Asymptotic decay width of excited hadronic clusters*

Serge Rudaz
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853
(Received 18 November 1974)

The principle of detailed balance is used to investigate the asymptotic behavior of the average decay width of hadronic clusters in the framework of models, such as the dual resonance and statistical bootstrap models, in which quasi-two-body decays are dominant, resulting in linear cluster decay chains. An expression for the width is derived in the high-mass limit, and its implications are discussed.

Cluster emission models have become popular in recent years in the interpretation of multiparticle production phenomena.[1,2] Such models generally consist of a two-step picture whereby one or many clusters (excited hadronic states; the more flamboyant term "fireball" is also commonly used) are produced through some dynamical mechanism (multiperipheral-like exchange, independent emission, single- and double-diffraction dissociation, etc.) and subsequently decay isotropically in their rest frame. Models of this type have been found to successfully reproduce the main features of both inclusive[1,2] and exclusive[3] data.

In this note we will not be concerned with the nature of the cluster production mechanisms; rather, taking the point of view that clusters have a real dynamical significance, we will investigate their asymptotic average decay width, assuming dominance of quasi-two-body decay modes, resulting in treelike decay chains for the clusters. Examples of models exhibiting such a behavior are the dual resonance[4] and statistical bootstrap[5,6] models (respectively, DRM and SBM). Both the DRM and the SBM give rise to an asymptotic density of states of the form[5,7]

$$\rho(m) \sim Cm^{\alpha}\exp(\beta m). \qquad (1)$$

A behavior of this type was first obtained by Hagedorn in the framework of his thermodynamic model.[8] The value of the exponent α is crucial: The "strong version" of the statistical bootstrap[5] and a ghost-free version of the DRM in four dimensions[9] give $\alpha=-3$; for such a value of α, it is found[5] that quasi-two-body decays dominate over many-body decays for large cluster masses.[10] This is not so for $\alpha=-\frac{5}{2}$, for example, corresponding to the thermodynamic[8] (weak statistical bootstrap) model, where many-body decays are important asymptotically. The value of β is determined from phenomenology; a currently popular number is $\beta^{-1}\simeq 160$ MeV.

Because of the similarity of their spectrum of states, many studies have been made of a possible relation between SBM and DRM.[5,11-13] Our calculation will incorporate general properties common to both models. Later in the paper, we will compare our results with those obtained in previous calculations, both in the framework of SBM[12,14,15] and in that of explicit dual models.[13,16-18]

The calculation of the partial decay width $\Gamma_c(M)$ for the process $H(M)\to h(m)+c(\mu_c)$ proceeds along the lines first suggested by Weisskopf[19] in his study of neutron evaporation from nuclei using statistical methods. That is, we will use the principle of detailed balance[20] to relate $\Gamma_c(M)$ to the cross section $\sigma_c(M;m)$ for the inverse process $h(m)+c(\mu_c)\to H(M)$, and assuming dominance of quasi-two-body decay modes, we will write the total width $\Gamma(M)$ as follows:

$$\Gamma(M)=\sum_c \Gamma_c(M). \qquad (2)$$

Here, the sum runs over the *light particle* species c, of mass μ_c, such as $c=\pi$, η, ρ, ω, etc., while H and h designate *clusters* of invariant masses M and m, respectively. The density of states describing the clusters H and h will be taken to have the form given in Eq. (1), with $\alpha=-3$, while for particle c we will use the form $g_c\delta(\mu-\mu_c)$, where the weight factor g_c is given by $g_c=(2S_c+1)(2I_c+1)2^{\lambda_c}$, with S_c and I_c the spin and isospin of c, respectively, and λ_c is 0 or 1 according to whether c is self-conjugate or not. This weight factor is already included in the density of states, Eq. (1), for the clusters H and h. Furthermore, we will neglect angular momentum and spin effects: This does not affect our general conclusions, which will hold for quantities averaged over initial, and summed over final, spins. The inclusion of angular momentum has been treated in detail by Wolfenstein[21] in the nuclear case, and was also studied by Hamer in a more recent publication.[22]

A simple application of the principle of detailed balance, namely

$$\rho_1 P(1\to 2)=\rho_2 P(2\to 1) \tag{3}$$

(where ρ and P are, respectively, density of states and transition probability, and 1 and 2 are state labels), then yields the result

$$\Gamma_c(M)=\frac{g_c}{8\pi^2 M^2\rho(M)} \times\int_{m_0}^{M-\mu_c} dm\,\rho(m)\sigma_c(M;m)\lambda(M^2,m^2,\mu_c^2)\,, \tag{4}$$

with $\lambda(x,y,z)=x^2+y^2+z^2-2xy-2xz-2yz$. Equation (4) was first written down, in a slightly different form, by Matthiae,[14] who worked within the Hagedorn thermodynamic model[8] and who used the same density of states for all three objects H, h, and c, namely Eq. (1) with $\alpha=-\frac{5}{2}$. Our approach and results, however, are different from his. Matthiae's calculation was redone by Nahm,[15] who used the strong SBM solution, $\alpha=-3$.

We will be interested in the large-M behavior of $\Gamma_c(M)$; but we must first discuss the behavior of the single-cluster formation cross section, $\sigma_c(M;m)$. There is no clear theoretical determination of the dependence of $\sigma_c(M;m)$ on the external mass m. However, on the strength of experimental results[23] on the scattering of hadronic systems (πp, 3π, 5π, etc.) off nucleons, as obtained through optical-model analyses[24] of coherent production processes in particle-nucleus collisions, one can infer that $\sigma_c(M;m)$ is, to a good approximation, independent of m. Thus we write

$$\Gamma_c(M)=\frac{g_c\sigma_c(M)Me^{-\beta M}}{8\pi^2} \times\int_{m_0}^{M-\mu_c} dm\,e^{\beta m}\,\frac{\lambda(M^2,m^2,\mu_c^2)}{m^3}\,. \tag{5}$$

Here, m_0 is a finite, fixed mass, which characterizes the onset of the continuum for the cluster $h(m)$; it may be taken to be $m_0\simeq 1.3$ GeV, although its exact value will not be important for our purposes. Notice that the constant C, appearing in Eq. (1), has been canceled out of Eq. (5), which is just as well, since its value is quite uncertain. Multiple integration by parts now leads to the result

$$\Gamma_c(M)=\frac{g_c\sigma_c(M)}{2\pi^2}\beta^{-3}e^{-x_c}[x_c^{\,2}+2x_c+2+O(1/M)]\,, \tag{6}$$

where we have defined $x_c=\beta\mu_c$ to simplify the notation.

As M gets large, therefore, we obtain

$$\Gamma(M)\sim\frac{\beta^{-3}}{2\pi^2}\sum_c g_c\sigma_c(M)e^{-x_c}(x_c^{\,2}+2x_c+2)\,. \tag{7}$$

Equation (7) is the central result of this paper: It was obtained under the assumption that the dominant decay mode of a heavy cluster is decay to a light particle plus another cluster of comparable mass. We have used a density of states abstracted from DRM and SBM, where the above assumption holds (we have accordingly neglected n-body decays, with $n>2$). We may also remark that, at least in the framework of the statistical bootstrap, this assumption leads to some quite distinctive implications for cosmology, in particular about the duration of the hadron era in the "big bang," and about galaxy formation.[10,25] We shall now proceed to discuss some of the implications of Eqs. (6) and (7).

To obtain an estimate of $\Gamma(M)$, we have still to specify the M dependence of σ_c. We first note that we expect σ_c to be actually independent of c [e.g., $\sigma(\pi N)\simeq\sigma(\rho N)$, etc.], again on the basis of experiment, and shall henceforth drop the subscript c.

A natural estimate of the single-cluster cross section follows from the application of the Freund-Harari conjecture,[26] namely that the *average effect* of the direct-channel resonances can be described, through duality, by the exchange in the crossed channel of the leading non-Pomeranchuk trajectories with intercept $\alpha_R(0)\simeq\frac{1}{2}$. We are thus led to

$$\sigma(M)=\frac{\sigma_0}{(M/M_0)}\,, \tag{8}$$

where σ_0 is a constant to be specified below, and $M_0=1$ GeV sets the scale. Putting this back into Eq. (7) gives a total width in M falling as M^{-1}, or alternately

$$\Gamma(M^2)\equiv M\Gamma(M)\to\text{constant}\,. \tag{9}$$

Such a result was obtained by Green and Veneziano,[16] and also by Chan and Tsou,[17] and by Chadha,[18] who used various methods of calculation, all within the DRM. In particular, Green and Veneziano arrive at the estimate $\Gamma(M^2)<1/\pi$ GeV^2. Restricting the light particles to the set $c=\{\pi,\ \eta,\ \rho,\ \omega\}$, we arrive, using Eqs. (7) and (8), at the asymptotic result

$$\Gamma(M^2)\to 0.12\ \text{GeV}^2\,, \tag{10}$$

where we have used $\sigma_0=24$ mb $\simeq 1.2 m_\pi^{\,-2}$, corresponding to a good parametrization of meson-baryon cross sections,[27] at least up to Serpukhov energies. This agrees very nicely with the Green-Veneziano result, but one should refrain from drawing premature conclusions, since the picture of cluster decay implied in our calculation leads to a multiplicity $n(M)\sim M$, while the dual model of Green and Veneziano gives rise to a form $n(M)\sim\ln M$.

Another possibility, albeit slightly extreme, is that the single-cluster cross section is a *constant*, implying a fully dual description of high-energy collisions, that is, the direct channel resonances somehow build up the crossed channel Pomeranchuk exchange. In that case, with $\sigma(M) = \sigma_0$, we obtain

$$\Gamma(M) \to \text{constant}. \tag{11}$$

One can again estimate $\Gamma(M)$ using Eq. (7): With $\sigma_0 \simeq 24$ mb, as before, and with $c = \{\pi, \eta, \rho, \omega\}$, one arrives at

$$\Gamma(M) \to 125 \text{ MeV} \tag{12}$$

asymptotically. If one also includes kaons among the light particles, this value is modified to

$$\Gamma(M) \to 166 \text{ MeV} \tag{13}$$

asymptotically. The result, Eq. (11), was also obtained by Frautschi[12] and by the Kiev group,[13] the former applying the full machinery of the SBM and the latter making use of a statistical approach to DRM. Both approaches now yield $n(M) \sim M$, and Frautschi estimates $\Gamma \to 180$ MeV.

The agreement between Frautschi's SBM results and ours is not surprising. Indeed, in the full statistical approach, one would write our sum over light particles c as an integral over a spectrum given by Eq. (1) (see, e.g., the paper of Nahm[15]); but the convergence of this sum is essentially guaranteed by the SBM sum rule,

$$\ln 2 = \frac{V}{(2\pi)^3} \int dm\, \rho(m) \int d^3p \exp[-\beta(m^2+p^2)^{1/2}].$$

Thus, our sum over a small number of discrete states is expected to give results agreeing with those of the SBM as studied by Frautschi.[12]

Besides its simplicity, our approach has the nice feature of allowing the calculation of the relative probabilities for decays such as $H \to \pi + h$ and $H \to \rho + h$, for example, *independently of the postulated behavior of the cross section* $\sigma(M)$.

As an example of this, let us consider a simplified model of the hadronic world where $c = \{\pi, \rho\}$ (in fact, we need not worry about including ω, η in the set c if we consider only clusters decaying into charged pions), and let us calculate the average number of ρ mesons to be found in five-pion and seven-pion decays of clusters which may be diffractively produced in reactions like the coherent process

$$\pi^\pm + \text{nucleus} \to (\text{cluster})^\pm + \text{nucleus}.$$

To this end, let us make the reasonable assumption that the last three pions of the cluster decay chain appear as a $\pi\rho$ system: This was used in a previous analysis of the coherent reaction $\pi^\pm A \to (5\pi)^\pm A$ and led to a good description of available data.[3]

On the basis of Eq. (6), and for large M, one easily obtains that

$$\begin{aligned} p(H \to \pi h) &\simeq 69\%, \\ p(H \to \rho h) &\simeq 31\%, \end{aligned} \tag{14}$$

where $p(V)$ is the probability of occurrence of the vertex V. It is now a simple matter to obtain $\langle n_\rho \rangle$, the average number of ρ mesons to be found in a multipion final state from cluster decay. Consider the five-pion decay of a cluster, with the possible decay chains $\pi\pi(\pi\rho)$ and $\rho(\pi\rho)$. Using the result, Eq. (14), one finds that the probability of occurrence of $\pi\pi(\pi\rho)$ is 61%, and that of $\rho(\pi\rho)$ is 39%. It follows that

$$\langle n_\rho(5\pi) \rangle \simeq 1.4. \tag{15}$$

The result of a recent experiment[28] on $\pi^\pm p \to p\, 3\pi^\pm\, 2\pi^\mp$, at $p_{\text{lab}} = 16$ GeV/c, is

$$\langle n_\rho(5\pi) \rangle^{\text{exp}} \simeq 1.2,$$

which agrees quite well with our crude estimate of Eq. (15).

Using the same method, one can easily calculate the average number of ρ mesons to be found in the seven-(charged)-pion decay of a cluster:

$$\langle n_\rho(7\pi) \rangle \simeq 1.8\text{–}1.9. \tag{16}$$

The large number of possible combinations in the analysis of a seven-pion event makes it unfortunately difficult, but presumably not impossible, to test this prediction in the near future.

Another interesting consequence of our analysis is the following prediction of the ratio of the widths of baryonic clusters to those of mesonic clusters,

$$\frac{\Gamma(\text{baryon} \to \text{baryon} + c)}{\Gamma(\text{meson} \to \text{meson} + c)} \simeq \frac{\sigma(\text{baryon} + c)}{\sigma(\text{meson} + c)} \simeq \tfrac{3}{2}, \tag{17}$$

as determined, for example, from a simple quark model. Such a trend indeed seems to obtain for the widths of high-lying states, although the large experimental uncertainties prevent us from making a more definite statement on this matter.

To conclude, we have presented in this paper a calculation of cluster widths, and given estimates on their asymptotic behavior which depended on the form of the single-cluster formation cross section. However, we were also able to calculate light-particle emission probabilities and related quantities, independently of the postulated behavior of $\sigma(M)$, and thereby to make some quantitative statements about the nature of cluster decay chains.

I wish to thank J. Dorn, P. Floyd, and C. Stevens for providing a stimulating atmosphere for the writing of a preliminary version of this work.

*Work supported in part by the National Science Foundation.

[1]Recent reviews include the following: A. Białas, in Proceedings of the IV International Symposium on Multiparticle Hadrodynamics, Pavia, 1973 (unpublished); E. L. Berger (CERN Report No. Th. 1737), in Proceedings of the 1973 International School of Subnuclear Physics, Erice, Italy, edited by A. Zichichi (to be published); A. Krzywicki, invited lecture at the Colloque sur la Physique Hadronique aux Energies des ISR, Marseille, 1973, Orsay Report No. LPTPE-LPTHE 73/33 (unpublished); H. Satz, Acta Phys. Pol. B5, 3 (1974); J. Ranft, and J. Kripfganz, in Proceedings of the V International Symposium on Many-Particle Hadrodynamics, Eisenach/Leipzig, DDR, 1974 (unpublished).

[2]S. Pokorski and L. Van Hove, Nucl. Phys. B60, 379 (1973); Acta Phys. Pol. B5, 229 (1974); C. J. Hamer, Phys. Rev. D 7, 2723 (1973); E. L. Berger, G. C. Fox, and A. Krzywicki, Phys. Lett. 43B, 132 (1973); A. Białas, K. Fiałkowski, and K. Zalewski, *ibid.* 45B, 337 (1973); E. L. Berger and G. C. Fox, *ibid.* 47B, 162 (1973); A. W. Chao and C. Quigg, Phys. Rev. D 9, 2016 (1974); M. Le Bellac, H. I. Miettinen, and R. G. Roberts, Phys. Lett. 48B, 115 (1974); E. L. Berger, *ibid.* 49B, 369 (1974); A. Białas, M. Jacob, and S. Pokorski, Nucl. Phys. B75, 259 (1974); E. L. Berger, *ibid.* B85, 61 (1975); C. J. Hamer, Phys. Rev. D 10, 1458 (1974). This list is by no means exhaustive: Further references may be found in the above papers.

[3]B. Margolis and S. Rudaz, Phys. Rev. D 9, 653 (1974); R. Harris, H. J. Lubatti, K. Moriyasu, W. J. Podolsky, H. H. Bingham, W. B. Fretter, W. R. Graves, F. C. Porter, and G. P. Yost, University of Washington, Seattle, Visual Techniques Laboratory Report No. VTL-PUB-19, 1974 (unpublished).

[4]G. Veneziano, Phys. Rep. 9C, 201 (1974); J. Scherk, New York University Report No. NYU-TR3/74 (unpublished).

[5]S Frautschi, Phys. Rev. D 3, 2821 (1971).

[6]R. Hagedorn and J. Ranft, Nucl. Phys. B48, 157 (1972); W. Nahm, *ibid.* B45, 525 (1972); C. J. Hamer and S. C. Frautschi, Phys. Rev. D 4, 2125 (1971); C. J. Hamer, *ibid.* 8, 3558 (1973); R. Hagedorn and I. Montvay, Nucl. Phys. B59, 45 (1973); I. Montvay, *ibid.* B53, 521 (1973); F. Csikor, I. Farkas, Z. Katona, and I. Montvay, *ibid.* B74, 323 (1974).

[7]S. Fubini and G. Veneziano, Nuovo Cimento 64A, 811 (1969); K. Huang and S. Weinberg, Phys. Rev. Lett. 25, 895 (1970).

[8]R. Hagedorn, Nuovo Cimento Suppl. 3, 147 (1965); R. Hagedorn and J. Ranft, *ibid.* 6, 169 (1968).

[9]R. C. Brower, Phys. Rev. D 6, 1655 (1972).

[10]S. Frautschi, Ann. N. Y. Acad. Sci. 224, 355 (1973).

[11]H. Satz, University of Bielefeld Report No. Bi 74/16 (unpublished); Nuovo Cimento Lett. 4, 910 (1972); Phys. Lett. 44B, 373 (1973); M. I. Gorenstein, V. A. Miransky, H. Satz, V. P. Shelest, and G. M. Zinovjev, Nucl. Phys. B76, 453 (1974), and references quoted therein.

[12]S. Frautschi, Caltech Report No. CALT-68-451, 1974 (unpublished).

[13]V. A. Miransky, V. P. Shelest, B. V. Struminsky, and G. M. Zinovjev, Phys. Lett. 43B, 73 (1973); M. I. Gorenstein, G. M. Zinovjev, V. I. Makarov, V. A. Miransky, and V. P. Shelest, Zh. Eksp. Teor. Fiz. Pis'ma Red. 15, 686 (1972) [JETP Lett. 15, 487 (1972)]; V. I. Makarov, V. A. Miransky, V. P. Shelest, and G. M. Zinovjev, Nuovo Cimento Lett. 8, 151 (1973).

[14]G. Matthiae, Nucl. Phys. B7, 142 (1968).

[15]W. Nahm, Nucl. Phys. B68, 111 (1974).

[16]M. B. Green and G. Veneziano, Phys. Lett. 36B, 477 (1971).

[17]Chan Hong-Mo and Tsou Sheung-Tsun, Phys. Rev. D 4, 156 (1971).

[18]S. Chadha, doctoral dissertation, University of Oxford, 1973 (unpublished).

[19]V. Weisskopf, Phys. Rev. 52, 295 (1937).

[20]See, e.g., M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1964).

[21]L. Wolfenstein, Phys. Rev. 82, 690 (1951).

[22]C. J. Hamer, Phys. Rev. D 9, 2512 (1974).

[23]For a review, see W. Beusch, Acta Phys. Pol. B3, 679 (1972); more recent results have been quoted in P. Mühlemann, K. Freudenreich, F. X. Gentit, G. Bellini, M. di Corato, G. Vegni, C. Bemporad, W. Beusch, E. Polgár, D. Websdale, P. Astbury, J. G. Lee, and M. Letheren, Nucl. Phys. B59, 106 (1973); also, W. C. Carithers, T. Celik, T. Ferbel, P. Mühlemann, D. Underwood, D. G. Ryan, and H. H. Williams, paper submitted to the XVII International Conference on High Energy Physics, Imperial College, London, England, 1974 (unpublished) (University of Rochester report), and many other papers.

[24]K. S. Kölbig and B. Margolis, Nucl. Phys. B6, 85 (1968).

[25]R. Carlitz, S. Frautschi, and W. Nahm, Astron. Astrophys. 26, 171 (1973).

[26]P. G. O. Freund, Phys. Rev. Lett. 20, 235 (1968); H. Harari, *ibid.* 20, 1395 (1968).

[27]J. L. Rosner, in a collection of reports on ISABELLE physics prospects, Brookhaven Report No. BNL 17522, 1972 (unpublished).

[28]H. Böttcher, K. Lanius, and Ch. Spiering, Institut für Hochenergiephysik, Akademie der Wissenschaften der DDR, Report No. PHE 74-2, 1974 (unpublished); see also K. Lanius, JINR Dubna Report No. E1-8061 (unpublished).