PHYSICAL REVIEW D VOLUME 11, NUMBER 7 1 APRIL 1975

Many-particle final states and the energy dependence of charge exchange and momentum transfer*

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(Received 21 October 1974)

An inequality restricting the energy dependence of two-body reactions which transfer any combination of charge and momentum is derived using the assumption that a class of many-particle production events fulfills the requirement of local compensation of quantum numbers.

In a recent paper by Krzywicki and myself[1] it was shown that a bound can be placed on the energy dependence of near-forward charge-exchange scattering using, primarily, two assumptions: First, it was assumed that a certain class of multiparticle production events obeys a condition described as "local compensation of charge"; and second, it was assumed that all other scattering processes occur only as the shadow of this class of multiparticle events. Roughly speaking, a set of events fulfills local compensation of an additively conserved quantum number if each particle carrying the value q is almost always surrounded by a small number of particles in the neighboring region of rapidity space carrying a total quantum number of $-q$. In the present paper we will give another derivation of the main result of Ref. 1 using a method which seems technically somewhat more convenient than the original. We will also derive new restrictions on the rate of shrinkage with energy of the forward peaks observed in two-body elastic scattering and charge-exchange scattering. More precisely, we will show that if $T(q_1,q_2)$ is the scattering amplitude for a two-body process exchanging charge q_1 and transverse momentum q_2, then as the energy in the center-of-mass system $\sqrt{s}$ is made progressively larger we have

$$\left|\frac{T(q_1,q_2)}{T(0,0)}\right| \leq a(q_1,q_2) \times s^{-(b_1q_1^2+b_2q_2^2-c_1q_1^4-c_2q_2^4-dq_1^2q_2^2+\cdots)}, \qquad (1)$$

where the energy-independent quantities $a(q_1,q_2)$, $b_i, c_i, d, \ldots$ are determined by multiparticle production data. Throughout the following discussion we will assume for convenience that all particles have the same mass, all spins are zero, and all forward scattering amplitudes (exchanging neither charge nor transverse momentum) are equal.

Let us begin by introducing a slight generalization of the function $Z(y)$ called a zone graph in Ref. 1. Consider a scattering event involving a collection of hadrons $\{h_i\}$,

$$h_1+h_2 \rightarrow h_3+\cdots\,. \qquad (2)$$

Assume y_i is the center-of-mass system rapidity of h_i, p_{1i} is the electrical charge of h_i, and p_{2i} is the c.m. component of transverse momentum of h_i in some arbitrary direction $\hat{e}$. Then a two-component zone graph $Z_j(y)$ can be defined by

$$Z_j(y) = -\sum_{i=1,2}\theta(y-y_i)p_{ji} + \sum_{i\geq 3}\theta(y-y_i)p_{ji}\,, \qquad (3)$$

where $\theta(y)$ is the usual step function given by 0 for negative arguments and 1 elsewhere.

Now we will repeat the procedure of Ref. 1 and divide the class of all final configurations of hadrons which could appear in (2) into two subsets, one called dense and the other called diffuse. Dense configurations will be those with all rapidity gaps between successive particles in rapidity space less than or equal to $\Delta(s)$, where $\Delta(s)$ is a certain monotonically increasing function of s bounded by $\mu \ln(s)$ for a constant $\mu \ll 1$. Diffuse configurations will be all those which are not dense. Let $\langle\ \rangle$ represent the operation of averaging over the ensemble of dense final configurations produced in (2). A set of quantities called zone moments can be constructed through the relation

$$M_{j_1\cdots j_m}(y_1,\ldots,y_m) = \langle Z_{j_1}(y_1)\cdots Z_{j_m}(y_m)\rangle\,,$$

and from these we can define a set of functions called zone correlations by taking a cluster decomposition[2]:

$$M_j(y)=C_j(y)\,,$$
$$M_{j_1j_2}(y_1,y_2)=C_{j_1j_2}(y_1,y_2)+C_{j_1}(y_1)C_{j_2}(y_2)\,,$$
$$M_{j_1j_2j_3}(y_1,y_2,y_3)=C_{j_1j_2j_3}(y_1,y_2,y_3)+C_{j_1}(y_1)C_{j_2j_3}(y_2,y_3)+C_{j_2}(y_2)C_{j_1j_3}(y_1,y_3)$$
$$+C_{j_3}(y_3)C_{j_1j_2}(y_1,y_2)+C_{j_1}(y_1)C_{j_2}(y_2)C_{j_3}(y_3)\,,$$

etc.

It is useful to recognize that these functions are simply related to density correlations[3] and inclusive correlations. If $D(y,p_1,p_2)$ is the random variable giving the density of particles with rapidity y, charge p_1, and transverse momentum component p_2 in direction $\hat{e}$, then

$$Z_j(y)=\sum_{p_1'}\int_{-\infty}^{\infty}dp_2'\int_{-(\ln s)/2}^{y}dy'p_j'\,D(y',p_1',p_2')-\sum_{i=1,2}\theta(y-y_i)p_{ji}\,.$$

The preceding equation implies

$$C_{j_1\cdots j_m}(y_1',\ldots,y_m')=\sum_{p_{11}'',\ldots,p_{1m}''}\int_{-\infty}^{\infty}dp_{21}''\cdots\int_{-\infty}^{\infty}dp_{2m}''\int_{-(\ln s)/2}^{y_1'}dy_1''\cdots$$
$$\times\int_{-(\ln s)/2}^{y_m'}dy_m''p_{j_11}''\cdots p_{j_mm}''\,G(y_1'',p_{11}'',p_{21}'',\ldots,y_m'',p_{1m}'',p_{2m}'')$$
$$-\delta_{1m}\sum_{i=1,2}\theta(y_1'-y_i)p_{ji}\,, \tag{4}$$

where $G(y_1'',p_{11}'',p_{21}'',\ldots,y_m'',p_{1m}'',p_{2m}'')$ is the mth order density correlation, which can be expressed as a combination of inclusive correlations of order m and lower.[2,3]

The assumption of local compensation of charge by dense final configurations given in Ref. 1 can be reformulated and extended to include transverse momentum as follows:

(i) We assume that as $s\to\infty$ the functions $C_{j_1\cdots j_m}(y_1,\ldots,y_m)$ approach s-independent limits.

(ii) If the variables $y_1,\ldots,y_m$ are divided into two nonempty sets S_1 and S_2 such that $|y_i-y_j|>d$ for all $y_i\in S_1$, $y_j\in S_2$, and d is made progressively larger, then $C_{j_1\cdots j_m}(y_1,\ldots,y_m)$ rapidly approaches 0 once d is larger than a certain energy-independent correlation length λ.

(iii) The functions $C_{j_1\cdots j_m}(y_1,\ldots,y_m)$ are translationally invariant in the region in which $y_1,\ldots,y_m$ are many correlation lengths from the boundaries of rapidity space: $C_{j_1\cdots j_m}(y_1,\ldots,y_m)=C_{j_1\cdots j_m}(y_1+\alpha,\ldots,y_m+\alpha)$.

(iv) If $y_1,\ldots,y_m$ are many correlation lengths from the boundaries of rapidity space, $C_{j_1\cdots j_m}(y_1,\ldots,y_m)$ is the same for all possible choices of h_1 and h_2.

Assumption (iv) combined with charge-conjugation invariance implies that if $y_1,\ldots,y_m$ are many correlation lengths from the boundaries of rapidity space, $C_{j_1\cdots j_m}(y_1,\ldots,y_m)=0$ if an odd number of j_i are equal to 1. Also rotational invariance around the axis formed by the momenta of h_1 and h_2 implies $C_{j_1\cdots j_m}(y_1,\ldots,y_m)=0$ if an odd number of j_i are equal to 2. By using these two results and considering simple models for the set of zone graphs which can appear in (2) it is possible to convince oneself that the formulation of local compensation given in Ref. 1 has nearly the same content as assumptions (i)–(iv) with one exception—the present assumptions seem to permit a more realistic probability distribution of the number of zones of each type which might occur in any event. [A charge zone is an interval on which $Z_1(y)\neq 0$ bounded by two successive points at which $Z_1(y)=0$, and a transverse momentum zone is defined similarly using $Z_2(y)$.]

Now suppose h_i^q, $i=1,2$, are a pair of hadrons with charges p_{1i}^q, and transverse momenta p_{2i}^q in direction $\hat{e}$, given by $p_{j1}^q=p_{j1}+q_j$, $p_{j2}^q=p_{j2}-q_j$, $j=1,2$, where q_1 is an integer and q_2 is an arbitrary real number. We assume the transverse momenta of h_1^q and h_2^q perpendicular to $\hat{e}$ are 0. If s is sufficiently large, we can assume without contradiction that the energy and longitudinal momentum of h_i^q are nearly the same as those of h_i, $i=1,2$. Let $Z_j(y)$ be the zone graph for a certain set of hadrons $h_3+\cdots$ produced by h_1+h_2 as before, and let $Z_j^q(y)$ be the zone graph obtained if $h_3+\cdots$ were produced by $h_1^q+h_2^q$. Consider the relation between $Z_2^q(y)$ and $Z_2(y)$. The $\hat{e}$ component of transverse momentum p_{2i}^q of h_i, $i\geq 3$, in the center-of-mass system of $h_1^2+h_2^q$ is obtained from p_{2i} by a rotation through an angle of nearly $2q_2/\sqrt{s}$, which leaves the longitudinal momentum of h_i almost unchanged. Therefore we have

$$p_{2i}^q=p_{2i}+x_iq_2\,, \tag{5}$$

where x_i is the usual Feynman variable, $2p_{\parallel i}/\sqrt{s}$, formed from the longitudinal momentum $p_{\parallel i}$ of h_i. Equation (5) combined with the information that longitudinal momenta are unchanged implies that for particles in the central region—far from the boundaries of rapidity space—values of rapidity are also nearly unchanged in going from the frame of h_1+h_2 to the frame of $h_1^q+h_2^q$. Then using the asymptotic form of energy and longitudinal momentum conservation, $\sum_{i\geq 3} x_i\theta(\pm x_i)=\pm 1$, combined with definition (3), we find that for values of y in the central region of rapidity space

$$Z_2^q(y)=Z_2(y)-q_2 . \tag{6}$$

Strictly speaking, (6) depends on the assumption that as s becomes large the probability approaches 0 for events yielding a total energy in the central region which is a finite fraction of $\sqrt{s}$. Multiparticle production data suggest that this additional assumption is very likely fulfilled. If this assumption is correct, then by letting $s\to\infty$, (6) can be made arbitrarily close to exact for an arbitrarily large fraction of rapidity space. In the following discussion we will assume that (6) holds exactly throughout rapidity space. The small errors introduced by this procedure will not affect the s dependence of the bounds we finally obtain.

The preceding argument and the results of Ref. 1 also imply that if y is in the central region,

$$Z_1^q(y)=Z_1(y)-q_1 . \tag{7}$$

We will assume that (7) holds throughout rapidity space, and this also will not introduce errors which would affect the s dependence of our final results.

A bound on the scattering amplitude $T(q)$ for the two-body process $h_1+h_2\to h_1^q+h_2^q$ can be derived from the assumptions we have given. Let A be the component of the scattering operator S, taking diffuse initial states to diffuse final states, and let B be the component of S taking diffuse initial states to dense final states. Unitarity of S implies there exists a unitary operator C such that

$$A=C(1-B^\dagger B)^{1/2} . \tag{8}$$

We argued in Ref. 1 that if we assume the production of diffuse final states by diffuse initial states occurs only as the shadow of dense production, then if B were 1, A would also equal 1, which suggests we should choose $C=1$. We then assumed it was reasonable to expand (8) in a power series and use only the two leading terms:

$$A=1-\tfrac{1}{2}B^\dagger B . \tag{9}$$

If U_j is the random variable defined by

$$U_j=\int_{-(\ln s)/2}^{(\ln s)/2} Z_j(y)\,dy\,, \quad j=1,2$$

it follows from (9) combined with the Cauchy-Schwarz inequality that

$$\left|\frac{T(q)}{T(0)}\right| \leq \int_{-\infty}^{\infty} du_1 \int_{-\infty}^{\infty} du_2 \{P(u)P[u-q\ln s]\}^{1/2} , \tag{10}$$

where $P(u)$ is the joint differential probability for $U_j=u_j$, $j=1,2$.

Standard results of probability theory yield the expansion

$$P(u)=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} d\alpha_1 \int_{-\infty}^{\infty} d\alpha_2 \exp\left[-i\sum_{j=1,2}\alpha_j u_j+\sum_{m_1,m_2}(i\alpha_1)^{m_1}(i\alpha_2)^{m_2}\frac{C_{m_1m_2}}{m_1!m_2!}\right], \tag{11}$$

where $C_{m_1m_2}$ are the cumulants of the joint probability distribution of U_1 and U_2 given by

$$C_{m_1m_2}=\int_{-(\ln s)/2}^{(\ln s)/2} dy_1\cdots\int_{-(\ln s)/2}^{(\ln s)/2} dy_{m_1+m_2}\, C_{1\cdots 12\cdots 2}(y_1,\ldots,y_{m_1+m_2}) . \tag{12}$$

The index 1 occurs m_1 times and 2 occurs m_2 times in the subscript of the argument of the integral in (12). A proof of (11) and (12) follows, for example, from arguments given in Refs. 2 and 3. Assumptions (i)–(iii) imply that as $s\to\infty$

$$C_{m_1m_2}\to \ln s\, c_{m_1m_2}+O(1)\,, \qquad c_{m_1m_2}=\int_{-\infty}^{\infty} dy_2\cdots\int_{-\infty}^{\infty} dy_{m_1+m_2}\lim_{s\to\infty} C_{1\cdots 12\cdots 2}(0,y_2,\ldots,y_{m_1+m_2}) .$$

For simplicity in the remaining discussion we will ignore the terms $O(1)$ in the asymptotic form of $c_{m_1m_2}$. This will also not affect the s dependence of the bounds to be derived. If we now define $v_j=u_j/\ln s$, $j=1,2$, as s becomes large (11) takes the form

$$P(u)=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} d\alpha_1\int_{-\infty}^{\infty} d\alpha_2 s^{-g(\alpha,v)} , \tag{13}$$

where

$$g(\alpha, v) = i \sum_{j=1,2} \alpha_j v_j - \sum_{m_1, m_2} (i\alpha_1)^{m_1} (i\alpha_2)^{m_2} \frac{c_{m_1 m_2}}{m_1! m_2!} .$$

Consider the properties of the function $g(\alpha, v)$. It is not difficult to show that if $c_{m_1m_2}$ is sufficiently well behaved as m_1 and m_2 are made large, and if in addition all $c_{m_1m_2}$ with $m_1 + m_2 \geq 4$ are sufficiently small, there are contours in the complex α_1 and α_2 planes, extending from $-\infty$ to ∞, along which $g(\alpha, v)$ is positive real, approaches ∞ at both ends and has a single minimum near $\alpha_1 = -iv_1/(2c_{20})$, $\alpha_2 = -iv_2/(2c_{02})$. The point at which this minimum occurs is the only point in the complex α_1 and α_2 planes at which $\partial g(\alpha, v)/\partial \alpha_i = 0$, $i = 1, 2$. The size of higher order $c_{m_1m_2}$, however, is one measure of the randomness of the random function $Z_j(y)$. We will assume that $Z_j(y)$ is sufficiently random that higher-order correlations are well behaved and contours with the properties we described also exist for $c_{m_1m_2}$ in the real world.

If this assumption is fulfilled, then as $s \to \infty$ the integral in (13) can be performed on the contours we have chosen and is dominated by contributions near the minimum of $g(\alpha, v)$. Then $P(u)$ assumes the asymptotic form

$$P(u) = \frac{f(v)}{\ln s} s^{-h(v)} , \tag{14}$$

where $f(v)$ is an energy-independent function of v_i, $i = 1, 2$, $h(v) = g(\alpha^0, v)$, and α^0 is defined by $\partial g(\alpha, v)/\partial \alpha_i|_{\alpha=\alpha^0} = 0$, $i = 1, 2$. Placing (14) in (10) and changing the integration variables to $v_i' = v_i - \frac{1}{2}q_i$, $i = 1, 2$, we have

$$\left|\frac{T(q)}{T(0)}\right| \leq \ln s \int_{-\infty}^{\infty} dv_1' \int_{-\infty}^{\infty} dv_2' [f(v' + \tfrac{1}{2}q) f(v' - \tfrac{1}{2}q)]^{1/2} \times s^{-h(v'+q/2)/2 - h(v'-q/2)/2} . \tag{15}$$

If $Z_j(y)$ is sufficiently random, $\frac{1}{2}h(v' + \frac{1}{2}q) + \frac{1}{2}h(v' - \frac{1}{2}q)$ will have a single minimum and $f(v' + \frac{1}{2}q) f(v' - \frac{1}{2}q)$ will not be singular at this point. It follows from the symmetry properties of $h(v)$ that the minimum of $\frac{1}{2}h(v' + \frac{1}{2}q) + \frac{1}{2}h(v' - \frac{1}{2}q)$ will occur at $v_i' = 0$, $i = 1, 2$, and will therefore be equal to $h(\frac{1}{2}q)$. As $s \to \infty$ the right-hand side of (15) will be given by contributions near $v_i' = 0$, $i = 1, 2$, and we obtain

$$\left|\frac{T(q)}{T(0)}\right| \leq a(q) s^{-h(q/2)}$$

for a certain s-independent function $a(q)$. Returning to (14) and defining $P'(v)$ to be the joint differential probability for the random variables $U_j/\ln s$ to assume the values v_j, $j = 1, 2$, we have

$$\left|\frac{T(q)}{T(0)}\right| \leq \frac{a'(q)}{\ln s} P'(\tfrac{1}{2}q) \tag{16}$$

for another s-independent function $a'(q)$. Alternatively, if we define a new pair of random variables

$$R_j = y_0^{-1} \int_{-y_0/2}^{y_0/2} Z_j(y) dy , \quad j = 1, 2$$

where y_0 is a constant, then the derivation leading to (16) can be reformulated to show that if y_0 is sufficiently large, we have the approximation

$$h(\tfrac{1}{2}q) = y_0^{-1} |\ln[P''(\tfrac{1}{2}q)]| + O(y_0^{-1}) , \tag{17}$$

where $P''(r)$ is the joint differential probability that $R_j = r_j$, $j = 1, 2$. When y_0 is large the first term on the right-hand side of (17) is $O(1)$.

The function $h(\frac{1}{2}q)$ can be calculated as a power series in $c_{m_1m_2}$, $m_1 + m_2 \geq 4$. If we assume higher correlations are sufficiently small that we need only the terms determined solely by c_{20}, c_{02}, and the linear terms in $c_{m_1m_2}$, $m_1 + m_2 = 4$, then

$$\left|\frac{T(q)}{T(0)}\right| \leq a(q) s^{-b(q)} \tag{18}$$

where

$$b(q) = \frac{q_1^2}{8c_{20}} + \frac{q_2^2}{8c_{02}} - \frac{q_1^4 c_{40}}{384 c_{20}^4} - \frac{q_2^4 c_{04}}{384 c_{02}^4} - \frac{q_1^2 q_2^2 c_{22}}{64 c_{20}^2 c_{02}^2} .$$

This result can be described in Regge-pole language by the following:

(1) The difference between the Pomeron intercept and the intercept of the leading charged trajectory is greater than $1/(8c_{20}) - c_{40}/(384 c_{20}^4)$.

(2) The leading t dependence of the Pomeron trajectory is bounded by $-t/(8c_{02}) - t^2 c_{04}/(384 c_{02}^4)$.

(3) The leading t dependence of the highest-lying charged trajectory is bounded by $-t[1/(8c_{02}) - c_{22}/(64 c_{20}^2 c_{02}^2)] - t^2 c_{04}/(384 c_{02}^4)$.

Thus the difference between the bound for the slope of the Pomeron and the bound for the slope of charged trajectories is determined by correlations between the zone graph for charge and the zone graph for transverse momentum. If the bounds we have derived are fairly close to observed values of these slopes and the higher correlations neglected in (18) actually are small, then c_{22} must be negative since charged trajectories have greater slope than the Pomeron. This in turn would imply, roughly speaking, that in those events in which charge is compensated somewhat less locally than average, transverse momentum tends to be compensated more locally, and vice versa.

Order-of-magnitude estimates can be made for c_{20} and c_{02} if we assume that parameters obtained from dense final states are at least approximately equal to those obtained from experiments including all inelastic final states produced in p-p collisions. We will arbitrarily assume the formulas $C_{11}(y_1, y_2) = A_1 \exp(-|y_1 - y_2|/\lambda_1)$ and $C_{22}(y_1, y_2) = A_2 \exp(-|y_1 - y_2|/\lambda_2)$. Approximate values of A_j and λ_j can then be gotten using (4) and the relation[3] between second-order density correlations and inclusive correlations:

$$G(y_1, p_{11}, p_{21}, y_2, p_{12}, p_{22}) = C^I(y_1, p_{11}, p_{21}, y_2, p_{12}, p_{22}) + \delta(y_1 - y_2)\delta(p_{21} - p_{22})\delta_{p_{11}p_{12}} C^I(y_1, p_{11}, p_{21}).$$

It follows from the definition of $Z_1(y)$ that $C_{11}(0,0)$ is just the mean-squared charge transfer across $y=0$; thus p-p scattering data at 102 GeV/c (Ref. 4) yields $A_1 \cong 0.9$. And from data on charge correlations in p-p scattering at 102 GeV/c we obtain[4]

$$\frac{A_1}{\lambda_1^2} = \int_{-\infty}^{\infty} dp_{21} \int_{-\infty}^{\infty} dp_{22} \left[2C^I(0,1,p_{21},0,-1,p_{22}) - C^I(0,1,p_{21},0,1,p_{22}) - C^I(0,-1,p_{21},0,-1,p_{22})\right] \cong 0.5.$$

Therefore we have $\lambda_1 \cong 1$ and $c_{20} \cong 2$. This implies that charge-exchange scattering amplitudes must fall faster than elastic amplitudes by at least $s^{-0.07}$.

For A_2 and λ_2 we find

$$\frac{A_2}{\lambda_2^2} = -\sum_{p_{11}p_{12}} \int_{-\infty}^{\infty} dp_{21} \int_{-\infty}^{\infty} dp_{22}\, p_{21} p_{22} \times C^I(0, p_{11}, p_{21}, 0, p_{12}, p_{22})$$

$$= -\tfrac{1}{2}\langle k_{\perp 1} \cdot k_{\perp 2}\rangle \left(\frac{1}{\sigma}\right) \frac{d^2\sigma}{dy_1 dy_2}\bigg|_{y_1=y_2=0},$$

$$\frac{A_2}{\lambda_2} = \frac{1}{2}\sum_{p_1} \int_{-\infty}^{\infty} dp_2\, p_2^2\, C^I(0, p_1, p_2)$$

$$= \frac{1}{4}\langle k_\perp^2\rangle \left(\frac{1}{\sigma}\right) \frac{d\sigma}{dy}\bigg|_{y=0},$$

where σ is the total inelastic cross section, $d^2\sigma/dy_1 dy_2$ is the inclusive cross section for a pair of particles at y_1 and y_2, $\langle k_{\perp 1} \cdot k_{\perp 2}\rangle$ is the mean inner product of two-component transverse momenta for a pair of particles at y_1 and y_2, $d\sigma/dy$ is the inclusive cross section for a single particle at y, and $\langle k_\perp^2\rangle$ is the mean squared two-component transverse momentum for a single particle at y. If we assume inclusive cross sections for all particles and for charged particles are related by

$$\frac{d^2\sigma}{dy_1 dy_2} = \left(\frac{3}{2}\right)^2 \frac{d^2\sigma_{cc}}{dy_1 dy_2}, \quad \frac{d\sigma}{dy} = \left(\frac{3}{2}\right) \frac{d\sigma_c}{dy}$$

and $\langle k_{\perp 1} \cdot k_{\perp 2}\rangle$ and $\langle k_\perp^2\rangle$ are the same for charged particles and for all particles, then 102-GeV/c p-p scattering data give[4,5]

$$\left(\frac{1}{\sigma}\right) \frac{d^2\sigma}{dy_1 dy_2}\bigg|_{y_1=y_2=0} \cong 8,$$

$$\left(\frac{1}{\sigma}\right) \frac{d\sigma}{dy}\bigg|_{y=0} \cong 2,$$

$$\langle k_{\perp 1} \cdot k_{\perp 2}\rangle|_{y_1=y_2=0} \cong -0.01\ (\mathrm{GeV}/c)^2,$$

$$\langle k_\perp^2\rangle|_{y=0} \cong 0.2\ (\mathrm{GeV}/c)^2,$$

which implies $A_2 \cong 0.2$, $\lambda_2 \cong 2$, and $c_{02} \cong 0.8$. The upper bound we deduce for the rate of shrinkage of elastic scattering amplitudes is approximately $s^{-0.2|t|}$.

Before arriving at any final judgement of the strength of the bounds we have derived it would of course be useful to have accurate experimental values for c_{20} and c_{02} obtained from the collection of all inelastic events produced by p-p scattering. We would hope that observed c_{20} might be somewhat smaller than our estimate of 2, yielding a better bound on the s dependence of charge exchange. If this does not turn out to be the case, a number of possibilities remain. It might happen, for example, that higher-order correlations contribute significantly and more terms than appear in (18) should be included. If this were true, then instead of trying to measure many-body correlations directly it would probably be easier to observe $P'(v)$ or $P''(r)$ and consider the bound given by (16) or (17). Another alternative, which appears to me less likely, is that U_1 is strongly correlated with other parameters of $Z_j(y)$ in such a way that these contribute to suppressing the value of charge-exchange scattering. In this case a better bound on the energy dependence of charge exchange might be gotten by replacing (10) by an overlap integral depending on more than just the 2 parameters we used. We should probably mention that an apparently stronger restriction on the s dependence of charge exchange was derived in Ref. 1, but it does not seem that this result should be taken too seriously since its numerical value depended appreciably on rather unrealistic assumptions concerning the probability distribution of the number of charge zones per event. In any case, the bound we have gotten for the rate of shrinkage of elastic scattering seems likely to be fairly good.

An important question which remains is to precisely what extent data taken from all events actually can be used as a measure of the properties of dense configurations. Now if local compensation of quantum numbers does hold in the sense we have assumed, then it can be shown that values of $C_{j_1j_2}(y_1, y_2)$ gotten by averaging over all events including diffuse will still be free of long-range correlations. Thus if the cross section for dense configurations is somewhat larger than for diffuse, $C_{j_1j_2}(y_1, y_2)$ obtained from all configurations in p-p collisions should be a satisfactory approximation to $C_{j_1j_2}(y_1, y_2)$ for dense configurations. On the other hand, simple models suggest that higher correlation functions might acquire long-range tails if obtained from the ensemble of all events. Thus the interpretation of data for these quantities could require more detailed calculations of the effect of averaging over both dense and diffuse configurations. Even if long-range effects are present in higher correlations, however, it might still be possible to obtain a useful bound on two-body scattering processes by measuring $P''(r)$ for all events and using (17), with y_0 chosen large enough to include the short-range contribution of correlation functions and small enough to avoid including too much of the long-range tails. We should also mention that if local compensation of quantum numbers is fulfilled in the sense we have assumed here, then we believe it can be shown that the average rapidity length of zones will approach energy-independent constants both in diffuse final configurations and dense configurations as $s\to\infty$. In addition, the average number of charge or transverse momentum zones per event should grow linearly with $\ln(s)$. Thus direct experimental numbers for these quantities provide another convenient test of the assumptions we have made.

Finally, it might be useful to point out that if we had introduced a fairly obvious generalization of assumptions (i)–(iv), bounds similar to those we derived could also have been obtained for the exchange of any other combination of additively conserved quantum numbers (strangeness, baryon number, etc.) in addition to charge and transverse momentum.

I would like to thank Carl Bromberg and Tom Ferbel for providing me with unpublished data and for comments on the manuscript. I am also grateful to André Krzywicki for numerous discussions during the early stages of this work.

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*Work supported in part by the U. S. Atomic Energy Commission.

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