PHYSICAL REVIEW D VOLUME 11, NUMBER 7 1 APRIL 1975

Deep-inelastic structure functions in an approximation to the bag theory*

R. L. Jaffe

Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 25 November 1974)

A cavity approximation to the bag theory developed earlier is extended to the treatment of forward virtual Compton scattering. In the Bjorken limit and for small values of ω ($\omega = |2p \cdot q/q^2|$) it is argued that the operator nature of the bag boundaries might be ignored. Structure functions are calculated in one and three dimensions. Bjorken scaling is obtained. The model provides a realization of light-cone current algebra and possesses a parton interpretation. The structure functions show a quasielastic peak. The spreading of the structure functions about the peak is associated with confinement. As expected, Regge behavior is not obtained for large ω. The "momentum sum rule" is saturated, indicating that the hadron's charged constituents carry all the momentum in this model. νW_L is found to scale and is calculable. Application of the model to the calculation of spin-dependent and chiral-symmetry-violating structure functions is proposed. The nature of the intermediate states in this approximation is discussed. Problems associated with the cavity approximation are also discussed.

I. INTRODUCTION

In the bag theory[1,2] the hadron is taken to be an extended region of space (the bag) containing quark fields. The quarks are confined to the bag by a universal pressure, B, but are only weakly coupled to one another (by vector gluons) inside. It seems clear that such a hadron should allow a parton interpretation and exhibit Bjorken scaling. Indeed these considerations were fundamental in motivating the model. The calculation of current matrix elements has so far proved prohibitively difficult—even in one space dimension where a quantum theory exists[1]—so a rigorous calculation of deep-inelastic structure functions and verification of scaling is not yet possible. In an earlier paper[2] a semiclassical approximation was developed which allowed us to estimate current matrix elements at or near zero momentum transfer. Here I wish to extend the approximation scheme of Ref. 2 to study deep-inelastic scattering structure functions.

Bjorken scaling is obtained in this approximation. However, this should not be taken as a derivation of scaling in the bag theory since the relation of my approximation to a true quantum theory of bags is not understood. Here a parton (or light-cone) interpretation of the bag will be developed and the lore of parton models (or light-cone algebra) carried over to help explicate the bag theory. One can see, for example, how confinement modifies the structure function of a free particle. Furthermore, the semiclassical calculations provide a framework in which more realistic approximations can and will be discussed.

The approximation developed in Ref. 2 and applied here ignores quantum fluctuations. Furthermore, it is necessary for me to treat the final state (in the electroproduction amplitude) in a way which violates momentum conservation. Nevertheless, the deep-inelastic phenomenology which emerges from these calculations supports the intuitive picture of scaling which originally motivated the bag theory. At present, it is impossible to gauge whether quantum fluctuations and a proper treatment of the final state (which are closely related) will obliterate this simple picture. Calculations in progress[3] will reflect on this question but are outside the scope of this paper.

The paper is organized as follows. Section II contains a brief review of the general bag theory[1] and of the semiclassical quark model based on the bag.[2] Readers familiar with the theory should ignore this section. In Sec. III I extend the semiclassical approximation to the Compton scattering problem and develop the general framework for structure-function calculations. Section IV is devoted to the one-dimensional problem where all calculations may be performed analytically as a check on some of the approximations made in Section III. The real three-dimensional problem is treated in Sec. V. W_1, νW_2, and (where appropriate) νW_3 are calculated explicitly, various sum rules and relations among structure functions are verified, and parton distribution functions are extracted from the structure functions. In Sec. VI several further applications of this approach are listed. In Sec. VII the principal results of the paper are summarized and the approximations forced upon us in Sec. III are discussed further. Appendixes A and B deal with the definition of the structure function in the bag theory, with gauge invariance, and with details of the three-dimensional calculation.

II. REVIEW OF THE CLASSICAL BAG THEORY AND SEMICLASSICAL QUARK BAG MODEL

Before proceeding, it is necessary to review several features of the general bag theory[1] and of the semiclassical quark model based on the bag.[2] A bag is a finite region of space which contains fields, which I will take to be the fields of colored quarks.[4] The quark field does not exist outside of the bag: It is confined (in a Lorentz-invariant way) by endowing the space it occupies with a constant, universal energy per unit volume, B. The Hamiltonian of the bag consists of a field term and a term arising from the pressure B,

$$E=\int_{\text{bag}} d^3x\, T_0^0(\text{fields})+BV\,. \tag{1}$$

A bag with fixed energy cannot become too large, lest the potential energy BV exceed E. Allowed classical motions are those for which the field pressure balances the universal pressure B at each point on the bag's surface. No energy or momentum flows across the bag's surface. Clearly, the shape of a bag cannot be fixed *a priori* independent of the field inside: The pressures in general would not balance. Formally, this is reflected by the fact that the variables which describe the bag's surface are dynamical variables of constraint which are determined by the field degrees of freedom.

Equations of motion for the bag are developed from an action principle in Ref. 1. An alternative beginning with the stress-energy tensor is described in Ref. 5. One obtains both equations of motion for the field inside the bag and boundary conditions at the bag's surface. For the case of a massless Dirac field, noninteracting within the bag, these are

$$(1/i)\gamma^\mu\,\partial_\mu q_\alpha(x)=0 \text{ inside the bag,} \tag{2a}$$

$$i\gamma^\mu n_\mu q_\alpha(x)=q_\alpha(x) \text{ on the surface,} \tag{2b}$$

$$n^\mu\partial_\mu\sum_\alpha \bar{q}_\alpha(x)q_\alpha(x)=2B \text{ on the surface.} \tag{2c}$$

Here α is an internal symmetry index, e.g., color or SU(3), and n_μ is the covariant normal to the spacetime hypertube swept out by the bag. Equations (2) determine both the field $q_\alpha(x)$ and the boundary of the bag.

Although the theory as it stands confines quarks to finite regions of space, it does not remove quark quantum numbers from the spectrum (a one-quark bag is allowed). To do this,[1] colored gluons must be added to the bag and coupled in a Yang-Mills fashion to the quarks. Then application of Gauss's law shows that, regardless of the strength of the quark-gluon coupling, only color-singlet bags exist. If the gluons are too strongly coupled they will presumably ruin Bjorken scaling. I will therefore assume that they are weakly coupled and ignore them for the moment, except to restrict my considerations to color singlets.[6]

Equations (2) admit exact, classical solutions where the bag's boundary is a sphere of fixed radius, R_0. In Ref. 2 these solutions were exploited to construct a semiclassical model of the hadron. This model is equivalent to solving Dirac's equation Eq. (2a) in a spherical cavity of fixed radius R_0, subject to the boundary condition Eq. (2b), except that the radius R_0 is fixed by the quadratic boundary condition Eq. (2c). The result is a set of quantum modes characterized by the quantum numbers of Dirac's equation,[7] j, κ, and m, and by frequencies $\omega_{n\kappa j}$. Equation (2c) not only fixes R_0 but also limits j to be $\frac{1}{2}$. The solutions are specified as follows: The Dirac field is

$$q_\alpha(\vec{x},t)=\sum_{n>0,\ \kappa=\pm1,\ m=\pm1/2} N(n\kappa)\{b_\alpha(n\ \kappa\ j=\tfrac{1}{2}\ m)\psi_{n\kappa j=\frac{1}{2}m}(\vec{x},t)+d_\alpha^\dagger(n\ \kappa\ j=\tfrac{1}{2}\ m)\psi_{-n-\kappa j=\frac{1}{2}m}(\vec{x},t)\}\,, \tag{3}$$

where $b_\alpha^\dagger$ and $d_\alpha^\dagger$ create quark and antiquark excitations with wave functions $\psi_{n\kappa jm}(\vec{x},t)$ in the bag. $N(n\kappa)$ is a normalization constant[8]

$$N(n\kappa)\equiv\left(\frac{\omega_{n\kappa}{}^3}{2R_0{}^3(\omega_{n\kappa}+\kappa)\sin^2\omega_{n\kappa}}\right)^{1/2}, \tag{4}$$

and the wave functions ψ are defined by (e.g., for $\kappa=-1$)

$$\psi_{n-1\frac{1}{2}m}(\vec{x},t)=\frac{1}{\sqrt{4\pi}}\begin{pmatrix} ij_0\left(\dfrac{\omega_{n-1}|\vec{x}|}{R_0}\right)U_m \\ -j_1\left(\dfrac{\omega_{n-1}|\vec{x}|}{R_0}\right)\vec{\sigma}\cdot\hat{x}\,U_m\end{pmatrix} \times e^{-i\omega_{n-1}t/R_0}\,. \tag{5}$$

$j_l(z)$ are spherical Bessel's functions; U_m are two-component Pauli spinors. The $\kappa=+1$ solutions are higher frequency and are not needed here. The allowed frequencies are determined by Eq. (2b) and satisfy

$$\tan\omega_{n\kappa} = -\frac{\omega_{n\kappa}}{\omega_{n\kappa}+\kappa}\ . \tag{6}$$

The lowest frequency is $\omega_{1-1}=2.04$. Equations (3)–(5) could be (and indeed have been[9]) developed without reference to the bag. The bag and B enter essentially via the quadratic boundary condition

$$4\pi B R_0^4 = \sum_{\alpha n\kappa m} \omega_{n\kappa}[b_\alpha^\dagger(n\kappa m)b_\alpha(n\kappa m) + d_\alpha^\dagger(n\kappa m)d_\alpha(n\kappa m)]\ , \tag{7}$$

which determines *the operator* R_0 in terms of the field inside. The *parameter*, R_0, which occurs in Eqs. (4) and (5) is interpreted as the expectation value of R_0 in the state in question. This is the heart of the semiclassical approximation and limits our attention to diagonal matrix elements. Finally the semiclassical Hamiltonian is given by a virial theorem

$$E = 4B\langle V\rangle = \tfrac{16}{3}\pi B R_0^3\ , \tag{8}$$

in conjunction with Eq. (7). Since the virial theorem applies only to bags at rest Eq. (8) is also the mass operator.

Since only the lowest mode ($n=1$, $\kappa=-1$) will be of interest, I will use a somewhat abbreviated notation:

$$\begin{aligned} &\epsilon \equiv \omega_{1,-1} = 2.04\ ,\\ &N \equiv N\ \ (n=1,\ \kappa=-1)\ ,\\ &\psi_m(\vec{x},t) \equiv \psi_{1,-1,\frac{1}{2},m}(\vec{x},t)\ ,\\ &b_\alpha(m) \equiv b_\alpha(n=1,\kappa=-1,m)\ . \end{aligned} \tag{9}$$

III. COMPTON SCATTERING IN THE CAVITY APPROXIMATION

In Ref. 2 the bag was treated as a spherical cavity of fixed radius. The quantum modes in the cavity were populated with colored quarks to construct hadrons with the correct quantum numbers. The radius is not a free parameter but fixed by the field excitation in accordance with Eq. (7). For brevity I shall refer to this as the "cavity approximation."

The cross section for inelastic lepton scattering is given by the imaginary part of the appropriate forward Compton scattering amplitude as shown in Fig. 1. Viewing the leptoproduction process as forward Compton scattering—that is, as the response of a hadron at rest to the correlated product of local currents—allows me to use the cavity

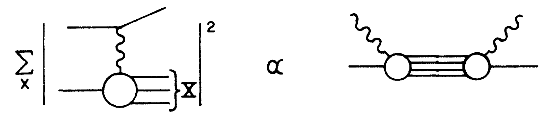

FIG. 1. The electroproduction cross section as the imaginary part of forward virtual Compton scattering.

approximation. The picture I have in mind is shown in Fig. 2. The virtual photon momentum in q_μ, with $\omega \equiv -2p\cdot q/q^2$, $\nu \equiv -p\cdot q$, and $\xi \equiv 1/\omega$. There the spherical cavity of radius R_0 is probed by currents at arbitrary points $(\vec{x}_1, t_1)$, $(\vec{x}_2, t_2)$ in its interior. The currents I use are the cavity currents

$$J_\mu^i(\vec{x},t) = \sum_a :\bar{q}_a(\vec{x},t)\gamma_\mu\lambda^i q_a(\vec{x},t):\ , \tag{10}$$

where λ^i are conventional SU(3) matrices and $q_a(\vec{x},t)$ are the fields of Eq. (3), with R_0 treated as a c number. The sum on conventional SU(3) is implicit; the index a denotes color degrees of freedom only.

Clearly, this approximation ignores the possible dynamical role of the bag's boundary. It is not possible to gauge the validity of the approximation. Some insight may be obtained by estimating the values of $|\vec{x}_1-\vec{x}_2|$ which are important in the Bjorken limit. It is well known[10] that the current correlation becomes lightlike in the Bjorken limit. As $q^2\to\infty$, $|\vec{x}_1-\vec{x}_2|^2-(t_1-t_2)^2\to 0$. Also the variable $P\cdot(x_1-x_2)$ is conjugate to ω,[11]

$$P\cdot(x_1-x_2)\sim\omega\ , \tag{11}$$

where P_μ is the target four-momentum. Combining these I obtain (in the target rest frame)

$$M|\vec{x}_1-\vec{x}_2|\sim\omega \tag{12}$$

in the Bjorken limit. When $|\vec{x}_1-\vec{x}_2|$ exceeds $2R_0$ the cavity approximation does not make sense: Both currents cannot act within the static bag.

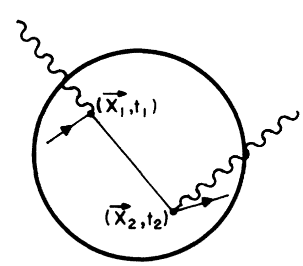

FIG. 2. Currents scattering off quanta in a static spherical cavity.

$|\vec{x}_1 - \vec{x}_2| = 2R_0$ corresponds to

$$\omega_{max} \cong 2MR_0 \cong 14\,. \tag{13}$$

I have used the value of R_0 for the proton found in Ref. 2: $R_0 = 1.4$ fm. Conversely, for ω substantially less than ω_{max} the currents are more closely correlated in coordinate space and may act within the bag relatively far from the boundaries. In this regime I propose to treat the boundary as a c number. One field operator in $J_\mu(\vec{x}_1, t_1)$ destroys a cavity quantum at $\vec{x}_1$ and t_1; one field operator from $J_\mu^i(\vec{x}_1, t_1)$ and another from $J_\nu^i(\vec{x}_2, t_2)$ form a cavity propagator,

$$\{q_\alpha(\vec{x}_1, t_1), \bar{q}_{\alpha'}(\vec{x}_2, t_2)\} \equiv S_{cav}(\vec{x}_1, \vec{x}_2; t_1 - t_2)\delta_{\alpha,\alpha'}\ , \tag{14}$$

which propagates the excitation to $(\vec{x}_2, t_2)$ where the quantum is recreated by the remaining field operator. In principle, the structure function may be directly calculated. This will be done for the case of a one-dimensional bag in the next section. In three dimensions the calculation of $S_{cav}(\vec{x}_1, \vec{x}_2, t_1 - t_2)$ seems prohibitively difficult. S_{cav} differs from the usual free-space, free-field anticommutator function $S(x_1 - x_2)$ only by virtue of the cavity boundary condition Eq. (2b). If the boundaries are indeed negligible in the Bjorken limit for small ω then it should be possible to replace S_{cav} by S within the bag.

In one dimension the structure functions can be computed using either propagator: The results are identical[12] to leading order in the Bjorken limit.[13] Stated simply, the light-cone singularity of the propagator is not altered by boundary conditions. This is an important check on the cavity approximation: If the propagator in the Bjorken limit depended on the boundaries it would be difficult to accept a treatment which ignores their dynamical structure. In three dimensions the free-space propagator must be used without the explicit verification that it gives the same result as the cavity propagator.

Many effects of using the cavity approximation may be estimated from general considerations. If the quarks were not confined their structure functions would be δ functions in $\xi \equiv 1/\omega$. The larger a bag, the more quanta it contains, the less important is the confinement. Therefore, one expects structure functions which go over into δ functions in the semiclassical (large quantum number) limit.

As $\omega \to \infty$ all values of $|\vec{x}_1 - \vec{x}_2|$ are probed. With the spatial integrals cut off at $r = R_0$ they are uniformly convergent and the structure functions will be found to be analytic in ξ even at $\xi = 0$. Since $F_1(\xi)$ (for electroproduction) is odd in ξ it will vanish at least like ξ for small ξ:

$$F_1(\xi) \underset{\xi \to 0}{\sim} \xi\,. \tag{15}$$

[$F_1(\xi)$ is the Bjorken limit of $W_1(q^2, \nu)$ which is defined conventionally as in Eq. (B11).] Conventional Regge theory leads to the expectation that $F_1(\xi)$ is not analytic at $\xi = 0$. Specifically, one expects

$$F_1(\xi) \underset{\xi \to 0}{\sim} |\xi|^{-\alpha(0)} \epsilon(\xi)\,,$$

with $\alpha(0) \leq 1$. There are several ways this might arise in refinements of the cavity approximation. In a realistic theory the bag's surface must fluctuate. Fluctuations will enhance the structure function for small ξ (small ξ corresponds to large $|\vec{x}_1 - \vec{x}_2|$) and concomitantly reduce it at larger values of ξ. Conserved-quantum-number sum rules such as Adler's[14] (which are valid in the cavity approximation, see below) fix the area under $F_1(\xi)$. Whether fluctuations are sufficient to induce the spatial integrals to diverge as $\xi \to 0$ depends on the magnitude of individual fluctuations and on the possibility of a divergence in the sum over all the fluctuations to which the photon might couple. Another (more conventional) possibility[15] is that $F_1(\xi)$ is enhanced at small ξ owing to high-momentum quark-antiquark pairs polarized out of the vacuum inside the bag by the colored gluons. In this case gluon-induced logarithmic violations of Bjorken scaling would have to be treated at the same time. In the cavity model pairs are associated with fluctuation in the bag's surface: If one tried to construct a state of three quarks and a color-singlet pair $[qqq(\bar{q}q)_0]$ the quadratic boundary condition Eq. (2c) could not be satisfied with a fixed radius[2]—the surface must fluctuate. In any case, the absence of Regge terms in the bag structure function, which would be ascribed to the absence of a sea of pairs in a parton language, is also associated with the absence of pairs in the cavity approximation.

Returning to the shape of $F_1(\xi)$ in the bag theory, it is well known[16] that present data require $F_1(\xi)$ to be large near $\xi = 0$ to saturate sum rules such as Adler's. Since the cavity structure functions saturate the sum rule but vanish at $\xi = 0$ they must exceed the present data for values of ξ away from zero:

$$F_1^{bag}(\xi) > F_1^{expt}(\xi) \text{ for } \xi \text{ not near zero.} \tag{16}$$

So far I have systematically ignored the state intermediate between absorption and emission of the virtual photon, which corresponds to the final state in lepton scattering. The cavity current scatters a quantum in the target into a highly excited mode in a cavity of the same radius (R_0). According to Eq. (7) this is not the radius appropriate to the new state. In fact, in three dimensions the intermediate state in general will not satisfy the quadratic boundary condition for any fixed radius because of the angular dependence introduced into Eq. (2c). So the intermediate state is not simple: It is a complicated superposition of fluctuating (perhaps many-hadron) states.

The approximation I have made is an adiabatic one: I assume the photon is reemitted before its influence propagates to the bag's boundaries. The validity of Eq. (2c) is a prerequisite for momentum conservation. If it is not satisfied there is unbalanced pressure on the cavity walls. Therefore, momentum is not, in general, conserved in the intermediate state. Energy is conserved since the model is time-translation invariant. The absence of momentum conservation destroys the spectral properties of the structure functions: They do not vanish below the physical threshold $2\nu=-q^2$. In practice this is a small effect. Outside the physical region the functions are an order of magnitude below their maximum. In principle it is a disturbing artifact of the approximation.

The quantitative formulation begins with the current correlation function, defined for the bag theory by

$$W_{\mu\nu}^{ij}\equiv\frac{M}{2\pi}\int_{-\infty}^{\infty}dt\int_{\text{bag}}d^3x_1\int_{\text{bag}}d^3x_2 e^{iq^0t-i\vec{q}\cdot(\vec{x}_1-\vec{x}_2)}\langle T|[J_\mu^i(\vec{x}_1,t),J_\nu^j(\vec{x}_2,0)]|T\rangle\,. \tag{17}$$

This expression is derived from the conventional definition of $W_{\mu\nu}^{ij}$ in Appendix A. In Eq. (17) the target state $|T\rangle$ is normalized to unity,

$$\langle T|T\rangle=1\,, \tag{18}$$

and the spatial integrals are restricted to the bag:

$$|\vec{x}_1|\le R_0,\quad |\vec{x}_2|\le R_0\,. \tag{19}$$

Parenthetically, one can see the sort of difficulty associated with a rigorous calculation of current matrix elements where the integrals in Eq. (17) would have operator-valued limits.

If the bag boundary receded to infinity, Eq. (17) would go over into the conventional $W_{\mu\nu}$ of local quantum field theory, where the structure function of a free Dirac particle is a δ function in ξ. Confinement eliminates frequencies from the δ function smearing out the structure function. Before evaluating $W_{\mu\nu}$ for the physical case of three dimensions, it is enlightening to study the one-dimensional analog.

IV. STRUCTURE FUNCTIONS IN ONE DIMENSION

A cavity approximation may be developed in one as well as three dimensions. Here the static bag is a line segment of length $2l$ centered, for convenience, at the origin. Equations analogous to Eqs. (2) have the following solutions:

$$q_\alpha(x,t)=\frac{1}{2\sqrt{l}}\sum_{n\ge 0} b_\alpha(n)\begin{pmatrix} e^{-in'\pi(t-x)/2l} \\ (-1)^n e^{-in'\pi(t+x)/2l}\end{pmatrix} + d_\alpha^\dagger(n)\begin{pmatrix} e^{in'\pi(t-x)/2l} \\ (-1)^n e^{in'\pi(t+x)/2l}\end{pmatrix}, \tag{20}$$

where $n'\equiv(n+\frac{1}{2})$. I use off-diagonal γ matrices:

$$\gamma^0=\begin{pmatrix}0&1\\1&0\end{pmatrix}\text{ and }\gamma'=\begin{pmatrix}0&-1\\1&0\end{pmatrix}. \tag{21}$$

The quadratic boundary condition fixes l:

$$4Bl^2=\pi\sum_{n\ge 0,\alpha} n'(b_\alpha^\dagger(n)b_\alpha(n)+d_\alpha^\dagger(n)d_\alpha(n))\,. \tag{22}$$

The virial theorem in two dimensions implies

$$M=2B(2l)\,, \tag{23}$$

so

$$M^2=4\pi B\sum_{n\ge 0,\alpha} n'(b_\alpha^\dagger(n)b_\alpha(n)+d_\alpha^\dagger(n)d_\alpha(n))\,. \tag{24}$$

The operators b_α and d_α obey canonical anticommutation relations

$$\{b_\alpha(n_1),b_{\alpha'}^\dagger(n_2)\}=\{d_\alpha(n_1),d_{\alpha'}^\dagger(n_2)\}=\delta_{\alpha\alpha'}\delta_{n_1,n_2}\,, \tag{25}$$

with all other anticommutators vanishing.

Unfortunately, the Compton amplitude vanishes in the Bjorken limit in one dimension for scattering off Dirac quanta. There is no transverse amplitude and the longitudinal one vanishes via the Callan-Gross[17] relation. Instead I use a scalar current,

$$J^i(x,t)\equiv\sum_a :\bar{q}_a(x,t)\lambda^i q_a(x,t):\,. \tag{26}$$

Only the sum over color indices is kept explicit. Conventional SU(3) indices are summed implicitly. The structure function is defined in analogy with Eq. (17),

$$W^{ij}=\frac{M}{2\pi}\int_{-\infty}^{\infty}dt\int_{-l}^{l}dx_1\int_{-l}^{l}dx_2\, e^{iq^0t-iq^1(x_1-x_2)}\langle T|[J^i(x_1,t),J^j(x_2,0)]|T\rangle\,. \tag{27}$$

The commutator may be expanded in terms of the fields

$$[J^i(x_1,t),J^j(x_2,0)]=\sum_a[\bar{q}_a(x_1,t)\lambda^i S_{\text{cav}}(x_1,x_2,t)\lambda^j q_a(x_2,0)-\bar{q}_a(x_2,0)\lambda^j S_{\text{cav}}(x_2,x_1,-t)\lambda^i q_a(x_1,t)]\,, \tag{28}$$

and the cavity propagator may be calculated explicitly from Eqs. (20) and (25),

$$S_{\text{cav}}(x_1,x_2,t)\equiv\{q(x_1,t),\bar{q}(x_2,0)\}=\frac{1}{4l}\sum_n\begin{pmatrix}(-1)^n e^{-in'\pi(t-x_1-x_2)/2l} & e^{-in'\pi(t-x_1+x_2)/2l}\\ e^{-in'\pi(t+x_1-x_2)/2l} & (-1)^n e^{-in'\pi(t+x_1+x_2)/2l}\end{pmatrix}\,. \tag{29}$$

The sum in Eq. (29) ranges over all positive and negative n. For the target state I will take a state of N_0 quanta in the mode $n=0$, N_1 in the mode $n=1$, etc. I assume that the color group is chosen such that these form a color singlet.

To reach the Bjorken limit define

$$q^1=q^0+M\xi \tag{30}$$

and let $q^0\to\infty$. Straightforward calculation then yields

$$\lim_{\text{Bj}}W^{ij}(q^0,\xi)=\frac{M}{4}\sum_{m,a}\left\{\sum_n\delta\left(q^0+\frac{\pi}{2l}(m-n)\right)\langle T|b_a^\dagger(m)\lambda^i\lambda^j b_a(m)|T\rangle\frac{\sin^2[\pi(m+\frac{1}{2})-Ml\xi]}{[\pi(m+\frac{1}{2})-Ml\xi]^2}\right.$$
$$\left.-\sum_n\delta\left(q^0-\frac{\pi}{2l}(m-n)\right)\langle T|b_a^\dagger(m)\lambda^j\lambda^i b_a(m)|T\rangle\frac{\sin^2[\pi(m+\frac{1}{2})+Ml\xi]}{[\pi(m+\frac{1}{2})+Ml\xi]^2}\right\}. \tag{31}$$

The index a denotes color; SU(3) indices are summed implicitly. The δ functions enforce energy conservation: W vanishes unless q^0 equals the energy difference of two cavity modes. Realistically these δ functions would be smoothed out by the finite widths of the intermediate states and by the experimental acceptance. Here I will define a "smeared" structure function $\overline{W}$ by averaging W over a Gaussian,

$$\overline{W}(\bar{q}^0,\xi)\equiv\int\frac{dq^0}{\Delta\sqrt{\pi}}e^{-(q^0-\bar{q}^0)^2/\Delta^2}W(q^0,\xi)\,,$$

where I assume $\pi/2l\ll\Delta$; i.e., many resonances are smeared. Finally the quantity Ml may be eliminated from Eq. (31) by using Eqs. (23) and (24),

$$Ml=\pi\sum_m N_m(m+\tfrac{1}{2})\equiv\pi\Lambda\,, \tag{32}$$

leaving the fundamental result of this section

$$\lim_{\text{Bj}}\overline{W}^{ij}(\bar{q}^0,\xi)=\frac{\Lambda}{2}\sum_{m,a}\left\{\frac{\sin^2[\pi(m+\frac{1}{2}-\Lambda\xi)]}{\pi^2(m+\frac{1}{2}-\Lambda\xi)^2}\langle T|b_a^\dagger(m)\lambda^i\lambda^j b_a(m)|T\rangle-\frac{\sin^2[\pi(m+\frac{1}{2}+\Lambda\xi)]}{\pi^2(m+\frac{1}{2}+\Lambda\xi)^2}\langle T|b_a^\dagger(m)\lambda^j\lambda^i b_a(m)|T\rangle\right\}. \tag{33}$$

Equation (33) displays Bjorken scaling explicitly:

$$\lim_{\text{Bj}}\overline{W}^{ij}(\bar{q}^0,\xi)=\overline{F}^{ij}(\xi)\,.$$

Crossing, implicit in Eq. (27), is now manifest:

$$\overline{F}^{ij}(\xi)=-\overline{F}^{ji}(-\xi)\,.$$

Several assertions of Sec. III are borne out explicitly: (1) As a number of quanta increases ($\Lambda\to\infty$ and the bag grows large) the structure function becomes a sum of δ functions appropriate to free quarks; (2) $\overline{F}^{ij}(\xi)$ vanishes as $\xi\to 0$ and is analytic there ["Regge" behavior in one dimension leads to the expectation $\overline{F}(\xi)\sim 1/\xi$ as $\xi\to 0$]; (3) $\overline{F}^{ij}(\xi)$ does not vanish for $|\xi|\geq 1$; however, the importance of the region $|\xi|>1$ decreases with increasing Λ. The importance of $|\xi|>1$ is some measure of the reliability of this treatment and is discussed further below.

The second term in Eq. (33) raises some deep questions. For certain choices of λ^i and λ^j (e.g., "neutrino" scattering quantum numbers, $\lambda^i=\lambda^-$, $\lambda^j=\lambda^+$) it causes $F^{ij}(\xi)$ to be *negative* in apparent violation of the positivity restrictions $F^{ij}(\xi)$. Also, it corresponds in the parton model to antipartons with negative distribution functions (see below). The second term corresponds to the z graph shown in Fig. 3(b). Note that these terms persist even if the simple product had been used rather than the commutator in Eq. (27). Usually such graphs would be excluded by momentum conservation for spacelike q^2. Lacking momentum conservation it is not possible to exclude them. They are negative as a consequence of Fermi statistics. The z graphs are large only near $\xi = 0$, where the cavity approximation is not applicable anyway.

Of course $F^{ij}(\xi)$ should be positive. Equation (33) can be negative because I have subtracted out the cavity-vacuum bubbles shown in Fig. 3(c). In a conventional field theory these graphs vanish for q^2 spacelike. In the cavity approximation they do not. Furthermore, in conventional field theories they are totally disconnected and therefore are to be subtracted out. Such questions regarding the vacuum cannot be studied in the semiclassical cavity approximation since the vacuum state is not within the scope of the approximation. I have discarded these graphs in this analysis and in the three-dimensional calculations in the next section. Whether this is a correct procedure can be determined only in a more realistic quantum treatment of the currents. Once the bubbles have been subtracted out $F^{ij}(\xi)$ is no longer necessarily positive.

To display the dependence of the structure function on Λ, $\overline{F}(\xi)$ is graphed in Fig. 4 for $N_0 = 1$, 2, 3, and 4 with all other modes unoccupied, and

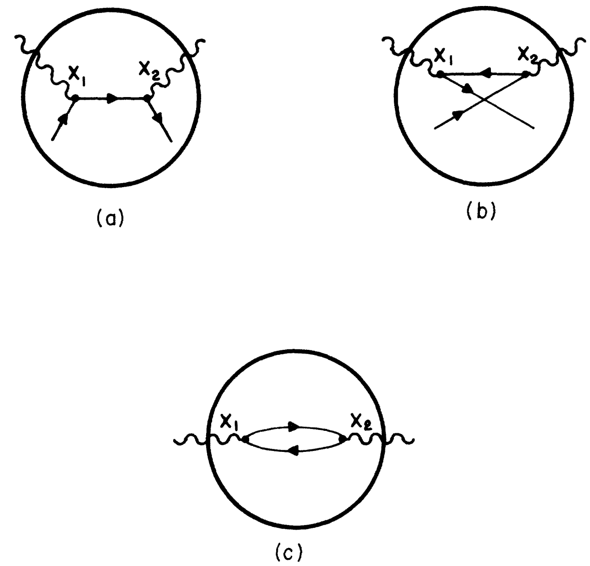

FIG. 3. Diagrams for Compton scattering in the cavity approximation to the bag.

$$\sum_a \langle T | b_a^\dagger(0)\lambda^i\lambda^j b_a(0) | T\rangle = \sum_a \langle T | b_a^\dagger(0)\lambda^j\lambda^i b_a(0) | T\rangle = 1\,.$$

The fact that $\overline{F}(\xi)$ vanishes at $\xi = 1$ for $N_0 = 3$ is not significant—it need not vanish in this approximation.

Sum rules place important constraints on the shape and magnitude of structure functions. More-

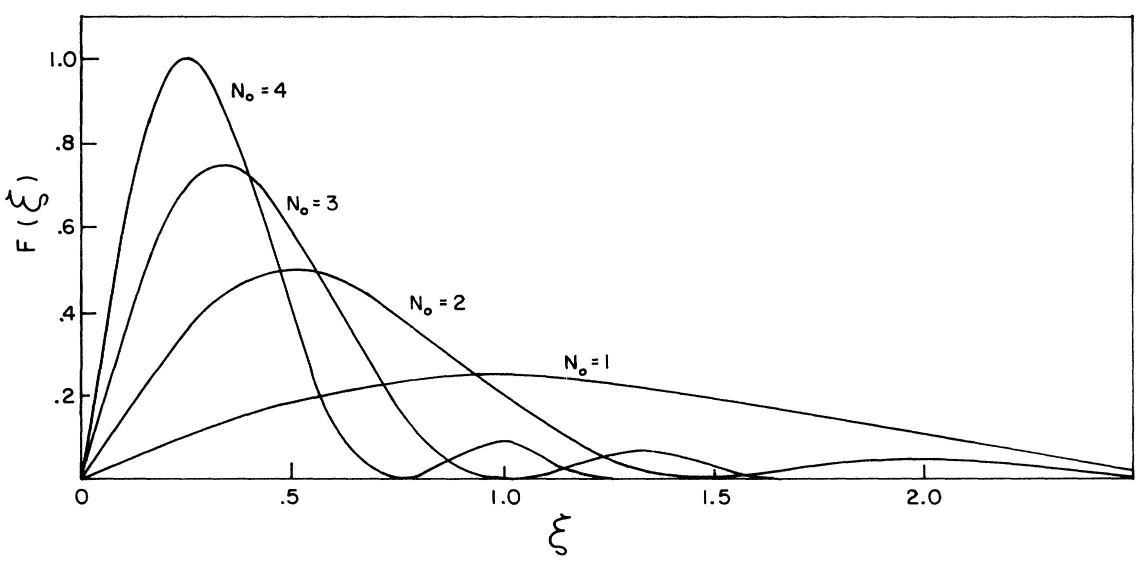

FIG. 4. Structure functions in one dimension as a function of the number of quanta in the target.

over, they provide some checks on both my approximations and algebra. From the definition, Eq. (27) of W^{ij}, it is easy to show that

$$\int_0^\infty d\xi\, \overline{F}^{[i,j]}(\xi) = \tfrac{1}{2} \sum_{a,m} \langle T | b_a^\dagger(m) [\lambda^i, \lambda^j]\, b_a(m) | T \rangle , \tag{34}$$

the archetype of the Adler and other conserved-quantum-number sum rules.[18] Likewise, it follows from Eq. (27) that

$$\int_0^\infty \xi\, d\xi\, \overline{F}^{\{+,-\}}(\xi) = \frac{1}{M} \int_{-l}^{l} dx\, T_0^0(\text{fields}) . \tag{35}$$

$T^{\mu\nu}$(fields) is the quark contribution to the stress-energy tensor, as distinct from the bag contribution (in my approximation there is no gluon term), and $\lambda^\pm = \frac{1}{2}(\lambda_1 \pm i\lambda_2)$. In one dimension the right-hand side of Eq. (35) is $\frac{1}{2}$. It is trivial to verify that Eq. (33) satisfies both sum rules.

In Eqs. (34) and (35) ξ must be integrated from zero to infinity in order to project out the equal-time commutator in Eq. (27). $F(\xi)$ is expected to be zero for $\xi > 1$, but in the cavity approximation it is not. A measure of the approximation is the saturation of the sum rules over the interval [0, 1]. Table I displays this information.

The sum rules guide the development of a parton interpretation of the structure function. The parton distributions must be normalized consistent with Eq. (34). This leads to the following assignments:

$$\begin{aligned} \overline{F}^{+-}(\xi) &= \tfrac{1}{2}[U(\xi) + \overline{D}(\xi)] , \\ \overline{F}^{-+}(\xi) &= \tfrac{1}{2}[D(\xi) + \overline{U}(\xi)] , \end{aligned} \tag{36}$$

where $U(\xi)$, $D(\xi)$, etc., would be the probability distributions of up and down quarks, etc., in an infinite-momentum frame. Purely on parton considerations a momentum sum rule can be derived:

$$2\int_0^\infty \xi\, d\xi\, \overline{F}^{\{+,-\}}(\xi) = \left(\frac{P(\text{fields})}{P} \right)_\infty .$$

The integral sums the longitudinal momenta of the partons. For the bag in the cavity approximation comparison with Eq. (35) yields

$$\left(\frac{P(\text{fields})}{P} \right)_\infty = 1 .$$

All the momentum is on the fields. My earlier[5] conjecture that the bag carried momentum in the infinite-momentum frame in proportion to the energy it carries in the rest frame is wrong.

Finally I turn to the question of replacing the cavity propagator, Eq. (29), by the free propagator. A review of the steps leading from Eq. (29) to Eq. (31) reveals that the diagonal terms in the cavity propagator do not contribute in the Bjorken limit. Effectively,

$$S_{\text{cav}}(x_1, x_2, t) \to \frac{1}{8l} \sum_n \left[(\gamma^0 + \gamma^1) e^{-in'\pi(t + x_1 - x_2)/2l} + (\gamma^0 - \gamma^1) e^{-in'\pi(t - x_1 + x_2)/2l} \right]$$

in the Bjorken limit. The sum on n yields a periodic (cavity) δ function

$$S_{\text{cav}}(x_1, x_2, t) \to \tfrac{1}{2}\left[(\gamma^0 + \gamma^1)\delta_P(t + x_1 - x_2) + (\gamma^0 - \gamma^1)\delta_P(t - x_1 + x_2) \right] , \tag{37}$$

where

$$\delta_P(\lambda) \equiv e^{-i\pi\lambda/4l} \sum_{m=-\infty}^{\infty} \delta(\lambda + 4ml) . \tag{38}$$

On the other hand, the free Dirac propagator in one dimension is merely

$$S(x_1, x_2, t) = \tfrac{1}{2}\left[(\gamma^0 + \gamma^1)\delta(t + x_1 - x_2) + (\gamma^0 - \gamma^1)\delta(t - x_1 + x_2) \right] . \tag{39}$$

$S_{\text{cav}}(x_1, x_2, t)$ of Eq. (37) yields the "discrete" structure function, Eq. (31). Substitution of $S(x_1, x_2, t)$ yields directly the smeared structure function, Eq. (33). This is expected: A discrete final state yields a discrete structure function; a continuum approximation yields a continuous structure function. S_{cav} and S yield the same structure functions in the Bjorken limit, supporting the contention that the boundaries are unimportant insofar as the internal propagation is concerned. In three dimensions the calculation from $S_{\text{cav}}(\vec{x}_1, \vec{x}_2, t)$ is quite difficult—I shall use $S(\vec{x}_1, \vec{x}_2, t)$ from the outset.

TABLE I. Saturation of the Adler and momentum sum rules over the physical region $0 \le \xi \le 1$ as a function of the number of quanta in the target, N_0, in one dimension.

N_0	$\dfrac{\int_0^1 d\xi\, F_1^{\bar{\nu}p - \nu p}(\xi)}{\int_0^\infty d\xi\, F_1^{\bar{\nu}p - \nu p}(\xi)}$	$\dfrac{\int_0^1 \xi\, d\xi\, F_1^{\bar{\nu}p + \nu p}(\xi)}{\int_0^\infty \xi\, d\xi\, F_1^{\bar{\nu}p + \nu p}(\xi)}$
1	45.1%	20.4%
2	85.2%	73.4%
3	92.6%	85.7%
4	94.5%	89.2%

V. STRUCTURE FUNCTIONS IN THREE DIMENSIONS

The starting point is Eq. (17) for $W_{\mu\nu}^{ij}$. I consider only vector currents here. The extension to chiral currents is straightforward and is quoted. Performing the commutator and introducing the *free* propagator

$$S(\vec{x}_1-\vec{x}_2,t)=-(2\pi)^{-3}\int d^4k\, \not k\epsilon(k^0)\delta(k^2)\times e^{-i\vec{k}\cdot(\vec{x}_1-\vec{x}_2)+ik^0t} \qquad (40)$$

I obtain

$$W_{\mu\nu}^{ij}=M(2\pi)^{-4}\sum_a\int d^4k\, k^\rho\epsilon(k^0)\delta(k^2)\int_{-\infty}^{\infty}dt\int_{\rm bag}d^3x_1\int_{\rm bag}d^3x_2 e^{i(k^0+q^0)t-(\vec{k}+\vec{q})\cdot(\vec{x}_1-\vec{x}_2)}$$
$$\times[S_{\mu\rho\nu\sigma}\langle T|\bar{q}_a(\vec{x}_1,t)\gamma^\sigma\lambda^i\lambda^j q_a(\vec{x}_2,0)-\bar{q}_a(\vec{x}_2,0)\gamma^\sigma\lambda^j\lambda^i q_a(\vec{x}_1,t)|T\rangle$$
$$-i\epsilon_{\mu\rho\nu\sigma}\langle T|\bar{q}_a(\vec{x}_1,t)\gamma^\sigma\gamma^5\lambda^i\lambda^j q_a(\vec{x}_2,0)+\bar{q}_a(\vec{x}_2,0)\gamma^\sigma\gamma^5\lambda^j\lambda^i q_a(\vec{x}_1,t)|T\rangle], \qquad (41)$$

where I have used

$$\gamma_\mu\gamma_\rho\gamma_\nu=S_{\mu\rho\nu\sigma}\gamma^\sigma+i\epsilon_{\mu\rho\nu\sigma}\gamma^\sigma\gamma^5,$$
$$S_{\mu\rho\nu\sigma}\equiv g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\rho}-g_{\mu\nu}g_{\rho\sigma}.$$

A priori, it is not obvious that $W_{\mu\nu}^{ij}$ given by Eq. (41) is a gauge-invariant symmetric tensor (when evaluated between spin-averaged target states). In Appendix B this is shown to be true to leading order in the Bjorken limit. In particular the antisymmetric term vanishes, $W_L(q^2,\nu)$ goes to zero in the Bjorken limit, and the coefficient of $g_{\mu\nu}$ is $W_1(q^2,\nu)$.

Equation (41) may be simplified considerably. Indeed, it may be reduced to a single integral over the angle between $\vec{k}$ and $\vec{q}$, which has to be performed numerically. The algebraic reduction is also carried out in Appendix B for the case in which the target consists only of quanta in the lowest cavity mode. Other more complicated target states present no additional difficulties. The result is as follows:

$$\lim_{\rm Bj}W_1^{ij}(q^2,\nu)\equiv F_1^{ij}(\xi)$$
$$=\frac{\Delta\epsilon^2}{4\pi(\epsilon-1)}\left\{\int_{\beta_-}^{\infty}\beta d\beta\left[t_{00}{}^2(\epsilon,\beta)+t_{11}{}^2(\epsilon,\beta)-\frac{2\epsilon}{\beta}(1-\Lambda\xi)t_{00}(\epsilon,\beta)t_{11}(\epsilon,\beta)\right]\right.$$
$$\left.\times\sum_{m,a}\langle T|b_a^\dagger(m)\lambda^i\lambda^j b_a(m)|T\rangle-(\epsilon\to-\epsilon;i\to j)\right\}, \qquad (42)$$

where

$$\beta_-=|\Lambda\xi|\epsilon|-\epsilon|, \qquad (43)$$

$$\Lambda\equiv\tfrac{4}{3}\sum_{m,a}\langle T|b_a^\dagger(m)b_a(m)|T\rangle, \qquad (44)$$

and

$$t_{00}(\epsilon,\beta)=(\epsilon^2-\beta^2)^{-1}[(\epsilon-1)j_0(\beta)-\beta y_0(\beta)], \qquad (45)$$

$$t_{11}(\epsilon,\beta)=(\epsilon^2-\beta^2)^{-1}[\beta j_0(\beta)-\epsilon j_1(\beta)], \qquad (46)$$

where j_0, j_1, and y_0 are spherical Bessel's functions. ϵ and the operators $b_a(m)$ are defined in Eq. (9). Equation (44) is a reminder of the approximations I have made: Λ occurs in the mass of the target and its radius, R_0, both of which have been treated as c numbers equal to their expectation values throughout this calculation.

Bjorken scaling is manifest in Eq. (42): The integrals are uniformly convergent and are only a function of ξ. Actual evaluation of $F_1^{ij}(\xi)$ was carried out on a computer. Structure functions for targets consisting of various numbers of quarks are shown in Fig. 5. The matrix elements are taken to be unity:

$$\sum_{m,a}\langle T|b_a^\dagger(m)\lambda^i\lambda^j b_a(m)|T\rangle=\sum_{m,a}\langle T|b_a^\dagger(m)\lambda^j\lambda^i b_a(m)|T\rangle=1.$$

Note the expected absence of Regge behavior $[F_1(\xi)\sim\xi$ as $\xi\to 0]$ and the failure of $F_1(\xi)$ to vanish for $\xi>1$. Neutrino and antineutrino structure functions are shown in Fig. 6.

Explicit calculation verifies that the structure functions obey the Adler and momentum sum rules:

$$\int_0^\infty d\xi[F_1^{\bar{\nu}p}(\xi)-F_1^{\nu p}(\xi)]=1, \qquad (47)$$

$$\int_0^\infty \xi d\xi[F_1^{\bar{\nu}p}(\xi)+F_1^{\nu p}(\xi)]=1. \qquad (48)$$

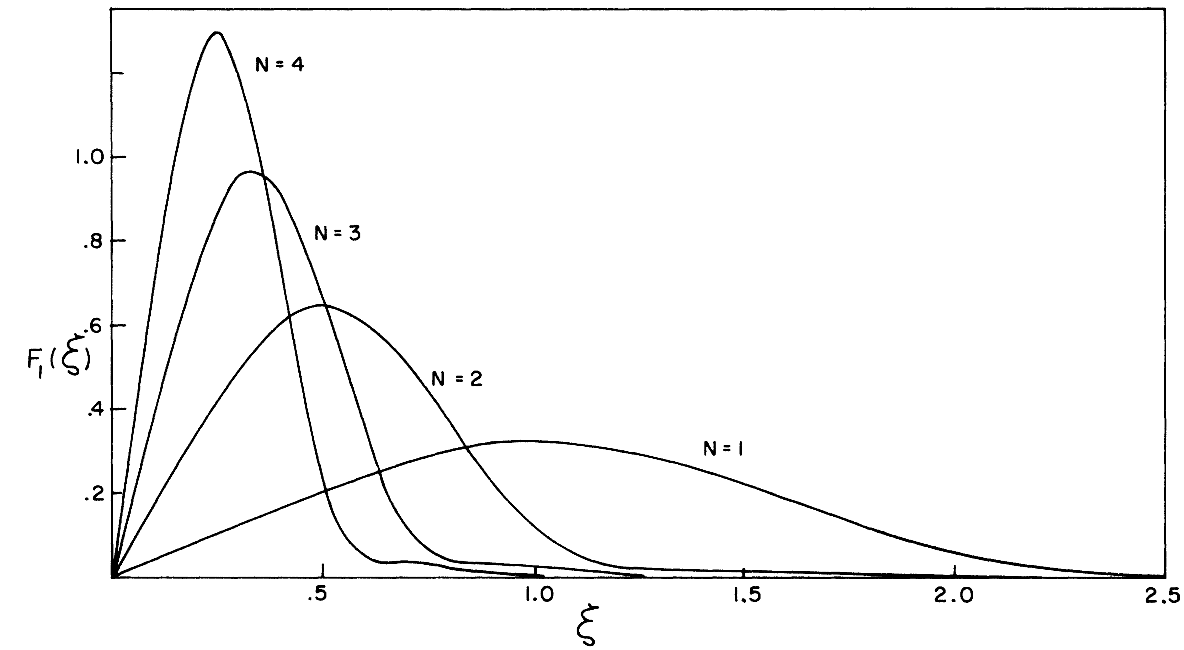

FIG. 5. Structure functions in three dimensions as a function of the number of quanta in the target. $N=3$ should correspond to physical electroproduction.

Again: *The parton interpretation of the momentum sum rule indicates that the bag carries no momentum in the parton model sense.*

For the purpose of discussing the dependence of the structure function on the quantum numbers of the current (i and j) consider the abbreviated notation

$$F_1^{ij}(\xi) = f_+(\xi)\langle\lambda^i\lambda^j\rangle - f_-(\xi)\langle\lambda^j\lambda^i\rangle \tag{49}$$

for Eq. (42), where

$$f_+(\xi) = -f_-(-\xi).$$

Since, as already mentioned, W_L vanishes in the Bjorken limit the Callan-Gross relation is obtained:

$$F_2^{ij}(\xi) = 2\xi F_1^{ij}(\xi). \tag{50}$$

Explicit calculation yields the expected form for $F_3^{ij}(\xi)$:

$$F_3^{ij}(\xi) \equiv \lim_{\mathrm{Bj}} \frac{\nu}{M^2} W_3(q^2, \nu)$$

$$= 2f_+(\xi)\langle\lambda^i\lambda^j\rangle + 2f_-(\xi)\langle\lambda^j\lambda^i\rangle. \tag{51}$$

Any structure function can be constructed from $f_\pm(\xi)$ which are graphed in Fig. 7. Effective parton distributions are found to be

$$U(\xi) = 2D(\xi) = 4f_+(\xi),$$

$$\overline{U}(\xi) = 2\overline{D}(\xi) = -4f_-(\xi).$$

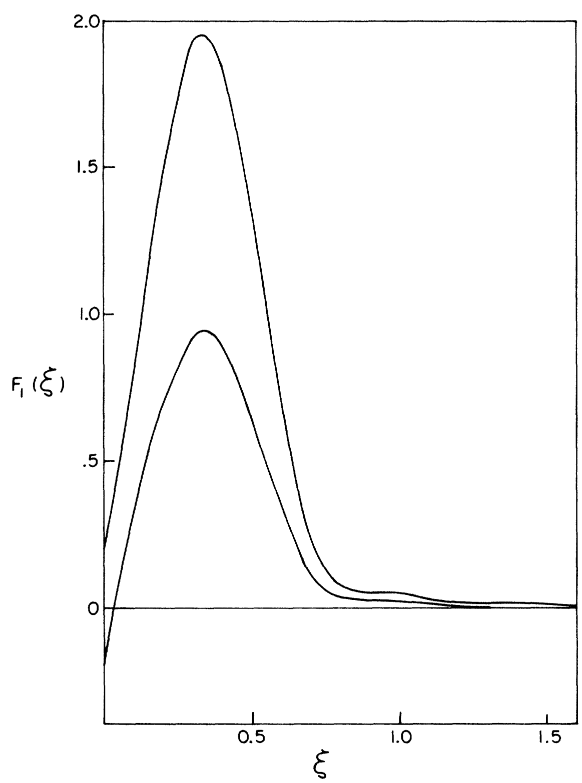

FIG. 6. $F_1(\xi)$ for $\overline{\nu}p$ (the upper curve) and νp scattering in three dimensions.

As in one dimension negative antiparton distributions are an artifact of the cavity and adiabatic approximations.

Also as in one dimension the saturation of the Adler and momentum sum rules in the physical region $0 \leq \xi \leq 1$ may be taken to be a measure of the extent of my approximations. Table II displays this information.

VI. FURTHER APPLICATIONS

Over the past few years much work has been done to relate deep-inelastic structure functions to other important characteristics of the hadrons. The model I have developed is approximate but well-defined. All of the various structure functions and associated sum rules which haunt the light-cone (or parton) literature may be evaluated in the cavity approximation to the bag theory. Sometimes this will yield predictions of as yet unmeasured quantities [e.g., (3) below]. Elsewhere, the insight developed in light-cone (LC) physics may lead to a better understanding of the bag model [e.g., (2) below]. Several illustrations follow.

(1) In Appendix B it is shown that $(\nu/M^2)W_L$ (W_L is the longitudinal structure function) scales in the Bjorken limit. $R = \sigma_L/\sigma_T$ therefore vanishes like $1/q^2$ in the model and the scaling function q^2R can be calculated explicitly.

(2) The chiral-symmetry-violating structure functions of neutrino scattering are sensitive to the boundary conditions (which violate chiral symmetry in the bag model[1]) so the cavity approximation is less reasonable. However, light-cone analysis relates them[19,20] to important static parameters of chiral-symmetry violation. Since chiral symmetry is rather obscure in the bag theory I believe interest outweighs caution in this case.

(3) Spin-dependent structure functions are calculable and are interesting in themselves.

(4) $J = 0$ fixed-pole sum rules[21] may be studied. The hypothesis of polynomial residues[22] can be tested via the CCN-RR[21,23] sum rule.

(5) Furthermore, the asymptotic parts of weak and electromagnetic radiative corrections to hadronic amplitudes may be calculated. For example, the log-divergent term in the Cottingham[24] formula is known to be related to a fixed pole in W_L.[25,20] Recently Chodos and Thorn have conjectured[26]—and proved under certain circumstances—that quark electromagnetic self-masses are finite in cavity models in which the quarks inside the cavity have no bare mass. This requires zero residue for the appropriate fixed pole in W_L. The approach I have outlined employs an idealized (free-field) propagator. The relation between this and the work of Chodos and Thorn is yet to be established.

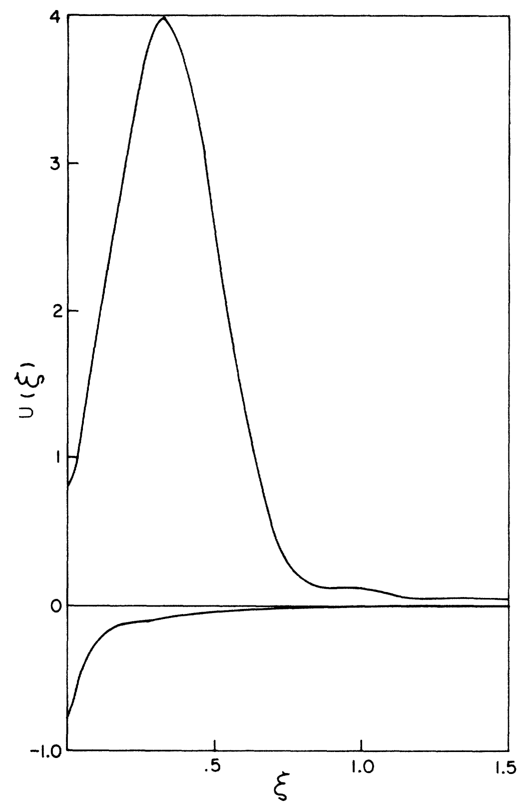

FIG. 7. Parton distribution functions: Upper curve is $U(\xi)$, lower is $\bar{U}(\xi)$.

(6) The fixed-pole sum rule for the σ term[27,19] (in pion-nucleon scattering) may be evaluated, although the PCAC identification of the pion with the divergence of the axial-vector current remains obscure.

(7) The structure functions of various targets (e.g., π, ρ, Δ^{++}, etc.) may be calculated, though the shapes should not be taken any more seriously than the shape of the nucleon structure function.

TABLE II. Saturation of the Adler and momentum sum rules over the physical region $0 < \xi \leq 1$ as a function of the number of quanta in the target, N, in three dimensions.

N	$\dfrac{\int_0^1 d\xi F_1^{\bar{\nu}p-\nu p}(\xi)}{\int_0^\infty d\xi F_1^{\bar{\nu}p-\nu p}(\xi)}$	$\dfrac{\int_0^1 \xi\, d\xi F_1^{\bar{\nu}p+\nu p}(\xi)}{\int_0^\infty \xi\, d\xi F_1^{\bar{\nu}p+\nu p}(\xi)}$
1	48.1%	26.0%
2	92.9%	85.2%
3	97.2%	92.7%
4	98.5%	94.9%

Since the pion is so light, the bound on ω developed in Sec. III becomes much more strict.

(8) Scattering off bags with some spin-zero constituents may be computed and compared with fermion bags.

(9) The approach to scaling may be studied explicitly. I have already done this in one dimension and find[13]

$$\lim_{\rm Bj} \overline{W}^{ij}(q^2, \nu) = \overline{F}^{ij}(\xi) - \frac{M^2}{q^2}\, \xi^2 \frac{d}{d\xi}\, \overline{F}^{ij}(\xi)\,.$$

This corresponds to early scaling in the variable referred to in the literature as ω(light cone),

$$\frac{1}{\omega_{\rm LC}} \equiv \xi_{\rm LC} = -\frac{\nu}{M}\left[1 - \left(1 + \frac{q^2M^2}{\nu^2}\right)^{1/2}\right] \cong \xi M - \frac{M^3}{2\nu}\, \xi^2\,.$$

Of course it is of great importance to improve upon the approximations I have been forced to make. In particular, the relation of my semiclassical and static treatment to a real, fluctuating quantum theory must be established.

VII. DISCUSSION AND SUMMARY

I have treated the bag as a cavity of fixed radius filled with quark fields. The forward Compton scattering of currents off the quanta in this cavity is a well-defined calculable process and has been the subject of this paper. Of course the bag is not a cavity. Its boundary must respond in a causal way to changes in the field degrees of freedom produced by local currents. This paper is predicated on the assumption that the response of the boundaries to the current does not play an essential role in the dynamics of Compton scattering in the Bjorken limit. Arguments were proposed to motivate this for small values of ω. When ω is greater than approximately 14 the picture is untenable. Little can as yet be said about Compton scattering for large values of ω. Regge behavior requires the integrals which define the structure functions to diverge as $\omega \to \infty$. As long as they are restricted to $r < R_0$ this does not seem possible. A physical picture quite different from the one I have discussed seems necessary to account for the Regge region. This is not to say Bjorken scaling does not hold at large ω, but it is not motivated from the naive treatment I have given.

The problems with large ω are inseparable from the question of boundary operator fluctuations. This, in turn, raises the more fundamental question of whether the bag theory in fact even possesses local currents at the quantum level. This is a difficult problem in any interacting theory. For the bag it is probably best studied in one dimension, where at least a quantum interpretation of the bag already exists.[1]

In principle, the Compton scattering from fields in a cavity is directly calculable. In practice, I have replaced the cavity quark propagator by the free Dirac propagator. To leading order in the Bjorken limit this should not matter. In one dimension the replacement may be studied explicitly and is found to be benign.

The principal phenomenological results of the calculation (in three dimensions) are summarized below:

(a) W_1 and $(\nu/M^2)W_i$ ($i = 2, 3$) scale in the Bjorken limit.

(b) W_L vanishes and $(\nu/M^2)W_L$ scales in the limit and is calculable.

(c) The structure functions satisfy the conventional quark light-cone algebra and in particular obey quantum-number sum rules such as Adler's.

(d) The "momentum sum rule" is satisfied and *all* the momentum is found to be on the quarks.

(e) The structure functions are peaked quasielastically at $\xi_0 = 1/N$ (N is the number of occupied modes in the target). The spread about ξ_0 is attributed to confinement. As N increases the bag grows large and the structure functions revert to those of a collection of free quarks.

(f) The structure functions are analytic in ξ at $\xi = 0$. For $F_1(\xi)$ in electroproduction this implies $\lim_{\xi\to 0} F_1(\xi) \sim (\xi)$. In Regge language, no $C = +1$ singularities with intercept greater than -1 are found.

(g) The shape of the structure functions is governed by (c) and (f) above. They lie somewhat above the data for values of ξ not near zero.

(h) $W_{\mu\nu}$ for electroproduction is gauge invariant to leading order in the Bjorken limit.

Some of the major problems with the cavity approximation as a phenomenology should also be noted.

(i) Momentum is not conserved in the intermediate state. Among the consequences of this are the failure of structure functions to vanish for $\xi > 1$, small violations of positivity for ξ near zero (where the approximation is surely not valid anyway), and an ambiguity regarding the treatment of quark bubble graphs such as Fig. 3(c).

(ii) $F_2^{en}(\xi)/F_2^{ep}(\xi)$ is $\frac{2}{3}$ everywhere. No mechanism for structure in this ratio is developed.

(iii) The experimental observation that the momentum sum rule is only 50% saturated is not explained.

ACKNOWLEDGMENTS

I would like to thank the following people for helpful conversations and advice regarding this work: R. Blankenbecler, S. Brodsky, A. Chodos, N. Christ, G. Farrar, R. P. Feynman, K. Johnson,

F. E. Low, and J. Mandula. Also the hospitality of everyone associated with the Aspen Center for Physics, where this work was begun, is gratefully acknowledged.

APPENDIX A: THE FORM OF $W_{\mu\nu}^{ij}$ IN THE CAVITY APPROXIMATION

The structure function is conventionally defined by

$$W_{\mu\nu}^{ij} = \frac{1}{4\pi} \int d^4x\, e^{-iq\cdot x} \langle P | [J_\mu^i(x), J_\nu^j(0)] | P \rangle , \qquad (A1)$$

where the states are covariantly normalized:

$$\langle P | P' \rangle = (2\pi)^3 2E \delta^3(\vec{P} - \vec{P}') . \qquad (A2)$$

Equation (A1) is not appropriate to a system like the cavity which is not translationally invariant. The center of the bag is at the spatial origin and the currents should be free to act anywhere within it. Also the normalization is not appropriate to cavity states.

Let us define states normalized to unity (still within the framework of conventional field theory),

$$|T\rangle \equiv [(2\pi)^3 2E\delta^3(0)]^{-1/2} |P\rangle , \qquad (A3)$$

and introduce another coordinate into Eq. (A1),

$$W_{\mu\nu}^{ij} = \frac{E}{4\pi^2\delta(0)} \int d^4x_1 \int d^4x_2 e^{-iq\cdot(x_1 - x_2)} \times \langle T | [J_\mu^i(x_1), J_\nu^j(x_2)] | T \rangle . \qquad (A4)$$

Nothing in Eq. (A4) depends on the average time $T \equiv \frac{1}{2}(t_1 + t_2)$, which may be integrated, leaving

$$W_{\mu\nu}^{ij} = \frac{M}{2\pi} \int_{-\infty}^{\infty} dt \int d^3x_1 \int d^3x_2 e^{iq^0 t - i\vec{q}\cdot(\vec{x}_1 - \vec{x}_2)} \times \langle T | [J_\mu^i(\vec{x}_1, t), J_\nu^j(\vec{x}_2, 0)] | T \rangle \qquad (A5)$$

E has been replaced by M since the target is taken to be at rest. Equation (A5) is carried over into the bag-cavity model by restricting the spatial integrals to the interior of the bag $|\vec{x}| \le R_0$. The result is Eq. (17) in the text. Equation (A5) could equally well be derived directly from a consideration of Compton scattering from a cavity.

APPENDIX B: EXPLICIT CALCULATIONS IN THREE DIMENSIONS, GAUGE INVARIANCE, ETC.

This appendix consists of two parts. In the first part the calculation of structure functions in three dimensions is illustrated using the coefficient of $g_{\mu\nu}$ in Eq. (41) as an example. In the second part gauge invariance and other tensor properties of $W_{\mu\nu}$ are studied.

1. Calculation of W_1

Consider the coefficient of $g_{\mu\nu}$ in Eq. (41). In Appendix B2 this will be shown to be $W_1(q^2, \nu)$. I assume that the target consists of a collection of quanta in the lowest cavity mode whose wave function is given by Eq. (5). The time integration in Eq. (41) may be done outright. The resulting δ function and the $\delta(k^2)$ allow the k^0 and $|\vec{k}|$ integrations to be performed. Substituting liberally from Eqs. (3)–(5) and introducing the notation of Eq. (9), I obtain

$$W_1^{ij}(q^0, \xi) = \frac{MN^2}{4\pi^2} \int d\Omega_k \left\{ |\vec{k}|^2 \left| \int \frac{d^3x}{4\pi} e^{-i(\vec{q}+\vec{k})\cdot\vec{x}} \left[j_0\!\left(\frac{\epsilon|\vec{x}|}{R_0}\right) \hat{k} - i j_1\!\left(\frac{\epsilon|\vec{x}|}{R_0}\right) \hat{x} \right] \right|^2_{|\vec{k}| = q^0 + \epsilon/R_0} \sum_{m,a} \langle T | b_a^\dagger(m) \lambda^i \lambda^j b_a(m) | T \rangle - (\epsilon \to -\epsilon;\ i \leftrightarrow j) \right\} . \qquad (B1)$$

Performing the angular part of the x integration and the azimuthal $\hat{k}$ integration one obtains

$$W_1^{ij}(q^0, \xi) = \frac{MN^2}{2\pi} \int_{-1}^{1} d\cos\theta_k \left\{ |\vec{k}|^2 \left| \int_0^{R_0} x^2 dx \left[j_0\!\left(\frac{\epsilon x}{R_0}\right) j_0(px) \hat{k} - j_1\!\left(\frac{\epsilon x}{R_0}\right) j_1(px) \hat{p} \right] \right|^2_{|\vec{k}| = q^0 + \epsilon/R_0} \times \sum_{m,a} \langle T | b_a^\dagger(m) \lambda^i \lambda^j b_a(m) | T \rangle - (\epsilon \to -\epsilon; i \leftrightarrow j) \right\} , \qquad (B2)$$

where

$$\vec{p} \equiv \vec{q} + \vec{k}, \quad p \equiv |\vec{p}| ,$$

and θ_k is the angle between q and k. So far this expression is exact. The x integrations may be done analytically without passing to the Bjorken limit. In the Bjorken limit

$|\vec{q}|\to\infty$ with $q^0=|\vec{q}|+M\xi$.

$|\vec{p}|$ is very large unless $\cos\theta_k\approx -1$. To display this dependence, it is best to change variables from $\cos\theta_k$ to $\beta_-\equiv R_0 p$. $\theta_k=\pi$ corresponds to $\beta=|MR_0\xi-\epsilon|\equiv\beta_-$, while $\theta_k=0$ implies $\beta\sim 0(|\vec{q}|)$. Performing, then, the x integrations and passing to the Bjorken limit,

$$\lim_{\mathrm{Bj}} W_1(q^0,\xi)=\frac{MN^2R_0^4}{2\pi}\int_{\beta_-}^{\infty}\beta\,d\beta\Big\{\Big[T_{00}{}^2(\epsilon,\beta)+T_{11}{}^2(\epsilon,\beta)-\frac{2}{\beta}(\epsilon-MR_0\xi)T_{00}(\epsilon,\beta)T_{11}(\epsilon,\beta)\Big]\times\sum_{m,a}\langle T|b_a^\dagger(m)\lambda^i\lambda^j b_a(m)|T\rangle-(\epsilon\to-\epsilon;\,i\to j)\Big\},\tag{B3}$$

where

$$T_{mn}=\int_0^1 z^2\,dz\,j_m(\epsilon z)j_n(\beta z);\tag{B4}$$

specifically

$$T_{00}=(\epsilon^2-\beta^2)^{-1}[\epsilon y_0(\epsilon)j_0(\beta)-\beta j_0(\epsilon)y_0(\beta)]\tag{B5}$$

and

$$T_{11}=(\epsilon^2-\beta^2)^{-1}[\beta j_1(\epsilon)j_0(\beta)-\epsilon j_0(\epsilon)j_1(\beta)].\tag{B6}$$

I have also used

$$\lim_{\mathrm{Bj}}\hat{p}\cdot\hat{k}=(1/\beta)(\epsilon-MR_0\xi)+\beta/2R_0q^0.\tag{B7}$$

The second term in Eq. (B7) was dropped from W_1 since it is of order $1/q^0$ compared to the other terms in Eq. (B3).

Bjorken scaling is now displayed explicitly: All of the terms in Eq. (B3) [and the term dropped from Eq. (B7)] remain convergent as the upper limit of the β integral is taken to infinity. The resulting convergent integral is a function only of ξ. To simplify W_1 further note that as a consequence of Eq. (6)

$$j_0(\epsilon)=j_1(\epsilon)=\frac{\epsilon}{\epsilon-1}y_0(\epsilon)\tag{B8}$$

and substitute for R_0 and M from Eqs. (4) and (7)

$$\lim_{\mathrm{Bj}} W_1^{ij}(q^0,\xi)\equiv F_1^{ij}(\xi)=\frac{\Lambda\epsilon^2}{4\pi(\epsilon-1)}\Big\{\int_{\beta_-}^{\infty}\beta\,d\beta\Big[t_{00}{}^2(\epsilon,\beta)+t_{11}{}^2(\epsilon,\beta)-\frac{2\epsilon}{\beta}(1-\Lambda\xi)t_{00}(\epsilon,\beta)t_{11}(\epsilon,\beta)\Big]\times\sum_{m,a}\langle T|b_a^\dagger(m)\lambda^i\lambda^j b_a(m)|T\rangle-(\epsilon\to-\epsilon;\;i\to j)\Big\}.\tag{B9}$$

This result was quoted in the text [Eq. (42)] where Λ and $t_{mn}(\epsilon,\beta)$ were defined.

2. Tensor structure of $W_{\mu\nu}$

Any component of $W_{\mu\nu}$ may be analyzed as the coefficient of $g_{\mu\nu}$ was treated above. First consider the coefficient of $\epsilon_{\mu\rho\nu\sigma}$ which violates parity conservation. Explicit calculation yields (using the fact that the target spin is averaged)

$$\tfrac{1}{2}(W_{\mu\nu}^{ij}-W_{\nu\mu}^{ij})=\frac{-MN^2}{4\pi}\epsilon_{\mu\rho\nu r}\epsilon_{rst}\int d\cos\theta_k\Big\{|\vec{k}|^2\int\frac{d^3x}{4\pi}e^{-i\vec{p}\cdot\vec{x}}\hat{x}_s\int\frac{d^3y}{4\pi}e^{i\vec{p}\cdot\vec{y}}\hat{y}_t\Big|_{|\vec{k}|=q^0+\omega/R_0}\times\sum_{m,a}\langle T|b_a^\dagger(m)\lambda^i\lambda^j b_a(m)|T\rangle+(\omega\to-\omega;\;i\to j)\Big\}.\tag{B10}$$

Both coordinate integrals are proportional to $\vec{p}$, and the resulting expression vanishes identically. Of course the coefficient of $\epsilon_{\mu\rho\nu\sigma}$ will not be zero either for chiral currents or in scattering from polarized targets.

Now, if $W_{\mu\nu}$ is to be gauge invariant it must possess the conventional decomposition

$$W^{ij}_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1^{ij}(q^2,\nu) + \frac{1}{M^2}\left(P_\mu + \frac{\nu}{q^2}q_\mu\right)\left(P_\nu + \frac{\nu}{q^2}q_\nu\right) W_2^{ij}(q^2,\nu). \tag{B11}$$

In the rest frame $q=(q^0,0,0,q^3)$, $P=(P^0,0,0,0)$, one expects

$$W_{00} = (1+\nu^2/M^2q^2)W_L\,, \tag{B12a}$$

$$W_{33} = (\nu^2/M^2q^2)W_L\,, \tag{B12b}$$

$$W_{03} = W_{30} = (\nu/Mq^2)(q^2+\nu^2/M^2)^{1/2}\,W_L\,, \tag{B12c}$$

$$W_{ij} = \delta_{ij}W_1, \quad i,j=1,2 \tag{B12d}$$

where

$$W_L \equiv (1+\nu^2/M^2q^2)W_2 - W_1\,. \tag{B13}$$

In part 1 of this appendix W_{ij} was calculated (only the $g_{\mu\nu}$ term in $W_{\mu\nu}$ have components in the $i,j=1,2$ subspace). Explicit calculation along the same lines yields

$$\lim_{\mathrm{Bj}} W^{ij}_{00} = -F_1^{ij}(\xi) + \frac{\Lambda\epsilon^2}{2\pi(\epsilon-1)}\left\{\int_{\beta_-}^{\infty}\beta\, d\beta[t_{00}{}^2(\epsilon,\beta)+t_{11}{}^2(\epsilon,\beta)]\sum_{m,a}\langle T|b_a^\dagger(m)\lambda^i\lambda^j b_a(m)|T\rangle - (\epsilon\to-\epsilon;\ i\to j)\right\}. \tag{B14}$$

W_{00} scales in the Bjorken limit.

Similar calculations yield

$$\lim_{\mathrm{Bj}} W_{33} = -\lim_{\mathrm{Bj}} W_{03} = \lim_{\mathrm{Bj}} W_{00}\,.$$

This is consistent with Eqs. (B12) to leading order in the Bjorken limit. Thus $W_{\mu\nu}$ in the rest frame possesses the tensor structure of Eq. (B11) and is therefore gauge invariant.

Furthermore, the scaling behavior of W_L has been exhibited. Since W_{00} scales, W_L must vanish in the Bjorken limit. This is the familiar Callan-Gross relation. In fact $\nu W_L(q^2,\nu)$ scales, as seen from comparison of Eqs. (B12a) and (B14):

$$\lim_{\mathrm{Bj}} \frac{\nu}{M^2} W_L(q^2,\nu) = \lim_{\mathrm{Bj}} (q^2/\nu)W_{00}$$

$$= -2\xi \lim_{\mathrm{Bj}} W_{00}\,.$$

Furthermore, W_L is calculable from Eq. (B14). This means $R\equiv\sigma_L/\sigma_T$ vanishes like $1/q^2$ in the Bjorken limit and is calculable.

*This work is supported in part through funds provided by the Atomic Energy Commission under Contract No. AT(11-1)-3069.

[1]A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974).

[2]A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, Phys. Rev. D 10, 2599 (1974).

[3]A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn (unpublished).

[4]For convenience I consider the fractionally charged color scheme of H. Fritzsch and M. Gell-Mann, in *Proceedings of the International Conference on Duality and Symmetry in Hadron Physics*, edited by E. Gotsman (Weizmann Science Press, Jerusalem, 1971). Other schemes such as the Han-Nambu model [M.-Y. Han and Y. Nambu, Phys. Rev. 139, B1006 (1965)] yield identical results with the minor proviso discussed in Ref. 1.

[5]R. L. Jaffe and K. Johnson, MIT Report No. MIT-CTP-435 presented at the Seminar on Quark and Parton Problems, Moscow, U.S.S.R., 1974 (unpublished). See also K. Johnson and C. B. Thorn, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974*, edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. I-251.

[6]Recently Low has shown that a relatively weak quark-gluon coupling within the bag can account for much of the phenomenology of high-energy forward hadron-hadron (bag-bag) scattering. See F. E. Low, M.I.T. report (unpublished).

[7]I use the Dirac formalism and matrices of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964). However, I employ the metric $-g_{00}=g_{ii}=1$ and therefore require $\{\gamma_\mu,\gamma_\nu\} = -2g_{\mu\nu}$.

[8]Since only $j=\frac{1}{2}$ is allowed the subscript j is dropped henceforth.

[9]P. N. Bogoliubov, Ann. Inst. Henri Poincaré 8, 163 (1967).

[10]R. A. Brandt, Phys. Rev. Lett. 23, 1260 (1969); L. S. Brown, in *Lectures in Theoretical Physics*, edited by W. E. Britten, B. W. Downs, and J. Down (Interscience, New York, 1969). The correlation is lightlike even in very pathological theories; see R. L. Jaffe, Phys. Rev. D 6, 716 (1972).

[11]This reflects the conjugate relation between $z^- \equiv (x-y)^-$ and q^+, light cone position and momentum. Briefly, the Bjorken limit is taken by letting $q^- \to \infty$ with q^+, P^+ fixed. Then $z^+ \to 0$ and $\omega = P^+/q^+$, $z \cdot P = z^- P^+$. The relation $z \cdot P \sim \omega$ follows from the uncertainty relation $z^- \sim 1/q^+$. The correlation between ω and $z \cdot P$ in light-cone analyses has been treated by this author, R. L. Jaffe, Ann. Phys. (N.Y.) 75, 545 (1973), and in parton and vector dominance language by A. suri and D. Yennie, *ibid.* 72, 243 (1972).

[12]A minor proviso is discussed in Sec. IV.

[13]Actually in one dimension the results are identical to all orders in an asymptotic expansion in $1/q^2$. The difference vanishes faster than any power of q^2.

[14]S. D. Adler, Phys. Rev. 143, 1144 (1965).

[15]This possibility was pointed out to me by F. E. Low.

[16]J. D. Bjorken and S. F. Tuan, Comments Nucl. Part. Phys. 5, 71 (1972).

[17]C. Callan and D. J. Gross, Phys. Rev. Lett. 22, 156 (1969).

[18]See J. Ellis and R. L. Jaffe, SLAC Report No. SLAC-PUB-1353 (unpublished), for derivations and discussions of these sum rules.

[19]R. L. Jaffe and C. H. Llewellyn Smith, Phys. Rev. D 7, 2506 (1973).

[20]D. J. Broadhurst, J. F. Gunion, and R. L. Jaffe, Phys. Rev. D 8, 566 (1973).

[21]D. Corrigan, J. M. Cornwall, and R. E. Norton, Phys. Rev. Lett. 23, 1141 (1970). For a review see D. J. Broadhurst, J. F. Gunion, and R. L. Jaffe, Ann. Phys. (N.Y.) 81, 88 (1973).

[22]T. P. Cheng and W.-K. Tung, Phys. Rev. Lett. 24, 851 (1970).

[23]R. Rajaraman and G. Rajasekaran, Phys. Rev. D 3, 266 (1971); 4, 2940(E) (1971).

[24]W. N. Cottingham, Ann. Phys. (N.Y.) 25, 424 (1963).

[25]R. Jackiw and H. J. Schnitzer, Phys. Rev. D 5, 2008 (1972); J. Gunion, *ibid.* 8, 517 (1973).

[26]A. Chodos and C. B. Thorn (unpublished).

[27]W.-C. Ng and P. Vinciarelli, Phys. Lett. 38B, 219 (1972).