Quantitative relationship between clock gravitational "red-shift" violations and nonuniversality of free-fall rates in nonmetric theories of gravity*

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When we assume conservation of energy (absence of perpetual motion of the first kind), a generally valid quantitative relationship is derived between possible violations of the clock gravitational "red-shift" effect, $\delta\nu/\nu = \delta U(1+\xi)/c^2$, $\xi \neq 0$ indicating violation, and nonuniversality of free-fall rates which can occur in nonmetric theories of gravity.

INTRODUCTION

In order to test the metric nature of gravitation, two kinds of experiments have been discussed and performed:

(1) Eötvös-type experiments in which the gravitational free-fall rates of different materials are compared,[1,2,3] and

(2) gravitational "red-shift" experiments in which clock rates are measured in various gravitational environments.[4,5,6]

Both types of experiments have been called foundational for metric theories of gravity because of the following property of such theories[7,8]: If all forms of laboratory (nongravitational) matter-energy are coupled to a second-rank tensor gravitational field, $g_{\mu\nu}$, in a universal way, producing the matter equation of motion in the presence of gravity,

$$T^{\mu\nu}|_{\nu} = 0 , \tag{1}$$

with $T^{\mu\nu}$ being the stress-energy tensor of matter, and the vertical line denoting covariant tensor derivative with respect to $g_{\mu\nu}$, then we have a metric theory of gravity with the following properties:

(1) All laboratory bodies, independent of internal composition, free-fall at identical rates in gravitational fields (the local equivalence of gravity and accelerated coordinate frames); and

(2) laboratory clock rates, independent of internal composition, are altered by weak gravitational fields as $\delta\nu/\nu = -\delta U/c^2$, where δU is the difference in Newtonian gravitational potential and c is the speed of light.

In recent years the Eötvös-type experiments have been improved to a most precise level, showing equality of free-fall rates to a part in 10^{12}.[3] On the other hand, atomic clocks stable to a part in 10^{15} have been developed, and the ability to "fly" these clocks in earth or solar orbits over which gravitational potentials change magnitudes more than in the terrestrial experiments has increased discussion of clock experiments as precision tests of gravity theory's foundations.[5]

Do these two types of experiments test different aspects of gravity, or is there a theoretical relationship between the outcome of one type of experiment and the outcome of the other? Indeed, by use of conservation-of-energy arguments and the assumption of universality of free fall (the equivalence principle) others have shown that the clock "red-shift" effect must follow.[9]

The purpose of this note is to explore the relationship between the two types of experiments, and by assuming conservation of energy, to derive a quantitative relationship between Eötvös-type experimental results and clock "red-shift" results which is generally valid in all nonmetric gravitational theories. Application to a few specific experimental cases is made.

CALCULATION

Considering the various clocks in nature, one can delineate two basic classes; clocks defined by oscillation (O clocks) and clocks defined by decay (D clocks). Speaking quantum mechanically, a quantum system with a set of discrete stationary states has a set of oscillation frequencies ω_{ab} related to each possible mixed quantum state $|a\rangle + |b\rangle$ or transitions between these stationary states. These ω_{ab} are the tick rates of O clocks. On the other hand, any quantum state with a nonzero Hamiltonian matrix element to continuum states, H_{if}, will decay into the continuum at a rate Γ_{if} given by Fermi's golden rule,

$$\Gamma_{if} = 2\pi |H_{if}|^2 \rho/\hbar , \tag{2}$$

where ρ is the density of final states and $\hbar$ is Planck's "constant."

All clock experiments are essentially comparisons between various ω_{ab} and/or Γ_{if} in nature, including, of course, the classical limits of these quantum-mechanical situations.

Consider now a thought experiment devised to obtain a quantitative relationship between violations of the universality of free fall and corresponding violations of the clock "red-shift" effect for O clocks in nonmetric theories of gravity. We will make no particular assumption about the field equations of the nonmetric theory of gravity.

Two identical quantum systems are in a uniform gravitational field, one system being at rest at altitude $z=0$ and in a ground state A, and the other system being in an excited quantum state B and at rest at altitude $z=h$. State B makes a transition to state A; the emitted quantum (not necessarily a photon) travels down and is totally absorbed by the identical system at $z=0$. It is found that the quantum is able to excite that system to a state B'. The system at altitude h is now dropped and assumed to accelerate in free fall at the rate g_A. Upon reaching altitude $z=0$ it has therefore acquired velocity $v_A{}^2=2g_Ah$. It then inelastically collides with the other system, leaving one in state A and the other in state B and the leftover kinetic energy given entirely to the system left in quantum state B, which now travels upward. This system is assumed to decelerate in free fall at rate g_B (remember we are considering the possibility of free-fall rates being body-dependent).

The system in state B must be just able to get back to altitude h; otherwise perpetual motion of the first kind will exist.

Figure 1 illustrates the sequence of events described above. At any locality the existence of nongravitational laws of physics with pertinent energy conservation laws is assumed. For the collision in this thought experiment energy conservation yields

$$\tfrac{1}{2}M_Av_A{}^2+M_{B'}c^2=M_Bc^2+\tfrac{1}{2}M_Bv_B{}^2\,, \tag{3}$$

or in terms of the gravitational accelerations (using $2gh=v^2$),

$$M_Ag_Ah+M_{B'}c^2=M_Bc^2+M_Bg_Bh\,. \tag{4}$$

Calling $g_B-g_A=\delta g$, the desired relationship is obtained:

$$(M_{B'}-M_B)/(M_B-M_A)=(gh/c^2)[1+(\delta g/g)(M/\delta M)]. \tag{5}$$

The left-hand side of Eq. (5) is just the clock gravitational frequency shift. Letting ξ parametrize the violation of the standard clock "red-shift" effect,

$$\delta\nu/\nu=(1+\xi)gh/c^2\,, \tag{6}$$

then this parameter is related to a violation of the universality of gravitational free fall by

$$\xi=(M/\delta M)(\delta g/g)\,. \tag{7}$$

To properly apply Eq. (7) requires some discussion. In realistic Eötvös-type experiments it is not the free-fall rates of different quantum states of a given system that are compared; instead various materials with as much difference as possible of some form of internal energy or configuration are used, such as aluminum and gold.[10] However, the δM in Eq. (7) for any particular clock is a rearrangement of some specific type of internal energy in that clock.

A reinterpretation of Eq. (7) to compare clock experiments and realistic Eötvös-type experiments therefore rests on two assumptions—that the ratio of $\delta g/g$ to $\delta M/M$ is a constant for any specific kind of rearrangement of energy or configuration in a system, and that the type of rearrangement in an atomic clock oscillating between its two quantum levels is compared to a rearrangement of energy or configuration of the same nature in the materials used in the Eötvös-type experiments.

The relation given by Eq. (7) is in agreement with recent work by Lightman and Lee[11] and Will[6] who have calculated $\delta g/g$ and $\delta\nu/\nu$ in a certain class of nonmetric theories of gravity to lowest order in the internal electrostatic energy content of bodies. One virtue of Eq. (7) is its claim to be generally valid to any order and for any form of internal energy as long as there is a conserved energy in the theory.

APPLICATIONS

Some examples which show the proper use of Eq. (7) in comparing free-fall experiments and clock experiments are now given. In each case I assume it has been shown that gold and aluminum bodies free fall at the same rates in gravitational fields to a part in 10^{12} (Ref. 3) and then calculate

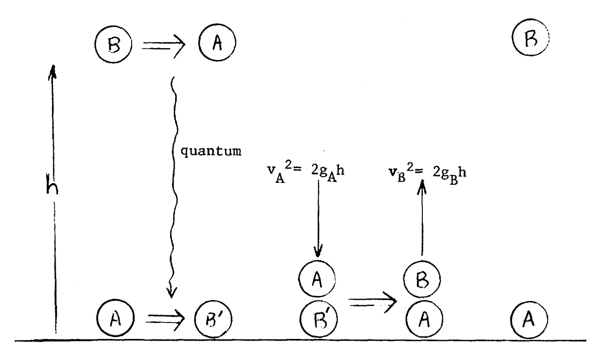

FIG. 1. The several steps are shown by which one quantum transition at altitude h ($B\to A$) excites another quantum transition ($A\to B'$), and the system then is brought back to its initial configuration by two free-fall operations interrupted by an inelastic collision.

the maximum clock "red-shift" effect violation compatible with this free-fall result for a variety of precision O clocks. Stated differently, how accurate must clock experiments be to test the metrical foundations of gravity at the same level of precision as the part in 10^{12} Eötvös-type experiments?

I. Molecular clock. The vibrating ammonia molecule provides the basis for a precision clock. The oscillations (transitions) between two molecular quantum states involve rearrangement of electrostatic interaction energy among the atomic electrons. If we assume that all electrostatic energy may have slight nonmetric coupling to gravity, we can compare the nuclear electrostatic energy contributions to the mass of aluminum and gold. This fractional difference is $|\delta M/M| \simeq 4\times 10^{-3}$, so by Eq. (7) $\xi \lesssim \frac{1}{4}\times 10^{-9}$. Assuming that only the electron electrostatic coupling is nonmetrical, then the electron electrostatic energies of gold and aluminum are to be compared giving $|\delta M/M| \simeq 10^{-7}$, which yields $\xi \lesssim 10^{-5}$.

II. Hyperfine clock. The hydrogen maser clock depends on oscillations (transitions) between hyperfine quantum states of hydrogen. The energy being rearranged here is magnetic. If one assumes that all magnetic interactions have a slight nonmetrical coupling to gravity, then the proper comparison is with the magnetic self-energy contributions to the masses of the protons and neutrons in the nuclei; the difference between the two nucleons' magnetic energy is estimated at 0.5 MeV.[12] The more neutron-rich gold differs from aluminum in magnetic energy by about $|\delta M/M| \simeq \frac{1}{2}\times 10^{-4}$, which yields a $\xi \lesssim 2\times 10^{-8}$.

DISCUSSION

If the assumptions of this calculation are upheld by nature's physical laws (primarily conservation of energy), it follows that atomic clock oscillators can only with difficulty compete with Eötvös-type experiments in searching for or putting limits on violations of the metrical foundations of gravitational law. Atomic clocks operationally stable to a part in 10^{15} inserted into a solar system orbit of about 0.1 astronomical units perigee would be necessary to detect a nonzero ξ of 10^{-7} or 10^{-8}, while Eötvös-type experiments can be improved by several orders of magnitude from their present levels of a part in 10^{12}.

On the other hand, clock experiments can now be viewed as testing even more fundamental foundations of physical law such as conservation of energy and should be pursued to their limiting accuracies.

Clocks based on decay processes (D clocks) offer the possibility of much larger violations of the clock "red-shift" effect while being compatible with a given level of null result in the Eötvös-type experiments. For example, consider a beta-decay clock which has its rate governed entirely by the weak-interaction Hamiltonian matrix element, while weak-interaction energy participates negligibly in the structure and mass-energy content of materials used in Eötvös-type and clock experiments. The search for any nonmetrical aspects of the gravity-to-weak interaction coupling could be most fruitful with beta-decay clocks.[13] However, there is the problem that D clocks have not been calibrated to anything near the accuracy of O-clock calibrations.

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[12]The neutron-proton electromagnetic mass difference is 1.3 MeV, and the neutron and proton magnetic moments differ, being −1.9 and +2.8 nuclear magnetons, respectively. Since the neutron is heavier we must assume that magnetic energies are comparable to electric energies in these nucleons. The resulting number 0.5 MeV is only an estimate though, as no adequate theory of the neutron-proton mass difference exists.
[13]If, for example, the weak-interaction Lagrangian density used a second cosmological-gravitation field $h_{\mu\nu}$ instead of $g_{\mu\nu}$, and $h_{\mu\nu}$ varied negligibly in the presence of ordinary matter, a weak-interaction D clock would have a $\xi = -2$ as $|H_{if}|^2$ would not change in gravitational potentials as (energy)^2 should, while the density of states factor in Eq. (2), $\rho = dn/dE$, would change in a gravitational potential in the opposite sense.