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$(3,\bar{3})+(\bar{3},3)$ chiral $SU(3)\times SU(3)$ symmetry breaking and the Cabibbo angle

S. Eliezer*
Physics Department, Imperial College, London SW7, England
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We consider the implications of a possible common origin for the strong symmetry breaking and the nonconservation of strangeness in weak interactions. Assuming that the strong SU(2) × SU(2)-symmetry-breaking contribution to the pion mass vanishes for a zero Cabibbo angle, then, in order to avoid large strangeness-changing terms, the chiral SU(3) × SU(3)-symmetry-breaking Hamiltonian cannot transform as a pure $(3,\bar{3})+(\bar{3},3)$ representation.

Some years ago, Gell-Mann, Oakes, and Renner[1] (GMOR) and, independently, Glashow and Weinberg[2] (GW) clarified the significance of the chiral SU(3)×SU(3) symmetry. They assumed that the symmetry-breaking Hamiltonian transforms as the $(3,\bar{3})+(\bar{3},3)$ representation so that the Hamiltonian describing the strong interaction can be written in the form

$$H=H_0+\epsilon_0 u_0+\epsilon_8 u_8\,, \tag{1}$$

where H_0 is the SU(3)×SU(3)-invariant part and where u_0 and u_i $(i=1,\ldots,8)$ are the scalar components of the $(3,\bar{3})+(\bar{3},3)$ representation. The ratio ϵ_8/ϵ_0 was found[1,2] to be near the value $-\sqrt{2}$, which corresponds to the limit of exact SU(2) ×SU(2) symmetry, which is realized through massless pions. Consequently Oakes[3] has suggested that the structure of SU(2)×SU(2)-symmetry breaking is obtained by rotating an invariant SU(2) ×SU(2) Hamiltonian by an angle 2θ about the seventh axis in SU(3) space, where θ is the Cabibbo angle.[4] This would imply a common origin for strong symmetry breaking and the nonconservation of strangeness in weak interactions. However, starting from the SU(2)×SU(2)-symmetric Hamiltonian $H(\theta=0)$

$$H(\theta=0)=H_0+\epsilon_0(u_0-\sqrt{2}\,u_8) \tag{2}$$

and applying the Cabibbo rotation[4]

$$\begin{aligned}H(\theta)&=e^{-2i\theta F_7}H(0)e^{2iF_7}\\&=H_0+\epsilon_0u_0+\epsilon_3u_3+\epsilon_6u_6+\epsilon_8u_8\,,\end{aligned} \tag{3}$$

where F_i $(i=1,\ldots,8)$ are the generators of the SU(3) group and

$$\frac{\epsilon_8}{\epsilon_0}=-\sqrt{2}\,(1-\tfrac{3}{2}\sin^2\theta)\,, \tag{4}$$

$$\frac{\epsilon_3}{\epsilon_0}=-\frac{\sqrt{3}}{\sqrt{2}}\sin^2\theta\,, \tag{5}$$

$$\frac{\epsilon_6}{\epsilon_0}=-\sqrt{6}\sin\theta\cos\theta\,, \tag{6}$$

we end up with a Hamiltonian completely different from that of Eq. (1). The Hamiltonian (3) involves two new terms: u_3, which breaks the isospin symmetry, and u_6, which violates strangeness conservation. While the introduction of the u_3 term in the Hamiltonian was welcomed by many authors,[5-11] the u_6 term is undesirable and therefore *was neglected without any theoretical justification*. If one accepts this *ad hoc* assumption, the use of the experimental value[12] $\sin\theta=0.27\pm0.02$ in Eqs. (4) and (5) gives

$$\frac{\epsilon_8}{\epsilon_0}\simeq-1.26\,,\quad\frac{\epsilon_3}{\epsilon_8}\simeq0.07\,. \tag{7}$$

While the result of ϵ_8/ϵ_0 is in agreement with the GMOR and GW analyses, the ϵ_3/ϵ_8 ratio is too big compared with estimates[6] obtained from n-p, Λ-p, and K^0-K^+ mass differences, which give $\epsilon_3/\epsilon_8\simeq0.02$. On the other hand, the hope to explain the $\eta\to3\pi$ decay rate with the large ϵ_3/ϵ_8 ratio[7] given above has been disproved recently,[13] thus only resulting in difficulties in explaining the SU(2)-breaking effects with a large ϵ_3u_3 term as given in Eq. (7).

To overcome the undesired result for ϵ_3/ϵ_8 given in Eq. (7) one may try to start with different Hamiltonians than the one given in Eq. (2). We shall make a Cabibbo rotation on the following two Hamiltonians suggested in the literature[14]:

$$H_a(\theta=0)=A(u_0-\sqrt{2}\,u_8)+B\left(\frac{\sqrt{2}}{\sqrt{3}}u_0+\frac{1}{\sqrt{3}}u_8+u_3\right), \tag{8}$$

$$H_b(\theta=0)=A(u_0-\sqrt{2}\,u_8)+B\left(u_3+\frac{1}{\sqrt{3}}u_8\right). \tag{9}$$

Imposing the strangeness conservation on the result, we get

$$H(\theta)=\epsilon_0u_0+\epsilon_3u_3+\epsilon_8u_8\,, \tag{10}$$

where

$$\epsilon_0^a=\frac{\sqrt{2}}{\sqrt{3}}B+A\,,\quad \epsilon_0^b=A\,, \tag{11}$$

$$\epsilon_8^a=\epsilon_8^b=\frac{1}{\sqrt{3}}B-\sqrt{2}A+\frac{3}{\sqrt{2}}\sin^2\theta\,, \tag{12}$$

$$\epsilon_3^a=\epsilon_3^b=B-\frac{\sqrt{3}}{\sqrt{2}}A\sin^2\theta\,. \tag{13}$$

The Hamiltonian (8) was motivated by an $SU(2)_L \times U(1)$ gauge theory,[15] and in quark language it states that $H(\theta=0)$ laks an $\mathfrak{N}$-quark mass term.[16] The Hamiltonian (9) suggests that the only chiral SU(2) symmetry-breaking terms when $\theta=0$ transform like a U-spin singlet, in analogy to the transformation law of the electromagnetic current. If we take as input the experimental value of the Cabibbo angle and possible values of[1,10] $-\sqrt{2}\leq\epsilon_8/\epsilon_0\leq -1.25$ we obtain

$$0.08\leq\frac{\epsilon_3^a}{\epsilon_8^a}\leq 0.14\,, \tag{14}$$

$$0.05\leq\frac{\epsilon_3^b}{\epsilon_8^b}\leq 0.24\,, \tag{15}$$

where the lower limit corresponds to $\epsilon_8/\epsilon_0=-1.25$. Thus, although the ratio ϵ_3/ϵ_8 is very sensitive to the input value of ϵ_8/ϵ_0 (but not to the value of $\sin\theta$), it is necessary to have $\epsilon_8/\epsilon_0\leq -1.25$ in order to obtain a desirable ϵ_3/ϵ_8 ratio.[6,10]

Since the already suggested "unrotated" Hamiltonians $H(\theta=0)$ are not very satisfactory and *the assumption of neglecting the strangeness-changing term* ($\sim u_6$) *in the "rotated" Hamiltonian* $H(\theta)$ *is completely unjustified*, we shall make an attempt to start with the most general Hamiltonian $H(\theta=0)$, taking into account that the resulting strong Hamiltonian $H(\theta)$ should be CP- and P-invariant, and to conserve charge and hypercharge. Thus,

$$H(\theta=0)=\epsilon_0u_0+\epsilon_3(0)u_3+\epsilon_6(0)u_6+\epsilon_8(0)u_8 +H_{\rm em}+H_w(\Delta s=0)\,, \tag{16}$$

where $H_{\rm em}$ and H_w are the usual electromagnetic and weak Hamiltonians. After performing the Cabibbo rotation on Eq. (16) to get

$$H(\theta)=\epsilon_0u_0+\epsilon_3(\theta)u_3+\epsilon_6(\theta)u_6+\epsilon_8(\theta)u_8 +H_{\rm em}+H_w(\Delta s=0)+H_w(\Delta s=1)\,, \tag{17}$$

where

$$\epsilon_3(\theta)=\epsilon_3(0)(1-\tfrac{1}{2}\sin^2\theta)+\epsilon_6(0)\tfrac{1}{2}\sin2\theta +\epsilon_8(0)\tfrac{1}{2}\sqrt{3}\sin^2\theta\,, \tag{18}$$

$$\epsilon_6(\theta)=-\epsilon_3(0)\tfrac{1}{2}\sin2\theta+\epsilon_6(0)\cos2\theta +\epsilon_8(0)\tfrac{1}{2}\sqrt{3}\sin2\theta\,, \tag{19}$$

$$\epsilon_8(\theta)=\epsilon_3(0)\tfrac{1}{2}\sqrt{3}\sin^2\theta-\epsilon_6(0)\tfrac{1}{2}\sqrt{3}\sin2\theta +\epsilon_8(0)(1-\tfrac{3}{2}\sin^2\theta)\,, \tag{20}$$

we require strangeness conservation [i.e., $\epsilon_6(\theta)=0$], which gives

$$\epsilon_6(0)=[-\tfrac{1}{2}\sqrt{3}\,\epsilon_8(0)+\tfrac{1}{2}\epsilon_3(0)]\tan2\theta\,. \tag{21}$$

At this point it is important to realize that if one does not require the existence of the strangeness-changing term u_6 in $H(\theta=0)$, namely $\epsilon_6(0)=0$, then solving Eqs. (18)–(20) one gets

$$\epsilon_3(\theta)=\epsilon_3(0)=\sqrt{3}\,\epsilon_8(\theta)=\sqrt{3}\,\epsilon_8(0)\,, \tag{22}$$

which results in a very sizable SU(2) breaking, and therefore this possibility should be neglected. In order to keep within the spirit of Oakes's original assumption,[3] we shall require that the pions are not massless due to the nonvanishing of the Cabibbo angle and

$$m_\pi^2(\theta=0)=0\,. \tag{23}$$

If $\epsilon_3\neq 0$ then the u_3 term mixes π_3 with π_8, but still this contribution to the pion mass is of order $\epsilon_3^2(0)/\epsilon_0$, which is believed to be one order of magnitude smaller than the usual electromagnetic interaction $-H_{\rm em}$ present in Eq. (17).[17] Therefore, the assumption given in Eq. (23) is equivalent to the relation

$$\epsilon_8(0)=-\sqrt{2}\,\epsilon_0\,, \tag{24}$$

which together with Eq. (21) in Eqs. (18) and (20) gives

$$\epsilon_3(\theta)=-\sqrt{2}\,\epsilon_0(\tfrac{1}{2}\sqrt{3}\sin^2\theta-\tfrac{1}{4}\sqrt{3}\sin2\theta\tan2\theta) +\epsilon_3(0)(1-\tfrac{1}{2}\sin^2\theta+\tfrac{1}{4}\sin2\theta\tan2\theta)\,, \tag{25}$$

$$\epsilon_8(\theta)=-\sqrt{2}\,\epsilon_0(1-\tfrac{3}{2}\sin^2\theta+\tfrac{3}{4}\sin2\theta\tan2\theta) +\epsilon_3(0)(\tfrac{1}{2}\sqrt{3}\sin^2\theta-\tfrac{1}{4}\sqrt{3}\sin2\theta\tan2\theta)\,. \tag{26}$$

Eliminating $\epsilon_3(0)$ from these equations we get a value for $\epsilon_8(\theta)/\epsilon_0$:

$$\frac{\epsilon_8(\theta)}{\epsilon_0}=-\sqrt{2}\,\frac{\left[1+\left(2+\dfrac{\sqrt{3}}{2\sqrt{2}}\dfrac{\epsilon_3(\theta)}{\epsilon_0}\right)\dfrac{\sin^2\theta}{\cos2\theta}\right]}{1+\left(\dfrac{\sin^2\theta}{2\cos2\theta}\right)} \tag{27}$$

$$\simeq-\sqrt{2}\left[1+\frac{\sqrt{3}}{2}\left(\sqrt{3}+\frac{\epsilon_3(\theta)}{\sqrt{2}\,\epsilon_0}\right)\frac{\sin^2\theta}{\cos2\theta}\right]<-\sqrt{2}\,. \tag{27$'$}$$

For any reasonable value of $\epsilon_3(\theta)$, ϵ_8/ϵ_0 is outside the allowed domains[18] in the GMOR and GW models. Therefore the result given in Eq. (27) suggests that if our scheme is accepted the chiral symmetry breaking cannot transform as a pure $(3,\bar{3})+(\bar{3},3)$ representation, but might contain other terms transforming like (8, 8) (Ref. 19) or $(6,\bar{6})+(\bar{6},6)$ (Ref. 20) under the group $SU(3)\times SU(3)$. It is important to mention that the analysis made in this paper, as well as the results obtained by Oakes[3] and others[21] making a Cabibbo rotation [which is a pure SU(3) rotation], *are independent* of the chiral-symmetry transformation properties of the Hamiltonian once Eq. (23) is assumed.[22]

To summarize, the existence of a common origin for the strong symmetry breaking, represented by the pion mass, and the nonconservation of strangeness in weak interactions, characterized by the Cabibbo angle, would suggest the relation

$$H(\theta)=e^{-2i\theta F_7}H(\theta=0)\,e^{2i\theta F_7}\ . \qquad (28)$$

In order to avoid a large strangeness-changing term ($\sim u_6$) in the Hamiltonian $H(\theta)$, one has to require the existence of this term in $H(\theta=0)$; otherwise one has to accept the *ad hoc* assumption of ignoring undesirable terms in $H(\theta)$. The conjectures $\epsilon_6(\theta)=0$ and $m_\pi^2(\theta=0)=0$ give the result (27), which is inconsistent with a pure chiral $(3,\bar{3})+(\bar{3},3)$ symmetry breaking. If the above suggested scheme [i.e., Eqs. (28) and (23)] is to be successful in generating the Cabibbo angle, then the Hamiltonian must be more complicated than a single $(3,\bar{3})+(\bar{3},3)$ term. This conclusion is supported by a recent report[23] discussing $\pi\pi$ s-wave scattering lengths. The indications from $\pi^+\pi^-\to\pi^0\pi^0$ results favor rather larger values of a_0^0 than those obtained from a pure chiral $(3,\bar{3})+(\bar{3},3)$-symmetry-breaking Hamiltonian. This $\pi\pi$ problem, together with the scheme suggested in this paper [which relates the Cabibbo angle to the SU(3)-symmetry breaking] may satisfactorily be obtained in a model where the symmetry-breaking Hamiltonian transforms, for example, as $(3,\bar{3})+(\bar{3},3)+(8,8)$ or $(3,\bar{3})+(\bar{3},3)+6,\bar{6})+(\bar{6},6)$ under chiral SU(3)×SU(3).

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*Present address: Physics Dept., University of Massachusetts, Amherst, Mass. 01002.

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which states that $H(\theta=0)$ lacks a $\mathcal{P}$-quark mass, we end with a vanishing Cabibbo angle, $\theta=0$.
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