Flux quantization and fractional charges of quarks*

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Two distinct objectives, (A) and (B), characterize this project of flux quantization and particle physics. (A) proposes that, instead of starting with electric point charges to derive magnetic and other properties of (elementary) particles, one may consider spinning, closed loops of quantized flux $\Phi_q = hc/e$ as the elementary constituents ("elementary loops") from which electric and other properties of particles are derived. The manifold of alternative forms of one single loop (which follow "fibrations" of ordinary space) represents a lepton. In terms of a heuristic model, the consistency of this program had been shown, and shown to imply the derivation of the electromagnetic coupling constant $e^2/\hbar c$. (B) relates the classification of particles and their conservation laws to the topology of flux loops and of their interlinkage. A magnetic field formed from nonintersecting "loopforms" implies topologically the forms of torus knots. The toroidal fibrations of ordinary three-space by two and three coaxially interlinked quarkloops represent mesons and baryons. The loops may independently spin about their common central symmetry axis and "whirl" about their common circular torus midline; effective spin (versus flux) orientation determines their equivalent electric charges. $\mathfrak{N}$, $\mathcal{P}$, and λ quarks correspond to fibrations of space in terms of loops of "winding numbers" (2, 1), (3, 1), and (3, 2), respectively; loops (3, ±2) have "unknotting numbers" ∓1 corresponding to strangeness ∓1. Electromagnetic interactions are, in the conventional way, understood as interactions of distant loops through their electromagnetic field, strong interactions as merging or creation of loop-antiloop pairs, as well as the various different types of exchanges of quark loops between the interacting particles, and weak interactions as crossing of loops over themselves or over those they interact with. This implies that strangeness-violating interactions are weak and parity-violating. The present objective (C) is to consider the existence of the two different types of spin-whirl motion, one of which has the same handedness as that of the fibration, in which case the effective motion of loopforms is subtractively composed of spin and whirl; such was assumed in (B) to characterize quarks. In the other type, i.e., opposite handedness of fibration and of spin-whirl motions, these motions contribute additively to effective motion of torus knots; loops of winding numbers (2, 1) then characterize electrons and muons. The riddle of fractional charges of quarks disappears in this theory. So, also, does the riddle of quark statistics (symmetric spin-isospin functions of baryons) because their linkage makes quark loops localized objects. The flux loops may properly be called "elementary loops." The next objective (D) concerns the probability amplitudes which characterize the distribution of the forms of a torus-knot flux loop. They are shown to be specified by an SO(4)-invariant formulation. To know which group representations to choose from is a prerequisite to a completion of objective (A), and to a quantitative specification of (B).

I. INTRODUCTION

The aim of this paper is to show that an electron as well as a quark may be considered as a superposition of a continuous manifold of "loopforms," i.e., of alternative forms which one quantized magnetic flux loop, an "elementary loop," may adopt. An analysis of the types of flux loops and of their modes of spinning and whirling gives a clue to the understanding of the relationship between a loop of an integer-charge lepton and a loop of a fractional-charge quark. It is shown that a consistent model exists which substantiates the claim that these elementary loops are the basic units of particle physics. Simple topological properties of various types of loops, and of linked pairs and triplets of loops, are shown to characterize leptons, and mesons and baryons. It is pointed out that the difference between electromagnetic, strong, and weak interactions is essentially one of the topology of interaction of flux loops.

A review of magnetic and electric properties of moving flux loops is given in Sec. II. We specify there the basic assumptions of the heuristic model of the electron and muon in terms of those flux loops. (These assumptions carry over to the simple quantum-mechanical model of a subsequent paper D.) Section II starts with the point-source model discussion which shows, for a muon and electron, the *same* Coulomb-type field arising as a consequence of their magnetic dipole fields, and proves the isotropy of that electric field. It then discusses the quasinonlocal effects implied by the *Zitterbewegung* and its implications for the model. This review of A (Ref. 1) [and B (Ref. 1)] is given in the context of the present new model for charged leptons shown in Fig. 3(d), which is to be superposed with its mirror image; this superposition coincides with the quasinonlocal dipole model of A and B, cf. Figs. 3(e).

Section III, which deals with the main topic of this paper, shows the motion of magnetic flux loops as implying electric fields of integer and of fractional charge, exactly the fields required for leptons and quarks, respectively.

Section IV goes into detailed discussions of the forms and motions of loops.

Section V recapitulates the all-important issue of A, i.e., the question of the electromagnetic coupling constant $e^2/\hbar c$, in the context of the present lepton model.

Section VI presents the interesting results about magnetic moments of hadrons. Not only is the dilemma of quark statistics disappearing in the loop model (because quark loops are localized objects, unlike quark particles), but also the size of the magnetic moments of quarks (and of hadrons) is directly determined.

Section VII discusses the angular velocities of spin and whirl, and it discusses the frequencies of the probability amplitudes (ψ functions) of mesons and baryons.

Section VIII presents some remarks on charge conservation.

Section IX indicates the group-theoretical classifications of loopform distributions which will be the subject matter of a later paper D.

II. REVIEW

A. Definition of the magnetic field

The Maxwell-Lorentz equations, which are assumed to form the background frame of the present theory, permit gauge and pseudogauge transformations, an expression of the charge conservation law. Whereas an ordinary gauge transformation is defined with the use of a single-valued gauge variable [denoted by $\varphi(x, y, z, ct)$ in A] and does not imply a physical change in the system under consideration, a pseudogauge transformation is defined through a gauge variable [denoted by $\vartheta(x, y, z, ct)$ in A and B] which is single-valued only modulo 2π and multiples of it. $\exp(i\vartheta)$ and $\partial_k \vartheta$ are single-valued functions of space and time. The latter gauge variable defines, through the singularity line of gradϑ, a quantized flux

$$\Phi_q = hc/e\ . \tag{1}$$

The defining equation is

$$A_k - (\hbar c/e)\partial_k \vartheta = \mathfrak{A}_k = 0\ , \tag{2}$$

i.e., the electromagnetic potentials A_k are defined by postulating the pseudogauge-invariant $\mathfrak{A}_k$ to be equal to zero, the simplest assumption indeed.[1] All magnetic flux is considered to arise from quantized flux.

Instead of introducing magnetic monopoles through such a pseudogauge transformation,[2] we proceed in a more conservative manner by introducing closed quantized flux loops only.

As regards the form of those quantized flux loops, one might perhaps be inclined to think of wide varieties of forms and introduce functionals to express their probability amplitude distribution. Instead, we shall postulate standardized forms, e.g., as a *first try,* the forms of a magnetic field line of a *point dipole* source and use its parameters, i.e., its flux orientation axis $\hat{\zeta}$ (cf. Fig. 2, and A), its azimuth α, and its size σ, essentially, to describe the manifold of loopforms. That these loopforms pass through the point which marks the "position" of the source, as if attached to it, is an assumption equivalent to the postulate of a point-dipole-source lepton whose magnetic field satisfies the Maxwell-Lorentz equations.

One might try to understand the magnetic field of a dipole source to arise from a distribution of *probabilities* of alternative forms which one quantized flux loop may adopt. Not only would such a procedure contradict certain principles of quantum mechanics, it would also imply the failure of a consistent program of representing the field of leptons in terms of quantized flux. It was recognized, however (see A), that a superposition of alternative forms (which a quantized flux loop may adopt) with complex probability *amplitudes* leads to a consistent heuristic theory. In a certain way such superposition is analogous to the construction of a quantum-mechanical path from alternative path histories, each characterized by a complex probability amplitude, in Feynman's space-time approach to quantum mechanics.[3] The recipe for obtaining these probability amplitudes is very different in our case of magnetic field from that of the case of quantum-mechanical paths.

What does "consistency" of the theory imply? It demands in the present context the possibility of describing a space-time *structure of an electromagnetic field (considered as the "observable")* in terms of a *probability-amplitude field which is the quantum-mechanical spinor field* of that internal structure. To be consistent, the probability-amplitude field should permit us to calculate the magnetic field and thus the equivalent electric field from quantized flux and satisfy the compatibility conditions that mc^2 is equal to the electromagnetic energy and that $\frac{1}{2}\hbar$ is equal to the electromagnetic angular momentum. Consistency obviously demands other points, too, matters to which we shall refer below. The phases of the probability amplitudes of the loopforms of a source lepton have nothing to do with the *pseudogauge field* ϑ whose $\nabla\vartheta$ singularities define the quantized flux. There is a continuous manifold of quantized flux loopforms, each with its defining ϑ field, and each represented by a complex probability amplitude.

The construction of a point-source dipole field (which satisfies the Maxwell-Lorentz equations), from probability amplitudes of flux loopforms, is in a certain sense trivial; the probability amplitudes have simply to be chosen so that the squares of their moduli correspond appropriately to the magnetic field intensities at the locations of the respective loopforms. Without further specification, the choice of probability amplitudes is, however, not yet determinate, in particular as we have not yet specified their phases.

B. Electric field from moving quantized flux

We take special relativistic covariance of Eq. (2) seriously. The basic definition of the potentials through (2), formed with relativistic four-vectors, necessarily implies, without further assumptions, that along with quantized flux there exists also an electric potential, provided the quantized flux loop moves. As explained in A and B and in Fig. 1 a magnetic field caused by a Bohr

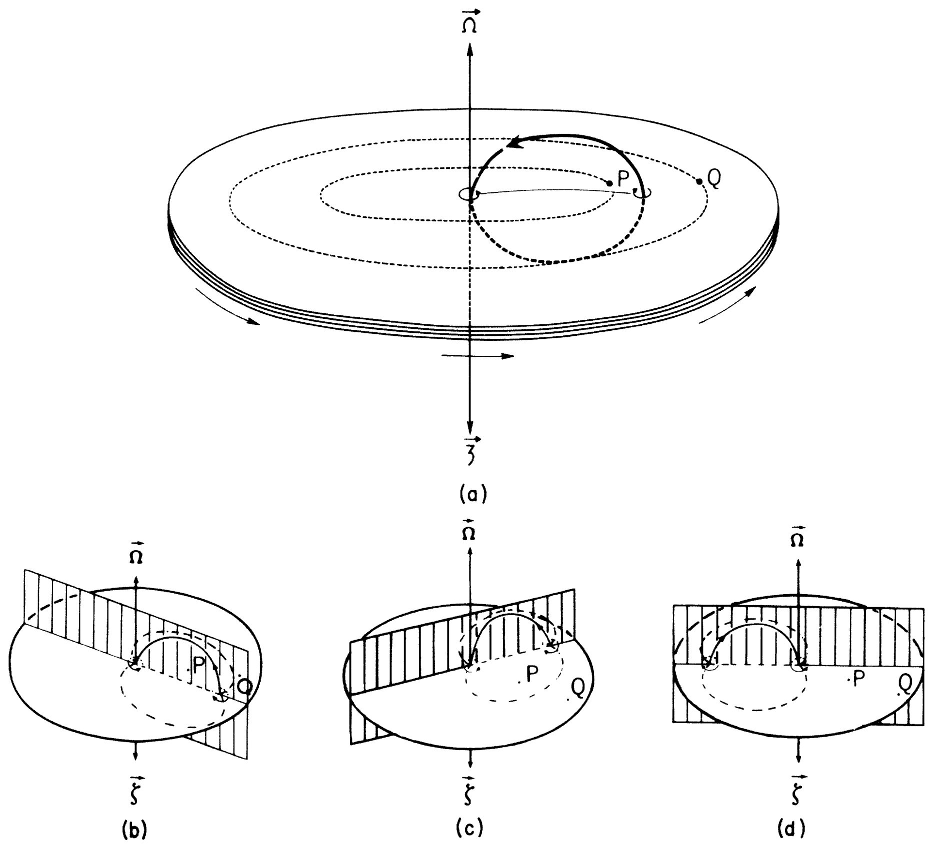


FIG. 1. (a) Illustration of the multivalued (single-valued modulo 2π) pseudogauge variable ϑ which defines a flux loop (heavily drawn, solid line above the equatorial plane, dotted line beneath it). In the equatorial plane a kind of Riemannian surface is illustrated, on which (not a complex variable but) the pseudogauge variable (phase function) ϑ is plotted. While the loop spins, the entire ϑ field has to spin along with it, one may represent this by plotting the equatorial ϑ values on the sheets of this surface (we have drawn only four of the infinitely many sheets) and by considering these sheets to spin with the flux loop, about the flux orientation axis ($\hat{\mu} \equiv \hat{\zeta}$). Accordingly, a point P in the equatorial plane, if P lies inside the flux-loop's "aphelion" (i.e., is linked with the flux loop), may leave a (dotted line) trace on these multivalued sheets, the trace moving from one level of the sheets to the next each revolution of the sheets. This point P experiences thus a unidirectional rate of change of ϑ. The spinning loop therefore contributes toward the electric potential at points P which are linked with the loop. A point Q, however, which lies outside the loop's aphelion, leaves a trace on one sheet on which it stays while the loop with the sheets spins. Q thus does not experience a cumulative change of the ϑ with time, and thus no contribution to the electric potential from that loopform. (b), (c), and (d) illustrate the spinning of the loop which is shown only above the equatorial plane; the —··· — line on the equatorial plane indicates the locus $\theta = -\frac{3}{2}\pi$; The —·— line, the locus $\theta = -\frac{1}{2}$; the shaded plane represents $\theta = 0$ outside the loop and $\theta = -\frac{2}{2}\pi$ inside the loop. The two little circular arrows around the loop indicate the gradient of θ.

or a muon magneton, and spinning about its "flux orientation axis" (axis of magnetic moment) with angular velocity $2m_ec^2/\hbar$ or $2m_\mu c^2/\hbar$, respectively, implies a potential $\pm e/r$, its sign depending on parallelism or antiparallelism of spin and magnetic moment. This statement refers to an oversimplified model of the dipole flux-loop model (Fig. 1), a model in which all flux loopforms have one common flux orientation axis which then is also the magnetic-moment axis and spin axis. Moreover, that value $\pm e/r$ is the electric potential calculated in the equatorial plane only (for the more complete calculation, cf. Sec. II C and Appendix A of the present paper).

Several short comments are appropriate here. The *Zitterbewegung* motion may be considered as a spinning motion, a neat model calculation by Huang[4] illustrates that analogy. There is no issue about the magnitude of the angular velocity, $\Omega = 2mc^2/\hbar$, which is obviously implied by a spinor eigenfunction proportional to $\exp[i(\pm\frac{1}{2}\alpha - \omega t)]$; cf. A, Eq. (8.22) and what follows.

To calculate the electric potential from the basic equation (2) is straightforward and simple. One uses the laboratory rest system (about which we shall comment in more detail below), and one considers the pseudogauge function ϑ connected with each flux loopform, $\vartheta(x, y, z, ct)$, as a function of time; this is time-independent if plotted in a coordinate system which participates in the spinning motion of the flux loopform, as illustrated in Fig. 1. One does not make a Lorentz transformation to calculate the electric from the magnetic field. By the way, the spinning motion is a rotational motion and it also implies linear velocities $> c$.

For a loopform the function $\vartheta(x, y, z, ct)$ [which determines the electric potential $V = (\hbar c/e)\partial\vartheta/\partial ct$] is to be considered a many-valued function of space because $\vec{A} = -(\hbar c/e)\nabla\vartheta$ has a singularity at the location of the flux loop. Figure 1 shows that loopform when its magnetic flux orientation points downwards and its spin upwards. That loopform is shown as a heavily drawn line, solid above the equatorial plane, dashed below it. The value of the function ϑ on the equatorial plane of the loopform may be plotted on an infinite set of sheets (like Riemannian sheets) which spin about the vertical axis together with the flux loop. If the ϑ values are fixedly marked on the sheets (changing by $\pm 2\pi$ from sheet to sheet), the spinning of the sheets indicates the time dependence $\partial\vartheta/\partial ct$ of ϑ at any space-fixed (not sheet-fixed) point on the equatorial plane. For a space-fixed point Q which traces a circle on one of these sheets, ϑ does not change in the time average. For a space-fixed point P, however, the function ϑ changes by $\pm 2\pi$ for each period $2\pi/\Omega$ of spinning because P leaves a trace climbing throughout the (infinite) stack of sheets. Such is the situation of that loopform if that loopform would carry the entire quantized flux $\Phi_q = 2\pi\hbar c/e$. If we consider a point P in the equatorial plane (Fig. 1), the fraction F of the flux Φ_q which is linked with P is, for the point-source model, equal to $F = (e^2/2mc^2)/r$ because the effective magnetic dipole field implies

$$\int_r^\infty B_{\text{eff}}\, 2\pi r\, dr = (e\hbar/2mc)2\pi/r\,.$$

As the phase ϑ changes by $\pm 2\pi$ for each passage of flux of the amount Φ_q we have

$$\begin{aligned} V_{\text{eff}} &= (\hbar c/e)(\partial\vartheta/\partial ct)_{\text{eff}} \\ &= (\hbar c/e)(1/c)(\pm 2\pi)F(\Omega/2\pi) \\ &= \pm e/r\,. \end{aligned} \tag{3}$$

It is interesting to note that in the calculation of the electric potential, the *product* of magnetic moment $e\hbar/2mc$ and angular velocity $2mc^2/\hbar$ enters. The mass cancels out, and that is a rigorous cancellation (as shown in A, Sec. V B). Muon and electron have thus the same electric field if they have similar structure. We will have to postpone the discussion of their mass ratio to the next article.

We proceed here in a way which is somehow the reverse of the usual way. In the latter one, the start is made with the Coulomb potential, an assumption of minimal coupling; one arrives thereby at the Bohr magneton in Dirac's theory of the electron. The present, reverse procedure seems, at a first glance, to be not as natural as the usual procedure, *except* if the above taken starting point of assuming a Bohr or muon magneton can be replaced by starting with the quantized flux $\Phi_q = hc/e$, in which case the Bohr magneton is calculated from Φ_q. It is indeed the point of our heuristic model (see A) that this can be done and that along with it the electromagnetic coupling constant (Sommerfeld's fine-structure constant) may be determined. We shall come back to this point below, but before that we have shortly to digress on angular distribution of flux orientation axes, and on quasinonlocality.

C. Angular distribution of flux orientation, isotropy of electric potential

It would be highly artificial if we would attempt to represent a pure quantum state of an electron (e.g., spin in $+z$ direction) by a manifold of loopforms, all of which have their "flux orientation axes" pointing in the $+z$ direction exactly, while azimuth and size are distributed over the entire accessible ranges. Also, of course, the electric potential would not be isotropic.

We therefore made the assumption that different flux orientation axes $\hat{\zeta}$ are represented by probability amplitudes proportional to $[1+\cos(\hat{\zeta},\hat{z})]^{1/2} = (1+\zeta_z)^{1/2}$; probabilities are then understood as per unit solid angle of $\hat{\zeta}$ directions [Fig. 2(b)]. It was then shown in A, Sec. IV, Eqs. (4.11) to (4.19), that a spinning flux loopform distribution which corresponds to a magnetic-moment-up state, i.e., whose *resultant* magnetic moment points in the $+z$ direction, has an *isotropic* electric potential. That potential is $\pm\frac{2}{3}e/r$, cf. Eq. (6).

The task was (cf. A), to find the effective value $V_{\text{eff}} = (\hbar c/e)(\partial\vartheta/\partial ct)_{\text{eff}}$ for a distribution of loopform orientations $\hat{\zeta}$ [with probabilities $\frac{1}{2}(1+\zeta_z)d\zeta_z$] when the total effective magnetic moment (in the z direction) is $=e\hbar/2mc$, and when each loopform spins about its axis $\hat{\zeta}$ with angular velocity $\Omega = 2mc^2/\hbar$. The choice of

$$\tfrac{1}{2}\mu_\zeta(1+\zeta_z)d\zeta_z\,,\quad \text{with } \mu_\zeta = e\hbar/2mc \tag{4}$$

as the contribution (per sheaf) of the loopforms, towards the effective magnetic moment (in the $\hat{z}$ direction), leads to a total magnetic moment of the required amount $e\hbar/2mc$ (cf. Appendix A).

In order to calculate the electric potential in this general case, we calculate the magnetic flux linked (because of the spinning) with an arbitrary fixed point $P(\vec{r})$ [laid in the $(-y,+z)$ plane of Fig. 2(a)], the flux arising from one of the sheaves of loopforms of given flux orientation $\hat{\zeta}$ ($\equiv\vec{\zeta}$) and magnetic moment μ_ζ (cf. Appendix A). Relevant is the flux through the entire region outside the circle which passes through $P(\vec{r})$ and which is centered on the $\hat{\zeta}$ axis; the circle's radius is $r\sin(\vec{r},\hat{\zeta})$. We find

$$\int_{\vec{r}}^{\infty}\!\!\int \vec{B}\cdot d\vec{S} = \oint_{\vec{r}} \vec{A}\cdot d\vec{r} = -\mu_\zeta(2\pi/r)\sin^2(\vec{r},\hat{\zeta})\,.$$

The ratio of flux linked with the loopforms to quantized flux is

$$\int_{\vec{r}}^{\infty}\!\!\int \vec{B}\cdot d\vec{S}/\Phi_q$$

so that for a dipole field of moment μ_ζ an elementary calculation gives

$$\begin{aligned}(\partial_0\vartheta)_{\text{eff}} &= \mp(2\pi/c)\left(\int_{\vec{r}}^{\infty}\!\!\int \vec{B}\cdot d\vec{S}/\Phi_q\right)\Omega/2\pi\\ &= \pm(2\pi/c)[\mu_\zeta(2\pi/c)\sin^2(\vec{r},\hat{\zeta})/(2\pi\hbar c/e)]\\ &\quad\times(2mc^2/2\pi\hbar)\,,\end{aligned} \tag{5}$$

cf. Fig. 2(a). The expression $\sin^2(\vec{r},\hat{\zeta})$ occurs because we postulated that spinning $\vec{\Omega}$ and flux orientation $\hat{\zeta}$ are parallel or antiparallel. The averaging of this expression is obtained by the multiplication with

$$-(1+\zeta_\zeta)(\tfrac{1}{2}d\zeta_z)(d\beta/2\pi)(d\alpha/2\pi)$$

and integration. With $\sin^2(\vec{r},\hat{\zeta})$ taken as integrand, one obtains the *isotropic* value $\frac{2}{3}$ so that we are led to the result (cf. Appendix A)

$$V_{\text{eff}} = \pm\tfrac{2}{3}e/r \tag{6}$$

for $\Omega = 2mc^2/\hbar$. The details of the calculation, in A [Eq. (4.18)], make it obvious that the result of isotropy depended entirely on the assumption of (4) and of coaxiality of flux orientation and spin. And, as we pointed out previously, it is this assumption too which permits the independent spinning of 2 or 3 quarks in a hadron.

From Eqs. (5.17) and (5.18) of A it is seen that an extended-source model presents the same factor $\sin^2(\vec{r},\hat{\zeta})$ to be averaged. Accordingly, the result of isotropy of V_{eff} holds there too; the r dependence of V_{eff} is different, of course.

D. Quasinonlocality of the source

The *point*-source model which we so far discussed has the difficulty (1) that any idea of distributing the amount of quantized flux over alternative loopforms implies a zero magnetic dipole moment. Even more serious is (2) the impossibility of combining a point-source model with any (not grossly singular) topological structural properties of magnetic flux loopforms. Both the concept of relating quantized flux hc/e to magnetic moment and the concept of topological structure of loops are the cardinal issues of the present theory.

The striking results of quantum electrodynamics have been obtained on the basis of an electron considered as a particle located at a point. As we are here to attempt to describe a *stationary single* particle, we may consider its mean position as determinate, whereas the position, related to it by the Pryce-Foldy-Wouthuysen-Tani transformation,[4] shows the features of a *Zitterbewegung* of the type of an angular velocity $2mc^2/\hbar$ of spinning, and of a linear extension of the order of $\hbar/mc$. This indeterminacy of the source's position implies a corresponding indeterminacy of the field lines, and may therefore be interpreted as quasisize $\hbar/mc$ of the core. This transformation is, however, not a point transformation. It is therefore a very daring attempt which needs to be interpreted with great caution when we make a model of a distribution of quantized flux loops as if they corresponded to an extended source. One

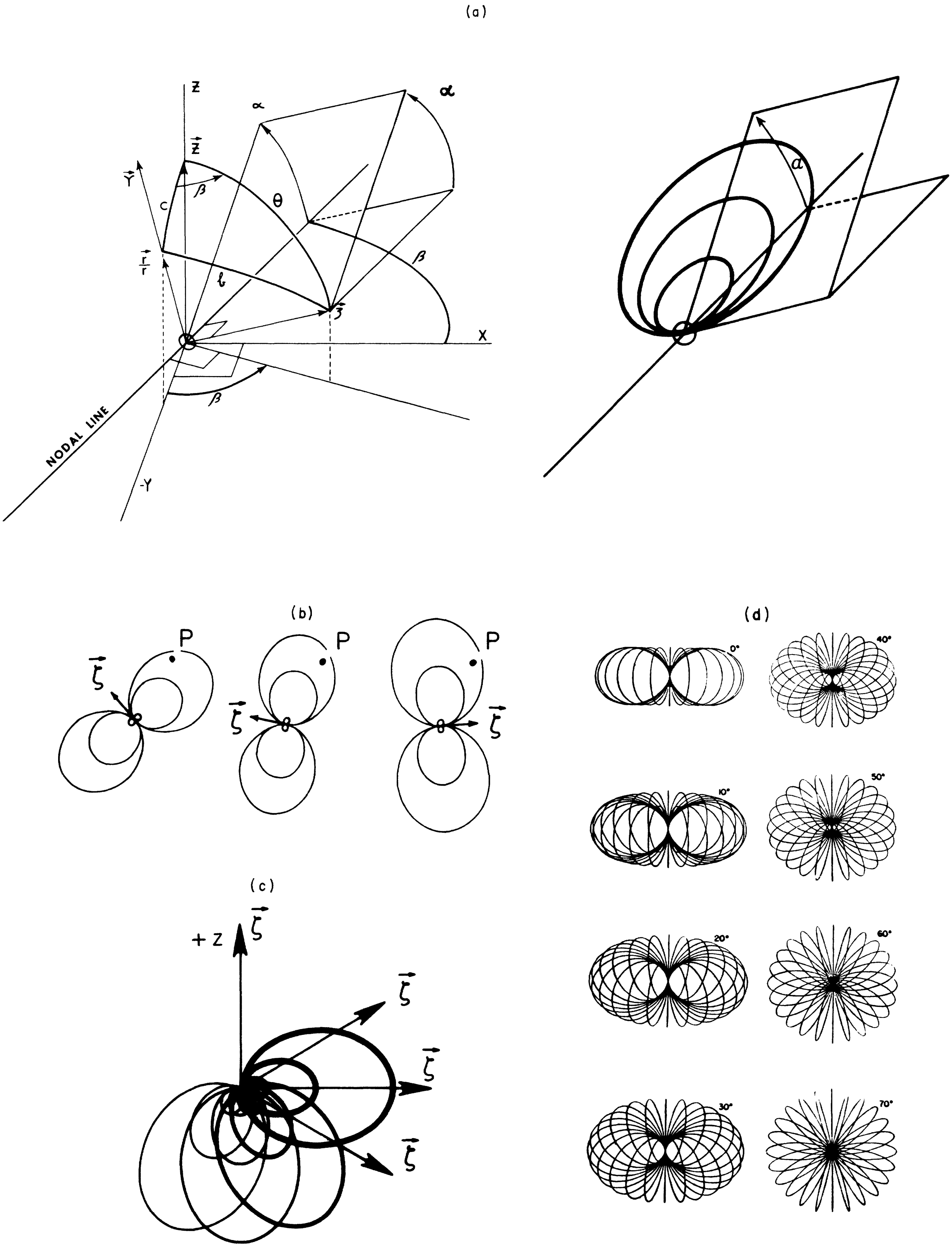


FIG. 2. *(Continued on following page)*

(e)

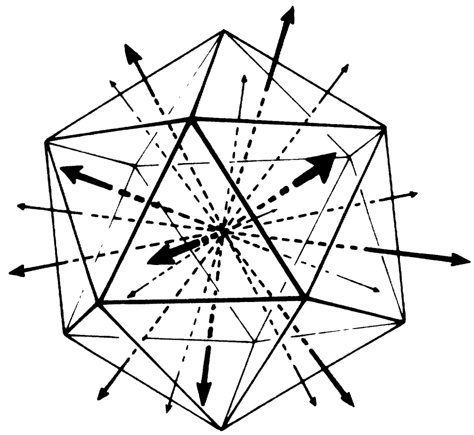


FIG. 2. (a) Diagram specifying the Euler angles β, θ, α which we used in denoting loopforms and their probability amplitudes. The Euler angles β, θ characterize a sheaf of loopforms of flux orientation $\hat{\zeta}$, in relation to a field point $P(\vec{r})$. (b) As examples, three sheaves of flux loopforms are illustrated as contributors to the resulting magnetic field of the dipole whose moment $\vec{\mu}$ is oriented in the $\vec{z}$ direction. (c) The strength of the arrows $\hat{\zeta}$ represent the square of the respective probability amplitudes at each orientation, i.e., $[1+\cos(\vec{\zeta},\vec{z})]$; the contributing loopform sheaves are generated by rotation of field lines associated with these orientations (three of those field lines are drawn to represent the generators of each sheaf). (d) A "sheaf" of loopforms of a magnetic dipole point source is shown here, for various inclinations of the flux orientation (symmetry) axis; this picture refers to loopforms of just one size σ; these figures have been given by Malcolm Correll [Am. J. Phys. 27, 588 (1959)]. (e) Different flux orientation axes, characterizing different bundles, are shown in three-dimensional view by the arrows. The "bundling" of loopforms into 20 bundles as regards different flux orientations $\vec{\zeta}$ is illustrated here in terms of the 20 sides of an icosahedron. The (square of the) magnitudes of the probability amplitudes are, as in Fig. 2(c), indicated by the strengths of the 20 arrows of flux orientation. The phases of the probability amplitudes are closely interrelated for loopforms which are close neighbors; this phase relationship was assumed to cease to exist for loopforms which have flux orientations $\hat{\zeta}$ or azimuths α which differ by more than a radian. Accordingly, we collect loopforms together into phase-related bundles. The grouping according to the 20 sides of an icosahedron in the space of flux orientation axes may be considered as a convenient way to illustrate that bundling: Indeed, the vectors corresponding to neighboring icosahedron surfaces are 0.976 rad, i.e., just about one radian apart. The z axis points in a direction perpendicular to the paper plane. This concept of a bundle is different from that used in fiber space topology.

should in this context also be reminded of the important recognition[5] that there is an equivalence between a local interaction plus an indefinite metric and a nonlocal interaction with a positive-definite metric, the latter one applying to our model. With this in mind, it would therefore seem difficult to precisely formulate the space-time structural and topological properties of loopforms without taking recourse to yet unfamiliar new mathematical techniques. We therefore investigate what such a "quasinonlocal," quasi-extended-source model has to say. Indeed, it is suggested that those quasinonlocal features are not only a matter pertaining to hadronic structure, but to leptonic structure, too. In order to determine the probability (amplitude) distribution for loopforms, we assumed the Maxwell-Lorentz equations to hold for empty space with a singularity at the location of the point source. When quasinonlocality is now expressed in terms of a quasiextended source, it will be assumed that this source distribution corresponds to the torus-knot loopform fields, the connection being given by the Maxwell-Lorentz equations, cf. Appendix D.

E. Relationship of effective (wing)-magnetic moment to quantized flux

We have, up to now when we told about the relationship of magnetic moment to electric charge, performed the calculation by starting with the effective magnetic flux and (Bohr) magnetic moment, and therefrom calculated the effective electric potential. Up to this point of the present exposition, we ignored the issue of effective flux versus quantized flux. That amounts to having ignored the fact that probability-amplitude superposition leads to a reduction of quantized flux to effective wing flux because such a reduction is caused by probability-amplitude interference. This issue is the central topic of A.

What is the point of this reduction? If we look at the order of magnitude of the magnetic moment of a source of linear extension $\hbar/mc$, carrying a total flux of the amount of $\Phi_q = hc/e$, we find that magnetic moment to be of the order of $(\hbar/mc) \times (hc/4\pi e)$, which is a few orders of magnitude too large to explain a Bohr (or correspondingly a muon) magneton, (with the respective m_e and m_μ). It was postulated that this consideration which was based on probability superposition (total Φ_q parceled out over the loopforms), is to be replaced by the aforementioned probability-amplitude superposition, which, because of partially destructive interference, reduces Φ_q to the effective wing flux Φ_{wing} and, correspondingly, the above magnetic moment to μ_{wing} (cf. Appendix B). In crude, simple terms it was assumed that flux loopforms have the phases of their probability amplitudes distributed in such a manner that neighboring loopforms have similar phases, but distant loopforms not. We therefore bundled loopforms together into neighboring bundles of loopforms, some N statistically independent bundles altogether. Thus, instead of an in-phase superposition of their amplitudes to a resultant amplitude proportional to N-bundle amplitudes, and to a probability proportional to N^2, the interfering superposition leads to a resultant amplitude proportional to $N^{1/2}$-bundle amplitudes and a probability proportional to N. We are coming back to this issue in Sec. V.

F. Summary of the project

The points about this proposal are the following:

(1) We realized that not only does such an interference reduction lead from quantized flux to effective wing flux, but that the same reduction brings the electromagnetic energy to mc^2 and the electromagnetic angular momentum to $\frac{1}{2}\hbar$. This means that consistency of a purely electromagnetic model of leptons may be achieved (cf. A).

(2) It was realized that there were no other choices, neither for Φ_q, which is hc/e, nor for the approximate size of the quasinonlocal source, i.e., $\hbar/mc$. That necessitated the reduction assumption, which, as pointed out earlier, was in any case to be anticipated as a necessity because probability amplitudes, not probabilities, are to characterize the model.

(3) But, most important, it was recognized (A, Sec. X) that the possibility of calculating the reduction factor N gives us the possibility of estimating the electromagnetic interaction constant $e^2/\hbar c$. This point will be restated below in Sec. V on the basis of the presently proposed loop model for leptons and on the basis of Appendix B. It might be noted that the constant $e^2/\hbar c$ was initially introduced in this theory as a *coupling* constant through Eq. (2), where it indicates the coupling between the electromagnetic potentials A_k, which characterize the "source" particle, with the pseudogauge field ϑ, which is a quantity pertaining to any "field" particle's wave function.

(4) We attempted to calculate that reduction factor and thereby the electromagnetic coupling constant by bundling flux loopforms together so that different loopform bundles were distinguished by some unit radian distance in the space of parameters: orientation, azimuth, and size of loopforms. In Appendix B an extension of those calculations to the present lepton model leads to revised (still very crude) numerical data.

(5) Instead of such rough heuristic counting, one will have to find a way of counting the number of participating modes, i.e., of wave modes which may describe probability-amplitude distributions for flux loopform parameters. This is the central job: to determine the lepton's probability amplitude functions, to be able to calculate, rather than estimate $e^2/\hbar c$, and to find the electron versus muon characterization of probability-amplitude functions (paper D).

That is one part of the project of relating particle physics to "elementary loops;" these flux loops should be considered the truly elementary constituents of matter. The other part is no less important: It is the study of the topology of the loopforms, i.e., of the fibrations and their motions. The different porperties of leptons, quarks, mesons and baryons are reflected in differences of topological structure and motion (cf. B and Refs. 6 and 7). A flux loop, if interlinked with one or two others, behaves as a quark.

The different types of interactions are to be found in (1) at-a-distance *interaction of electromagnetic* character caused by flux loops in motion, (2) crossing of loops over themselves or over those they interact with *(weak interactions)* (this is a matter of cobordism of knots), (3) merging or pair creation of loops *(strong interactions)* as well as quark loop exchanges between particles. These imply quark-antiquark annihilation or pair production when a $q\bar{q}$ meets another q from a hadron. Baryon number conservation is due to flux-loop (quark) conservation. Charge conservation is implicit in the present theory which is based on gauge invariance (Fig. 3.)

III. INTEGER AND FRACTIONAL CHARGE

Using a heuristic model we discussed leptons[1] and hadrons[1] in terms of quantized flux. The model takes advantage of the possibility of constructing a quantum state from a superposition (with complex probability amplitudes) of alternative forms of closed loops of quantized flux. In particular, an electron's or a muon's magnetic dipole field was constructed from such a superposition of "flux loopforms" (i.e., from a manifold of orientations, azimuths, and sizes which a quantized flux loop may adopt) so as to represent the Faraday dipole field lines corresponding to a Bohr or a muon magneton $e\hbar/2mc$. If this field is assumed to spin with *Zitterbewegung* angular velocity $2mc^2/\hbar$, a Coulomb-type electric field results.

With the *Zitterbewegung* the source appears to have a ("core") extension $\hbar/mc$. The heuristic assumption is made that such a "quasinonlocal source" might be discussed in terms of an extended-source model. Accordingly, after having discussed the properties of point-source models of the type represented in Figs. 1 and 2, one may discuss such extended-source models of the type shown in Figs. 3, 4, and 5. They not only permit relating quantized flux to effective flux, and thus effective magnetic moment and electric charge, they also permit that topological structures of quantized flux loops occur which have the forms of torus knots.

It is the concept of Seifert fibration [H. Seifert, Acta Math. 60, 147 (1933)] which defines the properties of fibration of ordinary three-space.

The objective of this paper is to show that the electric fields $\pm e/r$ of charged leptons and $\pm e/3r$, etc. of quarks arise from the two possible types of motion which a quantized flux loop may undergo. We should recall that it was assumed (see B) that the loops, i.e., the fibration of space by the magnetic lines, spin about the central straight doughnut axis with an angular velocity $2mc^2/\hbar$, and that they also whirl about the circular doughnut axis with the same angular velocity. The handedness of the fibration may be related to the handedness of the spin-whirl motion so as to bring about an additive or a subtractive effect on the "effective spin angular velocity," resulting in an electric potential $\pm e/r$ or $\pm e/3r$, respectively. These electron-muon loops and $\mathfrak{N}$ quark loops have winding numbers $(2, \pm 1)$ ($\pm$ depending on their handedness); a loop $(2, +1)$ is represented in Fig. 3(a), a one-parameter manifold of loopforms $(2, +1)$ covering a fibration of a torus surface is shown in perspective in Fig. 3(d).

The present discussion is in terms of a heuristic model, using classical concepts of closed loops and bundles of closed loops for a description of the topology and of the probability-amplitude distribution of quantized flux. Such a model has three important functions.

First, it takes advantage of the correspondence between classical and quantum-mechanical concepts which may be used interchangeably insofar

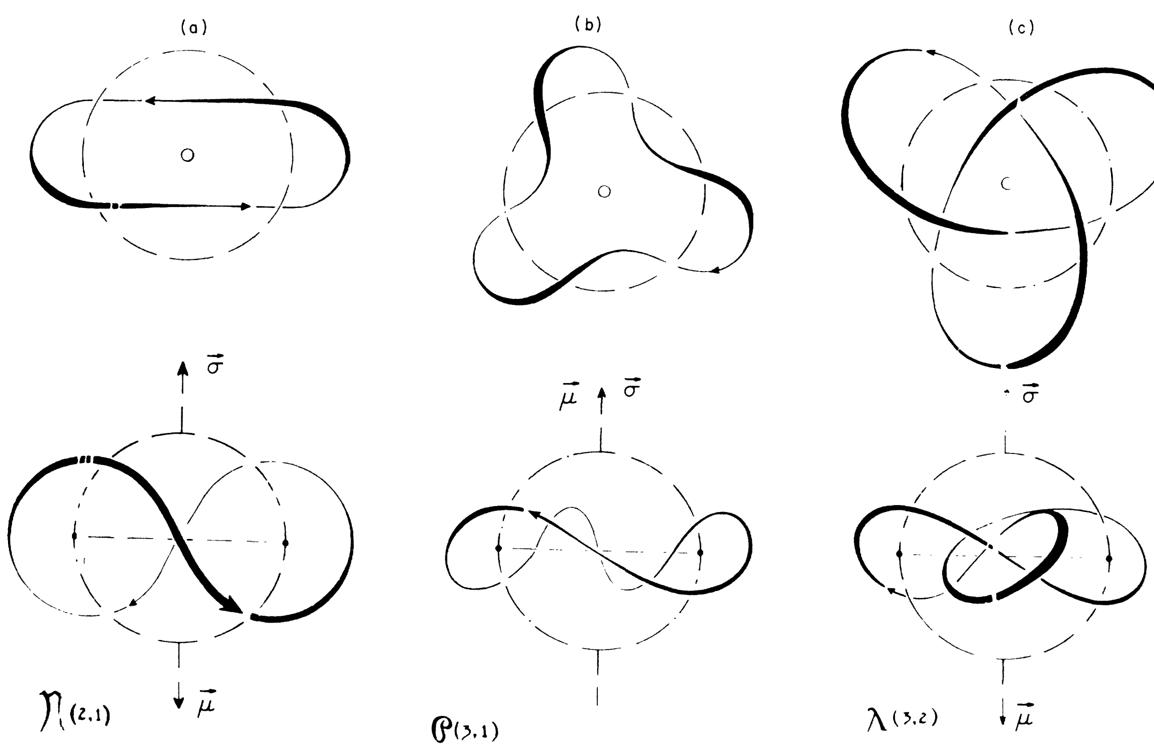


FIG. 3. *(Continued on following page)*

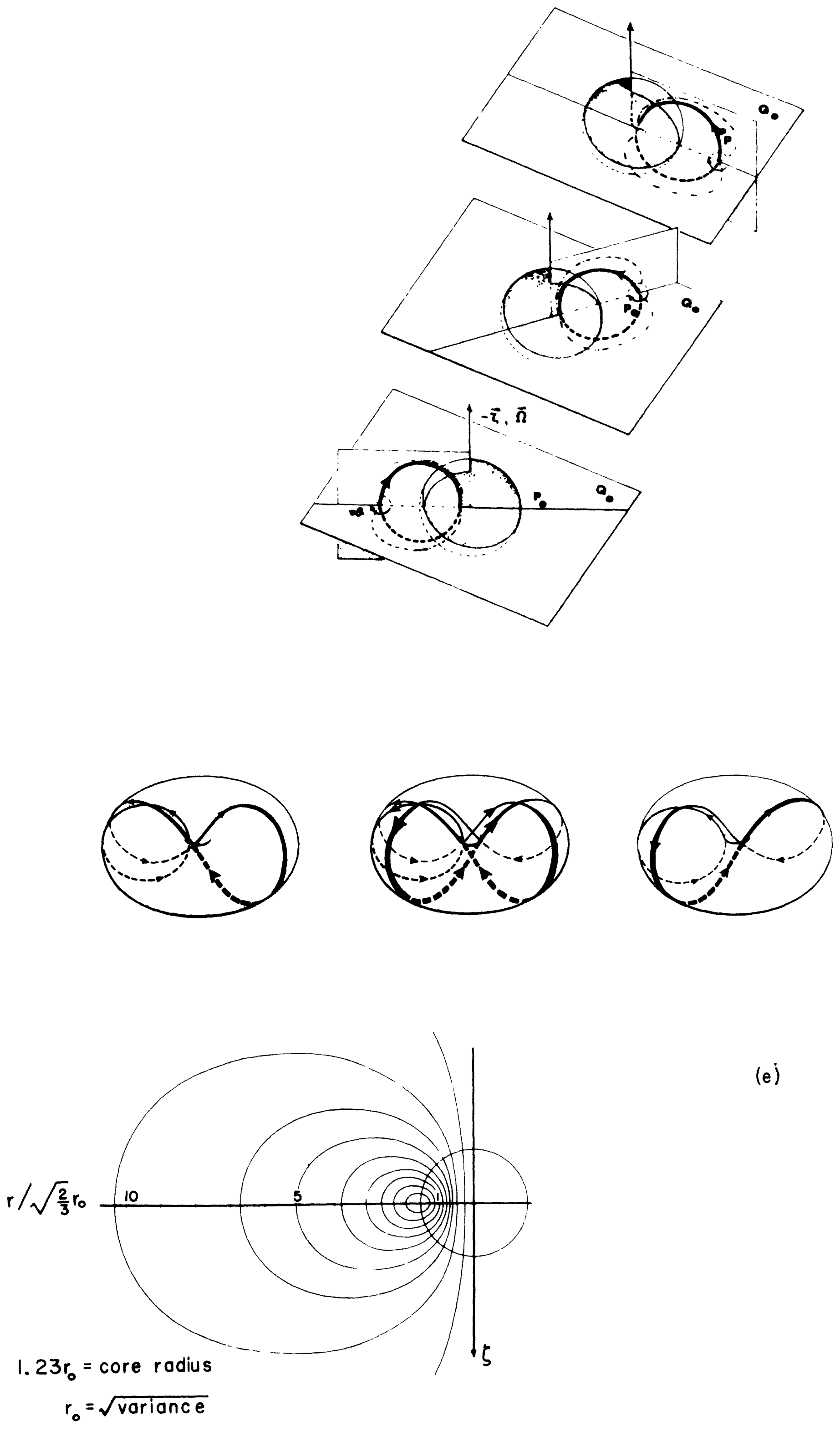


FIG. 3. (*Continued*)

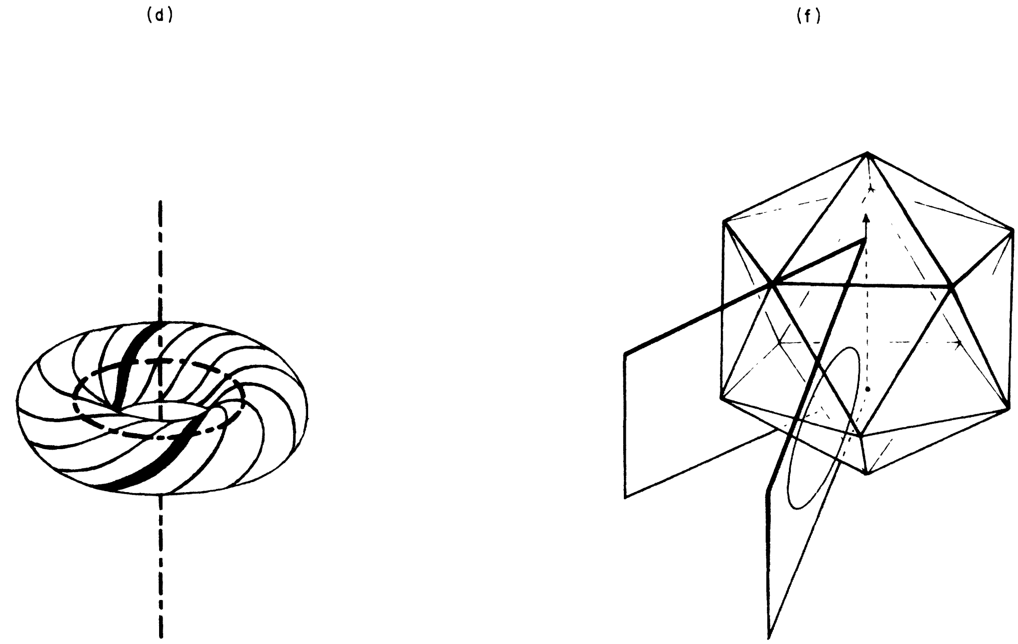


FIG. 3. (a)–(c) Forms of quarks in the spinning-top model; see B. These loops represent quarks only if interlinked with each other, e.g., in the manner of Fig. 4 for mesons, or Fig. 5 of B for baryons, and if the loops' spin and whirl motions contribute subtractively to the electric potential demanded by Eq. (2). The difference in winding numbers about the two —·— axes, i.e., $2-1=1$ ($\mathfrak{N}$), $3-1=2$ ($\mathcal{P}$), $3-2=1$ (λ), multiplied with the signature of spin with respect to magnetic moment, is proportional to the equivalent electric-charge contribution of the respective quarks. The present paper assigns loops of the type (a) also to electrons and to muons. In their case, spinning and whirling contribute additively to the electric potential, which is proportional to $2+1=3$ (instead of $2-1=1$ for $\mathfrak{N}$ quarks). The central spinning axis (straight line) is indicated by the vector ($\vec{\sigma}$) in the lower pictures and by (⊙) in the upper ones. The circular whirl axis (core equatorial line) is indicated by the circle in the upper pictures and, seen sidewise, by the horizontal center line in the lower pictures; the "core region" is symbolized by a sphere (circles in the pictures). Fig. 3(d) shows, in perspective, a one parametric (azimuth α) manifold of loop forms of type $(2,+1)$; it corresponds to a particular flux orientation $\hat{\xi}$ and particular size σ (characterized by the thickness of the torus). If that manifold is repeated, for the whole range of all sizes σ, throughout space, one gets a (left-hand-screw) fibration (a "sheaf of flux loopforms"). A superposition of that with the corresponding right-handed fibration results in a sheaf of magnetic field lines of an extended source dipole, as discussed all along in A, and in Sec. II C, Appendix A, and elsewhere in the present paper. (I should like to thank R. T. Barbee and J. Furlow for illustrations.) (e) For an extended source, the extension (of the order $\hbar/mc$) is indicated here by the diameter of the spheres (of the three upper drawings and of the lowest drawing). This diameter is the same as of the circular doughnut (whirl) axis (fourth drawings from top). The three upper drawings illustrate the spinning of one flux loopform around its flux orientation axis $\hat{\xi}$. These drawings show how a space-fixed point P which is linked with the spinning loopform experiences a unidirectional change of ϑ with time, and thus registers a contribution to the electric potential V. The next, triplet, drawing shows how one meridional loopform of the upper three drawings may be considered as a superposition of a left-handed torus loop $(2,+1)$, shown at left, and a right-handed torus loop $(2,-1)$, shown at right, the two torus loops and the resultant meridional loop all shown together in the center drawing. Both torus loops (loopforms) are spinning the same way about the straight center axis; they whirl in opposite ways about the circular torus axis. Accordingly, the center figure, and also the ones left and right of it, move as a whole, as if they were just only spinning, and that with an angular spinning velocity $(1\pm\frac{1}{2})2mc^2/\hbar = \left\{\begin{smallmatrix}3\\1\end{smallmatrix}\right\} mc^2/\hbar$, resulting in a Coulomb field of charge e or charge $\frac{1}{3}e$ depending on the relative motion of whirl and spin with respect to the handedness of the loop. The lowest figure represents the resulting meridional field which was discussed in the upper drawings, but now other loopforms of different sizes are incorporated in that figure, too. (f) Whereas the Figs. 3(a) to 3(e) referred to a single flux orientation, this figure illustrates again, as in Fig. 2(e), different flux orientations. It shows one loopform of a particular flux orientation $\hat{\xi}$ (indicated by the arrow), of a particular azimuth α and a particular size σ. The core of this extended-source model is not specifically drawn, the size of the icosahedron may be considered as representing the size of the core. The other plane which also passes through the axis $\hat{\xi}$ (arrow), is a reference plane, defining α as the angle between the two planes.

as quantum- or wave-mechanical states may be defined from a superposition of semiclassical structures by means of complex probability amplitudes. With some caution and experience one can easily see the limits of such analogies and avoid hasty conclusions.

Second, it permits checking on the consistency of the theory with itself and with the known data covering a vast range of particle physics and of electromagnetic, strong, and weak interactions.

Third, the heuristic model is the most reliable guide in trying to find a complete quantum-mechanical theory, a task which without the model would be prohibitively difficult.

Referring to the first four sections of B, we recall the requirement that manifolds of loopforms should be formed from Seifert fibrations of space,[6] i.e., the loops should be of the topological forms of torus knots, i.e., closed nonintersecting lines on a toroidal surface, characterized by a pair of integer winding numbers. This condition restricts the types of knots to those of winding numbers $[(0,\pm1)]$, $(1,0)$, $(1,\pm1)$, $(2,\pm1)$, $(1,\pm2)$, $(3,\pm1)$, $(1,\pm3)$, $(3,\pm2)$, $(2,\pm3)$ and those of higher winding numbers. The first number tells how often the loop winds about the circular doughnut axis in the direction indicated by the flux orientation axis $\hat{\zeta}$; it is therefore by definition always positive. The second number tells how often the loop goes around the central doughnut axis; it is taken as positive if the resulting loop is left-handed as shown in Fig. 3, negative for the mirror forms. We also gave some consideration to the question of which of those loops may have physical significance, and we showed why $\mathfrak{N}, \mathcal{P}, \lambda$ quarks should essentially be represented by loops (2, 1), (3, 1) (3, 2), respectively, and the "spinning top" model be adopted, cf. Appendix F.

The interlinkage of quark loops has been discussed in B of Ref. 1. We are here simply showing a linked-loop contribution toward a kaon in Fig. 4. One sees that the possibility exists for independent spinning of $\bar{\mathfrak{N}}$ and λ quarks, and, of course, their magnetic moments are independent too.

In the earlier papers we assumed an electron or a muon to be represented by alternative forms of one closed loop of quantized flux, i.e., loopforms of the forms of magnetic dipole field lines (Fig. 5 in A or Fig. 1 in B), which lines have winding numbers (1, 0). We are discussing the variant assumption, i.e., to represent an electron or a muon by loopforms of winding numbers (2, +1) plus their mirror forms (2, −1), i.e., both torus loops. The additive superposition of these two types of loopforms has the form of ordinary dipole field lines as in A; cf. Fig. 3(e). Such additive superposition is nothing different from the additive superposition, e.g., of loopforms of different flux orientation whose probability amplitudes are additively superposed too. (This superposition still counts as only two core traverses; it should by no means be confused with the structure of two separate loops of mesons, loops confined to separate toroidal regions.) The handedness of those loops, and the handedness of their respective simultaneous spinning-whirling motions are, however, assumed to be related to each other in such a manner that the resulting motion is *additively* composed of the spin and whirl contributions in the case of electrons or muons. As we assume that the spinning about the central doughnut axis and the whirling about the circular doughnut axis occur with the same angular velocity (an assumption always made in B for quarks anyhow), we recognize that for each spin period $2+1=3$ wings pass by a fixed point in space, e.g., on the doughnut surface on which the loopform is drawn. This is an obvious simple topological fact which is easily recognized with the help of torus-loop models [cf. Fig. 3(d)].

A few more remarks about electrons or muons follow. The signature of their charge is given by the relation of magnetic moment to effective spinning [as it is in the case of quarks, Figs. 3(a)–3(c); the word "effective spinning" is supposed to indicate the modified spinning when the effect of whirl motion

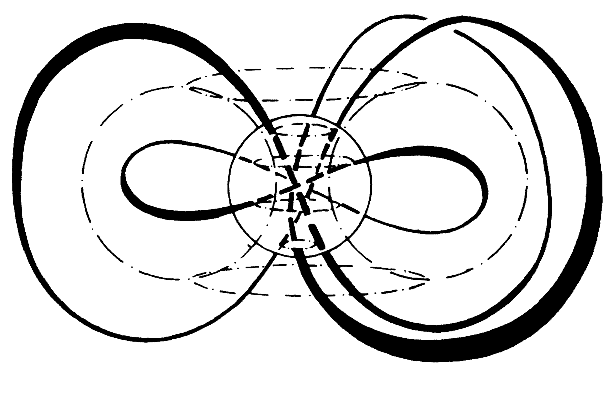


FIG. 4. A λ (3,2) loop and a $\bar{\mathfrak{N}}$ (2,−1) loop, contributing to a kaon. The inside of the doughnut is fibrated in the manner of a (2,1) loop, the outside as a (3,2) loop. The kaon is assumed to be a superposition of this fibration and a fibration in which (2,1) is outside, (3,2) inside; the superposition is the usual type of quantum-mechanical probability-amplitude superposition. In order not to overload the picture, neither the spinning axis nor the whirl axis are shown here. The core is symbolized by the sphere shown as a circle. See also Fig. 4 of B.

on apparent spinning is taken into consideration; we may ignore that specification "effective" except in the case of the alternative assumption mentioned below]. The mass ratio of electron to muon will be discussed in a future paper (D).

A field (2, +1) would imply a particle of intrinsic helicity. As an electron or muon has neither of these properties, we assumed that they correspond to equal, additive superpositions from (2, +1) and (2, −1) loopforms. For a one-electron state these two types of loopforms have the same spinning but opposite whirl so as to yield the same additive effects for generating the electric field. Under a P transformation, the superposition of these two moving loopforms goes over into itself.

For $\mathfrak{N}, \mathcal{P}, \lambda$ quarks, as was assumed in B, again with the spinning-top model, the winding numbers are (2, 1), (3, 1), (3, 2), respectively, and it was assumed that their handedness and the relationship between direction of spin and whirl are such as to imply a resulting motion which is *subtractively* composed of the spinning and the whirling contributions. Accordingly, for each period of spinning or of whirling motion $(2-1)=1$ $\mathfrak{N}$ wing, $(3-1)=2$ $\mathcal{P}$ wings, $(3-2)=1$ λ wing pass by a fixed point on the core equator. Below we shall develop the details and show the consistency of these remarks which lead to an understanding of the charges $\mp e, \mp e; -\frac{1}{3}e, +\frac{2}{3}e, -\frac{1}{3}e$ of electrons, muons, and $\mathfrak{N}, \mathcal{P}, \lambda$ quarks, and to appropriate approximations of their magnetic moments.

We have shown that $\pm\frac{1}{3}e$ and $\pm e$ result without *ad hoc* hypotheses, from subtractive and additive spin-whirl motion of (2, +1) and (2, −1) torus loops. This resolves one of the major puzzles of the conventional quark model. We have, however, not discussed why pairs of $(+\frac{1}{3}e, -\frac{1}{3}e)$ particles are not separately produced, but only such pairs of quark-antiquark loops, each of which interlinked with other quark(s) in hadrons. Nor have we given reasons why (3,1) loops and (3, 2) do not perform additive spin-whirl motions. This theory shows how integer-charge leptons and hadrons arise from quark loops; the restriction to integer charge has still to be proven.

IV. FORM AND MOTION OF LOOPS: SPINNING AND WHIRL

The flux quantization model of particle physics (see B) assumes that flux loopforms should be closed lines and that those which belong to a particular orientation $\hat{\zeta}$ should represent Seifert fibrations, i.e., be of a torus loopform. The two "singular" lines of the fibration are, in the case of the spinning-top model (B)[1] which we adopted, the central straight torus (doughnut) axis and the circular torus axis[6] indicated by the dash-dot-dash lines of Fig. 3(d).

The fibrations of winding numbers (2, 1), cf. Fig. 3, characterize an electron or a muon. We assumed that the motion consists of spinning and whirling with angular velocities about the central and about the circular axes, which are of the same amount, i.e., $2mc^2/\hbar$ (it will be explained below which mass is to be taken for m and why we assumed just these kinds of motions). As the spinning and whirling are assumed to be in such relation to the handedness of the (2, 1) lepton loop that spinning and whirling contribute additively to the effective spinning angular velocity, this effective velocity will be $(1+\frac{1}{2})=\frac{3}{2}$ times $2mc^2/\hbar$.

The quarks of a hadron are to be relegated to different toroidal regions: e.g., q inside a doughnut, $\bar{q}$ outside that doughnut, or vice versa in the case of a meson. Thus their independent motions are spinning and whirling about those two singular axes. Because of the form of any single torus knot, its whirling is equivalent to an additive or subtractive contribution to the spinning about the central axis, the amount depending on the ratio of the winding numbers. A loop of winding numbers (1, 1) would slide along itself with no physical effect if spinning and whirling contribute subtractively. For $\mathfrak{N}, \mathcal{P}, \lambda$ loops we assumed subtractive contribution of spin and whirl. Their winding numbers (2, 1), (3, 1), (3, 2) would thus, because of equality of angular velocity $2mc^2/\hbar$ of spin and whirl, behave as if they would only spin about the central axis with an effective angular velocity equal to $(1-\frac{1}{2})=\frac{1}{2}$, $(1-\frac{1}{3})=\frac{2}{3}$, $(1-\frac{2}{3})=\frac{1}{3}$ times $2mc^2/\hbar$, respectively. The combined spin-whirl motion, being equivalent to these effective velocities about the central axis, thus implies a relative motion of quark loops with respect to each other, a motion about the central axis which is, by rotational symmetry, also the "flux orientation" axis, i.e., the magnetic dipole axis of the quarks or exactly opposite to it.

Let us consider the spin-whirl motion in the case of an electron or muon. Does the spinning axis of a sheaf of lepton loopforms (note that there is no relative spinning to be considered here as there is only one loop for a lepton) coincide with the flux orientation axis, i.e., with the central straight axis of the fibration (again permitting parallelism or antiparallelism), or not? (The coincidence which will be shown to be the case is to be understood to be the relationship between spinning axis and flux orientation axis of a loopform or of a sheaf of loopforms. One lepton corresponds to an amplitude superposition of many different sheaves with different flux orientations and thus different spinning axes).

The above question had been brought under scrutiny in A, Eqs. (4.11) to (4.19) (cf. Sec. II C of the present paper). That was done for a magnetic dipole field [winding numbers (1, 0)]. For our field of winding numbers (2, 1) that calculation applies directly, too, because the circular component of the magnetic field (winding once around the central doughnut axis) does not contribute to the electric field, and the meridional field component, winding twice around the circular axis (2 wings) is assumed to have the same form and total strength as the dipole field; the superposition (2, +1) + (2, −1) is that meridional field.

The first result of Eq. (4.18) in A (cf. Sec. II C) was the recognition of the isotropy of the resulting electric field (i.e., the Coulomb field). This result stemmed from the assumption that magnetic moment and spinning axes were parallel, i.e., that the field of quantized flux loops was constructed from a probability-amplitude superposition of sheaves of loopforms, each spinning about its sheaf axis (flux orientation axis), i.e., magnetic moment parallel or antiparallel to the spinning axis (as already mentioned above). It was then seen [Eq. (A7′) or A, Eq. (4.18)] that it is not reasonable to deviate from the assumptions that "instantaneous magnetic moment is parallel or antiparallel to instantaneous spinning axis" because this only will lead to *isotropy* of the ensuing electric field. This is an important result of the heuristic model of alternative loopforms of an electron or muon; that result, as it holds in the heuristic model, will also have its counterpart in a final full-fledged quantum-mechanical model. Without that result we might have been at sea in guessing what relationships should be assumed between flux orientation and spinning axes.

Although that calculation is, without further discussion, directly referring to electron and muon loops only, we may do well to assume it to hold for linked quark loops too.

The second result of Eqs. (6) and (A7) is that for a simple dipole field from loopforms (1, 0) of magnetic moment $e\hbar/2mc$ and spinning angular velocity $2mc^2/\hbar$, an electric potential $V_{\text{eff}} = \pm\frac{2}{3}e/r$ arises. (That electric potential is a direct consequence of the basic equation which defines flux quantization, for spinning dipole field loopforms.) Now replacing this angular velocity $2mc^2/\hbar$ by the effective angular velocities, which are, for charged leptons and for $\mathfrak{N}, \mathcal{P}, \lambda$ quarks, equal to $\frac{3}{2}\times 2m_{\text{lepton}}c^2/\hbar$ and $(\frac{1}{2}, \frac{2}{3}, \frac{1}{3})\times 2m_{\text{hadron}}c^2/\hbar$ (cf. Sec. VII of the present paper), and replacing the magnetic moment $e\hbar/2mc$ by the magnetic moment per wing, i.e., $\frac{1}{2}e\hbar/2mc$ times the number of wings, i.e., $\mp\frac{2}{2}e\hbar/2m_{\text{lepton}}c$ and $(-\frac{2}{2}, +\frac{3}{2}, -\frac{3}{2})\times e\hbar/2m_{\text{hadron}}c$ (cf. Sec. VI of the present paper), we get the effective electric charges:

$$\frac{3}{2}\times(\mp\frac{2}{2})\times\frac{2}{3}e = \mp e$$

and

$$\frac{1}{2}\times(-\frac{2}{2})\times\frac{2}{3}e = -\frac{1}{3}e\,, \qquad (7)$$

$$\frac{2}{3}\times(+\frac{3}{2})\times\frac{2}{3}e = +\frac{2}{3}e\,,$$

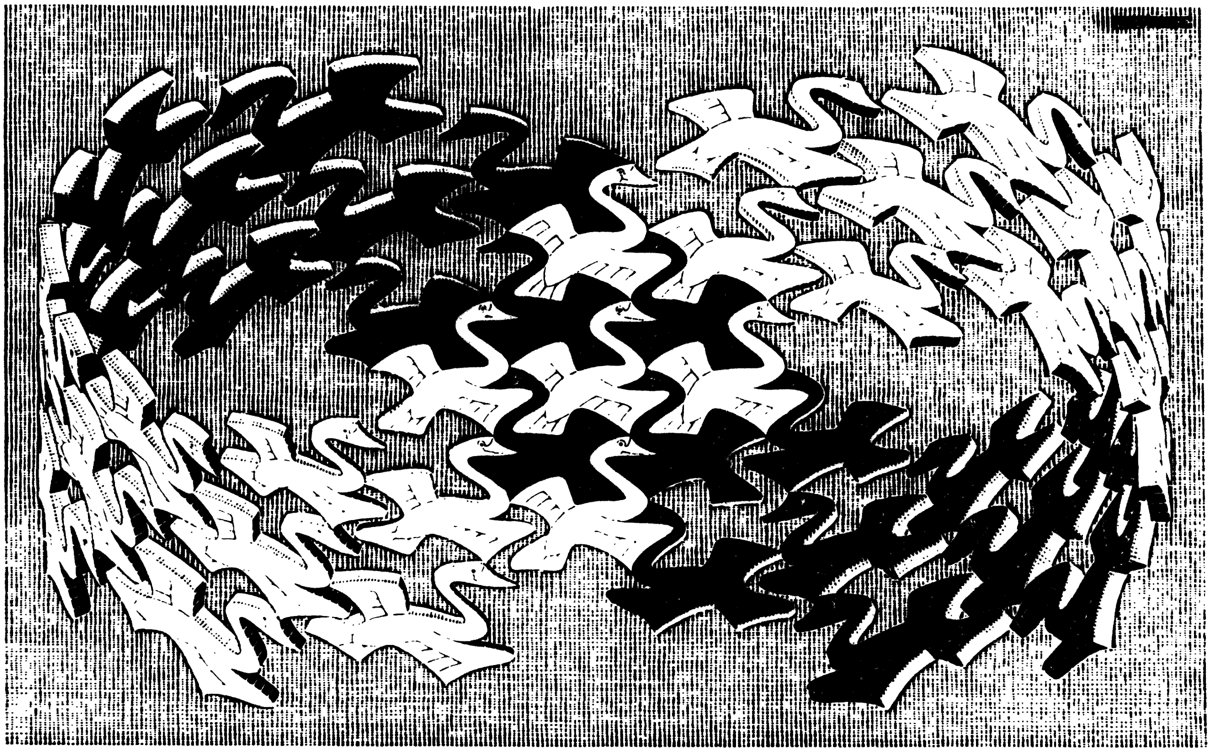
$$\frac{1}{3}\times(-\frac{3}{2})\times\frac{2}{3}e = -\frac{1}{3}e\,.$$

The justification for this calculation will be developed in the subsequent paragraphs and sections. We note again the cancellation of the mass m in the calculation of the electric field.

With the general assumption of coincidence of spinning and flux orientation axes we may then represent all loopforms (lepton or hadron loopforms) by their flux orientation axis ξ, their azimuths α_s, α_w, their size σ, and of course their winding numbers, unknotting numbers, handedness of the loop, handedness of whirling with respect to spinning, and parallelism or antiparallelism of magnetic moment to spinning.

Here we have to insert an important remark about handedness of a torus-loop fibration. Consider the regions of fibration adjacent to the central straight axis and to the circular torus axis. A little model shows that the fibration is either of the type of a right-handed screw relative to both axes or of the type of a left-handed screw. In Fig. 2 of B (Ref. 1) [which shows the same trefoil (3, +2) as that of Fig. 3(c) of the present paper], we pictured the neutrino and the quark loop as such *left-handed torus knots* (and antiquarks as right-handed ones). Those neutrino pictures (3, +2) of B show, however, that the outer parts of the wings slide through space (like a coasting three-bladed propeller) in a *right-hand helical* manner. The trefoil picture for the neutrino was therefore wrongly assigned; it represents an *antineutrino,* not a neutrino. The effective spinning motion of this trefoil of the antineutrino is a right-handed-screw helical motion with the effect that the propeller slices through space, producing no electric field whatever. As the handedness of the trefoil is invariant with respect to Lorentz transformations of any velocity $< c$, the helicity is invariant, too; i.e., the neutrino has no rest system and no rest mass. Apart from the translational motion, the neutrino's motion might be considered like other leptons' motion to have additive contributions of spin and whirl motion. (See Fig. 5 for illustrations of a trefoil and a lively picture of the electron flux loop.)

A loop has the alternative of being left-handed



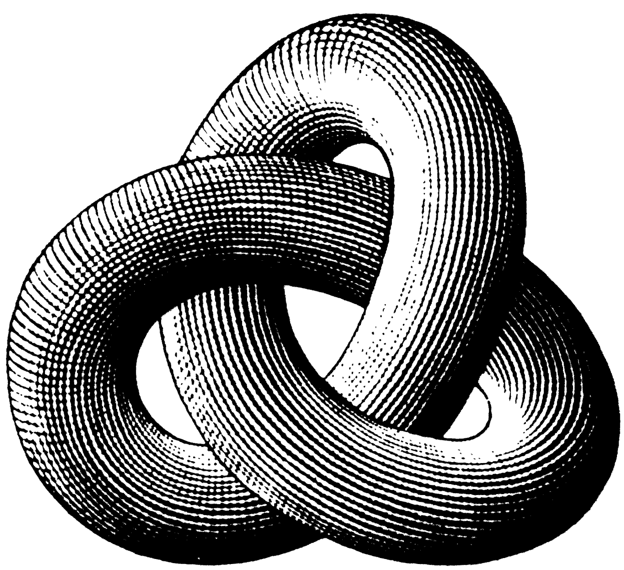


FIG. 5. Pictures from *The Graphic Works of M. C. Escher* as illustrations of a trefoil and a lively picture of the electron flux loop. We are grateful for permission to produce these figures to Koninklijke Uitgeverij. Erven J. J. Tijl N. V., Zwolle, Holland, and Ballantine Books, N.Y., *The Graphic Works of M. C. Escher*.

or right-handed, which in topological language is defined as the character of the fibration next to the singularity lines. This alternative relates even to loops $(2, \pm 1)$ or $(3, \pm 1)$ which are not knotted. Trefoils $(3, \pm 2)$ are of course also either left- $(3, +2)$ or right- $(3, -2)$ handed torus knots; as they are knotted, they have the "unknotting number" -1 or $+1$, respectively, designating strangeness -1 or $+1$ of the λ or $\bar{\lambda}$ quark, respectively.

As already mentioned, there is not only the alternative of right-handedness or left-handedness of fibration, but also the alternative of right-handedness or left-handedness of the spinning-whirling motion. If both are right-handed or both left-handed, spinning and whirling contribute subtractively to the effective spinning motion (quarks); if one is right-handed, the other left-handed, the contributions are additive (electron or muon). Magnetic moment parallel or antiparallel to effective spin implies positive or negative electric charge.

Loops of winding numbers (4, 1), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4), and higher ones whose winding numbers are also prime with respect to each other, can form Seifert fibrations, too. They might perhaps actually exist, perhaps superposed with the previously discussed loop alternatives. Without going into such speculative details, it should, however, be remarked that, apart from signatures, their properties (when spin and whirl contribute subtractively) may be characterized in Table I. It is of interest to note that charge again occurs in multiples of $\frac{1}{3}e$.

The assignment of handedness to λ quarks has been made the same as that to $\bar{\nu}$ in order to make the reaction $K^- \to \mu^- \bar{\nu}_\mu$ plausible, $\bar{\nu}$ having been assigned $(3, +2)$, i.e., left-handedness, to account for its right-handed intrinsic helicity.

Alternative assumption about loops. We may ask whether loops of winding numbers (1, 2), (1, 3), (2, 3) may have a special significance. For the symmetric-axes model (B, Figs. 6 ff.), loops (2, 1), (3, 1), (3, 2) on one axis are loops like (1, 2), (1, 3), (2, 3) on the other axis. For the spinning-top model the present alternative choice is distinctly different from the (2, 1), (3, 1), (3, 2) choice. But it is good to note the properties of that alternative loop choice. Even though we do not use the symmetric-axes model, the pictures of it in B illustrate the topological circumstances: The unwinding numbers and the handedness are not changed by going over to this alternative choice through reversal of winding numbers, we might thus associate the above winding numbers with $\mathfrak{N}, \mathcal{P}, \lambda$ quarks. But this alternative is only a curiosity.

If the assumption is made that the magnetic moment still is proportional to the number of "wings" (i.e., core traverses) for this alternative model, we should then assume the magnetic moments of the quarks $\mathfrak{N}(1, \pm 2), \mathcal{P}(1, \pm 3), \lambda(2, \pm 3)$, related to spin direction, to be given by $-1, +1, -2$ (which is not very encouraging). As we again suppose subtractive contributions from whirl and spin towards effective spinning, we find the effective spin angular velocities to be proportional to $(\frac{2}{1}-1) = +1$, $(\frac{3}{1}-1) = +2$, $(\frac{3}{2}-1) = +\frac{1}{2}$. The product of those two quantities is proportional to the effective electric charge, i.e., $(-1)(+1) = -1$, $(+1)(+2) = +2$, $(-2)(+\frac{1}{2}) = -1$. In other words, we get the same charge ratios for the quarks as in the regular model, but neither do the muon or electron fit in that alternative model, nor do we get appropriate magnetic moments.

We may also perform the counting of electric charge by noting, in a manner analogous to that indicated in the latter parts of Sec. III, that for each period of the spinning (not the effective spinning) or of the whirling motion $2-1=1$ $\mathfrak{N}$ wing, $3-1=2$ $\mathcal{P}$ wings, $3-2=1$ λ wing pass by a fixed point on the equator of the core. To get the electric field, these numbers have to be multiplied by the signatures of the magnetic field lines, i.e., $-1, +1, -1$, respectively.

V. ELECTROMAGNETIC COUPLING CONSTANT

We define intrinsic magnetic moment as proportional to the number of core traverses of a loop, which

TABLE I. Properties of some loops.

Winding numbers	(4, 1)	(4, 3)	(5, 1)	(5, 2)	(5, 3)	(5, 4)
Effective spin angular velocity in units of $2mc^2/\hbar$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{1}{5}$
Magnetic moment in units of $e\hbar/2mc$	$\frac{4}{2}$	$\frac{4}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$
Effective charge in units of e	1	$\frac{1}{3}$	$\frac{4}{3}$	1	$\frac{2}{3}$	$\frac{1}{3}$
Unwinding number (strangeness)	0	3	0	2	4	6

is equal to its number of wings. We thereby assume that the magnetic field of a loop or of a sheaf of loopforms has a meridional component (component in planes which pass through the flux orientation axis $\hat{\xi}$, i.e., the central symmetry axis of the torus loop which represents a loopform), equal to the magnetic field of a dipole of orientation $\hat{\xi}$ which corresponds to the flux of the loop. This neglects the inclination of the field lines with respect to the meridional planes in calculating the magnetic moment. The circular component (parallel to the circular torus axis) has no effect on the equivalent electric field under the motions discussed in Sec. IV of the present paper.

The magnetic moment per wing of an electron or muon of winding numbers (2, 1) should thereby be (with the identification of $e\hbar/2mc$ as intrinsic magnetic moment of the lepton) equal to $\frac{1}{2}e\hbar/2m_{\text{lepton}}c$, where the lepton mass $m = m_{\text{lepton}}$ is the mass of the electron or of the muon, respectively.

The argument of the important Sec. X of A implies the tentative assumption now that the reduction of quantized flux to wing flux (because of random phasedness of the probability amplitudes, leading to destructive interference) goes by the same reduction factor in the case of wings of loops (2, 1) as it went in the case of simple dipole field loops (1, 0) which have just one wing; cf. Appendix B.

We start with the distribution of the quantized flux

$$\Phi_q = 2\pi\hbar c/e \tag{1'}$$

over N bundles of loopforms. Because there are now two wings [winding numbers (2, 1)], that magnetic flux Φ_q is equivalent (after reduction) to the wing flux Φ_{wing} (cf. Appendix B), i.e., to half of the flux Φ_{eff} of a muon of a Bohr magneton $(e\hbar/2mc) \equiv 2\mu_{\text{wing}}$. This effective lepton flux Φ_{eff}, i.e., total effective flux through the core, is then [see A, Eq. (5.10)]

$$\begin{aligned} 2\Phi_{\text{wing}} &= \Phi_{\text{eff}} \\ &= 4\pi(e\hbar/2mc)/3.1r_0 \end{aligned} \tag{8}$$

where the factor 3.1 results from the elementary calculation of a Gaussian distribution of equivalent magnetization (cf. A, Sec. VC). Dividing these two equations we get (cf. Appendix B)

$$\begin{aligned} N/2 &= \Phi_q/2\Phi_{\text{wing}} \\ &= \Phi_q/\Phi_{\text{eff}} \\ &= 3.1(\hbar c/e^2)[r_0/(\hbar/mc)] \ . \end{aligned} \tag{9}$$

If we know to calculate the number N of statistically independent bundles of loopforms, and we may estimate $r_0/(\hbar/mc)$, which is of the order of magnitude unity, we may calculate the electromagnetic coupling constant $e^2/\hbar c$; with our new assumption about winding numbers (2, 1) instead of (1, 0), this implies some change in the calculation of N compared to that in A.[1] We note that the concept of a bundle [cf. Fig. 2(e)] differs from that used in fiber space topology.

VI. MAGNETIC MOMENTS OF HADRONS

One might tentatively assume that the reduction from quantized flux Φ_q to wing flux Φ_{wing}, a prime concern of paper A, is a matter pertaining equally to electron loops, muon loops, and to the (pairs and triplets) loop manifolds of mesons and of baryons. Accordingly we may assume that the reduction factor from quantized flux to the effective intrinsic wing flux of a quark is the same as for an electron or muon. It is, however, not a quark wing or an entire quark whose quantized flux is to be reduced to effective flux: First the full pair or triplet of quarks is composed, and then the entire field is reduced as in the case of a lepton. As the linear extension of the core of a hadron (so also its spinning frequency, Sec. VII of the present paper) is again not a matter of the individual quark but of the entire hadron, i.e., is of the order of $\hbar/m_{\text{hadron}}c$, the intrinsic magnetic moment per quark *wing* is to be assumed as equal to $\frac{1}{2}e\hbar/2m_{\text{hadron}}c$ just as for a lepton wing. This is an obvious assumption which then yields the baryon magnetic moments without the necessity of basing them with respect to each other.

This counting of wings (or core traverses) brings us to the intrinsic magnetic moments of $\mathfrak{N}, \mathcal{P}, \lambda$ quarks, i.e., to the values (−2, +3, −3) times $\frac{1}{2}e\hbar/2m_{\text{hadron}}c$.

These intrinsic magnetic moments may be considered as determining the electric fields; as we indicated in Sec. II of the present paper, the electric fields are determined by the products of the magnetic moments with the effective spinning frequencies $(\frac{1}{2}, \frac{2}{3}, \frac{1}{3}) \times 2m_{\text{hadron}}c^2/\hbar$, yielding the proper charges $\mp\frac{1}{3}e, \pm\frac{2}{3}e, \mp\frac{1}{3}e$ of $\mathfrak{N}, \mathcal{P}, \lambda$ quarks, respectively, and, as seen in Sec. IV, $\mp e$ for electron or muon (with m_{hadron} replaced by the respective m_{lepton}). There is in this theory no need for an *ad hoc* assumption relating the magnetic moment to the electric charge of a quark, but the approximate nature of the present theory had to be kept in mind.

When it comes to the calculation of the magnetic moments of a meson or of a baryon, we might naturally proceed in the customary way and assume a symmetric SU(6) function for the low-lying baryons. Before we justify this assumption by the calculation of magnetic moments and by calculating the ratio of proton to neutron magnetic moment and comparing the results with experimental data, we have to specify what such a symmetric SU(6) function means.

Whether we consider the conventional model in which quarks are considered as mass points, particles, or partons, or whether we consider our quark loop model, we should recall some facts pertinent to both. The quarks of a hadron behave quite nicely as independent objects, e.g., as regards spin, charge, magnetic moment, strangeness; their spin $\frac{1}{2}$ strongly suggests the Pauli exclusion principle to hold. (Nevertheless, a quark, not being an independent quantum particle, might differ basically from how otherwise we understand quantized particles to behave—thus the suggestion of parastatistics made by other groups, designed to avoid the following dilemma.)

The conventional SU(6) quark (considered as particles) model gets, however, into a well-known dilemma. The experimental data (in particular the ratio of proton to neutron magnetic moment) show a symmetric SU(6) function of spin and quark type to apply to the calculation of magnetic moments. As a lowest baryon state, e.g., the state of a proton, should have an S-state orbital wave function, the Pauli principle is violated by the symmetric SU(6) function.

The quark *loop* model is quite different: The two quarks of a meson or three of a baryon are localized objects. For a meson there is a region inside the doughnut and a region outside the doughnut (the doughnut surface is a variable surface of only topological significance, to separate the two quark regions). For a baryon there are three regions: inside, in the middle, and outside. It may be even more adequate to use a formulation in 6+1 or 9+1 dimensional space-time.

In writing down a SU(6) function for three quarks, consider the first entry, $\mathfrak{N}\uparrow$, in a triplet $\mathfrak{N}\uparrow\mathcal{P}\uparrow\mathcal{P}\downarrow$ as referring to the inner region (next to the circular axis), the second, $\mathcal{P}\uparrow$, in that example to the middle region, and the third, $\mathcal{P}\downarrow$, to the outer region (which also is closest to the central straight axis). A state which is symmetric in the second and third positions is

$$6^{-1/2}(2\mathfrak{N}\downarrow\mathcal{P}\uparrow\mathcal{P}\uparrow - \mathfrak{N}\uparrow\mathcal{P}\downarrow\mathcal{P}\uparrow - \mathfrak{N}\uparrow\mathcal{P}\uparrow\mathcal{P}\downarrow), \qquad (10)$$

and the state completely symmetric with respect to exchange of any of the three positions is

$$\begin{aligned}18^{-1/2}(&2\mathfrak{N}\downarrow\mathcal{P}\uparrow\mathcal{P}\uparrow - \mathfrak{N}\uparrow\mathcal{P}\downarrow\mathcal{P}\uparrow - \mathfrak{N}\uparrow\mathcal{P}\uparrow\mathcal{P}\downarrow\\ &+ 2\mathcal{P}\uparrow\mathfrak{N}\downarrow\mathcal{P}\uparrow - \mathcal{P}\uparrow\mathfrak{N}\uparrow\mathcal{P}\downarrow - \mathcal{P}\downarrow\mathfrak{N}\uparrow\mathcal{P}\uparrow\\ &+ 2\mathcal{P}\uparrow\mathcal{P}\uparrow\mathfrak{N}\downarrow - \mathcal{P}\downarrow\mathcal{P}\uparrow\mathfrak{N}\uparrow - \mathcal{P}\uparrow\mathcal{P}\downarrow\mathfrak{N}\uparrow). \qquad (11)\end{aligned}$$

As the three entries of each of these nine summands refer to three distinct spatial regions, there is no violation of the Pauli principle and thus no reason to reject this symmetric SU(6) function.

What are the reasons for the choice of the symmetric SU(6) function? A simple function, $\mathfrak{N}\uparrow\mathcal{P}\downarrow\mathcal{P}\uparrow$ for example, would be unphysical in that it would relegate one quark ($\mathfrak{N}\uparrow$) to the inner region exclusively; other, quite different states would then exist, too, e.g., $\mathcal{P}\downarrow\mathfrak{N}\uparrow\mathcal{P}\uparrow$. An antisymmetric function for the proton,

$$\begin{aligned}|p\uparrow\rangle = 6^{-1/2}(&\mathfrak{N}\uparrow\mathcal{P}\uparrow\mathcal{P}\downarrow - \mathfrak{N}\uparrow\mathcal{P}\downarrow\mathcal{P}\uparrow + \mathcal{P}\downarrow\mathfrak{N}\uparrow\mathcal{P}\uparrow\\ &- \mathcal{P}\uparrow\mathfrak{N}\uparrow\mathcal{P}\downarrow + \mathcal{P}\uparrow\mathcal{P}\downarrow\mathfrak{N}\uparrow - \mathcal{P}\downarrow\mathcal{P}\uparrow\mathfrak{N}\uparrow), \qquad (12)\end{aligned}$$

yields a magnetic moment (because the wing numbers for $\mathcal{P}, \mathfrak{N}$ are +3, −2) of

$$\begin{aligned}\langle p\uparrow|\mu|p\uparrow\rangle &= 3\times\tfrac{1}{6}[(-2+3-3)+(-2-3+3)]\\ &= -2 \text{ times } \tfrac{1}{2}e\hbar/2m_{\text{baryon}}c, \qquad (13)\end{aligned}$$

$$\begin{aligned}\langle n\uparrow|\mu|n\uparrow\rangle &= 3\times\tfrac{1}{6}[(3-2+2)+(3+2-2)]\\ &= +3 \text{ times } \tfrac{1}{2}e\hbar/2m_{\text{baryon}}c. \qquad (14)\end{aligned}$$

The ratio of p to n magnetic moments would be $-\frac{2}{3}$ instead of the observed value −1.47, and the signatures of the magnetic moments are wrong.

The symmetric SU(6) function, however, yields

$$\begin{aligned}\langle p\uparrow|\mu|p\uparrow\rangle &= 3\times\tfrac{1}{18}[4\times(+2+3+3)\\ &\qquad + 1\times(-2-3+3)\\ &\qquad + 1\times(-2+3-3)]\\ &= \tfrac{1}{6}(32-2-2)\\ &= 4.66 \text{ times } \tfrac{1}{2}e\hbar/2m_{\text{baryon}}c, \qquad (15)\end{aligned}$$

i.e.,

$$\langle p\uparrow|\mu|p\uparrow\rangle = 2.33e\hbar/2m_{\text{baryon}}c.$$

This is not far off from the observed value; it is interesting to get the numerical factor of the right order of magnitude. For the neutron we get

$$\begin{aligned}\langle n\uparrow|\mu|n\uparrow\rangle &= 3\times\tfrac{1}{18}[4\times(-3-2-2)\\ &\qquad + 1\times(+3+2-2)\\ &\qquad + 1\times(+3-2+2)]\\ &= \tfrac{1}{6}(-28+3+3)\\ &= -3.67 \text{ times } \tfrac{1}{2}e\hbar/2m_{\text{baryon}}c, \qquad (16)\end{aligned}$$

i.e.,

$$\langle n\uparrow|\mu|n\uparrow\rangle = -1.83e\hbar/2m_{\text{baryon}}c.$$

Thus the ratio of the proton's to the neutron's magnetic moment is $\approx +2.33/-1.83 = -1.28$ instead of the observed value −1.47. Let us note the magnetic moments of other, strange baryons[8]:

$$|\Lambda^0\uparrow\rangle = 12^{-1/2}(\mathcal{P}\uparrow\mathcal{N}\downarrow\lambda\uparrow - \mathcal{P}\downarrow\mathcal{N}\uparrow\lambda\uparrow - \mathcal{N}\uparrow\mathcal{P}\downarrow\lambda\uparrow + \mathcal{N}\downarrow\mathcal{P}\uparrow\lambda\uparrow + \text{perm.}),$$

$$\langle\Lambda^0\uparrow|\mu|\Lambda^0\uparrow\rangle = 3\times\tfrac{1}{12}[(3+2-3)+(-3-2-3)+(-2-3-3)+(2+3-3)]$$

$$=\tfrac{1}{4}(+2-8-8+2)$$

$$=-3 \text{ times } \tfrac{1}{2}e\hbar/2m_{\text{baryon}}c\,,$$

i.e.,

$$\langle\Lambda^0\uparrow|\mu|\Lambda^0\uparrow\rangle = -1.5e\hbar/2m_{\Lambda^0}c = -1.26e\hbar/2m_p c\,; \tag{17}$$

$$|\Sigma^+\uparrow\rangle = 18^{-1/2}(2\mathcal{P}\uparrow\mathcal{P}\uparrow\lambda\downarrow - \mathcal{P}\uparrow\mathcal{P}\downarrow\lambda\uparrow - \mathcal{P}\downarrow\mathcal{P}\uparrow\lambda\uparrow + \text{perm.}),$$

$$\langle\Sigma^+\uparrow|\mu|\Sigma^+\uparrow\rangle = 3\times\tfrac{1}{18}[4(3+3+3)+(3-3-3)+(-3+3-3)]$$

$$=\tfrac{1}{6}(36+3-3)$$

$$=6 \text{ times } \tfrac{1}{2}e\hbar/2m_{\text{baryon}}c\,,$$

i.e.,

$$\langle\Sigma^+\uparrow|\mu|\Sigma^+\uparrow\rangle = +3e\hbar/2m_{\Sigma^+}c = +2.35e\hbar/2m_p c\,; \tag{18}$$

$$|\Xi^-\uparrow\rangle = 18^{-1/2}(2\mathcal{N}\downarrow\lambda\uparrow\lambda\uparrow - \mathcal{N}\uparrow\lambda\uparrow\lambda\downarrow - \mathcal{N}\uparrow\lambda\downarrow\lambda\uparrow + \text{perm.}),$$

$$\langle\Xi^-\uparrow|\mu|\Xi^-\uparrow\rangle = 3\times\tfrac{1}{18}[4(2-3-3)+(-2-3+3)+(-2+3-3)]$$

$$=\tfrac{1}{6}(-16-2-2)$$

$$=-3.33 \text{ times } \tfrac{1}{2}e\hbar/2m_{\text{baryon}}c,$$

i.e.,

$$\langle\Xi^-\uparrow|\mu|\Xi^-\uparrow\rangle = -1.67e\hbar/2m_{\Xi^-}c = -1.19e\hbar/2m_p c\,. \tag{19}$$

These magnetic-moment evaluations result from the simplest model based on hypotheses as regards quark loop types, their magnetic moments, and other assumptions. It is premature at this time to suggest which of the simplifying assumptions of the model need to be modified to get closer to the observed values of the moments.

In particular, the inclinations of the flux loops where they traverse the core are of importance for the magnetic moment which they produce.

There is also the possibility that loops listed in Table I are admixed in lieu of the ordinary $\mathcal{N}(2,1)$, $\mathcal{P}(3,1)$, $\lambda(3,2)$ quark loops.

It might be noted that the simple fact that quark torus *loops* are localized objects in the present theory replaces the hypothesis of color of quarks when introduced to explain symmetric spin-isospin functions to account for the statistics of quarks if postulated as *particles*. Localization does not imply supernumerary meson or baryon states.

The magnetic moment of a singlet meson, e.g., $\bar{K}^0 = 2^{-1/2}(\lambda\downarrow\bar{\mathcal{N}}\uparrow - \lambda\uparrow\bar{\mathcal{N}}\downarrow)$, +perm. (Fig. 4) is, as it should be, zero because the two terms contribute $(+3+2)$ plus $(-3-2)$ core traverses (wings).

VII. SPINNING AND WHIRLING ANGULAR VELOCITIES

Spinning and whirling are assumed to be of equal angular velocities because there is, on the basis of topological considerations, a symmetry between the central and the circular axes even in the spinning-top model.[6] It would be very queer to make fundamentally different assumptions about spin and whirl frequencies. Starting with the consideration of electron or muon, it is difficult to assume any other but the *Zitterbewegung* frequency for a spin-$\frac{1}{2}$ object (see Huang, Ref. 4).

Spinning and whirling are subject to the fundamental assumption (see B) that the regions in which the different quarks of a hadron spin and whirl should be coaxial. That assumption is to be understood to refer to a set of loopforms belonging to *one* flux orientation $\hat{\zeta}$, i.e., to a sheaf, Fig. 2(d). Other loopforms and sheaves of loopforms have different flux orientations $\hat{\zeta}$.

The question arises as to what are to be the frequencies of the probability-amplitude functions when two quarks form a meson or three quarks a baryon.

One single time factor

$$\exp[-i(2m_{\text{meson}}c^2/\hbar)] \text{ or } \exp[-i(2m_{\text{baryon}}c^2/\hbar)] \tag{20}$$

was assumed to be attached to the probability amplitudes of the 2 or 3 quarks (the same one to whirling as to spinning) just as one single size $\hbar/m_{\text{meson}}c$ or $\hbar/m_{\text{baryon}}c$ characterizes the interlinked torus loops.

VIII. CHARGE CONSERVATION

Conservation of electric charge is a fundamental assumption, never violated. Furthermore, electric charges of particles are integer multiples of e. It was at the start of the development of the ordinary SU(3) quark model that it was recognized that it would be difficult to make assignments other than $q\bar{q}$ for mesons (bosons) and qqq for baryons (fermions). The demand for integral electric charge means then that different quarks may differ by 0 or $\pm e$ in charge (or a quark charge plus an antiquark charge are to add up to 0 or $\pm e$) and the sum of any three quark charges is to be an integer, too. This led to the assignments $-\frac{1}{3}e$, $+\frac{2}{3}e$, $-\frac{1}{3}e$ or their negatives unless one postulates more quarks than needed at that time.

As the theory of flux quantization is based on gauge invariance, charge conservation is implied by it. But for the consistency of the theory it had to be shown how these charges $\mp e, \mp e; \mp\frac{1}{3}e, \pm\frac{2}{3}e, \mp\frac{1}{3}e$ actually arise in the flux quantization model: A single flux loop (lepton) spins and whirls in an additive manner, a quark loop of a meson or baryon spins and whirls subtractively as regards the effect on the equivalent electric charge. The integral charge of the particle thus demands that only those spin-whirl motions of flux loops exist which satisfy this integer-charge condition. This means, as regards the simplest types of loops, there are for electron or muon, those of winding numbers $(2, +1)$ *plus* $(2, -1)$ with additive spin-whirl motion; similarly for neutrinos or antineutrinos, $(3, -2)$ or $(3, +2)$, respectively. For quarks the winding numbers are $(2, \pm 1)$, $(3, \pm 1)$, $(3, \pm 2)$ with subtractive spin-whirl motion, and perhaps some additional ones listed in Table I. Their charges, all multiples of $e/3$, make them fit into $q\bar{q}$ meson and qqq baryon models.

IX. REMARKS ABOUT GROUP-THEORETICAL CLASSIFICATION OF LOOPFORMS

In B we gave a most informative discussion of simple topological issues which are implied in the use of flux quantization in particle physics. The group-theoretical aspects of that program are trivial, such as the discussion of admissibility of symmetric spin-isospin functions.

In the present paper (C) we stayed essentially within the framework of the heuristic model of A: The quantum-mechanical probability amplitudes have been constructed, by amplitude superposition, from semiclassical concepts of alternative loopforms. The essential next step (D) (the topic of the following paper) is to introduce in a straightforward way probability-amplitude wave functions which characterize the loopform distributions. This may be done if one understands the group with respect to which that distribution is to be invariant, which is SO(4).

It is again the simple heuristic[9] loop model which permits a direct recognition of the group-theoretical aspect of flux quantization in particle physics. The groups which the model implies specify the type of probability-amplitude functions (wave functions)[10] appropriate to the respective particles. The correct knowledge of the probability-amplitude waves—which are representations of the group in question—may, in turn, specify the exact nature of the spinning and whirling motions[11] and may give a quantitative description of the various leptonic and hadronic loop motions.

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APPENDIX A: SUPERPOSITION

As the calculation of effective magnetic moment μ_{eff} and effective electric potential V_{eff} involves some quite fundamental assumptions (formulated in this heuristic model), we may discuss here those assumptions [even though they had been stated in A, Sec. IV, and properly carried through in Eqs. (4.1) to (4.19); the Eqs. (4.20) to (4.25) should be disregarded].

Let us consider an electron or a muon whose effective magnetic moment $\mu_{\text{eff}} = e\hbar/2mc$ points in the $+\hat{z}$ direction; that lepton state may be denoted by $|\mu_z^+\rangle$. The alternative loopforms contribute toward that state with complex probability amplitudes. We bundled the loopforms into a finite number of bundles $(\hat{\zeta})$ whose corresponding probability amplitudes $|(\hat{\zeta}))$ then are so different that the addition of those bundle amplitudes results in a total amplitude as if the different bundle *amplitudes* were random-phased. Accordingly, when it comes to probability contributions, the different bundle *probabilities* may simply be added; we desire that resulting probability to be normalized and that it correspond to a probability-amplitude distribution proportional to $(1+\cos\theta)^{1/2} = (1+\zeta_z)^{1/2}$. We consider the number of bundles large enough to permit the use of a continuous probability function of θ rather than adding bundles discretely (cf. A, Sec. IV).

We perform the discussion of this appendix in terms of the simple dipole loop model of winding numbers (1, 0) in order to clarify the points at issue in comparing them with A. This is permissible because the superposition of left-handed loopforms (2, +1) and right-handed ones (2, −1) to represent a nonhanded electron or muon field results in an effective field of exactly the type (1, 0) which we assumed for electrons or muons in A and B. This is permissible also because we are, in this appendix, not counting numbers of bundles of loopforms.

We used the simple assumption for probability amplitudes of sheaves of loopforms (sets of loopforms covering the full range of azimuths α and sizes σ)

$$\langle \mu_z^+ | (\hat{\zeta})) \propto (1+\cos\theta)^{1/2} = (1+\zeta_z)^{1/2} . \tag{A1}$$

This is an obvious choice (1) because of analogy to the same expression representing the probability amplitude $\langle \mu_z^+ | \mu_{\hat\zeta} \rangle$ for finding the magnetic moment in the $\hat{\zeta}$ direction when a measurement on a pure state $|\mu_z^+\rangle$ is performed; we shall come back to this point later in this appendix, and (2) because that assumption leads to an isotropic potential [cf. A, Eqs. (4.18) and (4.19); cf. (A7) below].

The probability amplitudes $|(\hat{\zeta}))$ of sheaves, or $|(\lambda))$ of bundles of loopforms do not form an orthogonal set; they may be normalized by

$$\sum_{(\hat\zeta)} \langle \mu_z' | (\hat{\zeta}))((\hat{\zeta}) | \mu_z'' \rangle = \sum_{(\lambda)} \langle \mu_z' | (\lambda))((\lambda) | \mu_z'' \rangle = \delta_{\mu_z' \mu_z''} . \tag{A2}$$

If there are altogether $\mathfrak{N}$ sheaves $(\hat{\zeta})$ or N bundles $(\lambda) \equiv (\hat{\zeta}, \alpha, \sigma)$, the summation may be written

$$\sum_{(\hat\zeta)} = \int_{-1}^{+1} \mathfrak{N}(\tfrac{1}{2} d\zeta_z) \quad \text{or} \quad \sum_{(\lambda)} = \int_{-1}^{+1} N(\tfrac{1}{2} d\zeta_z) \tag{A3}$$

and

$$\langle \mu_z^+ | (\hat{\zeta})) = \mathfrak{N}^{-1/2}(1+\zeta_z)^{1/2} \quad \text{or} \quad \langle \mu_z^+ | (\lambda)) = N^{-1/2}(1+\zeta_z)^{1/2} , \tag{A4}$$

so that

$$\sum_{(\lambda)} \langle \mu_z^+ | (\lambda))((\lambda) | \mu_z^+ \rangle = \int_{-1}^{+1} N^{-1}(1+\zeta_z) N(\tfrac{1}{2} d\zeta_z) = 1 . \tag{A5}$$

Let us review the argument leading to Eqs. (4) and (A1).

Quantum mechanics states that an experiment on a lepton whose magnetic moment $e\hbar/2mc$ is directed in the $+\hat{z}$ direction, when measurement in an arbitrary direction $\hat{\zeta}$ is made, yields the magnetic moment $e\hbar/2mc$ with a probability amplitude $\langle \mu_z^+ | \hat{\zeta} \rangle$, i.e., with probability $\langle \mu_z^+ | \hat{\zeta} \rangle \langle \hat{\zeta} | \mu_z^+ \rangle \propto (1+\zeta_z)$, i.e., with a positive probability even if ζ_z is negative, i.e., even if $\hat{z}$ and $\hat{\zeta}$ are in almost opposite directions.

With this in mind as an analogy, we asked what reasonable assumptions we could make about probability-amplitude contributions from loopforms of flux orientation $\hat{\zeta}$, towards the amplitude $|\mu_z^+\rangle$ of a state of effective magnetic moment $e\hbar/2mc$ pointing in the $\hat{z}$ direction. That superposition is a concept similar to Feynman's superposition to construct a quantum state.[3]

Both in the quantum-mechanical measurement issue and in this issue of assignment of probability amplitudes for alternative, contributing loopforms, we refer to a quantum state $|\mu_z^+\rangle$. In the measurement issue we ask for the probability amplitudes for the magnetic moment, pointing in the various $\hat{\zeta}$ directions. In the superposition issue we ask for the amplitudes of the contributing loopforms, of alternative flux orientations $\hat{\zeta}$. The two problems are in some way inverse to each other.

We may correspondingly *assume*

$$\mu_{\text{eff}} = \sum_{(\hat\zeta)} (e\hbar/2mc) \langle \mu_z^+ | (\hat{\zeta}))((\hat{\zeta}) | \mu_z^+ \rangle$$

i.e.,

$$\mu_{\text{eff}} = \int_{-1}^{+1} (e\hbar/2mc)(1+\zeta_z)(\tfrac{1}{2} d\zeta_z) = e\hbar/2mc . \tag{A6}$$

The integrand represents the probability weighted contribution of sheaves of flux loopforms of orientation $\hat{\zeta}$, towards μ_{eff}, in the $|\mu_z^+\rangle$ state. The analogy to the quantum-mechanical measurement situation indeed suggests that even for $\hat{z}$ and $\hat{\zeta}$ of almost opposite direction, there is still a positive contribution towards μ_{eff} of that state. [In A this assumption was stated after Eq. (4.15) with $\mu_\zeta = e\hbar/2mc$.] The inclusion of a further factor ζ_z into the integrand would be contrary to the analogy between the quantum-mechanical measurement issue and the probability-amplitude superposition issue.

Assuming instantaneous spinning and magnetic moment as parallel [Sec. II C, Eq. (5)], the effective electric potential is given by Eq. (4.19) of A:

$$V_{\text{eff}} = \sum_{(\hat{\zeta})} (\hbar c/e)(\pm 2\pi/c)[\mu_\zeta (2\pi/r)\sin^2(\vec{r}, \hat{\zeta})\Phi_q^{-1}] (\Omega_\zeta/2\pi)\langle \mu_z^+ | (\hat{\zeta}))((\hat{\zeta}) | \mu_z^+\rangle$$

$$= (\hbar c/e)(\pm 2\pi/c)(e\hbar/2mc)(2\pi/r)(e/2\pi\hbar c)(2mc^2/2\pi\hbar)\int_{-1}^{+1}\int_0^{2\pi}\int_0^{2\pi} \sin^2(\vec{r}, \hat{\zeta})(1+\zeta_z)(\tfrac{1}{2}d\zeta_z)(d\beta/2\pi)(d\alpha/2\pi)$$

$$= \pm\tfrac{2}{3}\, e/r\,. \tag{A7}$$

Again, the integrand (with the preceding factors) represents the probability weighted contribution of sheaves of flux loopforms of flux orientation $\hat{\zeta}$, towards V_{eff}, in the $|\mu_z^+\rangle$ state. The evaluation of the integral [cf. Fig. 2(a)] is obtained with $\cos b = \cos\theta\cos c + \sin\theta\sin c\cos\beta$,

$$\int\int\int (1-\cos^2 b)(1+\zeta_z) = \int\int\int (1-\cos^2\theta\cos^2 c - 2\cos\theta\cos c\sin\theta\sin c\cos\beta - \sin^2\theta\sin^2 c\cos^2\beta)(1+\zeta_z)$$

$$= \int_{-1}^{+1} [1-\cos^2\theta\cos^2 c - \tfrac{1}{2}(1-\cos^2\theta)(1-\cos^2 c)]\,(1+\zeta_z)(\tfrac{1}{2}d\zeta_z)$$

$$= \int_{-1}^{+1} [\tfrac{1}{2}+\tfrac{1}{2}\zeta_z^2 + (\tfrac{1}{2}-\tfrac{3}{2}\zeta_z^2)(r_z^2/r^2)]\,\tfrac{1}{2}d\zeta_z + \int \text{odd-power integrand}$$

$$= \tfrac{1}{4}[\zeta_z + \tfrac{1}{3}\zeta_z^3 + (\zeta_z - 3\times\tfrac{1}{3}\zeta_z^3)r_z^2/r^2]_{-1}^{+1} = \tfrac{2}{3}\,. \tag{A7$'$}$$

APPENDIX B: REDUCTION OF QUANTIZED FLUX TO WING FLUX AND EFFECTIVE FLUX

We recognized that the calculation of the magnetic moments $e\hbar/2mc$ of electron or muon (and accordingly of their electric potentials e/r) from quantized flux $\Phi_q = hc/e$ cannot be achieved by probability superposition of loopforms, because a total flux Φ_q, connected with a quasinonlocal source of extension $\approx \hbar/mc$, results in a magnetic moment which is 2 to 3 orders of magnitude too large. Superposition with complex probability amplitudes permits a "reduction" of flux and magnetic moment to the correct values $e\hbar/2mc$; the different phases of the probability amplitudes of the loopforms may imply a partially destructive interference which results in that reduction. Such a superposition should be understood in terms of a superposition of *modes* of probability-amplitude distributions *(wave functions)*.

Before the appropriate modes—wave functions—and their superposition formalism are known, it is useful to develop the heuristic model of A: We considered *neighboring* loopforms as having *phase-related* probability amplitudes, but considered loopforms whose flux orientations or azimuths differ by more than one radian as not phase related. In other words, we bundled loopforms together into some N different bundles whose probability-amplitude superpositions then leads to a reduction of effective flux and magnetic moment by a factor N. Now, however, with the new proposal of loopforms of type $(2, +1)$ and $(2, -1)$, new issues arise which we shall discuss in term of the heuristic model.

Even though the loopforms from which a lepton in a quantum state $|\mu_z^+\rangle$ is superposed are objects $|(\hat{\zeta}, \alpha_s, \alpha_w, \sigma)) \equiv |(\lambda))$, we may consider them, for the purpose of counting manifolds of bundles, simply as objects $|(\hat{\zeta}, \alpha, \sigma))$ because a whirl rotation can always be mapped onto a spin rotation as long as we characterize a flux loopform by those parameters of a smooth torus loop. Even statistical fluctuations of the forms of the torus loops should not invalidate that statement.

The question arises whether a loopform of winding numbers $(2, +1)$ and of a given set of parameters

$$\lambda \equiv (\hat{\zeta}, \alpha, \sigma) \equiv (\beta, \theta, \alpha, \sigma)$$

and the mirror loopform $(2, -1)$ of same parameter set λ may have different probability amplitudes and may have unrelated phases of amplitudes. The answer is no. If they had unrelated amplitudes, their superposition (which is a permitted operation) would no longer correspond to a Seifert fibration; instead, it would correspond to magnetic field lines of noninteger winding numbers, i.e., not to closed loops at all. But the hypothesis of sheaves of loopforms to represent a Seifert fibration is a basic assumption of the flux-loop model because the individual quantized flux loop without magnetic monopoles is a closed loop.

Consequently, in counting numbers of independent loopform bundles, pairs of $(2, +1)$ and $(2, -1)$ have to be counted as single units. The proposal of torus loops in the present paper (C) thus implies a different way of counting bundles, of course, affecting in particular Eq. (10.3) of A. The corresponding new counting is given in Eq. (9), Sec. V, and in Eqs. (B11a), (B11b), (B1a), and (B2) of the present appendix. We suggest that in order to count the manifold of bundles, the full range $0 \leq \alpha \leq 4\pi$ should be considered. This full range takes account of the double-valuedness of the probability-amplitude wave function over $0 \leq \alpha \leq 2\pi$. Counting bundles as we did in A, this would amount to about 12 as regards (α), to 20 as regards the orientation $(\hat{\xi}) = (\beta, \theta)$, and 1.67 as regards the size (σ). Accordingly

$$N = 12 \times 20 \times 1.67 = 400\,. \tag{B1a}$$

(One should also note that it was a too simple speculative assumption in A to assume $N = 207$. The relation of 207 to N and thus to $\frac{1}{137}$ will be discussed in paper D.) The procedure of obtaining a value for N in the heuristic model still has to be questioned; the numbers so obtained are crude estimates. There are substantial differences between this heuristic calculation of the reduction factor N and the calculation based on probability-amplitude wave functions (in paper D).

In Eq. (9) the ratio of the root mean square r_0 of the Gaussian distribution (of equivalent magnetization) to the length $\hbar/mc$ was not yet specified. An elementary calculation [cf. our Fig. 3(e), or A, Fig. 5], however, has shown

$$\text{core equatorial radius} = 1.23 r_0\,. \tag{B1b}$$

The core equator is the (topologically) singular line of the fibrations, i.e., the circular axis of the "doughnut" (the torus, ring) which, together with the straight central axis [cf. our Figs. 3(a)–3(e), Fig. 6, or B, Figs. 3, 5] constitute the singularities of the Seifert fibration.

Considering these topological facts and the basic assumption of spinning angular velocity $\Omega = 2mc^2/\hbar$ = *Zitterbewegung* frequency, we may ask: What reasonable assumption may be made as regards $r_0/(\hbar/mc)$? We remember that the outer loop regions spin with linear velocity $> c$, and, if any meaning can be attached to linear velocities all the way inside the core, they may mostly be lower than c. The reasonable assumption may be to assume that the linear velocity at the circular singularity line (core equator) is equal to c. Accordingly

$$(2mc^2/\hbar) \times 1.23 r_0 = c\,, \qquad r_0/(\hbar/mc) = 1/2.46\,. \tag{B1c}$$

This yields, by Eq. (9),

$$e^2/\hbar c \approx (3.1/2.46)(2/400) \approx 1/160\,. \tag{B2}$$

As we want the condition of velocity c at the core equator to hold for all lepton and quark loops irrespective of their winding numbers, we assume that the intrinsic spinning velocity $2mc^2/\hbar$ (not the effective velocities, e.g., $3mc^2/\hbar$) be considered in Eq. (B1c).

Still open is the question whether the parametrization of loop forms by $\beta, \theta, \alpha, \sigma$ provides for a sufficient characterization. One might have vibrations and fluctuations to consider, although it seems that the simple $\beta, \theta, \alpha, \sigma$ characterization not only avoids more elaborate assumptions, but, more important still, replaces the much more difficult characterization of loopforms by functionals. We consider a particle at rest; motions with constant velocity are simply taken care of by a Lorentz transformation; accelerated motion can no longer be handled by this simple loopform parametrization.

The connection of this model with QED can, however, only be established on a broader basis of formulation of loopforms in terms of functionals. That applies in particular to issues of interaction of leptons and hadrons with photons.

Reduction of quantized flux to effective flux, due to probability amplitude interference, may be formulated in various ways, cf. also paper D of this series.

We may now proceed to a reformulation of the reduction as outlined in A, Sec. IX [Eqs. (9.1) to (9.4) and (9.11) to (9.13) in A are to be changed; $\Phi_{q,z}$ and $\mu_{q,z}$ should not have been introduced there].

The quantized flux $\Phi_q = hc/e$ and the corresponding magnetic moment μ_q refer to one of the two wings [the number of wings equals the number of core traverses of the flux loop, and is equal to the first of the two winding numbers $(2, +1)$ or $(2, -1)$]. We shall denote the "reduced" values of Φ_q and of μ_q by Φ_{wing} and μ_{wing}, to distinguish them from the total "effective" values for the electron or muon, designated by $\Phi_{\text{eff}} = 2\Phi_{\text{wing}}$ and $\mu_{\text{eff}} = 2\mu_{\text{wing}}$. At this time we disregard the inclination of the core-traversing flux loop with regard to the central straight symmetry axis of the toroidal field. These Φ and μ are *then* connected by a relationship which for a Gaussian-type extended source was calculated in A, Eq. (5.10):

$$\Phi = 4\pi\mu / 3.1 r_0\,, \tag{B3}$$

where r_0 = root mean square of the Gaussian equivalent magnetization distribution of the source.

How is Φ_q related to the $\langle \mu_z' | \Phi_q | \mu_z'' \rangle$ and $((\lambda_1) | \Phi_q | (\lambda_2))$ matrix elements and to the matrix elements of Φ_q^2, of μ_q, of μ_q^2, and to the reduced and effective quantities defined below? In other words, how is the reduction formulated? For the basic entities we refer to the Appendix A and to A, Sec. VIII.

We note that Φ_q was defined as the flux which would result if the probability amplitudes of all the flux-loop bundles were in phase. With N denoting the effective total number of bundles, it is assumed for simplification that on each of the bundles falls the "bundle flux" $N^{-1}\Phi_q$ in that parcelling-out procedure. This bundle flux, in the language of Eqs. (B6), (B7), (B10), and (B11a) [cf. A, Eq. (8.9)], is to be identified with $((\lambda)|\Phi_q|(\lambda))$.

In order to carry through these simple calculations, we remember the obvious definitions of probability of a bundle (λ) [cf. A, Eqs. (8.11), (9.8), (9.9)],

$$|\langle \mu_z^{\pm}|(\lambda))|^2 = N^{-1}(1\pm\zeta_z), \tag{B4}$$

$$\sum_{\mu_z'} ((\lambda)|\mu_z'\rangle\langle\mu_z'|(\lambda)) = 2/N,$$
$$\sum_{(\lambda)} \langle\mu_z'|(\lambda))((\lambda)|\mu_z''\rangle = \delta_{\mu_z'\mu_z''}, \tag{B5}$$

and the summation over (λ) may be written as

$$\sum_{(\lambda)} = \int_{-1}^{+1} N\tfrac{1}{2}\,d\zeta_z\ . \tag{A3}$$

The furthergoing assumption

$$((\lambda_1)|\Phi_q|(\lambda_2)) = \Phi_q \sum_{\mu_z'} \tfrac{1}{2}((\lambda_1)|\mu_z'\rangle\langle\mu_z'|(\lambda_2)) \tag{B6}$$

contains the statements about the above-defined bundle flux

$$((\lambda)|\Phi_q|(\lambda)) = \Phi_q\,\tfrac{1}{2}(2/N)$$
$$= \Phi_q/N \tag{B7}$$

and implies the unreduced matrix elements

$$\langle\mu_z''|\Phi_q|\mu_z'''\rangle = \Phi_q \sum_{(\lambda_1)(\lambda_2)\mu_z'} \tfrac{1}{2}\langle\mu_z''|(\lambda_1))((\lambda_1)|\mu_z'\rangle \times\langle\mu_z'|(\lambda_2))((\lambda_2)|\mu_z'''\rangle$$
$$= \tfrac{1}{2}\Phi_q \sum_{\mu_z'} \delta_{\mu_z''\mu_z'}\delta_{\mu_z'\mu_z'''}$$
$$= \tfrac{1}{2}\Phi_q\delta_{\mu_z''\mu_z'''}, \tag{B8}$$

where

$$\tfrac{1}{2}\Phi_q = hc/2e\ . \tag{B9}$$

Instead of Eqs. (9.1) to (9.5) in A we introduce, following our Appendix A, for random-phased amplitudes $\langle\mu_z'|(\lambda)_R)$ the reduced matrix elements (subscript R stands for random phased)

$$\langle\mu_z'|\Phi_q|\mu_z''\rangle_{\rm red} = \Big\langle \sum_{(\lambda_1)_R(\lambda_2)_R} \langle\mu_z'|(\lambda_1)_R)((\lambda_1)|\Phi_q|(\lambda_2))((\lambda_2)_R|\mu_z''\rangle\Big\rangle_{\rm av}$$
$$= \sum_{(\lambda)} \langle\mu_z'|(\lambda))((\lambda)|\Phi_q|(\lambda))((\lambda)|\mu_z''\rangle\ . \tag{B10}$$

Thus, by (B7) and (B5)

$$\tfrac{1}{2}\Phi_{\rm eff} = \Phi_{\rm wing}$$
$$\equiv\langle\mu_z^+|\Phi_q|\mu_z^+\rangle_{\rm red}$$
$$= \Phi_q/N$$
$$= ((\lambda)|\Phi_q|(\lambda))$$
$$= \text{bundle flux} \tag{B11a}$$

[which could also have been calculated by integration from (A3), (B4), and (B7)]. Exactly the same set of equations (B6) to (B11) hold for the magnetic moments (assuming disregard of inclinations as a permissible approximation):

$$\tfrac{1}{2}\mu_{\rm eff} = \mu_{\rm wing}$$
$$\equiv\langle\mu_z^+|\mu_q|\mu_z^+\rangle_{\rm red}$$
$$= \mu_q/N$$
$$= ((\lambda)|\mu_q|(\lambda))$$
$$= \text{bundle moment}. \tag{B11b}$$

Let us now consider the quadratic quantities so as to be able to estimate the actual, i.e., reduced, values for electromagnetic energy and electromagnetic angular momentum.

(B8) implies

$$\langle\mu_z'|\Phi_q^{\,2}|\mu_z''\rangle = \tfrac{1}{4}\Phi_q^{\,2}\delta_{\mu_z'\mu_z''}\ . \tag{B12}$$

We may, with these unreduced matrix elements, go from $\langle\ |\ |\ \rangle$ to $(\ |\ |\ )$ in the same way as Eq. (B8) leads to Eq. (B6):

$$((\lambda_1)|\Phi_q^{\,2}|(\lambda_2))$$
$$= \sum_{\mu_z'\mu_z''} ((\lambda_1)|\mu_z'\rangle\langle\mu_z'|\Phi_q^{\,2}|\mu_z''\rangle\langle\mu_z''|(\lambda_2))$$
$$= \tfrac{1}{4}\Phi_q^{\,2}\sum ((\lambda_1)|\mu_z'\rangle\langle\mu_z'|(\lambda_2)) \tag{B13}$$

[cf. A, Eq. (8.9a)]. The reduction proceeds as in Eq. (B10) and we so obtain the square of the wing flux:

$$\langle\mu_z''|\Phi_q^{\,2}|\mu_z'''\rangle_{\rm red}$$
$$= \tfrac{1}{4}\Phi_q^{\,2}\sum_{\mu_z'(\lambda)} \langle\mu_z''|(\lambda))((\lambda)|\mu_z'\rangle\langle\mu_z'|(\lambda))((\lambda)|\mu_z'''\rangle$$
$$= \tfrac{1}{4}\Phi_q^{\,2}(2/N)\sum_{(\lambda)} \langle\mu_z''|(\lambda))((\lambda)|\mu_z'''\rangle$$
$$= (1/2N)\Phi_q^{\,2}\delta_{\mu_z''\mu_z'''} \tag{B14}$$

[one evidently has first to sum over μ_z' in order to be able to use Eqs. (B5)].

Equations analogous to (B12), (B13), and (B14) hold for $\langle\mu_z''|\mu_q^{\,2}|\mu_z'''\rangle$, etc., too. These are effective magnetic moments per wing. As, however, the two wings or two core traverses of a (2, +1) or or a (2, −1) loop have strictly one and the same probability amplitude, we may make an arithmetic addition of the two wing contributions and get for the reduced magnetic moments

$$\mu_{\rm eff}=2\mu_{\rm wing}\,,\quad (\mu^2)_{\rm eff}=4(\mu^2)_{\rm wing}\,, \tag{B15}$$

i.e., implying the same factor 2 as the unreduced moments (or magnetic fluxes). As (B11) and (B14) referred to wing flux, we get

$$\begin{aligned}(\mu^2)_{\rm eff}&=4(\mu^2)_{\rm wing}\\ &=4\langle\mu_z^+|\mu_q{}^2|\mu_z^+\rangle_{\rm red}\\ &=(2/N)\mu_q{}^2\,.\end{aligned} \tag{B16}$$

This gives us the relationship between $(B^2)_{\rm eff}$ and $(B_{\rm eff})^2$,

$$\begin{aligned}(B^2)_{\rm eff}/(B_{\rm eff})^2&=(\mu^2)_{\rm eff}/(\mu_{\rm eff})^2\\ &=(2/N)\mu_q{}^2/(2/N)^2\mu_q{}^2\\ &=\tfrac{1}{2}N\,.\end{aligned} \tag{B17}$$

It is most interesting to note that the reduction of quantized magnetic flux (or the corresponding magnetic moment) to the effective values, by Eq. (9) or (B11b), and the reduction of the quadratic quantities (B16) goes by the same factor $2/N$. This circumstance assures the consistency of the flux-quantization model in that we are now able to calculate the electromagnetic energy and angular momentum.

In Sec. VI of A we have calculated

$$\int\int_0^\infty\int[(B_{\rm eff})^2+(E_{\rm eff})^2](8\pi)^{-1}d^3r=\{0.138[(\hbar/mc)/r_0]^3+0.365[(\hbar/mc)/r_0]\}(e^2/\hbar c)mc^2 \tag{B18}$$

and

$$\left|\int\int_0^\infty\int\vec{\rm r}\times(\vec{\rm E}_{\rm eff}\times\vec{\rm B}_{\rm eff})(4\pi c)^{-1}d^3r\right|=0.47[(\hbar/mc)/r_0](e^2/\hbar c)\tfrac{1}{2}\hbar\,. \tag{B19}$$

Using the reduction factor given in (B17) for $(B^2)_{\rm eff}$, $(E^2)_{\rm eff}$, and $(\vec{\rm E}\times\vec{\rm B})_{\rm eff}$ and using $r_0/(\hbar/mc)=1/2.46$ from (B1c), we may now complete the energy and angular momentum calculations in two ways: (1) We might take $N=400$ from (B1a) and $e^2/\hbar c=1/160$ from (B2), i.e., (9), or (2) we might take $N=345$ (chosen so as to give the correct $e^2/\hbar c$ from Eq. (9) and take $e^2/\hbar c=\frac{1}{137}$.

By the second method we get

electromagnetic energy

$$\begin{aligned}&=(\tfrac{345}{2})(0.138\times2.46^3+0.365\times2.47)(\tfrac{1}{137})mc^2\\ &=172.5(2.06+0.90)(\tfrac{1}{137})mc^2\\ &=3.7mc^2\,,\end{aligned} \tag{B20}$$

electromagnetic angular momentum

$$\begin{aligned}&=(\tfrac{345}{2})(0.47\times2.46)(\tfrac{1}{137})\hbar/2\\ &=172.5(1.16)(\tfrac{1}{137})\hbar/2\\ &=1.5\hbar/2\,.\end{aligned} \tag{B21}$$

By procedure (1) we get the same values because it is $N\times e^2/\hbar c$ which determines the results and, by Eq. (9), that product has the same value for (1) as for (2).

The discrepancy of $3.7mc^2$ with the required mc^2 is considerable. One should, however, be aware of the fact that most of that $3.7mc^2$ is due to electromagnetic energy in the core and immediately next to the core. As the heuristic model is only a very crude approximation in that region, it would not make much sense to try to refine the heuristic model to get a closer estimate of the energy. Rather, one may do that with an adequate formulation of probability amplitudes in terms of probability-amplitude waves (in the next article), and may recognize that the present heuristic model gives the right order of magnitude of electromagnetic energy and angular momentum, a certainly not obvious result. [If, e.g., one would introduce instead of $r_0/(\hbar/mc)\approx\frac{1}{2}$ any other assumption for r_0, this would not only fail to be theoretically justifiable, it would also have the consequence that neither the electromagnetic energy nor the electromagnetic angular momentum would come close in any way to mc^2 or to $\frac{1}{2}\hbar$, respectively].

An explanatory note may be in order here to clarify what we mean by "calculation of electromagnetic energy and angular momentum." The Eqs. (B18) and (B19) show on their left-hand sides [after they are multiplied with $(B^2)_{\rm eff}/(B_{\rm eff})^2$ in Eq. (B17)] the electromagnetic energy and electromagnetic angular momentum. The right-hand sides of (B18) and (B19) [again after multiplication with $\frac{1}{2}N$, Eq. (B17)] express these quantities in terms of mc^2 and $\frac{1}{2}\hbar$, respectively. The theory is consistent if the factors multiplying mc^2 and $\frac{1}{2}\hbar$ respectively are one or at least of the order of magnitude unity.

APPENDIX C: STRONG INTERACTIONS AND "STRINGS" OF ALTERNATIVE LOOPFORMS

We have seen that some of the most troublesome difficulties of the conventional quark model of hadrons disappear if a quark is represented in terms of a closed quantized flux loop rather than in terms of a particle (e.g., a magnetic monopole). Strong interactions occur if such a quark loop merges with an antiquark loop or if they are created in pairs together. Such a reaction is a fast, i.e., really a strong, interaction, if these loops do not have to cut across each other or over themselves in that process. Otherwise the process is slow, a weak-interaction process, cf. B. The merging or pair creation implies a close approach, side by side, of the interacting loops along each other. Obviously, what happens, as a function of time, in the spatial regions between these adjacent loops concerns the strong-interaction process. That region may be characterized by a narrow two-dimensional ribbon whose edges are the two interacting loops, plus the three-dimensional surroundings of that ribbon. These spatial regions may correspond to the strings which have been proposed as connecting a quark particle with an antiquark particle in current quark-string models. With the proposed reformulation of the string model in terms of a "ribbon model," one might again find an appropriate representation of dual amplitudes.

The issue of "crosscutting" of loops is made responsible for weak interactions if such cutting across involves the loop itself or the loop with which it is interacting in a process of merging, or pair production.

It has also been pointed out in B that a strange quark (λ quark), having the form of a trefoil, i.e., a "knotted" torus knot of winding numbers (3, 2), can only undergo a slow merging with an $\mathfrak{N}$ quark which has the form of a simple unknotted torus knot (2, 1), because that implies a crosscutting of loops. But a λ quark may undergo a fast process with the mirror trefoil $\bar{\lambda}$ by merging or pair creation in associated production because a merger of a λ trefoil with a $\bar{\lambda}$ trefoil does not imply any cutting across of a loop cf. Fig. 6. Strangeness-violating processes are thereby *eo ipso* weak and they are parity violating, too.

It is interesting to note that it has been recognized that the electromagnetic properties of a

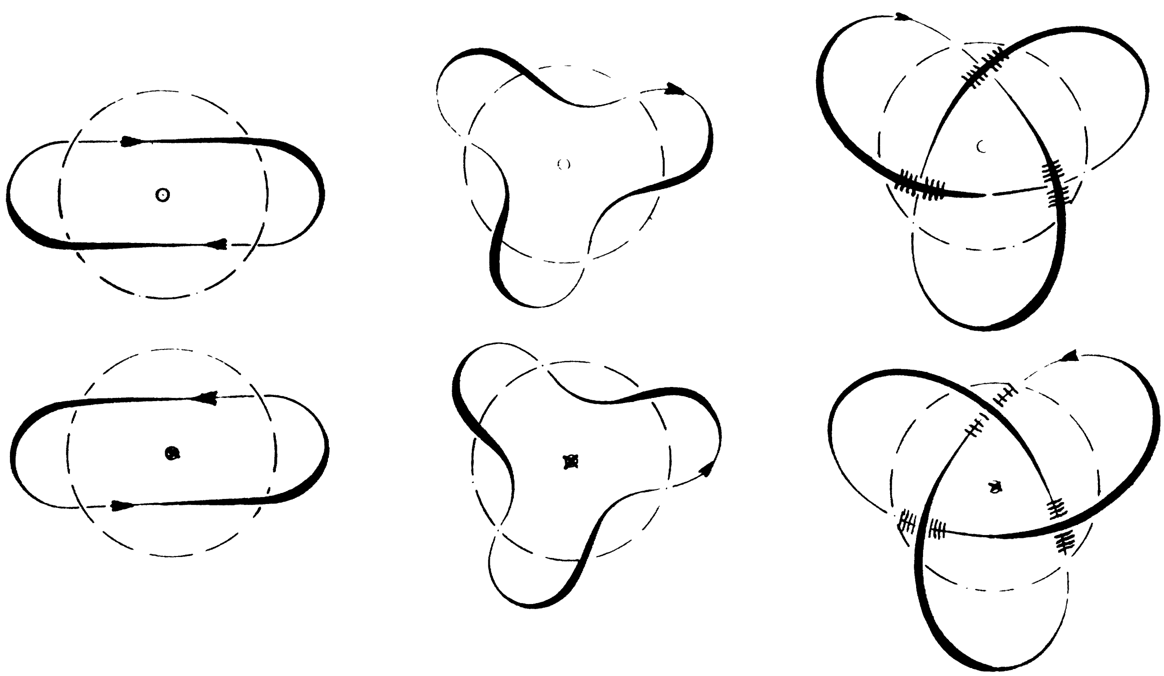

FIG. 6. Merging (pair annihilation) of a loop of quantized magnetic flux with an antiloop. Consider any one of the upper three loops as being laid on top of the respective lower loop. Starting from points where loop and antiloop thus make contact first, with antiparallel magnetic flux orientations, extinction through merging may occur, a process which proceeds all the way around the loop pairs. That is obviously possible in the case of the simple "unknotted" (2,1) and (3,1) loops on the left side of Fig. 6. In the case of the "knotted" loops (3,2), i.e., trefoils, one might ask whether the merging comes to a halt whenever the annihilation reaches the crosshatched regions. The other, not crosshatched, parts of the loops which seem to be preventing the merging of the crosshatched regions are, however, annihilating each other, too, so there is really nothing in the way of the crosshatched regions annihilating each other. Note that this unimpeded merging of a trefoil is possible only with a mirror trefoil, not with an identical trefoil or with an unknotted loop. Considering the latter, impeded, annihilations (which necessitate that a flux line cuts across itself or across the line with which it interacts) as a weak interaction, it is obvious that strangeness conservation may be understood topologically if the trefoils (i.e., λ quarks) are assigned strangeness ± 1 ("unknotting number") depending on their handedness.

quark resemble that of a lepton. That should indeed be the case in the present model of a quark and of a lepton, both as a quantized flux loop. The difference between a $\mu^+\mu^-$ pair annihilation or creation and a strong interaction implying a $\bar{q}q$ creation or annihilation is that in the latter case the $\bar{q}$ and the q are actually linked with one or more quarks.

APPENDIX D: FURTHER COMMENTS ABOUT FORMS OF LOOPS AND MAXWELL-LORENTZ EQUATIONS

To build up a magnetic field from alternative forms of one quantized flux loop (one in the case of leptons) means to take the closed loops of Faraday lines ($\mathrm{div}\vec{B}=0$) literally; they are the forms which a flux loop may adopt. As a flux loop is characterized by the singularity line of gradϑ (ϑ = pseudogauge variable), the motion (circular spinning) of that loop is a realistic physical phenomenon even if only indirectly measurable, as most physical quantities are.

As the basic assumption of the theory is to demand consistency with the Maxwell-Lorentz theory (the Maxwell-Lorentz equations relate the field to the distribution of charge and current), we want to add a few comments in regard to the assumptions about the equivalent distribution of charge and current in the present model. As regards the description in terms of mean position, the source is assumed to be a point dipole source. Its known electromagnetic field is then interpreted in terms of a superposition of a continuous manifold of forms of quantized flux loops, a superposition with complex amplitudes which also imply the above-discussed motion of the loopforms.

The quasinonlocal concept of ordinary position is related to mean position by the Pryce-Foldy-Wouthuysen-Tani transformation. Even though this is not a point transformation and thus does not, strictly speaking, permit a local space-time picture of moving flux loops, we try the use of an extended-source model. The structure of the source is assumed to correspond to a fibration of space: a toroidal form of the field of magnetic flux. The charge and current distribution, related by the Maxwell-Lorentz equations to the effective electromagnetic field, may then be determined from the distribution of flux loopforms. The charge and current distribution is limited to the inner region of the toroid because that is the region which corresponds to the point source before the Foldy-Wouthuysen transformation was applied.

A detailed mathematical formulation of the electromagnetic field (and, by the Maxwell-Lorentz equations, of the corresponding electric charge and magnetic moment distribution) due to the fibrations of ordinary position space may properly be given once we know what probability-amplitude superpositions of such fibrations apply to an electron or a muon, and to hadrons. That will be possible with the results of the next paper (D), which constructs probability-amplitude distributions of fibrations from bases of representation of SO(4), appropriate distributions which correspond to a core of size $\hbar/mc$ and have the proper asymptotic behavior outside the core. In the meanwhile we have given, in (A), Sec. V, a detailed mathematical description based on a simple approximation; we approximated the aforementioned magnetic-moment distribution by the spherical Gaussian distribution of magnetic moment over a region of linear size $\hbar/mc$.[12]

The different types of distributions, corresponding to charged and uncharged lepton, to mesons and baryons, have been discussed in B.

The parametrization of the loopforms, so successful in a description of a single particle, fails in collision processes. This serious limitation of the theory is, however, offset by the topological characterization of conservation of strangeness, of PC, and by such characterization of weak and strong interactions.

APPENDIX E: FOOTNOTES AND CORRECTIONS TO PAPERS A AND B

Omit Eqs. (4.20) to (4.25) in A; cf. Secs. II–IV of the present paper.

The magnetic dipole field Fig. 5, A, or Fig. 1, B is now to be considered as the resultant field due to a superposition of a right-handed and a left-handed torus-loop field; cf. Fig. 3(e) of the present paper.

The notation of Euler angles was chosen as (β, θ, α) instead of the more usual (α, β, γ). With that notation, (β, θ, α), we are now using the conventional definition of the Wigner D functions:

$$D^j_{nm}(\beta, \theta, \alpha) = d^j_{nm}(\theta)\exp(in\beta)\exp(im\alpha)\,.$$

This notation is to replace the notation in A, p. 327 (which used the same β, θ, α),

$$T^s_{mn}(\alpha, \theta, \beta) = P^s_{mn}(\theta)\exp(in\beta)\exp(im\alpha)\,.$$

Omit the formulas Eqs. (9.1) to (9.4), and (9.11), (9.12), and (9.13) of A because quantities Φ_{qz} and μ_{qz} should not have been introduced. Appendix B of the present paper simplifies and clarifies this reduction formalism.

Omit Sec. XIII of A, which advocated regionally differentiated reduction. In the new formalism of probability amplitudes in terms of wave functions (in paper D), we are replacing the heuristic bundle model by a model of "bundles" understood as wave-mechanical modes (some eigenfunctions). Every wave mode covers all spatial regions, and reduction depends on the statistical distribution of modes, no longer on a regional distribution of bundles. [One should also note that arguments in favor of regionally differentiated reduction based on consideration of $(B^2)_{\text{eff}}$ versus $(B_{\text{eff}})^2$ of A suffer from the impossibility of localizing energy and angular momentum distribution inside an elementary particle; only the integrated quantities have physical significance.] The present view about the reduction formalism simplifies the theory in many respects, and clears A of an undesirable complication.

The reduction factor is now redefined in Eq. (9); its numerical estimate in the present paper, Appendix B, should replace the numerical assumption made for it in A. N is to be calculated on the basis of probability-amplitude waves in the next paper (D).

Eq. (8.4) in A should have 20 columns and two rows corresponding to the counting of 20 independent flux orientations ($\hat{\xi}$).

It may be remarked that the basic structure of the theory has not been changed since the early beginnings, and that A presents a realistic way of setting up a heuristic model.

On the other hand, the topological forms of the various types of quantized flux loops have only gradually been developed. In particular, the neutrino trefoil assignment is to be found in Sec. IV of the present paper (C), the quark assignments were already given in B, together with the meson and baryon model, Figs. 4 and 5 in B, and some basic loop issues in the Appendix of A, whereas Figs. 13, 15, 16 of A are outdated, as well as the frequency and magnetic-moment estimates derived from those figures in A. As regards B, the third and fourth paragraphs of the subsection entitled "Magnetic Moment" (left column of page 451, lines 16 to 29) should be deleted. In B, the spinning-top model is the physically significant model.

APPENDIX F: NEW QUARKS

To give a more complete table of the simple types of loops, we may list loops with subtractive spin-whirl motion (assigned to quarks), and those with additive spin-whirl motion (assigned to leptons) in Table II. (The earlier comments in Sec. IV are supplemented in this appendix, which was written after the discovery of the new particles of 3.1, 3.7, and 4.2 GeV.)

We had, without having explained it in any way, assumed that the *total* electric charge of a lepton,

TABLE II. Loops with subtractive spin-whirl motion (assigned to quarks) and with additive spin-whirl motion (assigned to leptons).

	Quark loops subtractive motion of spinning and whirling											Lepton loops additive motion of spinning and whirling						
Winding numbers	(1,0)	(1,1)	(2,1)	(3,1)	(3,2)	(4,1)	(4,3)	(5,1)	(5,2)	(5,3)	(5,4)	(1,0)	(1,1)	(2,1)	(3,1)	(3,2)	(5,1)	(5,4)
Effective spin angular velocity in units of $2mc^2/\hbar$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	1	2	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{6}{5}$	$\frac{9}{5}$
Magnetic moment in units of $e\hbar/2mc$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{4}{2}$	$\frac{4}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{5}{2}$
Effective charge in units of e	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	1	$\frac{1}{3}$	$\frac{4}{3}$	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2	3
Unwinding number (strangeness)	0	0	0	0	1	0	3	0	2	4	6	0	0	0	0	1	0	6

meson, or baryon is integer.

With the assignments of loops to quarks, and with the assumptions made, what further loops might we expect to find in particle physics?

For the *additive* loops, because they refer to leptons which are represented by a single loop only, which therefore has to have integral charge, we find (2, 1), (5, 1), (5, 4), . . ., the first being the electron or muon (cf. paper D), the (5, 4) having strangeness 6; thus, if another charged lepton is to be found, one might first of all expect to find a (5, 1) loop of charge $2e$.

As regards the *subtractive* loops, we preferred the assignments (2, 1), (3, 1), (3, 2) to quarks because loopforms of those types, belonging to one sheaf (i.e., one flux orientation $\hat{\xi}$), are interlinked, whereas the (1, 0) are not.

We might expect quark-antiquark pair production of any ± charge, i.e., of types (4, 1), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4), besides the (2, 1), (3, 1), (3, 2). Pair production is a strong process, not only for those of zero strangeness, but for the others also because their pair production is, as associative production, strong (cf. the loop picture, Fig. 6). For the types of loops of opposite, nonzero strangeness, i.e., (4, 3), (5, 2), (5, 3), (5, 4), one may have reasons for expecting a long lifetime.

It is among the latter ones that candidates for the 3.1-GeV etc. particles are to be looked for. In connection with the quark-loop model the question of detection of K^0 variants whose λ quark is replaced by an equally charged but strangeness-3 quark, i.e., (4, 3), has come up.

It is also interesting to note that all the quark charges listed in the tables as well as charges of higher wound loops are multiples of $e/3$. Some of those new quarks might even be mixed in with the conventional three quarks.

The conventional SU(3) quark model is a scheme whose difficulties of fractional charge and of its quark statistics in baryons have disappeared with the loop model. The fact that this SU(3) model is so immensely fruitful [we may say because of the importance of the simple (2, 1), (3, 1), and the (3, 2) loops] should not lead us to expect that the next extension of the quark model would have to be confined to the SU(4) and all its complexities. It might be even simpler to look into the intrinsic properties of several quark loops following the simple $\mathcal{N} = (2, 1)$, $\mathcal{P} = (3, 1)$, $\lambda = (3, 2)$.

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†On leave from Univ. of Maryland.

[1]The present paper is self-contained. The references to preceding papers are: H. Jehle, Phys. Rev. D 3, 306 (1971), quoted as A; 6, 441 (1972), quoted as B. References to the literature are to be found there. A concerns leptons; B concerns hadrons. The paper D will specify the probability-amplitude waves. The appendixes A and B of the present paper are not self-contained; they refer to A, B, or to *Boulder Lecture Notes in Theoretical Physics, Vol. 10B: High Energy Physics and Fundamental Particles*, edited by W. E. Brittin and A. O. Barut (Gordon and Breach, New York, 1968), p. 673; *Boulder Lecture Notes in Theoretical Physics, Vol. 11A: Elementary Particle Physics*, edited by W. E. Brittin *et al.* (Gordon and Breach, New York, 1969), p. 493; *Boulder Lecture Notes in Theoretical Physics, Vol. 14A: Topics in Strong Interactions*, edited by A. O. Barut and W. Brittin (Colorado Assoc. Univ. Press, Boulder, 1972), p. 399; Int. J. Quantum Chem. 2S, 373 (1968); 3, 269 (1969); *Symmetries and Quark Models*, edited by Ramesh Chand (Gordon and Breach, New York, 1970), p. 97.

[2]Some early flux-quantization papers (it should be noted that London recognized the significance of flux quantization and took it as a starting point in his formulation of the theory of superconductivity): F. London, *Superfluids I* (Wiley, New York, 1950), Dover reprint; J. M. Blatt, *Theory of Superconductivity* (Academic, New York, 1964); B. S. Deaver and W. M. Fairbank, Phys. Rev. Lett. 7, 43 (1961); N. Byers and C. N. Yang, *ibid.* 7, 46 (1961); L. Onsager, *ibid.* 7, 50 (1961). Also see P. A. M. Dirac, Proc. R. Soc. A133, 60 (1931); Y. Aharonov and D. Bohm, Phys. Rev. 115, 485 (1959). Also see H. Fröhlich, *ibid.* 79, 167 (1950); Proc. R. Soc. A215, 291 (1952); M. R. Schafroth, Phys. Rev. 96, 1442 (1954); J. Bardeen, L. N. Cooper, and J. R. Schrieffer, *ibid.* 108, 1175 (1957); R. Doll and M. Nabauer, Phys. Rev. Lett. 7, 51 (1961); L. Cooper, Am. J. Phys. 28, 91 (1960).

[3]Superposition of probability amplitudes of alternative path histories: R. P. Feynman, Rev. Mod. Phys. 20, 367 (1948); R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965); J. R. Klauder, Ann. Phys. (N.Y.) 11, 123 (1960); in proceedings of the Lectures at the University of Bern, Switzerland, 1962 (unpublished). Also see C. W. Misner, Rev. Mod. Phys. 29, 497 (1957); W. E. Brittin and W. R. Chappell, *ibid.* 34, 620 (1962); S. S. Schweber, J. Math. Phys. 3, 831 (1962); I. Gel'fand and A. Yaglom, Fortschr. Phys. 5, 517 (1957).

[4]Nonlocality in relation to Pryce-Foldy-Wouthuysen-Tani transformation: W. C. Parke, Ph.D. thesis, George Washington Univ. 1967 (unpublished); K. Huang, Am. J. Phys. 20, 479 (1952). Also see E. Schrödinger, Sitzungsber. Preuss. Akad. Wiss., Phys.-Math. Kl. X, 418 (1930); A. O. Barut, H. Kleinert, and S. Malin, Nuovo Cimento 58A, 835 (1968); M. H. L. Pryce, Proc. R. Soc. A195, 62 (1948); L. L. Foldy and S. A. Wouthuysen, Phys. Rev. 78, 29 (1950); C. G. Bollini and J. J. Giambiagi, Nuovo Cimento 16, 202 (1960); 21, 107 (1961); H. Jehle and W. C. Parke, Phys. Rev. 137, B760 (1965); L. Castell, Nuovo Cimento 49A, 285 (1967);

A. Sankaranarayanan and R. H. Good, Jr., Phys. Rev. 140, B509 (1965); K. Rafanelli, *ibid.* 155, 1420 (1967); H. Bacry, J. Math. Phys. 5, 109 (1964); Yu. M. Shirokov, Zh. Eksp. Teor. Fiz. 21, 748 (1951); A. S. Wightman, Rev. Mod. Phys. 34, 845 (1962); M. Lorente and P. Roman, personal communication; P. Roman, *Advanced Quantum Theory*, (Addison-Wesley, Reading, Mass., 1965); S. S. Schweber, *Relativistic Quantum Field Theory* (Row, Peterson, New York, 1961); H. Volk and M. Alonso, *Quantum Mechanics: Principles and Applications* (Addison-Wesley, Reading, Mass., 1973).

[5]Nonlocality and positive-definite metric: W. Heisenberg, in proceedings of the Symposium on Basic Questions in Elementary Particle Physics, Max Planck Institut, München, 1971 (unpublished); H. P. Dürr, W. Heisenberg, H. Mitter, S. Schlieder, and K. Yamazaki, Z. Naturforsch. 14A, 441 (1959). Also see articles by H. P. Duerr, M. Fierz, J. Rzewuski, R. L. Ingraham, D. I. Blockintsev, J. P. Vigier, W. Guettinger, and other participants, in Proceedings of the International Symposium on Nonlocal Quantum Field Theory, Dubna, U.S.S.R., Dubna, 1967, Dubna Report No. P2-3590, 1967 (unpublished); W. Heisenberg, Nucl. Phys. 4, 532 (1957); H. P. Dürr and E. Rudolph, Nuovo Cimento 65A, 423 (1970); E. C. G. Sudarshan, Phys. Rev. 123, 2183 (1961); N. Nakanishi, Prog. Theor. Phys. Suppl. 51, 1 (1972); L. K. Pandit, Nuovo Cimento Suppl. 11, 157 (1959). The Heisenberg-Dürr field theory is of course based on local interaction.

[6]Topology of loops; fibrations and cobordism: H. Seifert and W. Threlfall, *Topologie* (Teubner, Leipzig, 1934), Chelsea reprint; Math. Ann. 104, 1 (1931); 107, 543 (1932); Can. J. Math. 2, 1 (1950); H. Seifert, Acta. Math. 60, 147 (1933); Sitzungsber. Preuss. Akad. Wiss. Phys.-Math. Kl. 27, 811 (1933); Q. J. Math. 1, 23 (1950); Proc. Am. Math. Soc. 1, 287 (1950); R. H. Fox and R. H. Crowell, *Knot Theory* (Blaisdell-Ginn, New York, 1963); R. H. Fox, in *Symposium on Topology*, edited by T. Fort (Prentice-Hall, Englewood Cliffs, N. J., 1962); R. H. Fox and J. W. Milnor, Osaka J. Math. 3, 257 (1966) (cobordism); I. V. V. Raghavacharyulu, personal communication; H. S. M. Coxeter, *Introduction to Geometry* (Wiley, New York, 1969); personal communication; W. S. Massey, *Introduction to Algebraic Topology* (Harcourt, Brace & World, New York, 1967); Ch. H. Giffen and N. Smythe, personal communication; K. Reidemeister, *Knotentheorie* (Springer, Berlin, 1932), Chelsea reprint, 1961; W. Drechsler, Max Planck Institut Report No. PAE/PTh 24, 1974 (unpublished); in proceedings of the XIV Internationale Universitätswochen für Kernphysik 1973 der Karl-Franzens-Universität Graz, at Schladming (unpublished).

[7]Dual strings and loops: There are many points in the present project which are related to the following papers: H. B. Nielsen and P. Olesen, Nucl. Phys. B61, 45 (1973); A. O. Barut and D. Foerster, ICTP, Trieste, Italy, report, 1963 (unpublished); C. Rebbi, CERN Report No. TH 1691, 1973 (unpublished); G. 't Hooft, personal communication; A. O. Barut and G. L. Bornzin, personal communication; T. Takabayasi, Prog. Theor. Phys. 46, 1528 (1971); 46, 1924 (1971); Y. Nambu, Phys. Rev. D 4, 1193 (1971); in proceedings of the Copenhagen Symposium (unpublished); C. Rebbi, in proceedings of Lectures at the International Centre for Theoretical Physics, Trieste, Italy, 1972 (unpublished); P. Goddard, J. Goldstone, C. Rebbi, and C. B. Thorn, Nucl. Phys. B56, 109 (1973); S. Mandelstam, lecture given at the American Physical Society Meeting, Chicago, 1974 (unpublished); De Vries, in *Boulder Lecture Notes in Theoretical Physics, Vol. 14A: Topics in Strong Interactions*, edited by A. O. Barut and W. Brittin (Ref. 1); L. Clavelli, personal communication; A. Patrascioiu, personal communication; Nuovo Cimento Lett. 10, 676 (1974); J. H. Schwarz, Sci. Amer. 232, 61 (1975); A. O. Barut, Int. Conf. Math. Phys., Warszawa, 1974 (unpublished); A. O. Barut and G. L. Bornzin, Acta Phys. Austr. Suppl. 11, 565 (1973); A. O. Barut and R. Raczka, personal communication.

[8]Baryon magnetic moments: B. T. Feld, *Models of Elementary Particles* (Blaisdell, Waltham, Mass., 1969); Particle Data Group, Rev. Mod. Phys. 43, S1 (1973); T. Hansl, A. Manz, J. Marraffino, W. Matt, R. Settles, and G. Wolf, personal communication; Ch. E. Roos and M. E. Webster, personal communication.

[9]Various models: J. C. Slater, Nature 117, 587 (1926); D. Finkelstein, J. Math. Phys. 7, 1218 (1966); Phys. Rev. 100, 924 (1955); H. J. Bhabha and H. C. Corben, Proc. R. Soc. A178, 243 (1941); L. H. Thomas, Nature 117, 514 (1926); Philos. Mag. 3, 1 (1927); E. J. Post, personal communication; H. Hönl, Z. Naturforsch. 3A, 573 (1948); H. Hönl and A. Papapetrou, Z. Phys. 112, 65 (1939); 112, 512 (1939); 114, 478 (1939); H. C. Corben, *Classical and Quantum Theories of Spinning Particles* (Holden-Day, San Francisco, 1968) also contains a comprehensive bibliography; H. C. Corben, Nuovo Cimento 47A, 482 (1967); J. Math. Phys. 5, 1664 (1964); Phys. Rev. Lett. 15, 269 (1965); Phys. Rev. 131, 2219 (1963); 134, B832 (1964); D. F. Bartlett, personal communication; J. Djuric, personal communication; W. Drechsler, personal communication; M. Böhm, H. Joos, and M. Krammer, Acta Phys. Austr. Suppl. 11, 3 (1973); H. Harari, in *Elementary Processes at High Energy*, edited by A. Zichichi (Academic, New York, 1971), Part A, p. 170; R. Hagedorn, CERN Report No. 71-12 (unpublished); G. Morpurgo, Annu. Rev. Nucl. Sci. 20, 105 (1970); J. L. Lopez, CRN/HE 74-13 (unpublished); J. C. Pati and A. Salam, Phys. Rev. Lett. 31, 661 (1973); R. P. Feynman, Science 183, 601 (1974); A. E. Ruark, personal communication; R. H. Dalitz, personal communication; D. B. Lichtenberg, *Unitary Symmetry and Elementary Particles* (Academic, New York, 1970).

[10]Spinors and related issues: F. Bakke and H. Wergeland, Ark. f.d. Fys. Sem. Trondheim No. 8 (1973); Physica 69, 5 (1973); E. Mendels and S. A. Wouthuysen, private communication; R. Haag, Z. Naturforsch. 7A, 449 (1952); W. C. Parke and H. Jehle, Phys. Rev. 137, B760 (1965); in *Boulder Lecture Notes in Theoretical Physics, Vol. 7A: Lorentz Group*, edited by W. E. Brittin *et al.* (Gordon and Breach, New York, 1965); W. L. Bade and H. Jehle, Rev. Mod. Phys. 25, 714 (1953); L. I. Infeld and B. L. van der Waerden, Sitzungsber. Preuss. Akad. Wiss. Phys.-Math. Kl. 27, 380 (1933); L. Infeld, Acta Phys. Polon. 3, 1 (1934); E. M. Corson, *Theory of*

Spinor and Relativistic Wave Equations (Blackie, Glasgow, 1953); R. Brauer and H. Weyl, Am. J. Math. 57, 425 (1935); H. Feshbach and F. Villars, Rev. Mod. Phys. 30, 24 (1958); V. Bargmann and E. P. Wigner, Proc. Natl. Acad. Sci. USA 34, 211 (1948); K. M. Case, Phys. Rev. 95, 1323 (1954); 107, 307 (1957); R. H. Good, Jr., Rev. Mod. Phys. 27, 187 (1955); E. A. Hylleraas, Z. Phys. 140, 626 (1955); *Mathematical and Theoretical Physics* (Wiley, N.Y., 1970); O. Klein, Z. Phys. 58, 730 (1929); W. Kofink, Math. Z. 51, 702 (1949); H. A. Kramers, *Quantum Mechanics* (North-Holland, Amsterdam, 1957); S. S. Schweber, *An Introduction to Relativistic Qauntum Field Theory* (Row, Peterson, Evanston, Ill., 1961); J. Leite Lopez, *Lectures on Symmetries* (Gordon and Breach, N. Y., 1969); J. Serpe, *Les Lois de Conservations* (Inst. Interuniv., Bruxelles, 1959); H. Weyl, *The Classical Groups* (Princeton Univ. Press, Princeton, N.J., 1939); O. Laporte and G. E. Uhlenbeck, Phys. Rev. 37, 1380 (1931); 37, 1552 (1931); N. Kemmer, Proc. R. Soc. A173, 91 (1940); R. J. Finkelstein, Rev. Mod. Phys. 36, 632 (1964); P. O. Löwdin, *ibid.* 39, 259 (1967).

[11]Issues about torsion and related questions: R. J. Finkelstein, J. Math. Phys. 1, 440 (1960); Ann. Phys. (N.Y.) 12, 200 (1961); 15, 223 (1961); 17, 379 (1962); 21, 408 (1963).

[12]Self-consistent field issues: R. J. Finkelstein, Phys. Rev. 75, 1079 (1949); Nuovo Cimento 1, 1113 (1955); R. J. Finkelstein, R. LeLevier, and M. Ruderman, Phys. Rev. 83, 326 (1951); R. J. Finkelstein, S. G. Gasiorowicz, and P. Kaus, Can. J. Phys. 32, 480 (1954); J. J. Loferski, thesis, Univ. of Pennsylvania, 1949 (unpublished); J. Schäfer, personal communication; F. A. Kaempffer, Phys. Rev. 99, 1614 (1955); E. Schrödinger, Nature 169, 538 (1952); R. J. Finkelstein, C. Fronsdal, and P. Kaus, Phys. Rev. 103, 1571 (1956); in this paper references to related work are given; H. P. Dürr, Ref. 5.

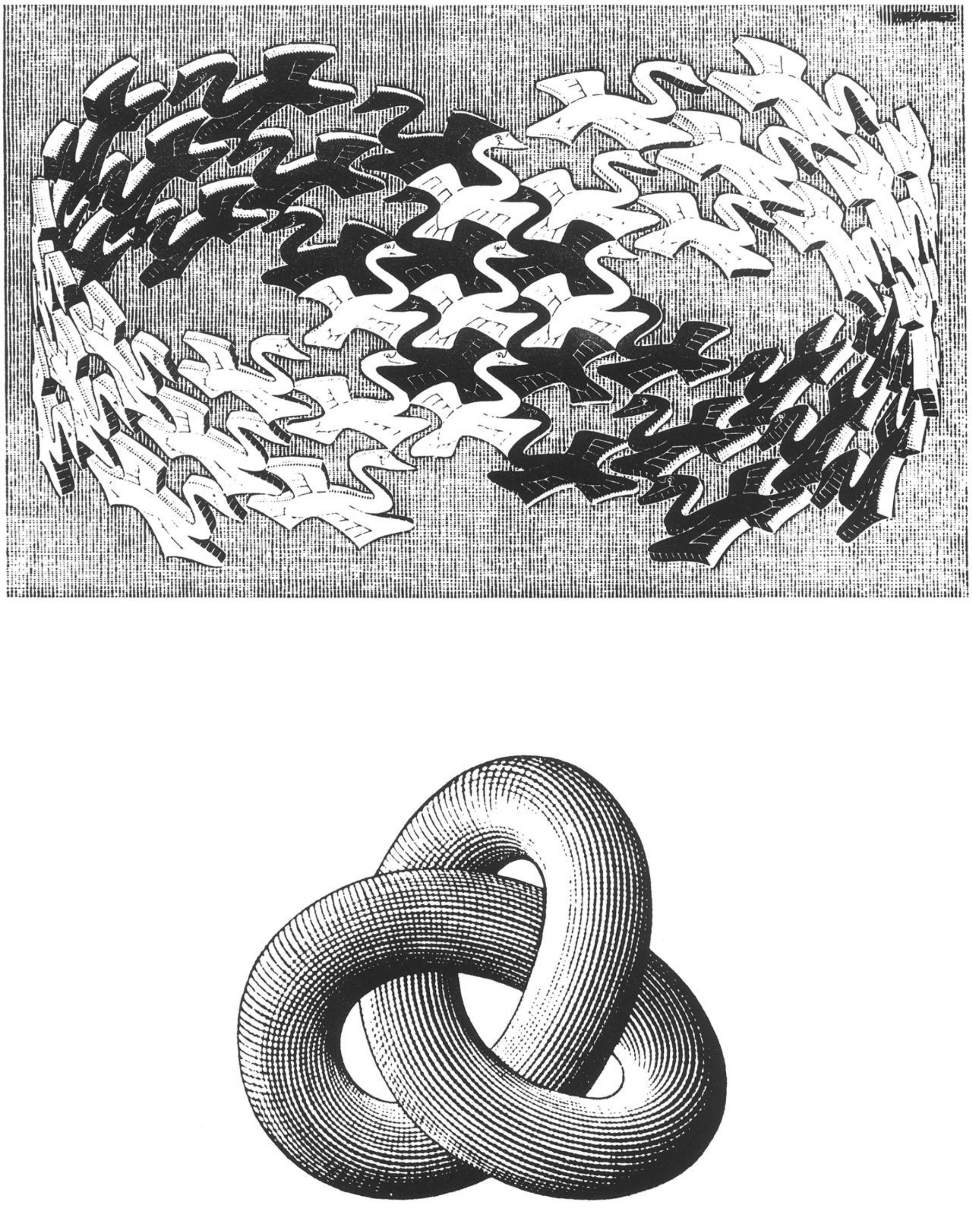

FIG. 5. Pictures from *The Graphic Works of M. C. Escher* as illustrations of a trefoil and a lively picture of the electron flux loop. We are grateful for permission to produce these figures to Koninklijke Uitgeverij. Erven J. J. Tijl N. V., Zwolle, Holland, and Ballantine Books, N.Y., *The Graphic Works of M. C. Escher*.