Value of the axial-vector coupling constant $g_{A,\mu}$ from muon nuclear capture experiments in hydrogen, deuterium, and helium

A. Vitale

Istituto di Fisica dell'Università di Bologna, and Istituto Nazionale di Fisica Nucleare, Sezione di Bologna, Italy

A. Bertin* and G. Carboni†

CERN, Geneva, Switzerland

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The results of muon capture experiments in hydrogen, deuterium, and ^3He were analyzed by comparing the observed capture rates to their theoretical expressions. From each experimental result, assuming conserved-vector-current, an allowed region of variation was obtained for the possible pairs of the axial-vector ($g_{A,\mu}$) and induced pseudoscalar ($g_{P,\mu}$) coupling constants. Assuming $g_{P,\mu}$ as given by one-pion exchange dominance, a limited range of variation for $g_{A,\mu}$ was then derived from each experimental result.

I. INTRODUCTION

Within the framework of the $V-A$ weak interaction coupling theory,[1] dropping second-class currents, the rate of the elementary process

$$\mu^- + p \to n + \nu_\mu, \qquad (1)$$

which occurs whenever a negative muon is captured by a nucleus, can be expressed as a function of four coupling constants, i.e., the polar-vector ($g_{V,\mu}$), axial-vector ($g_{A,\mu}$), induced-pseudoscalar ($g_{P,\mu}$), and weak-magnetism ($g_{M,\mu}$) coupling constants. Determining these form factors only from experiments on reaction (1) is a most appealing objective of muon physics.[2] This can be achieved provided several independent measurements are performed, the results of which are foreseen to depend on significantly different combinations of $g_{V,\mu}$, $g_{A,\mu}$, $g_{P,\mu}$, and $g_{M,\mu}$. Unfortunately, when muons are captured by high-atomic-number nuclei, the comparison between experimental results and theoretical predictions becomes more difficult, since a precise knowledge of the nuclear physics effects on the observable quantities for process (1) is generally missing. Therefore, the most favorable starting point to extract information on the four coupling constants is represented by the observation of muon nuclear capture in hydrogen, which is the only case in which the complications due to the nuclear structure are totally absent.

Experimental results on the nuclear capture rate of muons by hydrogen, in fact, were obtained by several groups,[3-8] yielding strong support to the $V-A$ character of the weak interaction of muons with matter.[2,7] In these measurements, however, reaction (1) was observed starting essentially from the singlet state of the muon-proton system, which means that the different experimental results actually refer to very similar combinations of the four coupling constants. To get information on $g_{V,\mu}$, $g_{A,\mu}$, $g_{P,\mu}$, and $g_{M,\mu}$ starting from experiments on reaction (1), one is then compelled to take into consideration muon capture from heavier nuclei, even if the present lack of knowledge of the nuclear physics effects represents an apparent drawback for the purpose of singling out the weak-interaction coupling.

We report here an analysis of the results presently available on the muon nuclear capture rates by hydrogen,[3-8] deuterium,[9,10] and ^3He nuclei,[11-13] which was performed by comparing the observed rates to their theoretical expressions given as a function of the coupling constants,[14-16] and looking at the range of values for $g_{A,\mu}$ and $g_{P,\mu}$ which are allowed by the different experimental results. Deuterium and ^3He were chosen as very light elements, where the complications of nuclear structure are expected to be substantially reduced. Furthermore, the analysis was suggested by the following recent advances of experimental investigation:

(i) The accuracy of the experimental results on muon capture in hydrogen and deuterium has sensitively improved in the past few years.[7,8,10]

(ii) It has recently been proved that about 5% of the muons stopped in helium form the $(\mu He)^+_{2S}$ metastable system, which has a fairly long lifetime even at pressures as high as 50 atm.[17] The experimental results on the muon nuclear capture by ^3He, which are referred to the theoretical value for the $1S$ ground state of the $(\mu He)^+$ muonic ion, can now be corrected for the 5% admixture of long-lived $2S$ states, where the capture process occurs at a rate which is smaller than in the $1S$ state by a factor of 8.

An outline of the experimental results, and of the physical conditions to which they refer, is given in Sec. II. The analysis and its results are presented in Sec. III, and discussed in Sec. IV. It will be seen that, due to the initial systems in which the capture processes were observed, and to the nature of the theoretical relations involved, the most significant information one obtains by the present analysis concerns the axial-vector coupling constant $g_{A,\mu}$ as determined from the different muon capture experiments.

II. GENERAL REMARKS ON THE RESULTS OF EXPERIMENT

A. Muon capture in hydrogen

A most important feature of the nuclear capture process of negative muons in hydrogen is that the rate at which reaction (1) takes place is strongly dependent on the total spin state of the muon-proton system[1,14] (see Table I). This is due to the fact that the capture rates are expressed as a function of the coupling constants through equations

TABLE I. Summary of the experimental results on the muon nuclear capture rates (λ) by protons, deuterons, and ^{3}He, and average values (λ_{av}) assumed for the present analysis. Some theoretical predictions (λ_{th}) are also presented for reference, as obtained for currently accepted values of the coupling constants.

$\mu^- + p \to n + \nu_\mu$				$\mu^- + d \to n + n + \nu_\mu$		$\mu^- + {}^3He \to {}^3H + \nu_\mu$	
Liquid H_2 target		Gaseous H_2 target					
Symbol	Rate (sec^{-1})	Symbol	Rate (sec^{-1})	Symbol	Rate (sec^{-1})	Symbol	Rate (sec^{-1})
λ_{bc} [a]	428 ± 85 [b]	λ_s [h]	651 ± 57 [i]	λ_d [k]	365 ± 96 [l]	λ_{st} [o]	1410 ± 140 [p]
λ_{bc}	450 ± 50 [c]	λ_s	686 ± 88 [j]	λ_d	445 ± 60 [m]	λ_{st}	1572 ± 46 [q]
						λ_{st}	1465 ± 67 [r]
$(\lambda_{bc})_{av}$	444 ± 43	$\lambda_{s,av}$	662 ± 48	$\lambda_{d,av}$	423 ± 51	$\lambda_{st,av}$	1529 ± 37
$(\lambda_{bc})_{th}$	522 [d]	$\lambda_{s,th}$	650 [d]	$\lambda_{d,th}$	312.7 [n]	$\lambda_{st,th}$	1438 [s]
λ_M [e]	515 ± 85 [f]	$\lambda_{t,th}$	14.82 [d]	$\lambda_{q,th}$ [k]	12 [n]		
λ_M	464 ± 42 [g]			$(\lambda_{st,th})_{\mu d}$	112.2 [n]		
$\lambda_{M,av}$	474 ± 37						
$\lambda_{M,th}$	493 [d]						

[a] λ_{bc} is the effective rate of muon capture in hydrogen bubble chambers, in which the nuclear capture proceeds from the $p\mu p$ muonic ion (rate λ_M; see e) and partially from the μp muonic atoms before they form the muonic molecule.
[b] See Ref. 3.
[c] See Ref. 4.
[d] See Ref. 14.
[e] $\lambda_M = 2\gamma_0 [(\frac{3}{4})\lambda_s + (\frac{1}{4})\lambda_t]$ is the capture rate in the ortho state of the $p\mu p$ muonic ion; here λ_s and λ_t are the capture rates in the singlet and triplet state of the muon-proton system, respectively, and γ_0 is a constant.
[f] See Ref. 5.
[g] See Ref. 6.
[h] Values referring to the singlet state of the μp muonic atom.
[i] See Ref. 7.
[j] See Ref. 8.
[k] λ_d is the capture rate in the doublet state of the muon-deuteron system, whereas λ_q is the one referring to the quartet state; $(\lambda_{st,th})_{\mu d}$ is the muon nuclear capture rate for a system of μd atoms in a statistical mixture of doublet and quartet states.
[l] See Ref. 9.
[m] See Ref. 10.
[n] See Ref. 15.
[o] λ_{st} is the capture rate in the statistical mixture of triplet and singlet states of the $(\mu\, {}^3He)^+_{1S}$ muonic ion.
[p] See Ref. 11.
[q] Corrected value from Ref. 12, allowing for a 5% admixture of $(\mu\, He)^+_{2S}$ states (original value: $\lambda_{st} = 1505 \pm 46$).
[r] See Ref. 13.
[s] See Ref. 16 (a typical value of $\lambda_{s,th}$ is reported in this table).

which are significantly different for the singlet and triplet hyperfine structure states, as follows from the $V-A$ character of the interaction.

A complete experimental outlook would be achieved if process (1) were observed in hydrogen starting separately from the triplet and from the singlet state. Nowadays, the results of experiment provide two different equations, which correlate the coupling constants to the observed rates of muon nuclear capture in the following initial systems:

(a) The $p\mu p$ molecular ion (liquid hydrogen targets). Reaction (1) proceeds at an effective rate

$$\lambda_M = 2\gamma_0(\tfrac{3}{4}\lambda_s + \tfrac{1}{4}\lambda_t) \tag{2}$$

[here $2\gamma_0 = 1.005 \pm 0.005$ (see Ref. 14) and γ_0 is the ratio of the muon density at either proton in the $p\mu p$ ion to the muon density at the proton in the μp atom in its ground state; λ_s and λ_t are the nuclear capture rates of muons by free protons in the singlet and triplet states, respectively, see Table I]. The $p\mu p$ systems are formed through the reaction

$$\mu p + p \to p\mu p\,, \tag{3}$$

which occurs at a rate $\lambda_{pp} = (2.25 \pm 0.13)\times 10^6\ \text{sec}^{-1}$ at the density of liquid hydrogen.[18] Equation (2) holds provided the $p\mu p$ molecular ions are all formed in the ortho state,[19] and if the ortho- to para-state transition rate is negligible compared to the free muon decay rate $\lambda_0 = 4.5\times 10^5\ \text{sec}^{-1}$, as is foreseen by theory.[20]

(b) The μp muonic atom (gaseous hydrogen targets). Stopping negative muons in low-density hydrogen yields two advantages. First, the formation of $p\mu p$ molecular ions is reduced to a small fraction of the μp atoms initially formed (5% at 8 atm and 293 °K) (see Ref. 7); reaction (1), therefore, is observed within the μp atom, which is a system free of μ-molecular complications. Second, the initial (statistical) fraction of triplet states in the μp atoms is promptly depopulated in favor of the lower-lying singlet states,[21] which means that process (1) is observed for a well-defined hyperfine structure state.

Although process (1) takes place in cases (a) and (b) for different total spin combinations of the muon-proton system, it is seen from Eq. (2), and from the theoretical values for λ_s and λ_t given in Table I, that also in case (a) the contribution of the singlet state to the nuclear capture process is dominant. As far as regards the determination of the different coupling constants, therefore, the two sets of measurements do not supply the possibility of considering two significantly independent equations. For more decisive results, with special regard to the determination of $g_{P,\mu}$, one still has to observe process (1) starting from the triplet state of the μp muonic atom, which remains an urgent experimental problem.

B. Muon capture in deuterium

As a second step to obtain information on the coupling constants from the experimental results available on very light nuclei, the case of muon capture by deuterons was considered, according to the process

$$\mu^- + d \to n + n + \nu_\mu\,. \tag{4}$$

Owing to the presence of two neutrons in the final state, and to the Pauli exclusion principle effects, reaction (4) is an almost pure Gamow-Teller transition,[22] i.e., more specifically related to $g_{A,\mu}$.

Also in this case, the rate at which the reaction occurs is very much dependent on the spin state of the muon-nucleon system (see Table I). The experimental results presently available pertain to the following muonic systems:

(a) the $p\mu d$ molecular ion (liquid target of deuterated hydrogen)[9];

(b) the μd muonic atom (gaseous target of deuterated hydrogen).[10]

Both these results are referred to the doublet spin state of the muon-deuteron system. Measurements of the rate of process (4) starting from other combinations of the total spin states are missing.

C. Muon capture in ^{3}He

The third process which was considered here is the muon nuclear capture by ^{3}He. For the present purpose, in fact, the results on the rate of the superallowed transition

$$\mu^- + {}^3\text{He} \to {}^3\text{H} + \nu_\mu \tag{5}$$

are quite relevant for the following facts:

(a) Fairly accurate estimates of the nuclear matrix elements are possible, so that the theoretical predictions for the rate of process (5) have an acceptable degree of accuracy.

(b) Process (5) takes place within the $(\mu\,^3\text{He})^+$ muonic ion in the 1*S* state, for which it is generally assumed that the initial statistical population of the hyperfine structure (singlet and triplet) states is conserved. Strong support to this basic assumption was given by Winston and Telegdi,[23] who calculated that Auger effects, which represent the only possible way of transition between the two hyperfine structure levels, are absent for the $(\mu\text{He})^+$ ion in the 1*S* state. Moreover, an upper limit of $4\times 10^4\ \text{sec}^{-1}$ for the rate of triplet-to-singlet transition was derived by Zavattini,[2] looking at the time distribution of the recoiling tritons from process (5) observed by Clay *et al.*[13] Even with such maximum transition rate, a correction of only 3% would be introduced in the theoretical value of the capture rate referring to the statistical mixture of the triplet and singlet states. (It should be noticed that, opposite to the case of hydrogen and deuterium, the capture rates in the triplet and singlet states differ by only about 30%.)

The results of the measurements on the rate of process (5) are listed in Table I. The only experiment which was performed with a target of pure helium is the one by Auerbach *et al.*[12] The result by these authors was corrected to allow for a 5% admixture of long-lived $(\mu\mathrm{He})^+_{2S}$ states,[17] where the nuclear capture rate (which goes like n^{-3}, where n is the principal quantum number) is depressed by a factor of 8 with respect to the $(\mu\mathrm{He})^+_{1S}$ state. This correction takes the original value $[\lambda_{st} = (1505 \pm 46)\ \mathrm{sec}^{-1}]$ to the value of (1572 ± 46) sec^{-1} listed in Table I. Since the result by Auerbach *et al.* is the most precise one on muon nuclear capture by ^{3}He, the correction seems worthwhile being retained.

As to the other counter experiment,[13] which for technical reasons was carried out in ^{3}He targets contaminated by xenon, no correction was introduced, since from recent results[24] it appears that minor xenon concentrations will neutralize the $(\mu\mathrm{He})^+_{2S}$ ion through the electron-transfer reaction

$$(\mu\mathrm{He})^+_{2S} + \mathrm{Xe} \to [(\mu\mathrm{He})^+_{2S}e^-] + \mathrm{Xe}^+ , \qquad (6)$$

followed by prompt Auger transitions to the $1S$ state. No correction was introduced also for the result by Folomkin *et al.*,[11] which was carried out in a gas-filled diffusion chamber, since this experiment too was not performed in a pure ^{3}He medium.

III. ANALYSIS AND RESULTS

The experimental results presented in Sec. II were treated in the following way:

(i) Among the many theoretical studies,[14-16] the analytical expressions for the capture rates in the various muonic systems were assumed as given by Pascual *et al.*[14-16]

(ii) By comparing each experimental rate with its theoretical expression, we can determine a region containing the possible pairs of values for two of the coupling constants, provided the other two are fixed by some suitable choice. Here we chose to look for the regions which the different capture rates observed allow for possible pairs of $g_{A,\mu}$ and $g_{P,\mu}$. Assuming the validity of the conserved vector current hypothesis (CVC),[25] the values

$$g_{V,\mu} = 0.968 , \qquad (7)$$

$$g_{M,\mu} = 3.6 \qquad (8)$$

were retained for $g_{V,\mu}$ and $g_{M,\mu}$ in units of the β-decay coupling constants

$$g_{V,\beta} = 1.1484 \times 10^{-11}\ \mathrm{MeV}^{-2} = g^0_{V,\mu} .$$

The data from the measurements on muon capture in hydrogen were considered first, analyzing separately those results which pertain to different physical conditions, and need therefore to be treated by different equations.

The average result $\lambda_{s,\mathrm{av}}$ from the experiments in gaseous hydrogen (see Table I) was compared to the theoretical expression for the muon nuclear capture in the singlet state of the muon-proton system. The average result $\lambda_{M,\mathrm{av}}$ from the counter experiments in liquid hydrogen was compared to the theoretical expression for the rate of muon capture in the $p\mu p$ molecular ion [see Eq. (2)]. Finally, the average result $(\lambda_{\mathrm{bc}})_{\mathrm{av}}$ from the bubble-chamber experiments was compared to the expression

$$(\lambda_{\mathrm{bc}})_{\mathrm{th}} = 0.17\lambda_s + 0.83\lambda_M \qquad (9)$$

to keep into account that a fraction of the nuclear capture events observed when muons are stopped in a hydrogen bubble chamber occurs within the μp muonic atoms before they form the $p\mu p$ molecule[14]; Eq. (9) is based on the value previously given for the rate λ_{pp} of reaction (3).

The regions so obtained, which contain the possible pairs of $g_{A,\mu}$ and $g_{P,\mu}$ in a $(g_{A,\mu}, g_{P,\mu})$ plane, are shown in Fig. 1. It is seen that all the experimental results are compatible with a range of $g_{A,\mu}$ values which is much reduced in size with respect to the one allowed for $g_{P,\mu}$. This has to be expected since the measured rates are determined by the capture process in the singlet state of the muon-proton system, which is not sensitive to the value of $g_{P,\mu}$.[1,14]

The dashed curve, which represents the pairs of values corresponding to the theoretical rate[14] for the muon nuclear capture in the triplet state, shows how important a measurement of such a rate would be, even with a limited accuracy, to obtain an intersection region with the results of the present analysis in the $(g_{A,\mu}, g_{P,\mu})$ plane, i.e., a limited range of variations for both coupling constants.

Given the poor significance of the results shown in Fig. 1 with respect to $g_{P,\mu}$, the further step undertaken was to look for the values of $g_{A,\mu}$ which are allowed if $g_{P,\mu}$ is fixed by a proper hypothesis. Disregarding the Goldberger-Treiman relation,[26] which predicts the value of the ratio $g_{P,\mu}/g_{A,\mu}$, we chose to fix $g_{P,\mu}$ as given by the one-pion exchange dominance (OPED),[27] i.e.,

$$g_{P,\mu}/g^0_{V,\mu} = -9.1\,, \tag{10}$$

a value which will be retained also for the analysis of the deuterium and ^{3}He experiments. Assuming the value (10), the following set of values for $g_{A,\mu}$ are obtained from Fig. 1:

$$(g_{A,\mu})_{H_2} = -(1.07 \pm 0.07) \tag{11}$$

(resulting from the bubble-chamber experiments),

$$(g_{A,\mu})_{H_2} = -(1.16 \pm 0.05) \tag{12}$$

(from the liquid hydrogen counter experiments), and

$$(g_{A,\mu})_{H_2} = -(1.22 \pm 0.05) \tag{13}$$

(from the experiments with gaseous hydrogen targets). From Eqs. (11)–(13) one obtains the world-average value

$$(g_{A,\mu})_{H_2,\mathrm{av}} = -(1.17 \pm 0.03)\,, \tag{14}$$

which is the quantity we shall compare to the corresponding values for $g_{A,\mu}$ suggested by the results of the deuterium and ^{3}He muon capture experiments.

Going to the case of deuterium, the two existing experimental results refer both to the doublet spin state of the muon-deuteron system, as was previously mentioned. For the present analysis, therefore, their weighted average was compared to the theoretical expression for λ_d (see Table I).[15] It should be noticed that theoretical uncertainties in the evaluation of λ_d can be as high as 10%, if the various inaccuracies due to the nuclear structure effects on the rate of reaction (4) are taken into account. Therefore, an additional uncertainty of 10% was folded with the experimental error on $\lambda_{d,\mathrm{av}}$ in Table I while comparing it to the theoretical expression.

This procedure led to the results shown in Fig. 2, where the region containing the possible pairs of $g_{A,\mu}$ and $g_{P,\mu}$ allowed by the measurements of the muon nuclear capture by deuterons is shown, together with some significant results obtained from the hydrogen experiments. A most interesting feature of these results is that no intersection exists between the regions obtained by analyzing the deuterium experiments and the hydrogen ones. This is true even if only the results obtained in gaseous hydrogen are considered, which supply the nearest region to the one suggested by the deuterium experiments. Assuming for $g_{P,\mu}$ the value (10), one gets from Fig. 2

$$(g_{A,\mu})_{D_2,\mathrm{av}} = -(1.39 \pm 0.10)\,, \tag{15}$$

which is in disagreement with the value (14).

Finally, the average result $\lambda_{st,\mathrm{av}}$ for the muon nuclear rate by ^{3}He (see Table I) was compared to the theoretical expression for the reaction proceeding from the statistical mixture of singlet and triplet states in the 1S level of the $(\mu\mathrm{He})^+$ ion.[16] Also in this case, the theoretical inaccuracies due to the different approximations by which the theoretical expression $\lambda_{st,\mathrm{th}}$ is obtained were folded with experiment; this correction introduced an important dispersion on the allowed pairs of $g_{A,\mu}$ and $g_{P,\mu}$, since the experimental error on the average capture rate is very small.

The region containing all possible pairs of $g_{A,\mu}$ and $g_{P,\mu}$ allowed by the muon nuclear capture experiments in ^{3}He is presented in Fig. 3, together with those obtained from the gaseous hydrogen and deuterium experiments. In correspondence with

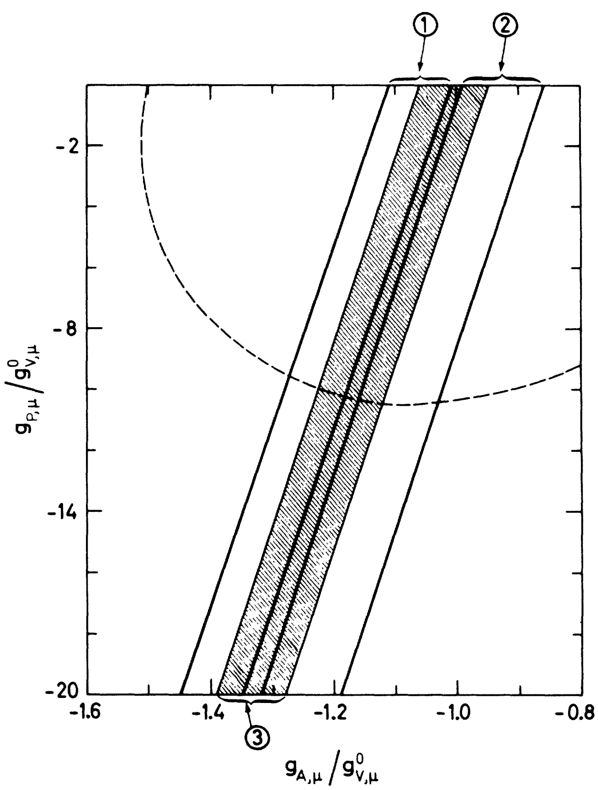

FIG. 1. Results obtained from the analysis of the experiments on the muon nuclear capture *in hydrogen*. The regions between the full lines contain the possible pairs of values for $g_{A,\mu}$ and $g_{P,\mu}$ which are allowed by different experiments. Region 1: experiments performed with gaseous hydrogen targets; Region 2: bubble-chamber experiments; Region 3: (dashed area): counter-experiments in liquid hydrogen. The dashed curve represents the pairs of values corresponding to the theoretical rate for the muon nuclear capture in the triplet state of the muon-proton system.

the OPED value for $g_{P,\mu}$ [see Eq. (10)], one gets from the ^{3}He muon capture experiments

$$(g_{A,\mu})_{^3\mathrm{He,av}} = -(1.27 \pm 0.06)\,, \qquad (16)$$

which is somewhat intermediate between the results (14) and (15).

IV. DISCUSSION AND CONCLUSIONS

The most striking result which has been obtained by the present analysis is the apparent inconsistency between the range of values for $g_{A,\mu}$ which are allowed by the muon nuclear capture experiments in hydrogen and deuterium, respectively, for reasonable values of $g_{P,\mu}$. Considering that the corresponding measurements were performed during a period of more than 10 years, and employing markedly varying techniques, this can at first sight be attributed to systematic experimental errors.

As far as regards the hydrogen experiments, however, the possibility of systematic errors of measurement is hard to maintain, since quite consistent experimental results were obtained by different techniques. In the case of deuterium, moreover, the most likely source of systematic misinterpretation of the experiments is due to the fact that, in the recent measurement in the μd muonic atoms,[10] the fraction of those atoms which are actually in the doublet state at the moment of capture is not well known. Nevertheless, for any value of such fraction smaller than unity, the range of allowed values for $g_{A,\mu}$ extracted from the deuterium experiments would shift towards even larger numbers than the one given by Eq. (15). This is made evident if one compares the experimental value $\lambda_{d,\mathrm{av}}$ in Table I to the theoretical rate $(\lambda_{st,\mathrm{th}})_{\mu d}$ referring to the statistical mixture of doublet and quartet states, instead of comparing it to the theoretical prediction for the capture rate in the doublet state.

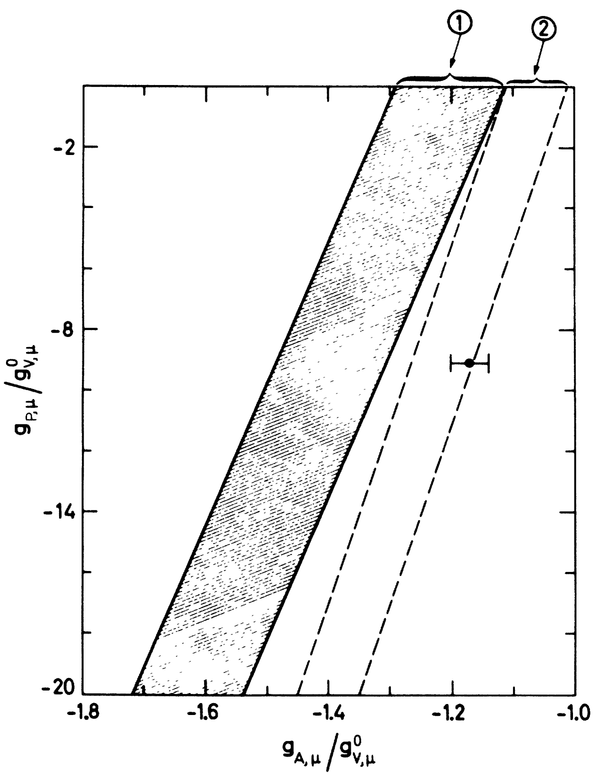

FIG. 2. The dashed area (Region 1) contains the possible pairs of values of $g_{A,\mu}$ and $g_{P,\mu}$ allowed by muon capture experiments *in deuterium*. The point in the figure represents the average value $(g_{A,\mu})_{H_2,\mathrm{av}}$ obtained (assuming $g_{P,\mu}$ as given by OPED) from the whole set of experiments in hydrogen. The dashed lines enclose the allowed region (Region 2) obtained by the separate analysis of the experiments of muon capture in gaseous hydrogen.

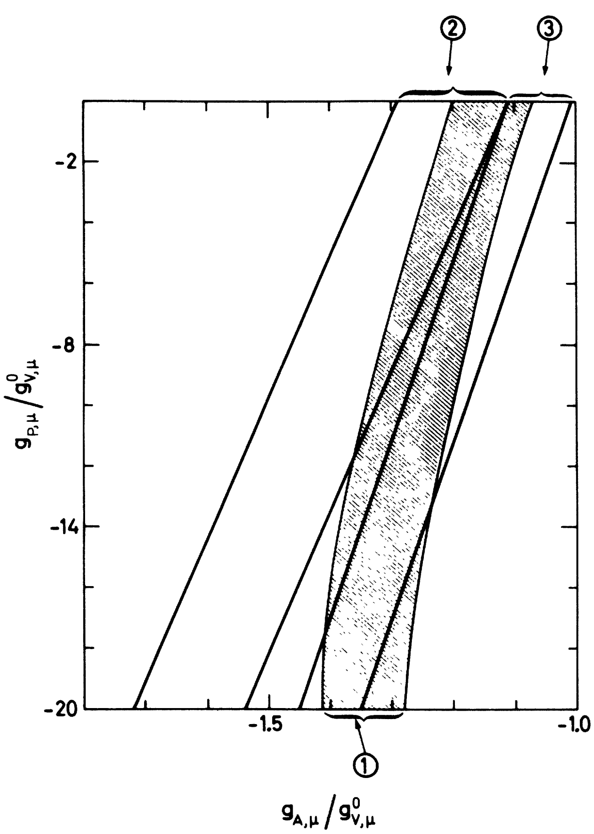

FIG. 3. The dashed area (Region 1) represents all possible pairs of values for $g_{A,\mu}$ and $g_{P,\mu}$ allowed by the average experimental result on the muon nuclear capture *by ^{3}He nuclei*. Also the possible values allowed by the deuterium experiments (Region 2) and by the gaseous hydrogen experiments (Region 3) are shown for reference.

A second possibility which can be taken into account is that the quoted inconsistency might be due to inadequacy of the theoretical predictions, which seems more likely to be present in the calculation of the muon nuclear capture rate by deuterons since this is based on a set of more complex assumptions. On the rate of process (4), however, a number of different calculations exist,[15] which show a substantial agreement among themselves. It is then difficult to call into account either wrong assumptions on the deuteron wave function (which is reasonably well known for the present purposes) or trivial computational errors, which would most likely have been evidenced out by the different approaches followed. As for the *n*-*n* final state interaction, its influence on the rate of capture of muons by deuterons is about 4% only.[15]

If one trusts the results of experiment, therefore, one is led to suspect that the present status of the theory is for some other reasons inadequate to account for the measured value of the muon nuclear capture rate by deuterons.

A second result of the present analysis is that, having corrected the experimental result on the capture rate of muons by ^{3}He nuclei, and taken into account the theoretical inaccuracies on the rate itself, we obtain a value of $g_{A,\mu}$ [see Eq. (16)] which is more compatible with the one obtained by analyzing the deuterium experiments than with the one suggested by the hydrogen ones.

One can now make the following concluding remarks:

(i) An analysis of the available experimental results on the muon nuclear capture rates by hydrogen, deuterium, and ^{3}He nuclei has been performed, with the aim of improving the present knowledge on the two coupling constants $g_{A,\mu}$ and $g_{P,\mu}$. As a result of the analysis, the possible values of $g_{A,\mu}$ allowed by the different experiments for reasonable choices of $g_{P,\mu}$ [i.e., those $g_{P,\mu}$ values which correspond to $g_{A,\mu}$ values in proximity of those suggested by the universal Fermi interaction[28] (UFI)] show an inconsistency. In particular, the value of $g_{A,\mu}$ extracted from the experimental results on muon capture in hydrogen [see Eq. (14)] is lower than the corresponding values obtained by analyzing the deuterium and ^{3}He experiments [see Eqs. (15) and (16)]; taken altogether, the χ^2 value for results (14), (15), and (16) is 6.

(ii) The above-mentioned inconsistency suggests, as an interesting possibility to be examined, that some so far unconsidered nuclear structure effects may modify the effective value of the axial-vector coupling constant $g_{A,\mu}$, even in very light nuclei. One such possible effect would be associated with the meson exchange graphs in nuclear physics[29]; the theoretical situation, however, still needs improvement on this specific point.

(iii) It is interesting to point out that the value for $g_{A,\mu}$ obtained from the present analysis of the muon capture experiments in gaseous hydrogen [see Eq. (13)] is in good agreement with the values supplied by the experimental results on the neutron β decay[30] and on the $(\pi\to\mu\nu)/(\pi\to e\nu)$ branching ratio.[31] This fact, provided one assumes that $g_{P,\mu}$, is in support of the muon-electron universality.

For this reason, the muon capture experiments in gaseous hydrogen represent a very effective tool of reference to investigate the nuclear physics effects on the processes of muon capture by other nuclei. From this standpoint, new and more accurate measurements of the muon nuclear capture by deuterons are necessary, since the nuclear structure plays such a disturbing role that even in a nucleus as light as ^{3}He the experimental accuracy is already beyond the theoretical one.

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*On leave of absence from Istituto di Fisica dell' Università di Bologna, and INFN, Sezione di Bologna, Italy.

†On leave of absence from Istituto di Fisica dell' Università di Pisa, and INFN, Sezione di Pisa, Italy.

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