{28} $$

where the subscript is ΔI and the superscript is ΔI_z. Upon sandwiching this Hamiltonian between the initial and final states in the basic amplitudes and using the Wigner-Eckart theorem we obtain

$$ A_{nl} = {}_l\langle n/2 \| H_{n/2} \| 0 \rangle \ , $$

the reduced matrix element for $\Delta I = n/2$. The $\Delta I = \frac{1}{2}$ rule states that either A_{10} or A_{11} or both are much greater than A_{30} and A_{31}.

So we have the following kinds of parameters:

(1) $\Delta I \neq \frac{1}{2}$ parameters (A_{30} and A_{31});
(2) $\Delta I = \frac{1}{2}$ parameters (A_{10} and A_{11}).

If, in any good fit to the data, one of the parameters A_{30} or A_{31} is not small compared to the larger of the parameters A_{10} and A_{11}, then the $\Delta I = \frac{1}{2}$ rule is not proved to be valid.

The experimentally determined quantities[2] are

$$R \equiv \frac{\rho_-}{\rho_0}\,\frac{|\Lambda_{-0}|^2+|\Lambda_{-1}|^2}{|\Lambda_{00}|^2+|\Lambda_{01}|^2} \equiv \frac{\rho_-}{\rho_0}\,\frac{R_-}{R_0} \equiv \frac{\rho_-}{\rho_0}\,\overline{R} = 1.793 \pm 0.029\ ,$$

$$\alpha_- \equiv 2\,\mathrm{Re}(\Lambda_{-0}^{*}\Lambda_{-1})/R_- = 0.647 \pm 0.013\ ,$$

$$\alpha_0 \equiv 2\,\mathrm{Re}(\Lambda_{00}^{*}\Lambda_{01})/R_0 = 0.651 \pm 0.045\ , \qquad (29)$$

$$\phi_- \equiv \sin^{-1}[\beta_-/(1-\alpha_-^{2})^{1/2}] = -6.5^\circ \pm 3.5^\circ\ ,$$

and

$$\phi_0 \equiv \sin^{-1}[\beta_0/(1-\alpha_0^{2})^{1/2}] = ?\ \text{(not yet measured)}\ ,$$

where the phase space factors ρ_n are defined as in Sec. II and $\beta_n \equiv 2\,\mathrm{Im}(\Lambda_{n0}^{*}\Lambda_{n1})/R_n$. As explained in the last section, a solution's prediction of the quantities $\gamma_n \equiv (|\Lambda_{n0}|^2 - |\Lambda_{n1}|^2)/R_n$ must be checked against the experimental signs. We see from the value of ϕ_- given above that $\gamma_- > 0$; the γ_0 sign has not been measured.

The equations for the experimentally determined quantities in terms of the parameters are rather long, so we only give them here for the special case of $\Delta I = \frac{1}{2}$:

$$\overline{R} = 2, \quad \alpha_0 = \alpha_1 = \frac{2A_{10}A_{11}f}{A_{10}^{\,2}+A_{11}^{\,2}}\ , \text{ and } \beta_0 = \beta_- = \frac{2A_{10}A_{11}g}{A_{10}^{\,2}+A_{11}^{\,2}}\ , \qquad (30)$$

where $f \equiv \cos(\delta_{11}-\delta_{10})$ and $g \equiv \sin(\delta_{11}-\delta_{10})$. (One can show that some $\Delta I \neq \frac{1}{2}$ solutions also satisfy these equations.) Since these equations contain only bilinear combinations of the basic amplitudes, we can only determine the amplitudes relative to some specified amplitude. We choose $A_{10} = 1$ and then determine all other amplitudes relative to it.

Three slightly different sets of low-energy pion-nucleon scattering phase shifts are reported by Nielsen and Oades.[7] We use the four phase shifts ($\delta_{2I,l}$ for $I = \frac{1}{2}$ and $\frac{3}{2}$ and $l = 0$ and 1) at the πN c.m. energy equal to the Λ mass (44.5 MeV πN laboratory kinetic energy) both as parameters to be varied in the fits and as experimental quantities themselves, using the three Nielsen-Oades phase shift sets in three separate fits.

Now we can obtain fits to the most recent data[2] starting from many random inputs and from various algebraic solutions to the equations $\overline{R} = 2$, $\alpha_0 = \alpha_-$, and $\phi_0 = \phi_-$. There are eight experimental numbers ($\overline{R}$, α_-, α_0, ϕ_-, δ_{10}, δ_{11}, δ_{30}, and δ_{31}. Also, one must check the sign of γ_-) and seven variable parameters ($\overline{A}_{11}$, $\overline{A}_{30}$, $\overline{A}_{31}$, δ_{10}, δ_{11}, δ_{30}, and δ_{31}, where $\overline{A}_{nl} \equiv A_{nl}/A_{10}$). The results of the fits are shown in Table V for the three Nielsen-Oades sets of πN phase shifts. We see that the $\Delta I = \frac{1}{2}$ and $\Delta I \neq \frac{1}{2}$ solutions are generally equally good. There is a preference for the $\Delta I = \frac{1}{2}$ solu-

TABLE V. $\Lambda \to N + \pi$ fits.

	Nielsen-Oades No. 1	$\Delta I = \frac{1}{2}$	$\Delta I \neq \frac{1}{2}$	Nielsen-Oades No. 2	$\Delta I = \frac{1}{2}$	$\Delta I \neq \frac{1}{2}$	Nielsen-Oades No. 3	$\Delta I = \frac{1}{2}$	$\Delta I \neq \frac{1}{2}$
A_{10}		1	1		1	1		1	1
A_{11}		0.374 ± 0.014	0.735 ± 0.018		0.373 ± 0.014	0.735 ± 0.018		0.373 ± 0.014	0.736 ± 0.01
A_{30}		0.0185 ± 0.0072	−0.356 ± 0.024		0.0185 ± 0.0072	−0.356 ± 0.024		0.0184 ± 0.0071	−0.357 ± 0.024
A_{31}		0.008 ± 0.015	0.368 ± 0.014		0.008 ± 0.015	0.387 ± 0.014		0.008 ± 0.015	0.388 ± 0.014
δ_{10}	8 ± 1	7.91 ± 0.97	8.05 ± 0.98	7.7 ± 1	7.63 ± 0.97	7.77 ± 0.98	7.1 ± 1	7.07 ± 0.47	7.19 ± 0.98
δ_{11}	−0.92 ± 0.2	−0.92 ± 0.20	−0.92 ± 0.20	−0.88 ± 0.2	−0.88 ± 0.20	−0.89 ± 0.20	−0.82 ± 0.2	−0.82 ± 0.20	−0.83 ± 0.20
δ_{30}	−4.45 ± 0.3	−4.45 ± 0.30	−4.45 ± 0.30	−4.45 ± 0.3	−4.45 ± 0.30	−4.45 ± 0.30	−4.45 ± 0.3	−4.45 ± 0.3	−4.45 ± 0.3
δ_{31}	−0.73 ± 0.1	−0.73 ± 0.10	−0.73 ± 0.10	−0.75 ± 0.1	−0.75 ± 0.10	−0.75 ± 0.10	−0.7 ± 0.1	−0.70 ± 0.10	−0.70 ± 0.10
χ^2		0.129	0.0656		0.0757	0.1109		0.0130	0.2172
Probability		0.719	0.797		0.782	0.738		0.908	0.641
	Experimental values				Predictions				
ϕ_-	−6.5 ± 3.5	−7.71 ± 0.92	−5.63 ± 0.77		−7.43 ± 0.92	−5.36 ± 0.77		−6.89 ± 0.91	−4.91 ± 0.76
ϕ_0	...	−7.38 ± 1.3	−160.8 ± 2.0		−7.1 ± 1.3	−161.3 ± 2.0		−6.6 ± 1.2	−162.4 ± 2.0
γ_-	0.76	0.756 ± 0.012	0.759 ± 0.011		0.746 ± 0.012	0.759 ± 0.011		0.757 ± 0.012	0.759 ± 0.011
γ_0	...	0.753 ± 0.041	−0.717 ± 0.042		0.753 ± 0.041	−0.719 ± 0.042		0.754 ± 0.041	−0.723 ± 0.041

tion when using the Nielsen-Oades set No. 3. Note that small changes in the πN phase shifts can shift the preference from one type of solution to another.

We conclude from Table V that the $\Delta I = \frac{1}{2}$ rule is not supported by the $\Lambda \to N\pi$ data. The predicted value of the unmeasured quantity ϕ_0 is quite different for the two kinds of solutions; perhaps a measurement of this quantity would help resolve the issue. However, we caution that we may not have found all possible solutions in our random fits. It is interesting to note that for the $\Delta I \neq \frac{1}{2}$ solution $A_{30}/A_{10} \cong -\frac{1}{3}$, $A_{31}/A_{11} \cong \sqrt{2}/3$ and $A_{30}/A_{31} \cong -1$.

V. OTHER DECAYS

Data for $\Sigma \to N\pi$ are available[2] only for $\Sigma^{\pm}$. The dominant Σ^0 decay mode is $\Lambda + \gamma$ and it swamps the nonleptonic channels $n + \pi^0$ and $p + \pi^-$. There are ten reduced matrix elements, $A_{nml} \equiv {}_l\langle n/2 \| A_{m/2} \| 1\rangle$, because $\Delta I = \frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ are allowed for Σ decays. Thus there are nine ratios to be determined, but there are only eight experimental quantities[2] to be fitted. One needs at least one experimental number for $\Sigma^0 \to N\pi$ in order to determine the decay parameters. We tried to fit the $\Sigma^{\pm}$ data by setting the $\Delta I = \frac{5}{2}$ amplitudes equal to zero, but failed to find a solution with a probability greater than 0.05. (This does not mean that a $\Delta I = \frac{1}{2}$ solution does not exist, because the inclusion or exclusion of small $\Delta I = \frac{3}{2}$ or $\frac{5}{2}$ parameters can make a large difference in the quality of fit for a $\Delta I = \frac{1}{2}$ solution. E.g., for the $\Delta I = \frac{1}{2}$ solution for $\Lambda \to N\pi$ in Table V, when one sets $A_{30} = A_{31} = 0$ then $\chi^2 \cong 25$ which corresponds to a probability of essentially zero. Almost all of the χ^2 comes from the fact that R is 1.850 ± 0.030 rather than the value of 2 mandated by $\Delta I = \frac{1}{2}$.)

There are no data available[2] for $\Omega \to \Xi\pi$.

The situation for $K \to 3\pi$ is complicated.[8] Suffice it to state here that there are six experimental numbers,[2] but there are almost twice that many parameters even under the assumption of CP conservation. The possibility of being able to determine anything definite about the $\Delta I = \frac{1}{2}$ rule on the basis of $K \to 3\pi$ data seems very remote, despite the many comments to the contrary in books and articles.

VI. CONCLUSION

We have shown that none of the nonleptonic decays of hadrons gives a definite verdict concerning the validity of the $\Delta I = \frac{1}{2}$ rule.

In $K \to 2\pi$ decays the $\Delta I = \frac{1}{2}$ solution is favored when the world average (two combined experiments) for ϕ_{00} is used as data. However, the $\Delta I \neq \frac{1}{2}$ solution is favored when the most recent value of ϕ_{00} is used. When semileptonic data are added, the $\Delta I \neq \frac{1}{2}$ solution is favored for both values of ϕ_{00}. A more precise measurement of ϕ_{00} is desirable.

In $\Xi \to \Lambda\pi$ decays the $\Delta I = \frac{1}{2}$ and $\Delta I \neq \frac{1}{2}$ solutions fit the data equally well, although neither fit is very good (probability $= 0.20$) apparently because of a discrepancy between the measured values of ϕ_0 and ϕ_-. More precise measurements of ϕ_0 and ϕ_- are desirable.

In $\Lambda \to N\pi$ decays the $\Delta I = \frac{1}{2}$ and $\Delta I \neq \frac{1}{2}$ solutions are variously favored, depending on which set of pion-nucleon scattering phase shifts are used for the final state interaction. The two solutions predict quite different values for the, as yet unmeasured, quantity ϕ_0. The $\Delta I = \frac{1}{2}$ solution predicts $\phi_0 \cong -7° \pm 1.5°$ and the $\Delta I \neq \frac{1}{2}$ solution predicts $\phi_0 = -161° \pm 2°$.

There are not enough experimental numbers to determine the parameters for $K \to 3\pi$, $\Sigma \to N\pi$, and $\Omega \to \Xi\pi$ decays, and it does not appear likely that there soon will be.

In conclusion, none of the nonleptonic decays of hadrons indicates the validity of the $\Delta I = \frac{1}{2}$ rule.

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