Models for hadron-hadron scattering at high energies and rising total cross sections

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The recent results of rising total cross sections for $\pi^{\pm}p$, $K^{\pm}p$, and pp scattering are analyzed by two simple analytic models for high-energy forward scattering which are derived from analyticity, crossing symmetry, and the unitarity constraints of the rigorous results. The numerical fit to the data for $p_L \geq 10$ GeV/c does not rule out the possibility that the crossing-odd amplitude may not be negligible at high energies.

The recent results[1] of an experiment at the Fermi National Accelerator Laboratory (Fermilab) prove that the total cross sections for pp, πp, and Kp scattering all show indeed a remarkable rise in the range of $50 \leq p_L \leq 200$ GeV/c, thus confirming beautifully the previous observations[2] made for pp at the CERN ISR. Already a number of theoretical models[3,4] have been put forth to explain such a rise in σ_{pp} seen at the CERN ISR. Among them, the works of Ref. 4, in particular, indicate the existence of a $\ln^2 s$ term in σ_T. However, in actuality these works predict either too fast an increase of σ_T or too rapid a decrease in $\sigma_{\bar{p}p}$ when compared with the Fermilab data. Thus, it will be interesting to develop a theoretical framework that can explain the seemingly general behavior of all σ_T, as well as the ratio $\alpha \equiv \mathrm{Re}F(s)/\mathrm{Im}F(s)$.

In this paper, we propose some simple analytic parametrizations for high-energy hadron-hadron scattering, which are derived from the "quasilocal" relations of analyticity and crossing symmetry with the constraints coming from the rigorous studies of analyticity, unitarity, and positivity. Such a quasilocal relation had been suggested some time ago[5] and several authors[6] have advocated it recently again.

As has been emphasized by many people,[7] most of the asymptotic statements in the literature involve an *additional* tacit assumption that the crossing-odd amplitude becomes negligible at high energies. But one can construct an example in which the crossing-odd amplitude can even grow in energy and yet is perfectly compatible with all the results of rigorous studies.[8]

Keeping such option in mind, we can then derive from the analyticity relation given by the Sommerfeld-Watson-Regge representation that

$$\mathrm{Re}[F_+(s,t)/s] = \left[\frac{\pi}{2}\frac{\partial}{\partial \ln s} + \frac{1}{3}\left(\frac{\pi}{2}\frac{\partial}{\partial \ln s}\right)^3 + \frac{2}{15}\left(\frac{\pi}{2}\frac{\partial}{\partial \ln s}\right)^5 + \cdots\right]\mathrm{Im}[F_+(s,t)/s]\,, \tag{1}$$

$$\frac{\pi}{2}\frac{\partial}{\partial \ln s}\mathrm{Re}[F_-(s,t)/s] = -\left[1 - \frac{1}{3}\left(\frac{\pi}{2}\frac{\partial}{\partial \ln s}\right)^2 - \frac{1}{45}\left(\frac{\pi}{2}\frac{\partial}{\partial \ln s}\right)^4 - \cdots\right]\mathrm{Im}[F_-(s,t)/s]\,, \tag{2}$$

where the crossing-even and -odd amplitudes are defined as $F_{\pm} = \frac{1}{2}(F_{AB} \pm F_{\bar{A}B})$ and normalized so that $s\sigma_T(s) = \mathrm{Im}F(s,0)$. In order to derive these,[9] we have assumed that the asymptotic behavior of $F_{\pm}$ is controlled by the rightmost singularities of $A(l,t)$, whose position is one in the forward direction for the reasons explained above. This is certainly possible even if the analytic continuation of the Froissart-Gribov relation down to $l=1$ may not coincide with the physical p-wave amplitude.[10]

The relations (1) and (2) are quasilocal due to the inherent asymptotic nature. They imply the convenience of using $\ln s$ as a natural variable in the high-energy region. Then, together with *de facto* rise of total cross sections, they are designed to describe as well the situations in which the singularities of $A(l,t)$ in the complex l plane are *not* necessarily just simple poles. In addition, these quasilocal relations have a number of interesting features: (a) F_+ is predominantly imaginary at high energies while F_- can be real[8]; (b) if $|F_+| \propto s(\ln s)^{\beta_+}$, with $\beta_+ \leq 2$, then we get from (1) that $\alpha_+ = \mathrm{Re}F_+/\mathrm{Im}F_+ \to \frac{1}{2}\pi\beta_+(\ln s)^{-1}$, again a well-known result[11]; (c) if, in addition, $|F_-| \propto s(\ln s)^{\beta_-}$ such that[8] $\beta_- - 1 \leq \frac{1}{2}\beta_+$, then it follows from (1) and (2) that $|\sigma_{\bar{A}B} - \sigma_{AB}| < (\sigma_{\bar{A}B} + \sigma_{AB})/\ln s$, the rigorous form of the Pomeranchuk theorem.[12]

Denoting $F_\pm(s,0) \equiv F_\pm(s)$ and allowing that $A(l,t)$ can possess harder singularities than simple poles, we may write

$$\mathrm{Im}[F_+(s)/s] = A_+ + B_+ \ln s + C_+ \ln^2 s + C_R s^{-1/2}, \quad (3)$$

$$\mathrm{Im}[F_-(s)/s] = B_- + \pi C_- \ln s - C_R s^{-1/2}, \quad (4)$$

which are compatible with the unitarity bounds of the rigorous studies. Here the C_R terms are introduced to represent the over-all contributions of the exchange-degenerate Reggeons. Also it is understood that $\ln s = \ln(s/\mathrm{GeV}^2)$ at this point, but the real scale will be determined by the fit to the experimental data. Then the *exact* analytic solutions of $\mathrm{Re}F_\pm$ can be obtained from (1) and (2);

$$\mathrm{Re}[F_+(s)/s] = (\pi/2)[B_+ + 2C_+ \ln s - (2/\pi)C_R s^{-1/2}], \quad (5)$$

$$\mathrm{Re}[F_-(s)/s] = -(2/\pi)[A_- + B_- \ln s + (\pi/2)C_- \ln^2 s + (\pi/2)C_R s^{-1/2}]. \quad (6)$$

Thus we obtain[13]

$$F_+(s)/is = \sigma_0 + C_+(\ln^2 s/s_+ - i\pi \ln s/s_+) + C_R(i+1)s^{-1/2}, \quad (7)$$

$$F_-(s)/is = iC_-(\ln^2 s/s_- - i\pi \ln s/s_-) + C_R(i-1)s^{-1/2}. \quad (8)$$

Depending on the absence of the C_- and C_R terms, we end up with different physical pictures:

(a) M model. This is the case of $C_- = 0$ and is the conventional and perhaps the most "moderate" line of pictures, e.g., the works of Cheng, Walker

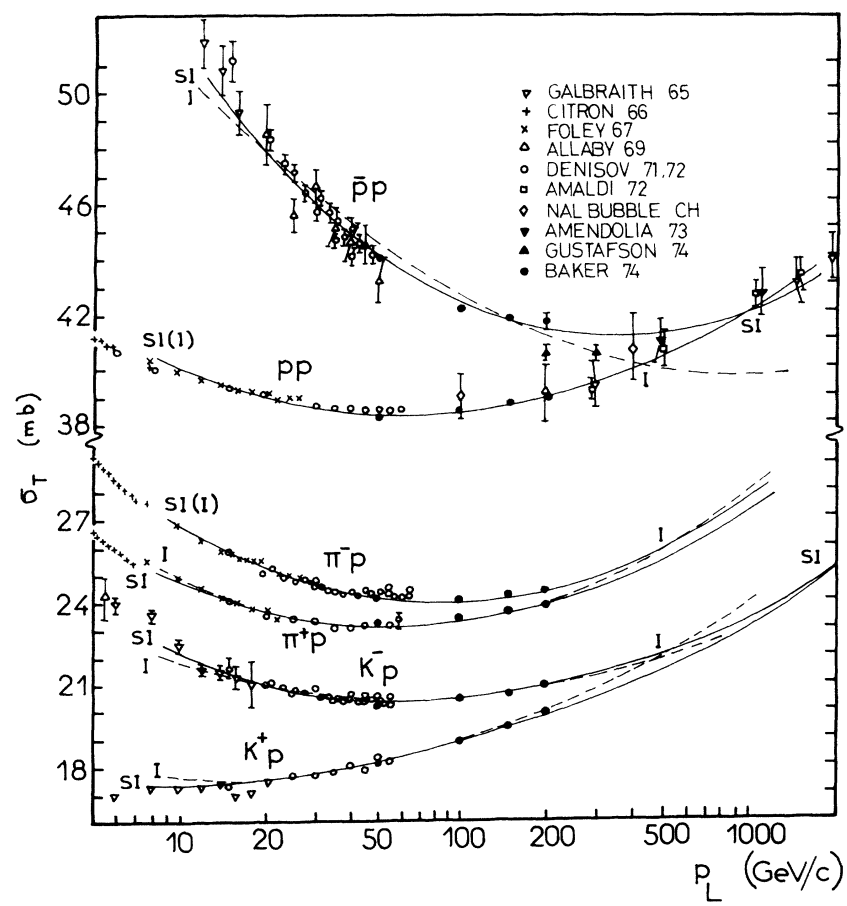

FIG. 1. Fits to σ_T. The data on $\sigma_T^{\pi p}$ for $30 < p_L < 60$ GeV/c are excluded.

TABLE I. Parameters used to fit σ_T and χ^2/number of data points, in the I model and in the SI model.

Parameters	I_{pp}	SI_{pp}	I_{Kp}	SI_{Kp}	$I_{\pi p}$	$SI_{\pi p}$
σ_0 (mb)	39.93	38.95	19.16	17.28	23.46	22.94
C_+ (mb)	0.53	0.50	0.38	0.24	0.63	0.54
s_+ (GeV^2)	508.8	193.8	61.9	14.6	140.3	111.6
C_- (mb)	0.45	0.14	0.17	0.013	0.076	−0.027
s_- (GeV^2)	820.7	551	794.7	73.3	984.5	500.2
C_R (mb GeV)	···	19.85	···	10.93	···	6.01
χ^2/data points	121/58	58/58	37/35	21/35	37/59	28/59

and Wu, and Bourrely and Fischer. But they give in general a faster increase of σ_T than the Fermilab data indicate.

(b) I model. This is the case of $C_R = 0$ so that even ρ does not contribute to F_-. This may appear to a traditional Reggeist improbable or even "improper." The parametrization of Lukaszuk and Nicolescu is a very special case of this picture, i.e., $C_+ = C_-$ and $s_+ = s_-$, which, however, gives a faster decrease of $\sigma_{\bar{p}p}$ than the actual data shown.

(c) SI model. This is the case of $C_- \neq 0$ and $C_R \neq 0$, i.e., the full structure of (7) and (8).

As it seems that the reality may lie somewhere between the full I model and full SI model, we have made a complete χ^2 fit (Fig. 1) to all available data[14] of σ_T with the two analytic parametrizations for πp, Kp, and pp scattering above $p_L \geq 10$ GeV/c. In the case of πp scattering, we have incorporated in addition the 18 data points[14] of the charge-exchange differential cross sections at $t = 0$ (Fig. 3). The ratios $\alpha = \mathrm{Re}F(s)/\mathrm{Im}F(s)$ are predicted and compared with the existing data. The parameters corresponding to the best fit in each picture are summarized in Table I along with the χ^2 value.

One can see that the over-all fit in both parametrizations is quite remarkable in all the cases, considering the simplicity of the models. They give a similar fit to σ_T (Fig. 1). Note from Fig. 2 that both the I and SI models can fit $\Delta\sigma_T$ equally well except the case of πp. Here the SI model appears to be definitely favored by the Fermilab data from the characteristic curvature of the $\ln(\Delta\sigma_T)$ vs $\ln p_L$ plot. In general, the I curves fall clearly faster than the trend of the data at high p_L. As we have excluded the Serpukhov data for πp, the fact that the I curve goes through their points in Fig. 2 is a pure accident. From Fig. 3, we see that the data of $d\sigma/dt$ at $t=0$ for $\pi^- p \to \pi^0 n$ can be fitted equally well by the two parametrizations. (See the *Added note.*) We note further that α_{pp} and $\alpha_{\bar{p}p}$ can be described more or less to the same fairness by both parametrizations. The only difficulty is in $\alpha_{\pi^- p}$ around $p_L = 58$ GeV/c. If the existing (Serpukhov) datum is indeed correct, then models with $\sigma_T \propto \ln^2 s$ including the M model will have a serious problem explaining this. The best SI-model value that we can come up with[15] is $\alpha_{\pi^- p}(p_L = 58\ \mathrm{GeV}/c) \cong 0.01$ instead of -0.08 ± 0.03 and $\alpha_{\pi^- p}(60) \cong 0$.

To conclude, we may say that though the actual data at this point in time are coarse, they appear, from the favored fit of the SI model, compatible

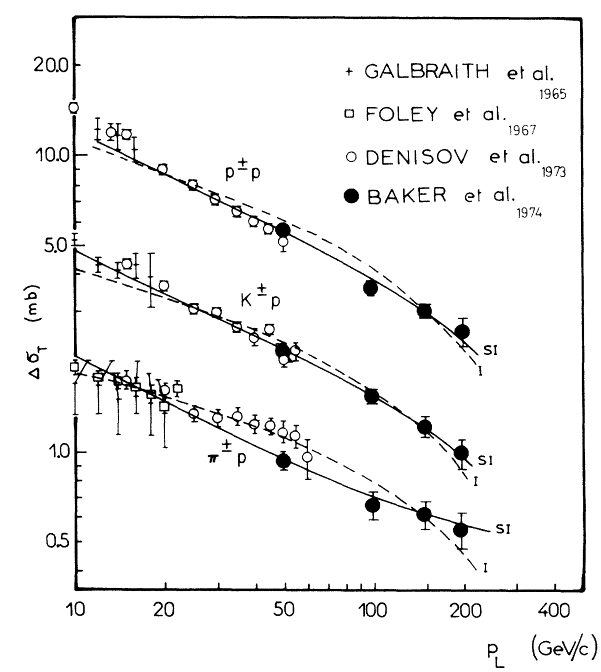

FIG. 2. Fits to $\Delta\sigma_T$. See the caption of Fig. 1.

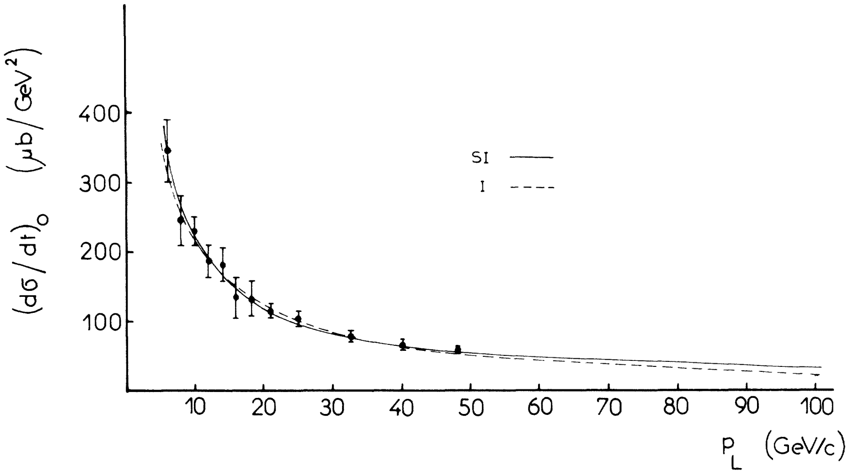

FIG. 3. $(d\sigma/dt)_0$ for $\pi^- p \to \pi^0 n$ as fitted by the two models.

with the possibility that $C_- \neq 0$ (see the *Added note*), thus suggesting various different physical consequences to anticipate: (1) The crossing-odd amplitude can be predominantly real at high energies, allowing the maximal growth of σ_T and $|\Delta\sigma_T|$ as permitted by the axiomatic asymptotic theorems[8]; (2) if $C_- > 0$, $\sigma_{\bar{A}B}$ and σ_{AB} must cross at some $s \geq s_-$ and $\alpha_{AB} \to -C_-/C_+$, a negative number; (3) if $C_- < 0$, then there is no crossover in the total cross sections. But $\Delta\sigma_T$ reaches a minimum at $s = (C_R/2\pi C_-)^2$ and $\alpha_{AB} \to C_-/C_+$, a positive limit. From the χ^2 fit, we see that the Kp and pp interactions are of the type (2) in either picture, while the πp interactions again *distinguish* between the two pictures, i.e., the I model is of the type (2) and the preferred SI model is of the type (3). In the case of SI model, the crossing occurs after $p_L \gtrsim 1000$ GeV/c for Kp and pp, while there is no crossing for πp and $(\Delta\sigma_T)_{\pi p}$ has a minimum at $p_L \cong 670$ GeV/c.

Added note. The new data[16] of the pion exchange reaction for $p_L = 20$–101 GeV/c from Fermilab, which became available to us after the completion of the present work, have been incorporated along with the earlier data at low energies in a subsequent work[17] and the conclusion reached in this paper is indeed found to be supported. In fact, it is shown that[17] the M-model fit with a single ρ-Regge pole, while it is statistically acceptable, is too steep to explain the trend of the Fermilab $\Delta\sigma_T$ at high energies as well as the low-energy data near $p_L = 10$ GeV/c and it is shown that the SI model can indeed fit $\Delta\sigma_T$ without any discrepancy at high- or low-energy ends and simultaneously with $(d\sigma/dt)_0$. We have received the 1974 analysis of Barger and Phillips,[18] again after the completion of this work, which concludes that there is also a discrepancy between the ρ-Regge model and $\Delta\sigma_T$, thus supporting our results.

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[9]Equation (2) is the more convenient form to deal with the possibility that $\mathrm{Im}[F_-(s)/s] \propto (\ln s)^{\beta_-}$, with $0<\beta_- \leq 1$. For a decreasing power form of $\mathrm{Im}[F_-(s)/s]$, it reduces to the result of Ref. 6.

[10]Then the p-wave amplitude should be given from the outset by the subtraction term, and the contour of the Sommerfeld-Watson-Regge transformation should be chosen so as not to include the point $l=1$. We thank Alan R. White for the clarifying discussions on this technicality.

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[13]We have set $\sigma_0 = A_+ - C_+\ln^2 s_+$, $B_+ = -2C_+\ln s_+$, $B_- = -\pi C_-\ln s_-$, and $A_- = \frac{1}{2}\pi C_-\ln^2 s_-$. In general we can allow one more parameter $\sigma_1 = (2/\pi)A_- - C_-\ln^2 s_-$, but from the data of $\Delta\sigma_T$, $\sigma_1 = 0$ seems to be supported *a posteriori*. See R. Oehme, Phys. Rev. D 9, 2695 (1974), for another reason.

[14]Experimental data for σ_T are quoted from the summary by the Particle Data Group, Phys. Lett. 50B, 1 (1974) (excluding the Serpukhov data for πp), as well as Ref. 1 and Ref. 2. $\Delta\sigma_T$ at low energies is from work by K. J. Foley *et al.*, Phys. Rev. Lett. 19, 330 (1967). $(d\sigma/dt)_0$ at low energies are from I. Mannelli *et al.*, *ibid.* 14, 408 (1965); A. V. Stirling *et al.*, *ibid.* 14, 763 (1965); and V. N. Bolotov *et al.*, presented to the XVI International Conference on High-Energy Physics, Chicago-Batavia, Illinois, 1972 (unpublished). Exclusion of this last data and inclusion of the new data (Ref. 16) of $(d\sigma/dt)_0$ support more favorably the conclusion of this paper (see the *Added note*).

[15]After the completion of this work, we received the revised Serpukhov data for $\alpha_{\pi^- p}$: V. D. Apokin *et al.*, presented to the XVII International Conference on High Energy Physics, London, 1974 (unpublished). Their corrected data shows now $\alpha_{\pi^- p}(p_L \cong 60\ \mathrm{GeV}/c) \cong 0$ in agreement with our prediction.

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