PHYSICAL REVIEW D VOLUME 11, NUMBER 7 1 APRIL 1975

Relativistic Green's-function approximation to the low-energy nucleon-nucleon interaction

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(Received 19 August 1974)

From a unitary approximation to the nucleon-nucleon Green's function we derive the phase shifts for the Yukawa model and the nonlinear σ model. The results agree with the experimental data except for the lowest waves: ${}^1S_0$, ${}^3S_1$, ${}^3P_0$ where, however, we get good scattering lengths.

INTRODUCTION

In order to explain the low-energy nucleon interaction[1] various authors have derived unitary approximations from field-theoretical models. On one side, Schrödinger's equation was used with potentials reproducing the relativistic one-particle-exchange contribution.[2] On the other, numerical solutions of the Bethe-Salpeter equation were tried[3] for the pion exchange and the $J=0$ waves and some semirelativistic equations were developed.[4] The first method gives numerically good results but depends on a very large number of free parameters, and the off-shell extension is arbitrary. The second one in the ladder approximation has no solution for the physical value of the coupling constant.

The field-theoretical input is usually inserted only at the lowest order α in the nucleon-boson coupling and the integral equations merely provide a unitarization method. The same procedure can be extended to the next order α^2, but the computational difficulties are such that it seems better to use a field-theoretical approximation to the S matrix such as the $[1/1]$ Padé approximation (P.A.) which can be derived from a variational principle (Lippmann and Schwinger) using a perturbative (Cini-Fubini) ansatz.

The $[1/1]$ P.A. to the S matrix improves, for the Yukawa theory, the unitarized Born approximation which reproduces correctly the very high waves.[5] The introduction of two-pion forces seems to describe the lower waves, but the wrong threshold behavior of the Born term in the ${}^1S_0$ and the ${}^3(J-1)_J$ waves produces some spurious effects in the $[1/1]$ P.A. phase shifts.

This anomalous behavior of the Born term is related to the pseudoscalar nature of the pion. Indeed, a virtual pion cannot be emitted by a real nucleon at rest, and one can check that the one-pion-exchange amplitude vanishes at zero energy. The S waves have zero scattering lengths and behave as P waves. If we want to avoid the introduction of other fields to restore good threshold behavior[6] we have to imagine a coupled-channel system of nucleons with both parities. This picture can be justified rigorously using, in a suitable basis, the relativistic nucleon-nucleon Green's function.[7] These negative-parity states are related to the structure of the Lorentz group and seem to play a crucial role in a Yukawa theory, as has been shown using the Bethe-Salpeter equation.

A partial calculation involving only the states with nucleons of the same parity (physical or not) was done for the Yukawa model and the nonlinear σ model.[8] The results were satisfactory except for the S waves above threshold. The complete calculation was first performed for $J=0$ (see Ref. 9) and showed no sign inversion of the ${}^3P_0$ wave.

A generalization[7] of the formalism of Goldberger *et al.*[10] has made it possible to perform the partial-wave expansion for any J. Following this scheme we have in this paper analyzed once more the Yukawa model and the nonlinear σ model. We confirm the results of Ref. 9 for the $J=0$ waves and find an excellent stability of the solution for the $J \geq 2$ waves ($J \geq 1$ in the nonlinear σ model). The only open problem is the inversion of the ${}^1S_0$, ${}^3S_1$, ${}^3P_0$ waves. In a forthcoming paper we shall investigate models to explain the S-wave physics above threshold. Including vector mesons we should obtain a short-range repulsion effect. A calculation based on the Bethe-Salpeter equation has been performed recently.[11] The results disagree with those obtained in Ref. 9 for $J=0$ due to a different renormalization procedure. We have chosen the same renormalization prescription as in Ref. 9 and found exactly the same results for $J=0$. In this paper we give also the other phase shifts for the Yukawa and the nonlinear σ model.

In Sec. I we shall review the Yukawa model and the nonlinear σ model with a special emphasis on the renormalization conditions, and in Sec. II we shall discuss the results and the physical relevance of the models we have investigated. Some details of the calculation will be given in an appendix.

I. THE MODELS AND THE RENORMALIZATION CONDITIONS

A. The Yukawa model

The interaction Lagrangian for the Yukawa model reads

$$\mathcal{L}_I = -ig\bar{N}\gamma_5\vec{\tau}N\cdot\vec{\pi} - \tfrac{1}{4}\lambda(\vec{\pi}\cdot\vec{\pi})^2 . \tag{1.1}$$

The constant λ does not affect a second-order calculation. We have used the standard renormalization conditions for the pion and nucleon propagators. For the πNN vertex we have expanded the amputated Green's function through

$$\Gamma_i(Q;p,p') = \gamma_5\tau_i(V_0 + \not{Q}V_Q + \tfrac{1}{2}\not{P}V_P + \tfrac{1}{2}[\not{Q},\not{P}]V_{PQ}) , \tag{1.2}$$

where

$$Q = p' - p\ ,\quad P = p' + p\ ,$$

and the only divergent amplitude V_0 is renormalized by

$$\bar{u}(p')\Gamma_i(p'-p;p,p')u(p)|_{(p'-p)^2=\mu^2} = g\,\bar{u}(p')\gamma_5\tau_i u(p) , \tag{1.3}$$

where $u(p)$ and $u(p')$ are the Dirac spinors associated with the nucleons. If we consider the nonphysical nucleon parity states[7] the vertex amplitude still has a pole at $t=\mu^2$. Such a pole can be canceled by the introduction of the nucleon self-energy graphs. These graphs should be absent if we amputate the Green's function using the exact nucleon propagator. Consequently, we have amputated the Green's function using a free nucleon propagator to get the same contribution for the physical amplitude and to cancel the pion-pole contribution of the πNN vertex.

B. The nonlinear σ model

The effective Lagrangian for the one-loop diagrams in the nonlinear σ model reads

$$\mathcal{L}_I = -i\frac{m}{f_\pi}\bar{N}\gamma_5\vec{\tau}N\cdot\vec{\pi} - \frac{m}{2f_\pi^{\,2}}\bar{N}N\vec{\pi}\cdot\vec{\pi} - \frac{\mu^2}{8f_\pi^{\,2}}(\vec{\pi}\cdot\vec{\pi})^2 + \frac{1}{2f_\pi^{\,2}}(\vec{\pi}\cdot\partial_\mu\vec{\pi})^2 , \tag{1.4}$$

where f_π is the pion decay constant, and m and μ are the nucleon and pion masses, respectively.

This model is not renormalizable, but we can obtain its Green's functions by expanding the linear σ-model Green's functions for a large σ mass.

The divergent graphs of the nucleon-nucleon Green's function are the pion and nucleon propagators, the pion-nucleon vertex, and graph (f) of Fig. 1.

The propagators are renormalized using the standard prescriptions. For the vertex we have now two divergent amplitudes V_0 and V_Q. The first one is renormalized through the Ward identity[12]

$$f_\pi\Gamma_j(0;p,p) = \frac{1}{2i}\tau_j[\gamma_5, S^{-1}(p)]_+ , \tag{1.5}$$

where $S^{-1}(p)$ is the exact inverse nucleon propagator. If we now impose Eq. (1.3) we get

$$g = \frac{m}{f_\pi} + g_0\left(\frac{m}{f_\pi}\right)^3 + O\left(\frac{m}{f_\pi}\right)^5 , \tag{1.6}$$

where g_0 is an arbitrary constant connected to V_q. Our choice for g_0 and consequently for V_Q is to require

$$g = \frac{m/f_\pi}{[1-2g_0(m/f_\pi)^2]^{1/2}} . \tag{1.7}$$

With such a choice, a $[1/1]$ P.A. to the S matrix gives a residue at the pion pole equal to the physical coupling constant $\alpha = g^2/4\pi$.

Graph (f) of Fig. 1 is logarithmically divergent and its subtraction point a^2, which affects only the $J \leq 1$ waves, is fixed by a best-fitting condition. It is found that the results do not depend critically on this parameter.

II. DISCUSSION OF THE RESULTS

Before comparing the theoretical results with the experimental data[13] one should estimate the energy range where a theory which includes one- and two-pion exchange contributions is expected to work. According to a suggestion we relate the impact parameter to the range of the n-pion exchange force. We obtain for the pion kinetic energy T

$$T \leq T_L^{(n)} , \tag{2.1}$$

with

$$T_L^{(n)} = \tau_0 n^2 L(L+1) , \tag{2.2}$$

where L is the orbital angular momentum, τ_0 is a constant, and $T_L^{(n)}$ is the energy below which the

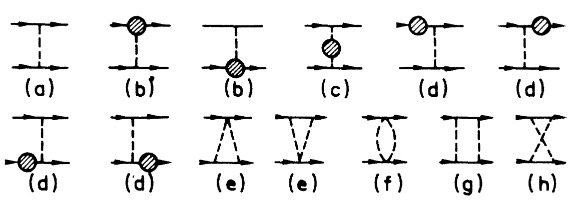


FIG. 1. Feynman diagrams contributing to the four-nucleon Green's function (up to one loop) in the Yukawa model and the nonlinear σ model.

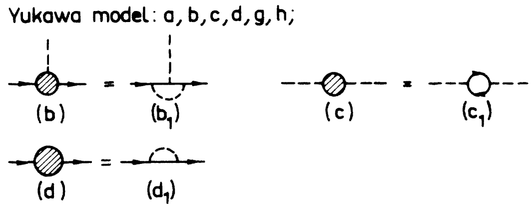


FIG. 2. Relevant portions of the Feynman diagrams for the Yukawa model.

contribution of more-than-n-pion exchange is negligible. From (2.1) we can derive the following ratios:

$$T_P : T_D : T_F : T_G : T_H = 1:3:6:10:15 . \qquad (2.3)$$

These are roughly verified by our results. In fact, we cannot expect our results to hold above the first inelastic threshold in the S channel. The detailed results can be summarized as follows.

Higher partial waves $J \geq 2$. For such waves we find excellent agreement of the results with the results of Ref. 8 and with the experimental data. Some quantitative disagreement appears only for the $^3P_2$ wave which is somewhat lower than the experimental data.

Low waves $J = 0, 1$. In the case of the nonlinear σ model we find excellent P and D waves for $J = 1$. The $^3S_1$ wave exhibits a bound-state behavior with a qualitatively correct scattering length. Again for the nonlinear σ model the $^1S_0$ scattering length has the correct sign and magnitude, but above threshold the phase shift is monotonically increasing. The $^3P_0$ wave is correct for very low energy (≤ 30 MeV), but we do not find the change of sign we can expect with more sophisticated models accounting for hard-core and spin-orbit effects.

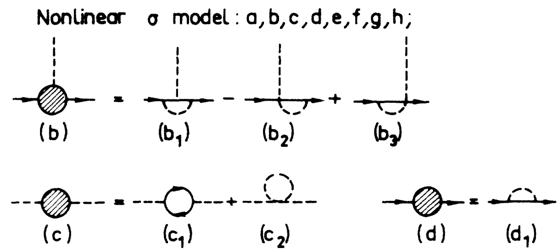
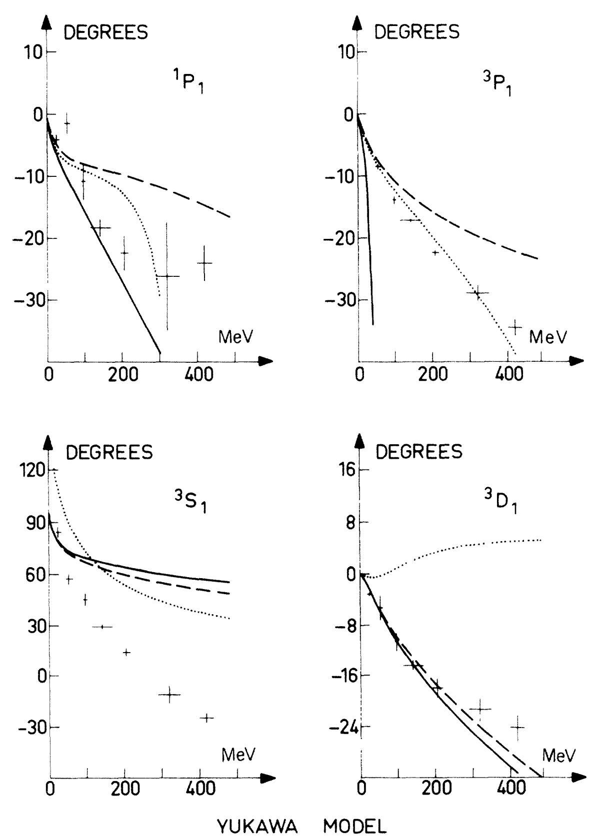


FIG. 3. Relevant portions of the Feynman diagrams for the nonlinear σ model.

For the Yukawa model the situation is similar for the $J = 0$ waves. For the $^3P_1$ wave we find a disagreement between these results and the results of Ref. 8. The reason is the inclusion in the calculation of unphysical S states which are of large amplitude and strongly coupled to the physical ones.

CONCLUSION

If we keep in mind that our results are derived from models without free parameters we can con-



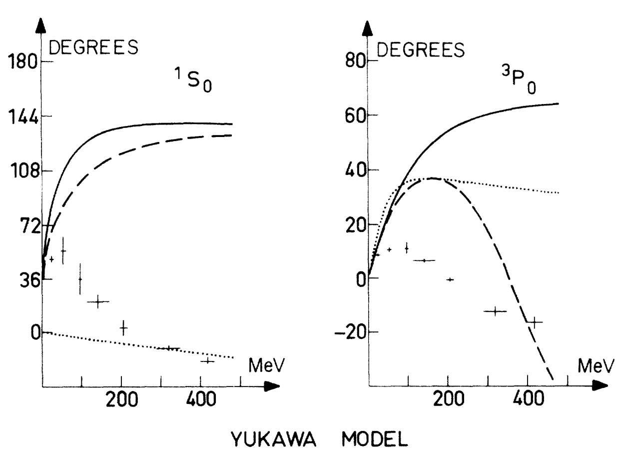


FIG. 4. *(Continued on following page)*

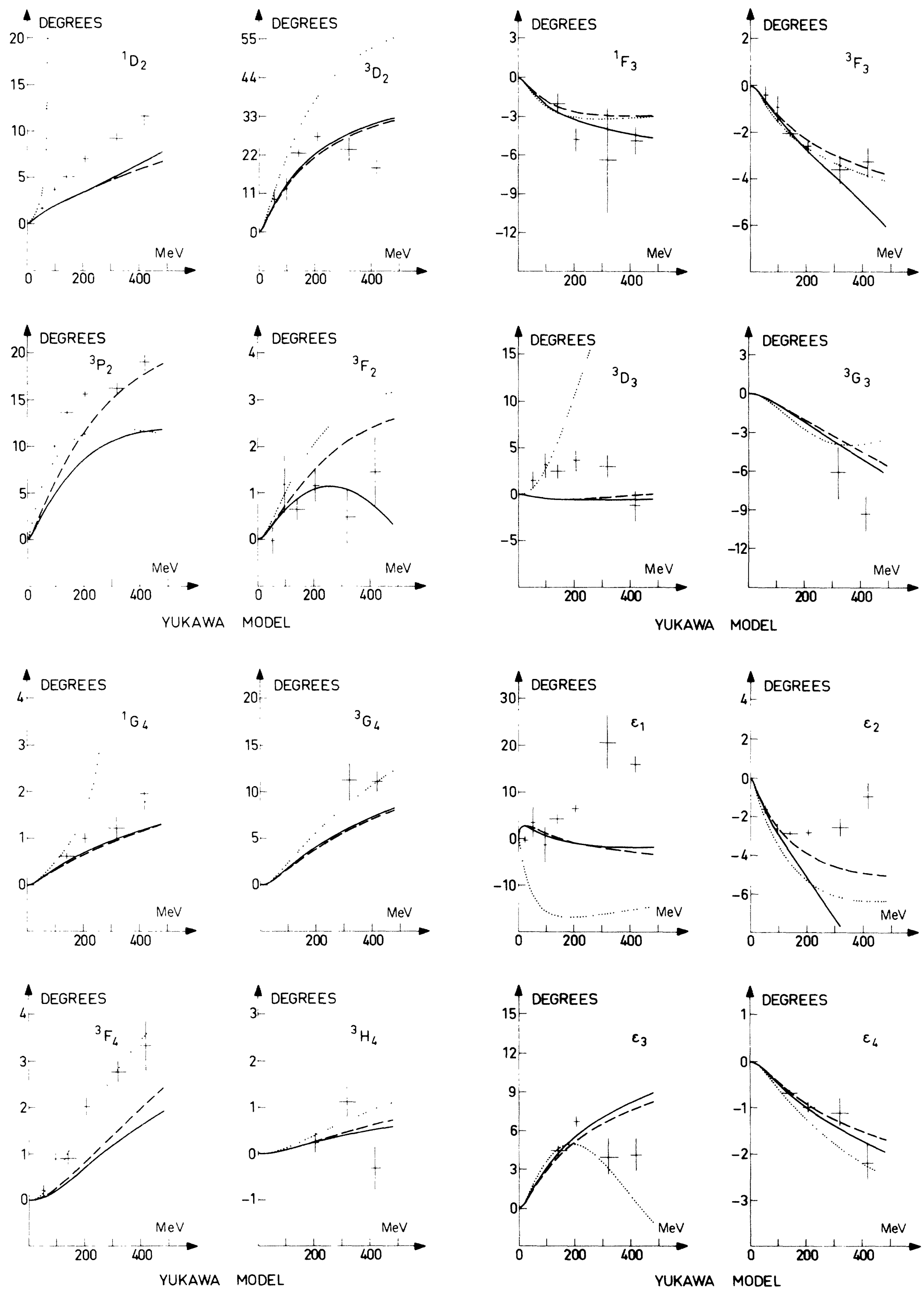


FIG. 4. The phase shifts with $J \leq 4$, as a function of E_{lab}, computed from the Yukawa model and compared with the experimental data. The curves are identified as follows. [1/1] complete Green's function Padé approximant: continuous line; [1/1] partial Green's function Padé approximant (i.e., using only the $|+\rangle$ and $|-\rangle$ states): dashed line; [1/1] S-matrix Padé approximant: dotted line. The values of the parameters for the Yukawa model are $m = 938$ MeV, $\mu = 138$ MeV, $g^2/4\pi = 15$.

clude that the phase shifts obtained agree for $J \geq 1$ with the experimental data, especially if we use the nonlinear σ model. This agreement appears within an energy range depending on the orbital angular momentum.

Two ways are now open for a future investigation. On the one side, we can use more sophisticated models including, for instance, the vector mesons to which the S-wave inversion is usually associated in potential theory; this will be the object of a subsequent paper. On the other, we can look for a better approximation by computing a [1/1] operator Padé approximant with a suitable procedure for choosing discrete momenta.[14]

ACKNOWLEDGMENTS

We would like to thank Professor D. Bessis for constructive criticism of the manuscript.

APPENDIX

Let G be the four-nucleon Green's function and let $T_{\beta\alpha}$ be defined by

$$T_{\beta\alpha} = \bar{u}(p_1')\otimes\bar{u}(p_2')\Gamma_\beta G\Gamma_\alpha u(p_1)\otimes u(p_2)\,, \tag{A1}$$

where p_1, p_2 (p_1', p_2') are the incoming (outgoing) nucleon momenta and where, according to Ref. 7, the Γ_α are defined by

$$\Gamma_+ = 1\otimes 1,\ \Gamma_- = \gamma_5\otimes\gamma_5,\ \Gamma_e = \frac{1}{\sqrt{2}}(1\otimes\gamma_5 + \gamma_5\otimes 1)\,,$$
$$\Gamma_0 = \frac{1}{\sqrt{2}}(1\otimes\gamma_5 - \gamma_5\otimes 1)\,. \tag{A2}$$

With this choice T_{++} represents the physical amplitude, the other $T_{\alpha\beta}$ representing transitions between states with nucleons of negative parity. The various diagrams which have to be taken into account in the Yukawa model and in the nonlinear σ model are drawn in Figs. 1–3. We shall now give for the Yukawa model the decomposition of each graph over the basis of the generalized Fermi invariants $\langle O_i\rangle$ and $\langle O_i^5\rangle$ defined in Ref. 7.

Accordingly, we introduce the following vectors:

$$\Delta = p_1 - p_1' = p_2' - p_2;\quad \pi_1 = \frac{p_1 + p_1'}{2}\,;\quad \pi_2 = \frac{p_2 + p_2'}{2}\,. \tag{A3}$$

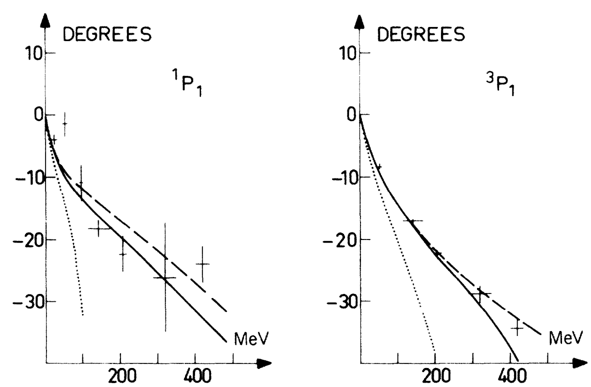
The Green's functions for the various graphs are as follows.

Born:

$$G = (\mathcal{P}_1 - 3\mathcal{P}_0)\frac{g^2}{t-\mu^2}\gamma_5\otimes\gamma_5\,. \tag{A4}$$

Pion self-energy:

$$G = -(\mathcal{P}_1 - 3\mathcal{P}_0)\tfrac{1}{4}g^4 H(t)\gamma_5\otimes\gamma_5\,. \tag{A5}$$



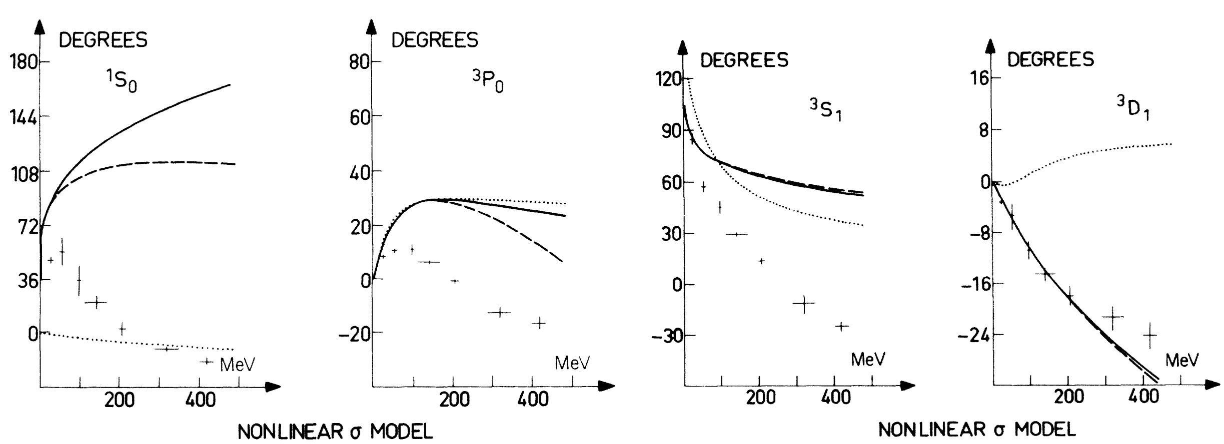


FIG. 5. *(Continued on following page)*

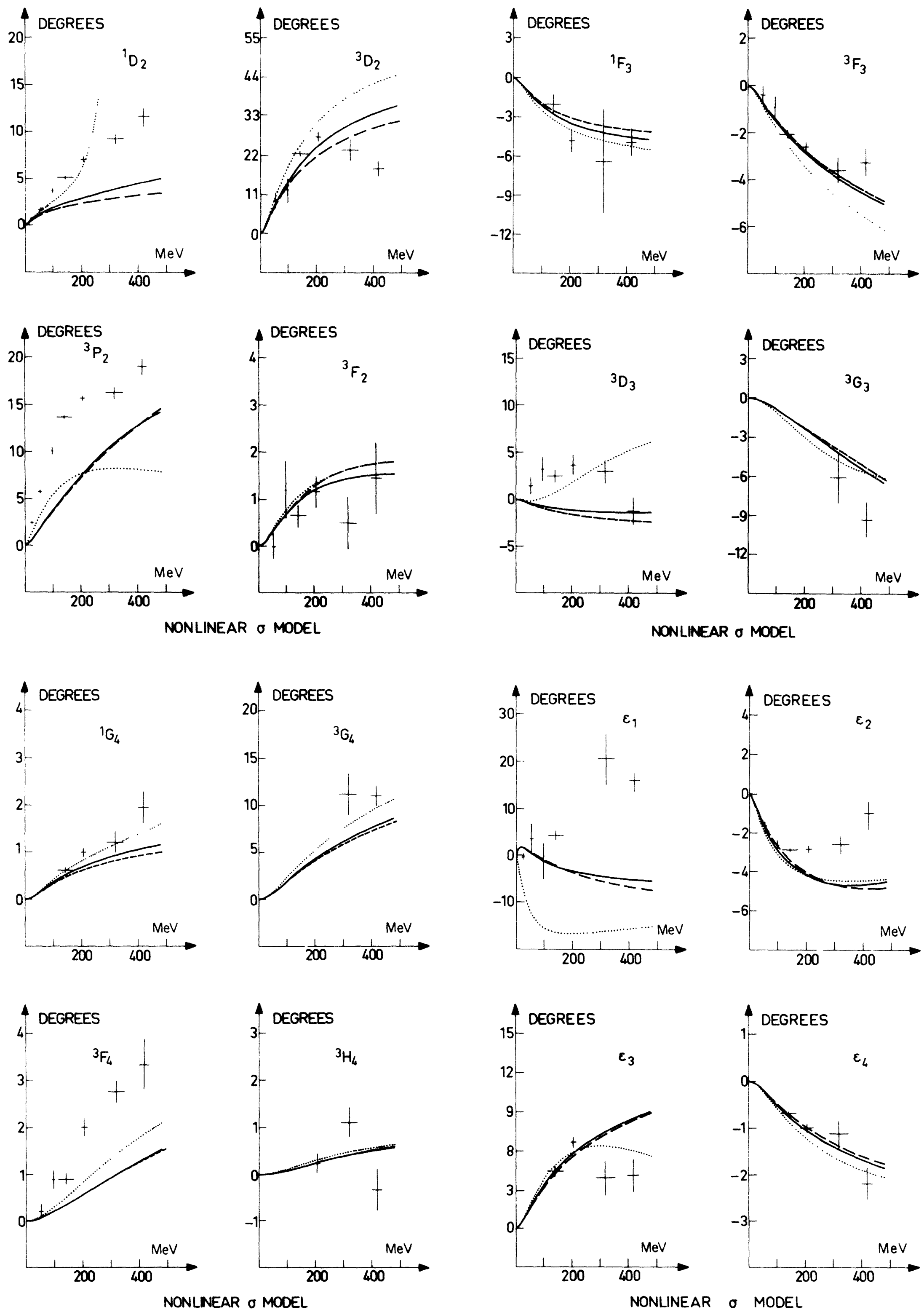


FIG. 5. The phase shifts with $J \leq 4$ computed from the nonlinear σ model and compared with the experimental data. The identification of curves is the same as in Fig. 4. The values of the parameters for the nonlinear σ model are $m = 938$ MeV, $\mu = 138$ MeV, $f_\pi = 93.8$ MeV, $g^2/4\pi = 15$, $a^2 = -1000\mu^2$.

Nucleon wave-function renormalization:

$$G=(\mathcal{P}_1-3\mathcal{P}_0)\frac{g^2}{t-\mu^2}\left[4O_s\gamma_5\otimes\gamma_5-\frac{O_s}{m}(\gamma_5\Delta\!\!\!/\otimes\gamma_5-\gamma_5\otimes\gamma_5\Delta\!\!\!/)\right]. \tag{A6}$$

Vertex:

$$G=(\mathcal{P}_1-3\mathcal{P}_0)\frac{g}{t-\mu^2}\{2V_0\gamma_5\otimes\gamma_5+V_Q(\gamma_5\otimes\gamma_5\Delta\!\!\!/-\gamma_5\Delta\!\!\!/\otimes\gamma_5)+V_{PQ}(\gamma_5\otimes\gamma_5[\Delta\!\!\!/,\not{\pi}_2]-\gamma_5[\Delta\!\!\!/,\not{\pi}_1]\otimes\gamma_5)\}. \tag{A7}$$

Box:

$$G=\left(\frac{g^2}{4\pi}\right)^2(\mathcal{P}_1+9\mathcal{P}_0)\{\tfrac{1}{2}B_0\gamma_\mu\otimes\gamma^\mu+\tfrac{1}{4}B_1(\not{\pi}_2-\not{\pi}_1)\otimes(\not{\pi}_2-\not{\pi}_1)+\tfrac{1}{4}B_2(\not{\pi}_2+\not{\pi}_1)\otimes(\not{\pi}_2+\not{\pi}_1)$$
$$-\tfrac{1}{2}B_3[(\not{\pi}_2-\not{\pi}_1)\otimes(\not{\pi}_2-m)-(\not{\pi}_1-m)\otimes(\not{\pi}_2-\not{\pi}_1)]-B_4(\not{\pi}_1-m)\otimes(\not{\pi}_2-m)+\tfrac{1}{4}B_5\Delta\!\!\!/\otimes\Delta\!\!\!/\}, \tag{A8}$$

where $\mathcal{P}_I$ is the projector over the state of isospin I; the amplitudes V_0, V_Q, V_{PQ}, H, O_s, B_k are the same as in Appendix F of Ref. 8.

The decomposition of $T_{\beta\alpha}$ on the invariants $\langle O_i\rangle$, $\langle O_i^5\rangle$ is given by (i) Table I for the Born graph; (ii) Table II for the nucleon wave-function renormalization; (iii) Table III for the vertex where the amplitudes V_{++}, V_{+-}, V_{--} read

$$V_{++}=V_0-2mV_Q+(t-4m^2)V_{PQ},\quad V_{--}=V_0+2mV_Q+(t-4m^2)V_{PQ},\quad V_{+-}=V_0+tV_{PQ}. \tag{A9}$$

For the box we have

$$T_{\beta\alpha}=\left(\frac{g^2}{4\pi}\right)^2(\mathcal{P}_1+9\mathcal{P}_0)\sum_{i=1}^{8}[\langle O_i\rangle(A_i)_{\beta\alpha}+\langle O_i^5\rangle(A_i^5)_{\beta\alpha}], \tag{A10}$$

where $(A_i)_{\beta\alpha}$, $(A_i^5)_{\beta\alpha}$ are linearly related to the six basic amplitudes B_k (which are given analytically in Ref. 8) by

$$(A_i)_{++}=a_i(m,m),\quad (A_i)_{+-}=b_i(m,m),$$
$$(A_i)_{-+}=b_i(-m,-m),\quad (A_i)_{--}=a_i(-m,-m),$$
$$(A_i^5)_{+e}=\frac{1}{\sqrt2}[c_i(m,m)+d_i(m,m)],\quad (A_i^5)_{+o}=\frac{1}{\sqrt2}[c_i(m,m)-d_i(m,m)],$$
$$(A_i^5)_{-e}=\frac{1}{\sqrt2}[c_i(-m,-m)+d_i(-m,-m)],\quad (A_i^5)_{-o}=-\frac{1}{\sqrt2}[c_i(-m,-m)-d_i(-m,-m)],$$
$$(A_i^5)_{e+}=-\frac{1}{\sqrt2}[c_i(m,-m)+d_i(-m,m)],\quad (A_i^5)_{e-}=-\frac{1}{\sqrt2}[c_i(-m,m)+d_i(m,-m)],$$
$$(A_i^5)_{o+}=-\frac{1}{\sqrt2}[c_i(m,-m)-d_i(-m,m)],\quad (A_i^5)_{o-}=\frac{1}{\sqrt2}[c_i(-m,m)-d_i(m,-m)], \tag{A11}$$
$$(A_i)_{ee}=-\tfrac{1}{2}[a_i(m,-m)+a_i(-m,m)+b_i(m,-m)+b_i(-m,m)],$$
$$(A_i)_{eo}=-\tfrac{1}{2}[a_i(m,-m)-a_i(-m,m)-b_i(m,-m)+b_i(-m,m)],$$
$$(A_i)_{oe}=-\tfrac{1}{2}[a_i(m,-m)-a_i(-m,m)+b_i(m,-m)-b_i(-m,m)],$$
$$(A_i)_{oo}=-\tfrac{1}{2}[a_i(m,-m)+a_i(-m,m)-b_i(m,-m)-b_i(-m,m)].$$

The amplitudes not quoted are zero and the a_i, b_i, c_i, d_i are defined by

$$a_1(m_1,m_2)=\frac{1}{8}\left[\tfrac{1}{2}(s-u-4m^2)B_1+\tfrac{1}{2}(s-u+4m^2)B_2-\left(1-\frac{m_1+m_2}{2m}\right)(s-u+8\epsilon m^2)B_3-8(m-m_1)(m-m_2)B_4\right]$$
$$a_2(m_1,m_2)=\frac{1}{8}\left[-\frac{t}{2}(B_1+B_2)+\left(1-\frac{m_1+m_2}{2m}\right)tB_3\right],$$
$$a_3(m_1,m_2)=\frac{1}{8}\frac{t}{2}(B_1-B_2),$$

TABLE I. Born-term contribution to the Green's function in the basis of positive- and negative-parity states.

$$T_{\beta\alpha}=(\mathcal{P}_1-3\mathcal{P}_0)\frac{g^2}{t-\mu^2}\quad \begin{array}{c} \\ (+)\\ (-)\\ (e)\\ (o)\end{array}\begin{array}{cccc} (+) & (-) & (e) & (o)\\ \langle O_5\rangle & \langle O_1\rangle & \frac{1}{\sqrt{2}}\langle O_1^5\rangle & -\frac{1}{\sqrt{2}}\langle O_5^5\rangle \\ \langle O_1\rangle & \langle O_5\rangle & \frac{1}{\sqrt{2}}\langle O_1^5\rangle & \frac{1}{\sqrt{2}}\langle O_5^5\rangle \\ \frac{1}{\sqrt{2}}\langle O_1^5\rangle & \frac{1}{\sqrt{2}}\langle O_1^5\rangle & \langle O_5\rangle+\langle O_1\rangle & 0 \\ -\frac{1}{\sqrt{2}}\langle O_5^5\rangle & \frac{1}{\sqrt{2}}\langle O_5^5\rangle & 0 & \langle O_5\rangle-\langle O_1\rangle \end{array}$$

$$a_4(m_1,m_2)=\frac{1}{8}\left[4B_0-\tfrac{1}{2}(s-u-4m^2)B_1+\tfrac{1}{2}(s-u+4m^2)B_2-4m^2\left(1-\frac{m_1+m_2}{2m}\right)B_3\right],$$

$$a_5(m_1,m_2)=\frac{1}{8}\left[\tfrac{1}{2}(s-u-4m^2)B_1+\tfrac{1}{2}(s-u+4m^2)B_2-\left(1-\frac{m_1+m_2}{2m}\right)(s-u)B_3\right],$$

$$a_6(m_1,m_2)=a_7(m_1,m_2)=0\,,$$

$$a_8(m_1,m_2)=\tfrac{1}{8}m(m_2-m_1)B_3\,.$$

For the b_i we find

$$b_1(m_1,m_2)=\frac{1}{8}\,\frac{s-u}{2}\,(B_2-B_1)\,,\quad b_2(m_1,m_2)=\frac{1}{8}\,\frac{s+u}{2}\,(B_2-B_1)\,,$$

$$b_3(m_1,m_2)=-\tfrac{1}{2}B_0-b_1(m_1,m_2)\,,\quad b_4(m_1,m_2)=-b_2(m_1,m_2)\,,$$

$$b_5(m_1,m_2)=b_1(m_1,m_2)-m_1m_2B_4-m^2B_5\,,\quad b_6(m_1,m_2)=-\tfrac{1}{8}m(m_1+m_2)B_3\,,$$

$$b_7(m_1,m_2)=\tfrac{1}{8}m(m_1-m_2)B_3\,,\quad b_8(m_1,m_2)=0\,.$$

For the c_i and d_i we get

$$c_1(m_1,m_2)=d_1(m_2,m_1)=\tfrac{1}{8}[2\epsilon m m_2B_3-4m_2(m_1-m)B_4]\,,$$

$$c_2(m_1,m_2)=d_2(m_2,m_1)=\tfrac{1}{8}mm_2B_3\,,$$

$$c_3(m_1,m_2)=d_3(m_2,m_1)=\tfrac{1}{8}[-m^2B_2+m(m-m_1)B_3]\,,$$

$$c_4(m_1,m_2)=d_4(m_2,m_1)=\tfrac{1}{8}[-2B_0+\tfrac{1}{2}u(B_2-B_1)]\,,$$

$$c_5(m_1,m_2)=-d_5(m_2,m_1)=c_1(m_1,m_2)\,,$$

$$c_6(m_1,m_2)=-d_6(m_2,m_1)=c_2(m_1,m_2)\,,$$

$$c_7(m_1,m_2)=-d_7(m_2,m_1)=\tfrac{1}{8}[m^2B_1-m(m-m_1)B_3]\,,$$

$$c_8(m_1,m_2)=-d_8(m_2,m_1)=\tfrac{1}{8}[-2B_0+\tfrac{1}{2}s(B_1-B_2)]\,,$$

TABLE II. Nucleon's wave-function renormalization contribution to the Green's function in the basis of positive- and negative-parity states.

$$T_{\beta\alpha}=(\mathcal{P}_1-3\mathcal{P}_0)\frac{2g^2O_S}{t-\mu^2}\begin{bmatrix} 0 & 2\langle O_1\rangle & \frac{1}{\sqrt{2}}\langle O_1^5\rangle & -\frac{1}{\sqrt{2}}\langle O_5^5\rangle \\ 2\langle O_1\rangle & 4\langle O_5\rangle & \frac{3}{\sqrt{2}}\langle O_1^5\rangle & \frac{3}{\sqrt{2}}\langle O_5^5\rangle \\ \frac{1}{\sqrt{2}}\langle O_1^5\rangle & \frac{3}{\sqrt{2}}\langle O_1^5\rangle & 2\langle O_1\rangle+2\langle O_5\rangle & 0 \\ -\frac{1}{\sqrt{2}}\langle O_5^5\rangle & \frac{3}{\sqrt{2}}\langle O_5^5\rangle & 0 & 2\langle O_5\rangle-2\langle O_1\rangle \end{bmatrix}$$

TABLE III. Vertex contribution to the Green's function in the basis of positive- and negative-parity states.

$$T_{\beta\alpha} = (\mathcal{P}_1 - 3\mathcal{P}_0)\frac{g}{t-\mu^2}\begin{bmatrix} 2V_{++}\langle O_5\rangle & 2V_{+-}\langle O_1\rangle & \frac{V_{++}+V_{+-}}{\sqrt{2}}\langle O_1^5\rangle & -\frac{V_{++}+V_{+-}}{\sqrt{2}}\langle O_5^5\rangle \\ 2V_{+-}\langle O_1\rangle & 2V_{--}\langle O_5\rangle & \frac{V_{--}+V_{+-}}{\sqrt{2}}\langle O_1^5\rangle & \frac{V_{--}+V_{+-}}{\sqrt{2}}\langle O_5^5\rangle \\ \frac{V_{++}+V_{+-}}{\sqrt{2}}\langle O_1^5\rangle & \frac{V_{--}+V_{+-}}{\sqrt{2}}\langle O_1^5\rangle & (V_{++}+V_{--})\langle O_5\rangle + 2V_{+-}\langle O_1\rangle & 0 \\ -\frac{V_{++}+V_{+-}}{\sqrt{2}}\langle O_5^5\rangle & \frac{V_{--}+V_{+-}}{\sqrt{2}}\langle O_5^5\rangle & 0 & (V_{++}+V_{--})\langle O_5\rangle - 2V_{+-}\langle O_1\rangle \end{bmatrix}$$

where $\epsilon=-1$ for the direct box and $\epsilon=+1$ for the crossed box.

Finally, the crossed box is given by the same formulas provided that the isospin coefficients are changed ($1\to 5$ for isospin 1, $9\to -3$ for isospin 0), and that we change the B_K's through

$$B_0(s,t)\to -B_0(u,t)\,,\quad B_1(s,t)\to -B_2(u,t)\,,$$
$$B_2(s,t)\to -B_1(u,t)\,,\quad B_3(s,t)\to -B_3(u,t)\,,$$
$$B_4(s,t)\to B_4(u,t)\,,\quad B_5(s,t)\to -B_5(u,t)\,.$$

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