PHYSICAL REVIEW D VOLUME 11, NUMBER 7 1 APRIL 1975

(8,8) chiral-symmetry breaking and the K_{l4}-decay form factors

C. V. Sastry and T. N. Tiwari
Department of Mathematics, Regional Engineering College, Rourkela 769008, Orissa, India

(Received 2 April 1974; revised manuscript received 23 July 1974)

The form factors for K_{l4} decay as well as the s-wave scattering lengths for $K\pi$ scattering have been calculated using the (8, 8) representation of broken chiral symmetry. The results compare favorably with available experimental data and theoretical estimates.

I. INTRODUCTION

Chiral symmetry, first proposed by Gell-Mann[1] more than a decade ago, has proved to be a good approximation for the study of strong interactions. Since so far we have no well-established, exact theory for the strongly interacting particles, we have to extract as much information as we can from the approximate theories, such as chiral symmetry. Therefore, it is important to know the SU(3)⊗SU(3) structure of the symmetry-breaking part of the Hamiltonian density. For many years, the most widely used form of the symmetry-breaking Hamiltonian density was a single representation of SU(3)⊗SU(3), viz., the $(3,\overline{3})\oplus(\overline{3},3)$ representation,[2] and it was fairly successful. However, some recent experimental data have shown the above-mentioned form of symmetry breaking to be unsatisfactory,[3] and interest has shifted to the search for alternate models of symmetry breaking. A very promising alternative seems to be the (8, 8) representation of broken SU(3)⊗SU(3), which has been used by many authors for various calculations.[4] In this paper we report the calculation of the K_{l4}-decay form factors and $K\pi$ scattering lengths using the (8, 8) representation of broken chiral symmetry.

Broken SU(3)⊗SU(3) symmetry has been used to study the effects of symmetry breaking in K_{l4} decay.[5] But these calculations were performed by using the techniques of perturbation theory around the SU(3)⊗SU(3)-symmetric limit and neglecting terms of second and higher order in the symmetry-breaking parameter. However, these perturbative calculations suffer from the drawback that the possibility of an enhancement from the higher-order symmetry-breaking terms cannot altogether be ruled out. In fact, such an enhancement has been shown to occur in the case of K_{l3} decay.[6] Therefore, although the experimental data available at present on the K_{l4}-decay form factors are few in number as well as uncertain due to the presence of large experimental errors, it is important to calculate these form factors by other methods. In the absence of precise experimental data, only such nonperturbative calculations can show us whether the first-order perturbation calculations for K_{l4} decay are really justified.

It was shown by Deshpande[7] that the general low-energy theorems on the K_{l3}-decay form factors can be derived on the basis of the $(3,\overline{3})\oplus(\overline{3},3)$-symmetry-breaking model along with the hypothesis of partial conservation of axial-vector current (PCAC). His results are valid to second order in momenta, but to all orders in the symmetry breaking, provided the PCAC hypothesis is taken to be an exact rule in K_{l3} decays. Recently Bose and Narayanaswamy have extended Deshpande's method to calculate the K_{l4}-decay form factors using the $(3,\overline{3})\oplus(\overline{3},3)$ model.[8] In this paper we follow essentially a similar approach using the (8, 8) representation of broken SU(3)⊗SU(3) to calculate these form factors. Therefore, we assume that the symmetry-breaking Hamiltonian density H' belongs to the (8, 8) representation of SU(3)⊗SU(3) and is given by

$$H' = z_0 + d z_8 . \tag{1}$$

Here d is the symmetry-breaking parameter. The quantities z_0 and z_8 are, respectively, the even-parity SU(3) singlet operator and the $T=Y=0$ member of the SU(3) octet of operators in the SU(3) decomposition of the (8, 8) representation. These operators are written as the products of the SU(3)⊗SU(3) currents, as follows[9]:

$$\begin{aligned} z_0 &= \frac{1}{2\sqrt{2}}\,(V-A)^{\mu}_{\beta}(V+A)_{\mu\beta} , \\ z_8 &= (\tfrac{3}{5})^{1/2} d_{8\alpha\beta}(V-A)^{\mu}_{\alpha}(V+A)_{\mu\beta} , \end{aligned} \tag{2}$$

where

$$\alpha, \beta = 1, 2, 3, \ldots ,$$
$$\mu = 0, 1, 2, 3 .$$

The K_{l3} matrix element plays a crucial role in the analysis of the K_{l4} form factors. Bose and Narayanaswamy have calculated even this K_{l3} matrix element using the $(3,\bar{3})\oplus(\bar{3},3)$ model. But in this paper we shall not employ the (8, 8) model to calculate this K_{l3} matrix element. Instead, we shall employ an approximate expression[5] for it, derived by Dashen and Weinstein[10] and valid to the first order of the symmetry-breaking parameter, regardless of the model of symmetry breaking.

II. K_{l4} DECAY FORM FACTORS

The axial-vector K_{l4} form factors are defined by

$$\langle \pi^+(p)\pi^-(q)|A_\mu^{K^-}|K^+(k)\rangle = \left(\frac{i}{m_K}\right)[(p+q)_\mu F_1+(p-q)_\mu F_2+k'_\mu F_3]\,, \quad (3)$$

where m_K is the mass of the K meson and $k' = (k-p-q)$. The off-shell $K\pi$ scattering amplitude is given by

$$T^{K^+\pi^+}(-q,p,k,k') = \frac{(m_\pi{}^2-p^2)(m_\pi{}^2-q^2)(m_K{}^2-k^2)(m_K{}^2-k'^2)}{F_\pi{}^2F_K{}^2m_\pi{}^2m_K{}^2} \times \int d^4x\,d^4y\exp(-ip\cdot x-iq\cdot y+ik\cdot z)\langle T(D_5^{\pi^-}(x)D_5^{\pi^+}(y)D_5^{K^+}(z)D_5^{K^-}(0))\rangle_0 = A^+-A^-\,, \quad (4)$$

where $A^\pm(s,u,t;p^2,q^2,k^2,k'^2)$ are the t-channel charge-conjugate eigenamplitudes. We assume PCAC:

$$D_5^i(x)\equiv\partial_\mu A_\mu^i(x)=F_i m_i{}^2\phi_i(x)\,, \quad (5)$$

where F_i are the meson decay constants. The decay constants for the π and K mesons will be denoted by F_π and F_K, respectively. The amplitudes $A^\pm$ satisfy the following crossing relations:

$$\begin{aligned} A^\pm(s,u,t;p^2,q^2,k^2,k'^2) &= \pm A^\pm(u,s,t;p^2,q^2,k'^2,k^2) \\ &= \pm A^\pm(u,s,t;q^2,p^2,k^2,k'^2)\,, \end{aligned} \quad (6)$$

where $s=(k-q)^2$, $u=(k-p)^2$, $t=(p+q)^2$. The charges Q^i and Q_5^i, defined by

$$\begin{aligned} Q^i &= \int V_0^i(x)d^3x\,, \\ Q_5^i &= \int A_0^i(x)d^3x\,, \end{aligned} \quad (7)$$

satisfy the well-known $SU(3)\otimes SU(3)$ current commutation relations. By using these commutation relations, along with the symmetry-breaking Hamiltonian defined in Eqs. (1) and (2), we get the following commutation relations to be used later:

$$\begin{aligned} [Q_5^{\pi^-}, D_5^{K^+}] &= \left(\frac{3\sqrt{10}+d}{5d}\right)D^{K^0}\,, \\ [Q_5^{K^+}, D_5^{\pi^-}] &= \left(\frac{3\sqrt{10}+6d}{5d}\right)D^{\bar{K}^0}\,. \end{aligned} \quad (8)$$

In obtaining these relations the $I=\frac{1}{2}$ part only of the commutator has been taken into account. These relations are analogous to the relations[8]

$$\begin{aligned} [Q_5^{\pi^-}, D_5^{K^+}] &= \left(\frac{2\sqrt{2}-c}{3c}\right)D^{K^0}\,, \\ [Q_5^{K^+}, D_5^{\pi^-}] &= \left(\frac{2\sqrt{2}+2c}{3c}\right)D^{\bar{K}^0}\,, \end{aligned} \quad (8')$$

derived for the $(3,\bar{3})\oplus(\bar{3},3)$ model; c being the symmetry-breaking parameter.

By standard reduction techniques, the K_{l4} form factors can be related to the three-particles-on-shell limit of the $I=\frac{3}{2}$ $K\pi$ amplitude:

$$\begin{aligned} A^+(s,u,t;m_\pi{}^2,m_\pi{}^2,m_K{}^2,k'^2)-A^-(s,u,t;m_\pi{}^2,m_\pi{}^2,m_K{}^2,k'^2) &= \frac{(m_K{}^2-k'^2)}{F_Km_K{}^2}\langle\pi^+(p)\pi^-(q)|D_5^{K^-}(0)|K^+(k)\rangle \\ &= \frac{(k'^2-m_K{}^2)}{F_Km_K{}^3}\{k'\cdot(p+q)F_1+k'\cdot(p-q)F_2+k'^2F_3\}\,. \end{aligned} \quad (9)$$

By considering various on-shell limits of the amplitudes defined in Eq. (4) and using the relations given in Eq. (8), we get the following relations:

$$A^+(k^2,m_K{}^2,m_\pi{}^2;m_\pi{}^2,0,k^2,m_K{}^2)=A^-(k^2,m_K{}^2,m_\pi{}^2;m_\pi{}^2,0,k^2,m_K{}^2)\,, \quad (10)$$

$$A^+(m_K{}^2,k^2,m_\pi{}^2;0,m_\pi{}^2,k^2,m_K{}^2)-A^-(m_K{}^2,k^2,m_\pi{}^2;0,m_\pi{}^2,k^2,m_K{}^2) = \frac{-i(m_K{}^2-k^2)}{F_\pi F_K m_K{}^2}\left(\frac{3\sqrt{10}+d}{5d}\right)\langle\pi^-(q)|D^{K^0}|K^-(q-k)\rangle\,, \quad (11)$$

$$A^+(p^2, m_\pi^2, m_K^2; p^2, m_\pi^2, m_K^2, 0) = A^-(p^2, m_\pi^2, m_K^2; p^2, m_\pi^2, m_K^2, 0)\,, \tag{12}$$

$$A^+(m_\pi^2, p^2, m_K^2; p^2, m_\pi^2, 0, m_K^2) - A^-(m_\pi^2, p^2, m_K^2; p^2, m_\pi^2, 0, m_K^2) = \frac{-i(m_\pi^2 - p^2)}{F_\pi F_K m_\pi^2}\left(\frac{3\sqrt{10}+6d}{5d}\right)\langle \pi^-(q)|D^{\bar{K}^0}|K^-(p+q)\rangle\,. \tag{13}$$

Now we utilize the Weinberg-Khuri technique[11] and expand the amplitudes $A^\pm$ in powers of the invariants, retaining terms up to quadratic in the invariants and satisfying crossing symmetry. We write these expansions as

$$A^+ = A + B(s+u) + Ct + D(s+u)^2 + Esu + Ft^2 + Gt(s+u) + H(p^2+q^2)(k^2+k'^2)\,, \tag{14}$$

$$A^- = \alpha(s-u) + \beta t(s-u) + \gamma(s^2-u^2) + \delta(s-u)(p^2+q^2) + \delta'(p^2-q^2)(k^2-k'^2)\,. \tag{15}$$

Assuming the form factors to be constant, one can derive various relations among the form factors and expansion coefficients $A, B, C, \ldots$; $\alpha, \beta, \gamma, \ldots$. These relations are given in Eq. (13) of Ref. 8. Next we come to the K_{l3} matrix element $\langle \pi^-(q)|D^{K^0}|K^-(q-k)\rangle$ given in Eq. (11). Instead of calculating this matrix element in the (8, 8) model and thus risking the introduction of furhter model-dependent uncertainties in the K_{l4}-decay calculations, we shall employ an approximate expression for it, given by Dashen and Weinstein.[10] This expression is valid to first order of the symmetry-breaking parameter, as mentioned earlier, regardless of the model of symmetry breaking. An essential point of the calculation of Dashen and Weinstein is that the meson pole dominance of the divergence of the axial-vector current is valid up to symmetry-breaking terms of second order. According to Dashen and Weinstein,

$$\begin{aligned}\langle \pi^-(q)|D^{K^0}|K^-(q-k)\rangle &\simeq i(a_0 + k^2 a_1)\,,\\ \langle \pi^-(q)|D^{K^0}|K^-(p+q)\rangle &\simeq i(a_0 + p^2 a_1)\,.\end{aligned} \tag{16}$$

The quantities a_0 and a_1 are given by

$$\begin{aligned}a_0 &= (m_K^2 - m_\pi^2)\,,\\ a_1 &= \frac{1}{2}\left(\frac{F_K}{F_\pi} - \frac{F_\pi}{F_K}\right)\,.\end{aligned} \tag{17}$$

These expressions, given in Eqs. (16) and (17) and used in the calculations mentioned in Ref. 5, are valid to first order of the symmetry breaking parameter d, and the corrections are of third order in d.

Combining the relations (16) with the Eqs. (10)-(13) and comparing the coefficients, we get the following relations:

$$\begin{aligned}&A + Bm_K^2 + Cm_\pi^2 + Dm_K^4 + Fm_\pi^4 + m_\pi^2 m_K^2(G+H)\\ &\quad = -(\alpha m_K^2 + \beta m_\pi^2 m_K^2 + \gamma m_K^4 + \delta m_\pi^2 m_K^2 + \delta' m_\pi^2 m_K^2)\\ &\quad = \frac{(3\sqrt{10}+d)m_K^2 a_0}{10 d m_K^2 F_\pi F_K}\,,\end{aligned} \tag{18}$$

$$\begin{aligned}&B + 2Dm_K^2 + Em_K^2 + (G+H)m_\pi^2\\ &\quad = \alpha + \beta m_\pi^2 + m_\pi^2(\delta+\delta')\\ &\quad = \frac{(3\sqrt{10}+d)(a_1 m_K^2 - a_0)}{10 d m_K^2 F_\pi F_K}\,,\end{aligned} \tag{19}$$

$$D = \gamma = \frac{-(3\sqrt{10}+d)a_1}{10 d m_K^2 F_\pi F_K}\,, \tag{20}$$

$$\begin{aligned}&A + Bm_\pi^2 + Cm_K^2 + Dm_\pi^4 + Fm_K^4 + (G+H)m_\pi^2 m_K^2\\ &\quad = -(\alpha m_\pi^2 + \beta m_\pi^2 m_K^2 + \gamma m_\pi^4 + (\delta+\delta')m_\pi^2 m_K^2)\\ &\quad = \frac{(3\sqrt{10}+6d)m_\pi^2 a_0}{10 d m_\pi^2 F_\pi F_K}\,,\end{aligned} \tag{21}$$

$$\begin{aligned}&B + 2Dm_\pi^2 + Em_\pi^2 + (G+H)m_K^2\\ &\quad = \alpha + \beta m_K^2 + (\delta+\delta')m_K^2\\ &\quad = \frac{(3\sqrt{10}+6d)(a_1 m_\pi^2 - a_0)}{10 d m_\pi^2 F_\pi F_K}\,,\end{aligned} \tag{22}$$

$$D = \frac{-(3\sqrt{10}+6d)a_1}{10 d m_\pi^2 F_\pi F_K}\,. \tag{23}$$

From Eqs. (20) and (23) one gets

$$\frac{m_K^2}{m_\pi^2} = \frac{3\sqrt{10}+d}{3\sqrt{10}+6d}\,,$$

which gives the following expression for the symmetry-breaking parameter d[12]:

$$d = -\frac{3\sqrt{10}\,(m_K^2 - m_\pi^2)}{6m_K^2 - m_\pi^2}\,. \tag{24}$$

Using the average values of the observed meson masses in the above relation, we get $d = -1.48$. Finally, eliminating the constants $A, B, C, \ldots$ and $\alpha, \beta, \gamma, \ldots$ from the relations (18)-(23) we obtain the following expressions for the K_{l4}-decay form factors:

$$F_1 = \frac{-m_K(3\sqrt{10}+d)}{5dF_\pi(2m_K^2-m_\pi^2)}\,(2a_0+3a_1m_\pi^2)\,,$$

$$F_2 = \frac{m_K}{F_\pi}\left[\frac{a_0}{m_K^2-m_\pi^2} - \frac{a_1(3\sqrt{10}+6d)}{5d}\right]\,, \tag{25}$$

$$F_3 = \frac{-m_K(3\sqrt{10}+d)}{5dF_\pi(2m_K^2-m_\pi^2)}\,[a_0+a_1(m_\pi^2+m_K^2)]\,.$$

For numerical calculations, we take the average values of the observed meson masses. For the decay constants, we take $F_K/F_\pi = 1.28$ and $F_\pi = 0.95m_\pi$. Then Eq. (17) gives

$$a_0 = 0.2272\ \text{GeV}^2\,,\qquad a_1 = 0.2494\,. \tag{26}$$

Using these values of a_0 and a_1 in Eqs. (25), we get the following values for the K_{l4}-decay form factors:

$$F_1 \simeq 4.08\,,\qquad F_2 \simeq 3.88\,,\qquad F_3 \simeq 2.55\,. \tag{27}$$

It is interesting to compare these results with those obtained for the $(3,\overline{3})\oplus(\overline{3},3)$ model. Wienke and Deshpande[13] have calculated the K_{l4} decay low-energy axial-vector form factors using the techniques of current algebra and the $(3,\overline{3}) \oplus (\overline{3},3)$ model. Their average values are

$$\left.\begin{aligned}\overline{F}_1 &= 3.97\\ \overline{F}_2 &= 4.01\end{aligned}\right\}\ \text{for}\ F_K/F_\pi = 1.33\,,\qquad \left.\begin{aligned}\overline{F}_1 &= 4.22\\ \overline{F}_2 &= 4.11\end{aligned}\right\}\ \text{for}\ F_K/F_\pi = 1.23\,. \tag{28}$$

The results of Bose and Narayanaswamy[8] for the same model are $F_1 \simeq 3.56$, $F_2 \simeq 3.4$, and $F_3 \simeq 2.5$. Lane has calculated[5] the K_{l4} form factors in the $(3,\overline{3})\oplus(\overline{3},3)$ model, using the perturbative treatment and the model-independent approximate expressions given in Eqs. (16) and (17) for the K_{l3} form factors. His results are $F_1 = F_2 = 3.95$ and $F_3 = -10.1\,(k'\cdot p)/(m_K^2 - k'^2)$. On the other hand, Weinberg obtained $F_1 = F_2 = 3.7$ from current-algebra calculations.[14] By fitting the experimental data, Berends *et al.* find[15] the following values for F_1 and F_2:

$$F_1 = 5.6 \pm 0.6\,,\qquad F_2 = 5.5 \pm 1.2\,. \tag{29}$$

By comparing the various numerical estimates of the K_{l4}-decay form factors given above, it is observed that the values of these form factors obtained from the (8, 8) model are only slightly different from those given by other calculations. The present results are similar to those of Wienke and Deshpande and somewhat higher than those of Bose and Narayanaswamy[8] for the $(3,\overline{3})\oplus(\overline{3},3)$ model and those of Weinberg[14] for current algebra, but almost the same as those of Lane[5] for the perturbation calculations.

The currently available experimental data on the K_{l4}-decay form factors are not only scant but also uncertain due to the presence of large experimental errors. Consequently, no serious comparison of theoretical calculations with the experimental data is possible at this stage; all such comparisons should be regarded as tentative at best. However, a glance at the various values of the form factors given above indicates that the present calculations of the K_{l4} form factors using the (8, 8) model of broken chiral symmetry are in better agreement with the available experimental data than other theoretical calculations. Thus, the (8, 8) model seems to be a promising alternative to the popular $(3,\overline{3})\oplus(\overline{3},3)$ model of Gell-Mann, Oakes, and Renner.

III. $K\pi$ SCATTERING LENGTHS

Weinberg has estimated the $\pi\pi$ scattering lengths[11] using low-energy theorems from SU(2) $\otimes$ SU(2) current algebra and the PCAC (partial conservation of axial-vector current) self-consistency conditions. For this purpose it was assumed that a linear expansion of the amplitudes in Mandelstam invariants is approximately valid up to threshold. Griffith used similar techniques to estimate the s-wave $K\pi$ and KK scattering lengths[16] in the $(3,\overline{3}) \oplus (\overline{3},3)$ model. In this section we estimate the s-wave $K\pi$ scattering lengths in the (8, 8) model following an approach similar to that of Griffith.

There are two independent s-wave scattering lengths in the $I=\frac{1}{2}$ and $\frac{3}{2}$ states, to be denoted here by $a^{1/2}$ and $a^{3/2}$, respectively. The momenta of the incoming (outgoing) K and π will be denoted by p and q (p' and q'), respectively. Other useful kinematical quantities are $s=(p+q)^2$, $t=(p'-p)^2$, and $u=(p-q')^2$. We shall extrapolate to threshold a linear expansion of the amplitude $A^I(s,t,u;q^2,q'^2,p^2,p'^2)$ in terms of the invariants $s,t,\ldots,$ assuming that there are no $J^P=0^+$ bound states and that unitarity effects do not lead to rapid variations of the amplitude at low energies.

Now we write the definite isospin s-channel amplitudes in terms of amplitudes with definite t-channel charge-conjugation properties satisfying the crossing relations

$$A^{3/2} = A^+ - A^-\,,\qquad A^{1/2} = A^+ + 2A^-\,, \tag{30}$$

where

$$A^{\pm}(s,t,u;q^2,q'^2,p^2,p'^2)$$
$$=\pm A^{\pm}(u,t,s;q'^2,q^2,p^2,p'^2)$$
$$=\pm A^{\pm}(u,t,s;q^2,q'^2,p'^2,p^2). \quad (31)$$

Using the t-channel crossing symmetry given by Eq. (31) and the kinematical condition $s+t+u=q^2+q'^2+p^2+p'^2$, we write the linear expansions with constant coefficients as follows:

$$A^{+}(s,t,\ldots)=A_1+B_1(s+u)+C_1t+D_1(p^2+p'^2),$$
$$A^{-}(s,t,\ldots)=A'(s-u). \quad (32)$$

Here the subscript 1 has been attached to the expansion coefficients in order to distinguish them from those of Sec. II.

In order to evaluate the expansion coefficients, we consider low-energy limits in current-commutator identities of the form

$$F^{ij}(s,t,\ldots)=q'^{\mu}q^{\nu}T^{ij}_{\mu\nu}-q'^{\mu}\int d^4x\, e^{iq'\cdot x}\langle p'|\delta(x_0)[A^i_\mu(x),A^j_0(0)]|p\rangle$$
$$+i\int d^4x\, e^{iq'\cdot x}\langle p'|\delta(x_0)[A^i_0(x),\partial^\nu A^j_\nu(0)]|p\rangle, \quad (33)$$

where

$$F^{ij}=-i\int d^4x\, e^{iq'\cdot x}\langle p'|T\{\partial^\mu A^i_\mu(x),\partial^\nu A^j_\nu(0)\}|p\rangle,$$
$$T^{ij}_{\mu\nu}=-i\int d^4x\, e^{iq'\cdot x}\langle p'|T\{A^i_\mu(x),A^j_\nu(0)\}|p\rangle, \quad (34)$$

are Fourier transforms of axial-vector current and divergence time-ordered products. Together with the normalization condition

$$\langle \pi^i|\partial^\mu A^i_\mu|0\rangle=m_i^2F_i/\sqrt{2}$$

and

$$F_\pi=\sqrt{2}\,MG_A/g_{\pi NN},$$

we assume the following relation between the off-mass-shell boson-boson amplitude A^{ij} and the amplitude F^{ij} for low-energy applications:

$$A^{ij}(s,t,\ldots)=\frac{(q'^2-m_i^2)(q^2-m_j^2)}{(m_i^2F_i/\sqrt{2})(m_j^2F_j/\sqrt{2})}F^{ij}(s,t,\ldots). \quad (35)$$

Now we take various low-energy limits in Eq. (32) for, say, $A^{3/2}=A(K^+\pi^+\to K^+\pi^+)$.

(1) $q\to 0$ (or $q'\to 0$). Keeping the other three particles on their mass shells (Adler's PCAC consistency condition), we have

$$A^{3/2}\to A^{3/2}(m_K^2,m_\pi^2,m_K^2;0,m_\pi^2,m_K^2,m_K^2)=0,$$

which gives

$$A_1+2m_K^2(B_1+D_1)+m_\pi^2C_1=0. \quad (36)$$

(2) $q',q\to 0$ with K's on the mass shell. In this case

$$A^{3/2}\to A^{3/2}(m_K^2+2p\cdot q,0,m_K^2-2p\cdot q;0,0,m_K^2,m_K^2)$$
$$=[A_1+2m_K^2(B_1+D_1)]-4(p\cdot q)A'+O(q^2,q'^2,q\cdot q')$$
$$=(2i/F_\pi^2)\langle K^+(p)|[A^{\pi^-},\partial^\mu A^{\pi^+}_\mu]|K^+(p)\rangle$$
$$+2(p\cdot q)/F_\pi^2+O(q\cdot q')$$

and

$$A'=-1/2F_\pi^2, \quad (37)$$

$$A_1+2m_K^2(B_1+D_1)=(6/F_\pi^2)\langle K^+(p)|\{[1+(\tfrac{2}{5})^{1/2}d]z_0+[\tfrac{6}{5}+(\tfrac{2}{5})^{1/2}d]z_8\}|K^+(p)\rangle. \quad (38)$$

Evaluating the right-hand side of Eq. (38) in the low-energy limit $p'\to 0$ we get, with the value $d=-1.48$ obtained in the previous section,

$$A_1+2m_K^2(B_1+D_1)=\frac{(6/F_\pi^2)\{(\frac{1}{2})^{1/2}[1-\frac{3}{5}(\frac{2}{5})^{1/2}]+(\frac{1}{5})^{1/2}[1-(\frac{1}{2})^{1/2}]d\}m_K^2}{\sqrt{2}-(\frac{1}{5})^{1/2}d}$$
$$=4.278\left(\frac{m_K^2}{F_\pi^2}\right). \quad (39)$$

(3) $p\to 0$ (or $p'\to 0$). Keeping the pions on the mass shell and using low-energy limits for kaons, we have

$$A^{3/2}\to A^{3/2}(m_\pi^2,m_K^2,m_\pi^2;m_\pi^2,m_\pi^2,m_K^2,0)=0,$$

which gives

$$A_1 + 2m_\pi^2 B_1 + m_K^2(C_1 + D_1) = 0\,. \tag{40}$$

(4) $p, p' \to 0$.

$$A^{3/2} \to A^{3/2}(m_\pi^2 + 2p\cdot q,\, 0,\, m_\pi^2 - 2p\cdot q;\, m_\pi^2, m_\pi^2, 0, 0) = A_1 + 2m_\pi^2 B_1 - 4(p\cdot q)A'$$
$$= (2i/F_K^2)\langle \pi^+(q)\,|\,[A^{K^-}, \partial^\mu A_\mu^{K^+}]\,|\,\pi^+(q)\rangle + 2(p\cdot q)/F_K^2\,,$$

and in the low-energy limit $q' \to 0$ we have

$$A' = -1/2F_K^2\,, \tag{41}$$

$$A_1 + 2m_\pi^2 B_1 = \frac{(6/F_K^2)\{(\frac{1}{2})^{1/2}[1 - (\frac{1}{10})^{1/2} d] - (\frac{1}{5})^{1/2}[(\frac{1}{10})^{1/2} - \frac{1}{2}d]\}m_\pi^2}{\sqrt{2} - 2(\frac{1}{5})^{1/2} d}$$
$$= 26.85\left(\frac{m_\pi^2}{F_K^2}\right)\,. \tag{42}$$

Comparing Eq. (37) and (41), we observe that simultaneous π and K low-energy limits would require the approximately valid equality $F_\pi = F_K$.

The expansion coefficients A_1, B_1, C_1, D_1, and A' can be obtained by solving five independent equations: (36), (37), (39), (40), and (42). The s-wave scattering lengths are given by

$$-8\pi(m_K + m_\pi)a^I = A^I((m_K + m_\pi)^2, 0, (m_K - m_\pi)^2; m_\pi^2, m_\pi^2, m_K^2, m_K^2)\,. \tag{43}$$

At threshold

$$A_{\rm th}^+ = A_1 + 2(m_K^2 + m_\pi^2)B_1 + 2m_K^2 D_1$$
$$= \frac{26.85(4\pi)m_\pi}{(m_K^2/m_\pi^2) - 1}\,L\,, \tag{44}$$

$$A_{\rm th}^- = 4\pi m_K m_\pi A'$$
$$= -8\pi m_K L\,, \tag{45}$$

where

$$L = m_\pi / 4\pi F^2 \simeq 0.11 m_\pi^{-1}\,.$$

Making use of these values of $A_{\rm th}^{\pm}$ in Eq. (30) and the average values of meson masses, we get

$$a^{1/2} = 2\left(\frac{m_K}{m_K + m_\pi}\right)L - \frac{13.43}{(m_K^2/m_\pi^2) - 1}\left(\frac{m_\pi}{m_K + m_\pi}\right)L$$
$$\simeq 2\left(\frac{m_K}{m_K + m_\pi}\right)L - 0.24L$$
$$\simeq 0.19 m_\pi^{-1}\,, \tag{46}$$

$$a^{3/2} = -\left(\frac{m_K}{m_K + m_\pi}\right)L - \frac{13.43}{(m_K^2/m_\pi^2) - 1}\left(\frac{m_\pi}{m_K + m_\pi}\right)L$$
$$\simeq -\left(\frac{m_K}{m_K + m_\pi}\right)L - 0.24L$$
$$\simeq -0.14 m_\pi^{-1}\,. \tag{47}$$

These results for the $K\pi$ scattering lengths are similar to the theoretical estimates of Griffith[16] for the $(3, \bar{3}) \oplus (\bar{3}, 3)$ model, which are

$$a^{1/2} = 2\left(\frac{m_K}{m_K + m_\pi}\right)L \simeq 2L \simeq 0.22 m_\pi^{-1}\,,$$
$$a^{3/2} = -\left(\frac{m_K}{m_K + m_\pi}\right)L \simeq -L \simeq -0.11 m_\pi^{-1}\,. \tag{48}$$

We observe that these expressions can be obtained from Eqs. (46) and (47) if the rather small second term on the right-hand sides of the latter expressions is neglected.

ACKNOWLEDGMENTS

The authors wish to express their thanks to Professor S. P. Misra for many helpful discussions and comments. One of us (T.N.T.) is grateful to the Ministry of Education, Government of India for the financial support.

[1]M. Gell-Mann, Phys. Rev. 125, 1067 (1962); Physics (N.Y.) 1, 63 (1964).

[2]M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968); S. Glashow and S. Weinberg, Phys. Rev. Lett. 20, 224 (1968).

[3]See, for example, A. Sirlin and M. Weinstein, Phys. Rev. D 6, 3588 (1972); T. P. Cheng and R. Dashen, Phys. Rev. Lett. 26, 594 (1971); G. Altarelli, N. Cab-

ibbo, and L. Maiani, Phys. Lett. 35B, 415 (1971).
[4]See, for example, H. Genz, J. Katz, and H. Steiner, Phys. Rev. D 7, 2100 (1973); J. Brehm, Nucl. Phys. B34, 269 (1971); R. Dashen, Phys. Rev. D 3, 1979 (1971); H. Genz and J. Katz, *ibid.* 6, 908 (1972).
[5]S. P. de Alwis, Phys. Rev. D 1, 2131 (1970); K. D. Lane, *ibid.* 2, 2703 (1970).
[6]S. P. de Alwis, Phys. Rev. D 2, 1346 (1970).
[7]N. G. Deshpande, Phys. Rev. D 2, 569 (1970).
[8]S. Bose and P. Narayanaswamy, Prog. Theor. Phys. 47, 237 (1972).
[9]A. Sirlin and M. Weinstein, Phys. Rev. D 6, 3588 (1972).
[10]R. Dashen and M. Weinstein, Phys. Rev. Lett. 22, 1337 (1969).
[11]S. Weinberg, Phys. Rev. Lett. 17, 616 (1966); N. N. Khuri, Phys. Rev. 153, 1477 (1967).
[12]It may be observed that the expression for d obtained here is not the same as could be obtained from the expression for z given by Genz and Katz in Ref. 4 because of the approximate nature of Eq. (8). However, the closeness of the values gives a justification for the approximation involved. The quantities d and z may be related by the equation $d=(\frac{5}{8})^{1/2}/z$.
[13]B. R. Wienke and N. G. Deshpande, Phys. Rev. D 5, 725 (1972).
[14]S. Weinberg, Phys. Rev. Lett. 17, 336 (1966).
[15]F. A. Berends, A. Donnachie, and G. C. Oades, Nucl. Phys. B3, 569 (1967).
[16]R. W. Griffith, Phys. Rev. 176, 1705 (1968).