PHYSICAL REVIEW D VOLUME 11, NUMBER 9 1 MAY 1975

Spin content of crossed-channel contributions and interior dispersion relations

Richard J. Jacob
Department of Physics, Arizona State University, Tempe, Arizona 85281

Gerald E. Hite
Universität Kaiserslautern, Kaiserslautern, West Germany
(Received 5 August 1974)

A method of using interior dispersion relations is presented to measure the relative importance and spin content of crossed-channel contributions to two-body scattering amplitudes in the physical region. As an illustration of the method, the $B^{(-)}$ amplitude of πN scattering is considered. It was found that in the physical region this amplitude receives contributions essentially only from the two lowest crossed-channel spin states. The higher of the two states ($J=3$) was found to be adequately described by the $g(1680)$ meson, while the lower ($J=1$) could not be described by the $\rho(760)$ meson alone, but could be described by a sum of the $\rho(760)$ and the $\rho'(1600)$ meson.

I. INTRODUCTION

The frequent discovery of new resonances, e.g., the $\rho'(1600)$ meson,[1] which should contribute as crossed channel exchanges to two-body scattering amplitudes, makes it useful to be able to extract crossed-channel contributions to physical scattering amplitudes. The most acceptable methods involve the usage of dispersion relations.[2,3]

In the dispersion relation approach, the full amplitude or a crossed-channel partial-wave amplitude is written in a representation containing contributions in the form of integrals from both the direct (s) and crossed (t) channels. The so-called discrepancy function is calculated from the difference of the amplitude and the integral over the direct channel and is therefore equal to the contribution of the crossed-channel. In other words, the t-channel contribution is found by subtracting the s-channel cut from the amplitude. If sufficient data are available to evaluate the integral over the s-channel cut, the t-channel contribution can be isolated and studied over the range of t values for which the amplitude is known.

Clearly the philosophy of the dominance of nearby cuts implies in the physical region, i.e., $t \leq 0$, where the amplitude is known that the main contribution comes from the s-channel cut and not from the t-channel cut. Consequently, the isolation of the t-channel contribution depends on a delicate cancellation between the amplitude and the integral over the s-channel cut. This means that such methods can best be applied to reactions like πN scattering where sufficient data are available to give reasonably accurate amplitude determinations.

Previous calculations of this nature have either used backward dispersion relations for the full amplitude or dispersion relations for individual t-channel partial-wave amplitudes. The cut structure is the same for the full amplitude and each t-channel partial-wave amplitude. Each of these two methods has a serious drawback or difficulty.

The use of a backward dispersion relation for the full amplitude[2] gives a discrepancy function which is the sum of the contributions from all t-channel partial waves. Consequently, it is not possible to learn which partial waves are important.

In working with a dispersion relation for a t-channel partial wave amplitude,[3] a difficulty is encountered in the elimination of the s-channel contribution. In order to evaluate this contribution, the discontinuity of the partial-wave amplitude must be known along the s-channel cut, i.e., $t \leq 0$. This discontinuity can be calculated only for those values of t for which a fixed-t dispersion relation for the full amplitude can be written, i.e., $0 \leq -t \leq L$, $L_{\pi N}^{\text{el}} \simeq \frac{1}{2} m_N^2$. Thus, only a portion of the s-channel cut, i.e., $0 \leq -t \leq L$, can be removed. Unfortunately, the remaining part of the s-channel cut, which begins at $t=-L$, is the "nearby" cut for the t region where the amplitude is known, i.e., $0 \leq -t \leq L$. Consequently, this contribution would not be expected to be negligible and the isolation of the t-channel contribution would not be possible.

In this paper we present and illustrate, by an application to πN scattering, a method using interior dispersion relations[4,5] which allows complete elimination of the s-channel contribution, as in the case of backward dispersion relations for elastic scattering, but which has an extra degree of freedom and can measure the spin content of the t-channel contribution.

II. DESCRIPTION OF METHOD

Most two-body scattering reactions for which sufficient experimental data are available are of the type

$$a + b \to c + d, \tag{2.1}$$

where $m_b = m_d$. We will accordingly limit our consideration to such reactions and will call reactions in which $m_a = m_c$ elastic (irrespective of quantum-number exchange) and all others inelastic. For such reactions it is always possible to work with amplitudes which are either even or odd under crossing, i.e., $\nu \equiv s - u \to -\nu$. If the parameter a is defined as

$$a \equiv -\phi/t^2 = -4(p_t p_t' \sin\theta_t)^2/t\,, \tag{2.2}$$

where ϕ is the Kibble boundary function, interior dispersion relations can be written for all amplitudes which then are to be considered functions of t and the parameter a. This parameterization results in curves in the Mandelstam plane described by

$$\begin{aligned} 2s &= \Sigma - t + \nu(t, a), \\ 2u &= \Sigma - t - \nu(t, a), \end{aligned} \tag{2.3}$$

where $\Sigma = m_a{}^2 + m_b{}^2 + m_c{}^2 + m_d{}^2$ and $\nu(t, a) = [(4p_t p_t')^2 + 4at]^{1/2}$. Clearly for a equals zero, these curves describe for the elastic case the backward direction, the resultant dispersion relations being the familiar backward dispersion relations,[2] whereas for the inelastic case the curves describe the boundary of the physical region, the resultant dispersion relations being the boundary dispersion relations.[6]

For negative values of a, such dispersion relations receive contributions from the s channel only from physical scattering angles. In the t-channel such curves remain within the boundaries of the appropriate Lehmann ellipse for a values greater than a certain value characteristic of the reaction under consideration, e.g., $a_{\pi N}^{\mathrm{el}} \geq -0.7\ \mathrm{GeV}^2$. In general, there is a large range of a values for which the amplitude and its discontinuities can be expanded in partial-wave series.

The value of using interior dispersion relations (IDR) lies in the following kinematic relationship:

$$Z_t(a, t)^2 = 1 + 4ta/(4p_t p_t')^2\,. \tag{2.4}$$

Since the discrepancy function D is equal to an integral in t of the t-channel discontinuity of the amplitude which can be expanded as a sum of products of partial-wave amplitudes which only depend on t, and polynomials in Z_t, i.e., Legendre polynomials or their derivatives which owing to crossing must be even in Z_t, the a dependence will factor out of the integrals and D is seen to be a polynomial in a. In particular, the partial-wave amplitude with the lowest spin value will give a constant in a, the next higher partial-wave amplitude will give a linear polynomial in a, etc. The fact that the order of the a dependence for each partial wave is simply related to the value of the spin means that the order of D as a function of a gives a simple measure of which partial-wave contributions are important.

Since the resonances of lowest mass, i.e, those nearest the physical s-channel region, are expected to dominate and it is an experimental fact that they have low spin values, the discrepancy function can be expected to have a rather simple a dependence. Assuming this were the case and also that the important partial-wave amplitudes could be expressed as a sum of resonances or some other useful expansion, including perhaps nonresonant background contributions, the highest non-negligible derivative with respect to a of D could be fitted as a function of t to yield the parameters for the highest non-negligible partial-wave amplitude. Using these parameters and the next-lowest derivative of D, the parameters of the next-lower partial-wave amplitude could be determined, etc. In this way, all the important parameters could be determined. Thus, for example, in a resonance dominance model, D would yield not only the masses and coupling constants, but also the value of the spin of the resonances. This situation is illustrated by the application in the next section.

III. APPLICATION OF METHOD TO πN SCATTERING

In this section we apply the method to the invariant πN amplitude, $B^{(-)}$, which has unit t-channel isospin. This amplitude has the advantage, at least in the region of small t values, that an unsubtracted fixed-t dispersion relation can be written, and that the better known direct-channel contributions, such as the $(J=\frac{1}{2})N$ and $(J=\frac{3}{2})\Delta$, are more important for $B^{(-)}$ than for other πN amplitudes. The IDR for $B^{(-)}$ is

$$\mathrm{Re}\,B^{(-)}(s,t) = B^{N(-)}_{\mathrm{IDR}} + \frac{1}{\pi}\,\mathrm{P}\int_{s_0}^{\infty} \mathrm{Im}\,B^{(-)}(s',t')\left(\frac{1}{s'-s} + \frac{1}{s'-u} - \frac{1}{s'-a}\right)ds' + \frac{1}{\pi}\,\mathrm{P}\int_{4\mu^2}^{\infty} \mathrm{Im}\,B^{(-)}(t', z_{t'})\,\frac{dt'}{t'-t}\,. \tag{3.1}$$

Here, a is held constant at a value in the range $-(m+\mu)^2 < -a < 35\mu^2$,[5] and m and μ are nucleon and pion masses, respectively. The nucleon Born term is

$$B^{N(-)}_{\rm IDR}=G^2\left(\frac{1}{m^2-s}+\frac{1}{m^2-u}-\frac{1}{m^2-a}\right),\quad G^2/4\pi=14.7\,.$$

For πN scattering the constant a curves in the s-t plane are such that $z_s=\cos\theta_s=-(s+a)/(s-a)$ and that, at $t=0$, all curves pass through the threshold point $s=s_0=(m+\mu)^2$.

Equation (3.1) expresses $\operatorname{Re}B^{(-)}$ clearly as being due to contributions from the direct channel (the cut $s_0\leq s\leq\infty$ and the nucleon pole) and the crossed-channel (the cut $4\mu^2\leq t<\infty$). Consequently, the contribution to $\operatorname{Re}B^{(-)}$ from the t channel can be isolated and the discrepancy function written as

$$\begin{aligned}D(t,a)&=\frac{1}{\pi}\,\mathrm{P}\int_{4\mu^2}^{\infty}\operatorname{Im}B^{(-)}(t',z_{t'})\,\frac{dt'}{t-t'}\\&=\operatorname{Re}B^{(-)}-B^{N(-)}_{\rm IDR}-\frac{1}{\pi}\,\mathrm{P}\int_{s_0}^{\infty}\operatorname{Im}B^{(-)}(s',t')\left(\frac{1}{s'-s}+\frac{1}{s'-u}-\frac{1}{s'-a}\right)ds'\,.\end{aligned}\tag{3.2}$$

We have, in effect, removed from the real part of the full amplitude the part due to the direct-channel dynamics.

Except near threshold, we can use partial-wave amplitudes to evaluate $\operatorname{Re}B^{(-)}$. Near threshold, we make use of the fact that for $-t\leq 26\mu^2$, the fixed-t (Ft) dispersion relation

$$\operatorname{Re}B^{(-)}(s,t)=B^{N(-)}_{\rm F\it t}+\frac{1}{\pi}\,\mathrm{P}\int_{s_0}^{\infty}\operatorname{Im}B^{(-)}(s',t)\left(\frac{1}{s'-s}+\frac{1}{s'-u}\right)ds'\tag{3.3}$$

is valid, and the discrepancy $D(t,a)$ can then be written in the form

$$D(t,a)=\frac{G^2}{m^2-a}+\frac{1}{\pi}\,\mathrm{P}\int_{s_0}^{\infty}\frac{ds'}{s'-a}\left[\operatorname{Im}(B(s',t)-B(s',t'))\,\frac{2s'+t-\Sigma}{t-t'}+\operatorname{Im}B(s',t')\right],\tag{3.4}$$

where $t'=-4q(s')^2[1-z_s(s',a)]$.

Equation (3.4) shows that the t dependence of $D(t,a)$ is determined by the product of $(2s'+t-\Sigma)$ and the derivativelike term $\operatorname{Im}(B(s',t)-B(s',t'))/(t-t')$, the latter of which is strongly dependent upon the higher partial-wave amplitudes. The relative importance of the high partial waves with respect to the N and Δ is illustrated by the fact that saturation of Eq. (3, 4) by these poles results in a negative value for $\partial D(t,a)/\partial t$,[5] whereas inclusion of higher partial waves results in a positive value. Consequently, the uncertainty in our results will not be given as much by that of the Δ contribution, which is well known experimentally, as by the lesser known $J=\frac{5}{2},\frac{7}{2},\frac{9}{2}$ partial-wave amplitudes. This difficulty, which is characteristic of all backward dispersion relation ($a=0$) calculations, underlines the importance of accurate amplitude analysis and cautions us against a too quantitative interpretation of such results. In this calculation we have used the recent Lovelace-Ahmehed πN phase shifts.[7]

The advantage of using IDR to analyze the spin content of the contributions is apparent when the t-channel partial-wave expansion,[8]

$$\frac{B^{(-)}}{4\pi}=\sum_{J=1,3,\ldots}\frac{(2J+1)(p_tp_t')^{J-1}P_J'(z_t)f_-^J(t)}{[J(J+1)]^{1/2}}\tag{3.5}$$

is substituted into Eq. (3.2) to give

$$\frac{D(t,a)}{4\pi}=\frac{1}{\pi}\left[\frac{3}{\sqrt{2}}\int_{4\mu^2}^{\infty}\operatorname{Im}f_-^1(t')\,\frac{dt'}{t'-t}+7\sqrt{3}\int_{4\mu^2}^{\infty}\operatorname{Im}f_-^3(t')\,\frac{dt'}{t'-t}+a35\sqrt{3}\int_{4\mu^2}^{\infty}\frac{\operatorname{Im}f_-^3(t')}{(4p_tp_t')^2}\,\frac{t'dt'}{t'-t}+\cdots\right].\tag{3.6}$$

Thus, a partial wave of spin J will contribute an a dependence of order $\frac{1}{2}(J-1)$ to $D(t,a)$.

The a dependence of $D(t,a)$ (Fig. 1) is seen to be linear (to within ~3% over the range $0\leq -a\leq 35\mu^2$) and furthermore, the t dependence of $\partial D(t,a)/\partial a$ can be explained and fit by a single $g(3^-)$ meson of mass 1680 MeV/c^2.[9] Using the fit parameters, the complete g meson contribution, which at $t=a=0$ is -19.6 (GeV)$^{-2}$, was subtracted out of $D(t,a)$ to obtain the $J=1$ contribution as shown in Fig. 2 (dashed line).

The dotted curve in Fig. 2 (which contains the ρ and g contribution) has been drawn to illustrate the t dependence expected if the $J=1$ contribution were due solely to the $\rho(770)$ meson. It is apparent that the $\rho(770)$ is not adequate to explain the t behavior. We therefore conclude that a heavier $\rho'(1^-)$ meson contributes in this reaction.

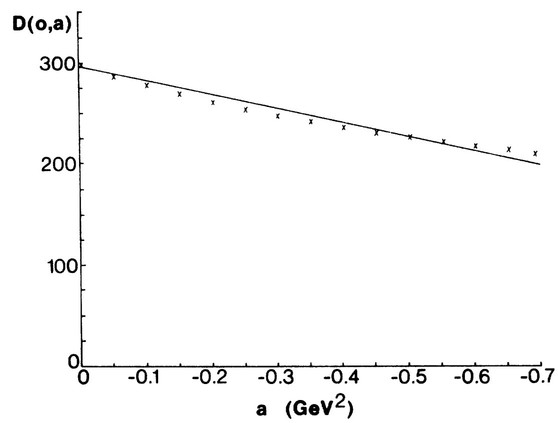

FIG. 1. $D(0, a)$ as calculated from Eq. (3.4) (xxxx) and from the best fit with ρ, ρ', and g contributions (——).

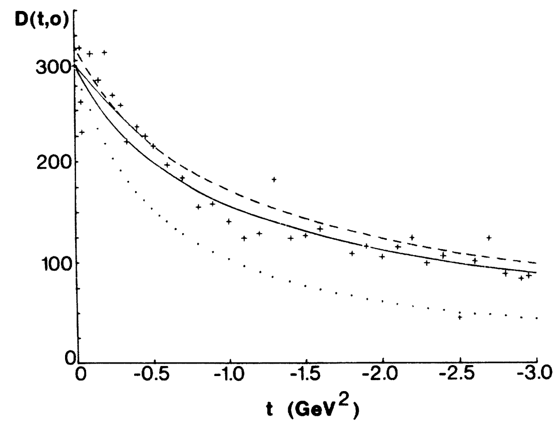

FIG. 2. $D(t, 0)$ as calculated from Eq. (3.2) (++++) and from Eq. (3.4) (short solid line). The long solid line is the best fit with ρ, ρ', and g contributions for $m'_\rho = 1600$ MeV/c^2. The dashed line is the $J=1$ (ρ and ρ') contribution alone, and the dotted line represents the extent to which the ρ and g contribute to the best fit.

Since the uncertainties in $D(t, a)$ are due mainly to the uncertainties in the higher s-channel partial waves at lower energies and since we have subtracted out, in a model-dependent way, the $J=3$ contributions, a quantitative determination of the mass of the ρ' is precluded in our present analysis. Indeed, reasonable fits are obtained for ρ' masses between 1300 and 1900 MeV/c^2. In Fig. 2 we show the fit obtained for $m'_\rho = 1600$ MeV/c^2, that being the mass most commonly attributed to the ρ'.[1,10] At this mass, we obtain

$$(G_{\rho\pi\pi} G_{\rho N\bar{N}} m'^2_\rho)/(G'_{\rho\pi\pi} G'_{\rho NN} m_\rho^2) = 1.05\,, \qquad (3.7)$$

where we have taken the narrow-width approximation

$$f^1_{-\rho}(t) = \frac{G_{\rho\pi\pi} G_{\rho N\bar{N}}}{m_\rho^2 - t}\,, \qquad (3.8)$$

and similarly for ρ'. The effect, for negative values of t, of including resonance widths gives a correction of order $(\Gamma/m_R)^2$, which being on the order of a few percent is well within expected errors.

IV. CONCLUSION

In this paper we have presented a dispersion relation method of isolating and studying crossed-channel contributions to the full amplitude in the physical direct-channel region. As an example of the power of this method, we considered the crossed-channel contributions to the $B^{(-)}$ amplitude of πN scattering and were able to show that (i) only spin 1 and 3 contributions are necessary, (ii) the spin 3 contribution is adequately accounted for by the $g(1680)$ meson, and (iii) the spin 1 contribution cannot be due solely to the $\rho(770)$, but requires the addition of a heavier ρ'.

ACKNOWLEDGMENTS

We wish to thank Dr. Darragh Nagle and the staff for their hospitality at the Los Alamos Scientific Laboratory, where most of the work was performed.

[1]A. Benvenuti, D. Cline, R. Rutz, D. D. Reeder, and V. R. Scherer, Phys. Rev. Lett. 27, 283 (1971); H. H. Bingham, W. B. Fretter, W. J. Podolsky, M. S. Rabin, A. H. Rosenfeld, G. Smadja, G. P. Yost, J. Ballam, G. B. Chadwick, Y. Eisenberg, E. Kogan, K. C. Moffeit, P. Seyboth, I. O. Skillicorn, H. Spitzer, and G. Wolf, Phys. Lett. 41B, 635 (1972); G. Barbarino, M. Grilli, E. Iarocci, P. Spillontini, V. Valente, R. Visentin, F. Ceradini, M. Concersi, L. Paoluzi, R. Santonico, M. Nigro, L. Trassatti, and G. T. Zorn, Nuovo Cimento Lett. 3, 689 (1972); R. Diebold, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 3, p. 1; R. R. Burns, P. E. Condon, J. Donahue, M. Mandelkern, L. R. Price, and J. Schultz, Phys. Rev. D 8, 1286 (1973); J. T. Carroll, in proceedings of the SLAC Summer Institute on Particle Physics, 1973 (unpublished); B. Hyams, C. Jones, P. Weilhammer, W. Blum, H. Dietl, G. Grayer, W. Koch, E. Lorenz, G. Lütjens, W. Männer, J. Mussburger, W. Ochs, V. Stierlin, and F. Wagner (CERN-Munich Collaboration), discussed by I. Butterworth in proceedings of the Second International Conference on Elementary Particles, Aix-en-Provence, 1973 [J. Phys. (Paris) Suppl. 34, C1-196 (1973)].

[2]D. Amati and S. Fubini, Annu. Rev. Nucl. Sci. 12, 359 (1962); P. S. Isaev and V. A. Meshcheryakov, Zh. Eksp. Teor. Fiz. 43, 1339 (1962) [Sov. Phys.—JETP 16, 951 (1963)]; D. H. Lyth, Rev. Mod. Phys. 37, 709 (1965); H. Goldberg, Phys. Rev. 171, 1485 (1968); B. Dutta-Roy, I. R. Lapidus, and M. J. Tausner, Phys. Rev. 177, 2529 (1969); C. Lovelace, R. M. Heinz, and A. Donnachie, Phys. Lett. 22, 332 (1966); H. Banerjee and S. Mallik, Nuovo Cimento Lett. 1, 87 (1969); J. Engels, G. Höhler, and B. Peterson, Nucl. Phys. B15, 365 (1970); J. Engels, Nucl. Phys. B25, 141 (1971); J. Baacke and H. Kleinert, Phys. Lett. 35B, 159 (1971); Nucl. Phys. B42, 301 (1972); G. E. Hite and R. Jacob, Nucl. Phys. B55, 587 (1973).

[3]G. Höhler, O. R. Strauss, and H. Wunder, Karlsruhe report, 1968 (unpublished).

[4]G. E. Hite, R. Jacob, and F. Steiner, Phys. Rev. D 6, 3333 (1972); G. E. Hite and F. Steiner, Nuovo Cimento 18A, 237 (1973).

[5]G. E. Hite and F. Steiner, Phys. Rev. D 8, 3205 (1973).

[6]G. E. Hite and R. Jacob, Phys. Rev. D 5, 422 (1972).

[7]S. Almehed and C. Lovelace, Nucl. Phys. B40, 157 (1972); CERN Report No. CERN TH 1408, 1971 (unpublished).

[8]V. Singh, Phys. Rev. 129, 1889 (1973).

[9]The (3^-) meson is quite well established. See Particle Data Group, Rev. Mod. Phys. 45, S93 (1973) for references.

[10]I. Bars and K. Lane, Phys. Rev. D 8, 1169 (1973); D. C. Peaslee, *ibid.* 221 (1973); E. Leader and B. Nicolescu, *ibid.* 7, 836 (1973); A. Bramòn and M. Greco, Nuovo Cimento Lett. 3, 693 (1972); C. Y. Huang, S. Oneda, and S. Matsuda, Phys. Rev. D 9, 3206 (1974).