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U(3)⊗U(3) symmetry model with isospin breaking

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Mesonic and baryonic interactions are studied in a model of chiral U(3)⊗U(3) symmetry. The scalar and pseudoscalar mesons are assumed to belong to the $(3,3^*)\oplus(3^*,3)$ representation. Nonets of vector and axial-vector fields are introduced as gauge fields. The baryons are assumed to transform as the $(3,3^*)\oplus(3^*,3)$ representation. Explicit and spontaneous symmetry breakings are introduced and masses and various couplings are obtained. Most of the parameters are fixed by using some particle masses as input. We find that the Lagrangian is approximately invariant under SU(2)⊗SU(2) while for the vacuum SU(3) is a better symmetry than SU(2)⊗SU(2). The scalar-meson masses are predicted to be $m_{S\pi}=949$ MeV, $m_{S_K}=1025$ MeV, $m_{S\eta}=1068$ MeV, and $m_{S\eta'}=694$ MeV. The $V\to PP$, $A\to VP$, $S\to PP$, and the three-body decay models $A\to 3P$ are calculated, which compare well with the experiments. The meson-meson scattering lengths and effective ranges are calculated and compared with experiments and the current-algebra results. The mesonic decay constants and the K_{l3} form factors are calculated, and we have $F_K/F_\pi=1.04$, $F_{S_K}/F_\pi=-0.14$, $f_+(0)=0.99$, $\xi=-0.24$, $\lambda_+=0.021$, and $\lambda_-=-0.022$. The BBP coupling constants and some of the meson-baryon scattering lengths, however, do not agree with the experiments. The nonelectromagnetic isospin-violating effects are studied and it is found that a single value of a parameter characterizing the strength of these interactions does not explain the following three results: (i) the electromagnetic mass differences of pseudoscalar mesons, (ii) the $\eta\to 3\pi$ decay parameters, and (iii) the baryon electromagnetic mass differences.

I. INTRODUCTION

Effective Lagrangian models with broken chiral symmetry incorporating current algebra and partially conserved axial-vector current (PCAC) have been employed by several authors[1,2] to study the mesonic and baryonic interactions in the past few years. In most of these works, however, a few dynamical quantities are calculated, taking some others as input. It would be desirable to have a unified framework in which all quantities of interest, including particle masses if possible, are calculable. The general guidelines for the construction of such models were discussed by Gasiorowicz and Geffen.[1] A model of this type for mesonic interactions based on chiral SU(3)⊗SU(3) symmetry broken down to $SU(2)_I\otimes U(1)_Y$ was studied by Bhargava and Dass.[3] In this paper we report a study of mesonic and baryonic interactions based on chiral U(3)⊗U(3) symmetry broken down to $U(1)_{I_3}\otimes U(1)_Y\otimes U(1)$.

In addition to the usual term breaking the chiral SU(3)⊗SU(3) symmetry explicitly to $SU(2)_I\otimes U(1)_Y \otimes U(1)_{V_0}$ and transforming as $(3, 3^*)\oplus(3^*, 3)$ (see Refs. 4 and 5) we have a term violating the isospin symmetry explicitly.[6] This term has been chosen to transform as $(3, 3^*)\oplus(3^*, 3)$. The need for a term breaking SU(2) symmetry explicitly has been emphasized several times in the past in connection with the electromagnetic mass differences[7] and the $\eta\to 3\pi$ decay.[8] Indeed, if the only SU(2)-violating effects are those coming from the interactions of the photon the electromagnetic mass differences of pseudoscalar mesons should obey the relation[9]

$$m_{K^+}{}^2-m_{K^0}{}^2=m_{\pi^+}{}^2-m_{\pi^0}{}^2\,,$$

which disagrees with the experiments. Again, in the case of η decay, Sutherland[10] noted that the amplitude vanishes in the soft-pion limit if PCAC is assumed. To explain the η decay it was therefore proposed that a term transforming as the third component of an isovector should be added to the Hamiltonian.[8]

The plan of the paper is as follows: In Sec. II we write down the Lagrangian for mesons. Some of the scalar fields are assumed to have nonzero vacuum expectation value (VEV) and we perform a polar decomposition of fields following Kibble[11] and Bardeen and Lee.[12] Various steps in getting the effective Lagrangian are then discussed. Most of the parameters are fixed by using the experimental values of some of the masses. In Sec. III we study the mesonic decays and low-energy meson scattering in Sec. IV. The structure of weak currents, meson-decay constants, and the K_{l3} form factors are discussed in Sec. V. The isospin-breaking effects in the mass differences and the $\eta\to 3\pi$ decays are studied in Sec. VI. Baryonic masses, coupling constants, and the meson-baryon scattering lengths are calculated in Sec. VII. The last section contains some concluding remarks.

II. THE MESONIC LAGRANGIAN AND MASSES

A. Fields

The basic symmetry group of the model is chiral $U(3)\otimes U(3)$, a general transformation of which is represented by the unitary operator

$$\mathfrak{U}=\exp\left[-i\sum_{k=0}^{8}(\epsilon_k^L Q_k^L+\epsilon_k^R Q_k^R)\right] .$$

The meson matrix

$$M=\frac{1}{\sqrt{2}}\sum_{k=0}^{8}\lambda_k(u_k+iv_k)$$

of scalar and pseudoscalar meson fields u_k and v_k belonging to the $(3,3^*)\oplus(3^*,3)$ representation transforms as

$$\mathfrak{U}M\mathfrak{U}^\dagger=U_L M U_R^\dagger ,$$

where

$$U_L=\exp\left(i\sum_{k=1}^{8}\epsilon_k^L\lambda_k/2\right)\times\exp(i\epsilon_0^L\lambda_0/2)$$
$$\equiv \hat{U}_L U_L^0 ,\ \text{etc.}$$

The symmetry is extended to a gauge symmetry by introducing gauge fields $X_\mu^{L,k}, X_\mu^{R,k}(k=0,1,\ldots,8)$ transforming as

$$\mathfrak{U}\hat{X}_\mu^L\mathfrak{U}^\dagger=\hat{U}_L\hat{X}_\mu^L\hat{U}_L^\dagger-\frac{1}{ig}\hat{U}_L\partial_\mu\hat{U}_L^\dagger ,$$

$$\mathfrak{U}X_\mu^{L,0}\mathfrak{U}^\dagger=X_\mu^{L,0}+\frac{1}{g_0}\partial_\mu\epsilon_0^L ,$$

where

$$\hat{X}_\mu^L=\tfrac{1}{2}\sum_{k=1}^{8}X_\mu^{L,k}\lambda_k ,$$

with similar equations for $\hat{X}_\mu^R$ and $X_\mu^{R,0}$.

B. The Lagrangian

We take the Lagrangian density to be the following:

$$\mathfrak{L}=\mathfrak{L}_M+\mathfrak{L}_{VA}+\mathfrak{L}_{\text{nonmin}}+\mathfrak{L}_{\text{SB}} ,$$
$$\mathfrak{L}_M=-\tfrac{1}{2}\{D_\mu M^\dagger D_\mu M\} -\tfrac{1}{2}\mu_0^2(W_2+\nu_1W_4+\nu_2W_2^2) , \tag{1}$$

where

$$D_\mu M=\partial_\mu M-ig(\hat{X}_\mu^L M-M\hat{X}_\mu^R) -\tfrac{1}{2}ig_0\lambda_0(X_\mu^{L,0}-X_\mu^{R,0})M ,$$
$$W_2=\{M^\dagger M\},\quad W_4=\{M^\dagger MM^\dagger M\} ,$$

and $\{\ \}$ indicates the trace operation in the internal symmetry space.

$$\mathfrak{L}_{VA}=-\tfrac{1}{4}\{X_{\mu\nu}^L X_{\mu\nu}^L+X_{\mu\nu}^R X_{\mu\nu}^R\} -\tfrac{1}{2}m_0^2\{X_\mu^L X_\mu^L+X_\mu^R X_\mu^R\} -\tfrac{1}{12}m_A^2\{X_\mu^L-X_\mu^R\}^2 ,$$

where

$$X_\mu^L=\tfrac{1}{2}\sum_{k=0}^{8}X_\mu^{L,k}\lambda_k =\hat{X}_\mu^L+\tfrac{1}{2}\lambda_0X_\mu^{L,0} ,$$

$$X_{\mu\nu}^L=\partial_\mu X_\nu^L-\partial_\nu X_\mu^L-ig[X_\mu^L,X_\nu^L] ,$$

and we have similar expressions for X_μ^R and $X_{\mu\nu}^R$:

$$\mathfrak{L}_{\text{nonmin}}=ih_0\{X_{\mu\nu}^L D_\mu M D_\nu M^\dagger+X_{\mu\nu}^R D_\mu M^\dagger D_\nu M\} -\tfrac{1}{4}h_1\{X_{\mu\nu}^L X_{\mu\nu}^L MM^\dagger+X_{\mu\nu}^R X_{\mu\nu}^R M^\dagger M\} -\tfrac{1}{2}h_2\{X_{\mu\nu}^L M X_{\mu\nu}^R M^\dagger\} ,$$

$$\mathfrak{L}_{\text{SB}}=\tfrac{1}{2}\lambda(\det M+\det M^\dagger)+\tfrac{1}{2}\{A(M+M^\dagger)\} ,$$

where

$$A=\frac{1}{\sqrt{2}}(a_0\lambda_0+a_8\lambda_8+a_3\lambda_3) =\begin{pmatrix} a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{pmatrix} . \tag{2}$$

In $\mathfrak{L}_{\text{SB}}$ the first term is not invariant only under the ninth axial transformations. The second term breaks the symmetry further to $SU(2)_I\otimes U(1)_Y\otimes U(1)_{V_0}$ and to $U(1)_{I_3}\otimes U(1)_Y\otimes U(1)_{V_0}$ in the two cases $a=b$ and $a\neq b$, respectively.

C. Field mixings, renormalizations, and particle masses

For the time being we ignore the isospin-breaking effects and take $b=a$ ($a_3=0$) up to Sec. VI. Assuming $\eta=\langle M\rangle_0$, we can perform the following decomposition[12]:

$$M=e^{i\chi}e^{i\phi}(\eta+\Sigma)e^{i\phi}e^{-i\chi} , \tag{3}$$

where

$$\eta=\frac{1}{\sqrt{2}}(\eta_0\lambda_0+\eta_8\lambda_8) \equiv\begin{pmatrix}\alpha & 0 & 0\\ 0 & \alpha & 0\\ 0 & 0 & \gamma\end{pmatrix} ,$$

$$\Sigma=(\sigma_0\lambda_0+\sigma_1\lambda_1+\sigma_2\lambda_2+\sigma_3\lambda_3+\sigma_8\lambda_8)/\sqrt{2} ,$$

$$\chi=\begin{pmatrix} 0 & i\sigma_K/2(\alpha-\gamma)\\ i\sigma_K^\dagger/2(\gamma-\alpha) & 0\end{pmatrix} ,$$

$$\sigma_K = \begin{pmatrix} \sigma_{K^+} \\ \sigma_{K^0} \end{pmatrix},$$

$$\phi = \frac{1}{\sqrt{2}} \sum_{k=0}^{8} \phi_k \lambda_k / f_k ,$$

and

$$f_k = \begin{cases} 2\alpha, & \text{for } k=1,2,3 \\ \alpha+\gamma, & \text{for } k=4,5,6,7 \\ 2(\alpha+2\gamma)/3, & \text{for } k=8 \\ 2(2\alpha+\gamma)/3, & \text{for } k=0. \end{cases}$$

Here the σ's and ϕ's are scalar and pseudoscalar fields with zero VEV's.

Equation (3) expresses M as a gauge transform of $(\eta + \Sigma)$. Performing the same transformation on the gauge fields and defining

$$S_L = e^{i\chi} e^{i\phi},$$
$$S_R = e^{i\chi} e^{-i\phi},$$

we have new gauge fields $Y_\mu^{L,k}$ and $Y_\mu^{R,k}$ given by

$$\hat{X}_\mu^L = S_L \hat{Y}_\mu^L S_L^\dagger - \frac{1}{ig} S_L \partial_\mu S_L^\dagger , \tag{4}$$

with similar expression for $\hat{X}_\mu^R$ and

$$X_\mu^{V,0} = Y_\mu^{V,0},$$
$$X_\mu^{A,0} = Y_\mu^{A,0} - \left(\frac{3}{2}\right)^{1/2} \frac{1}{g_0} \operatorname{Tr}(\partial_\mu \phi). \tag{4'}$$

The fields χ and ϕ in the exponentials are the "would-be" Goldstone bosons which would get eliminated in the absence of a gauge-noninvariant mass term for the gauge fields and $\mathcal{L}_{SB}$. When the mass term is present but $\mathcal{L}_{SB}$ is not included we have invariance under the transformations with constant parameters and the fields χ and ϕ remain massless, as expected from the Goldstone theorem.

The Lagrangian has linear terms in the fields σ_0 and σ_8. In order that VEV's of these fields may vanish we must have, in the phenomenological tree approximation, the coefficients of σ_0 and σ_8 equal to zero. This gives

$$\mu_0^2(\eta + 4\nu_1\eta^3 + 4\nu_2\eta\{\eta^2\}) = A + \lambda\eta^{-1}\det\eta . \tag{5}$$

The vector and the axial-vector fields

$$Y_\mu^{V_k} = \tfrac{1}{2}(Y_\mu^{L_k} + Y_\mu^{R_k}),$$
$$Y_\mu^{A_k} = \tfrac{1}{2}(Y_\mu^{L_k} - Y_\mu^{R_k})$$

have mixings with scalar and pseudoscalar fields coming from the terms of the type $Y_\mu^V \cdot \partial_\mu \sigma$ and $Y_\mu^A \cdot \partial_\mu \phi$. To remove these mixings we write

$$Y_\mu^{V_k} = \mathcal{Y}_\mu^{V_k} - C_{kl}^V \partial_\mu \sigma_l$$

and

$$Y_\mu^{A_k} = \mathcal{Y}_\mu^{A_k} - C_{kl}^A \partial_\mu \phi_l , \tag{6}$$

and determine the C's from the requirement that there be no mixing of type $\mathcal{Y}_\mu^V \cdot \partial_\mu \sigma$ and $\mathcal{Y}_\mu^A \cdot \partial_\mu \phi$. We now define renormalized vector and axial-vector fields V_μ^k and A_μ^k by the equations

$$V_\mu^k = Z_{V_k}{}^{1/2} \mathcal{Y}_\mu^{V_k}, \quad k=1,\ldots,7$$
$$A_\mu^k = Z_{A_k}{}^{1/2} \mathcal{Y}_\mu^{A_k}, \quad k=1,\ldots,7 \tag{7}$$

and determine the Z's from the requirement that the fields V_μ and A_μ have standard kinetic energy coefficients. We obtain, using particle symbols as subscripts,

$$Z_\rho{}^{-1} = 1 + (h_1 + h_2)\alpha^2,$$
$$Z_{K^*}{}^{-1} = 1 + \tfrac{1}{2} h_1(\alpha^2+\gamma^2) + h_2\alpha\gamma,$$
$$Z_{A_1}{}^{-1} = 1 + (h_1 - h_2)\alpha^2,$$
$$Z_{K_A}{}^{-1} = 1 + \tfrac{1}{2} h_1(\alpha^2+\gamma^2) - h_2\alpha\gamma,$$

and the squared masses of these particles are

$$m_\rho{}^2 = m_0{}^2 Z_\rho ,$$
$$m_{K^*}{}^2 = [m_0{}^2 + \tfrac{1}{2} g^2(\alpha-\gamma)^2] Z_{K^*},$$
$$m_{A_1}{}^2 = [m_0{}^2 + 2g^2\alpha^2] Z_{A_1},$$
$$m_{K_A}{}^2 = [m_0{}^2 + \tfrac{1}{2} g^2(\alpha+\gamma)^2] Z_{K_A}.$$

The mixing problem for the eighth and zeroth components in various multiplets is discussed in the appendix.

The renormalized scalar and pseudoscalar fields are defined by equations similar to Eq. (6). The renormalization constants and masses for the pseudoscalar and scalar mesons are

$$Z_\pi = 1 + 2g^2\alpha^2/m_0{}^2,$$
$$Z_K = 1 + g^2(\alpha+\gamma)^2/2m_0{}^2,$$
$$m_\pi{}^2 = Z_\pi(a/\alpha),$$
$$m_K{}^2 = Z_K(a+c)/(\alpha+\gamma),$$
$$Z_{S_\pi} = 1, \quad Z_{S_K} = 1 + g^2(\alpha-\gamma)^2/2m_0{}^2,$$
$$m_{S_\pi}{}^2 = \mu_0{}^2(\nu_0 - 12\nu_1\alpha^2) - \lambda\alpha ,$$
$$m_{S_K}{}^2 = Z_{S_K}(a-c)/(\alpha-\gamma),$$

where we have defined $\nu_0 = 1 - 4\nu_2\{\eta^2\}$.

Most of the parameters were determined through experimental masses. The crucial parameter $\delta = \alpha/\gamma$, characterizing the relative strengths of the breaking of SU(3) and of SU(3)⊗SU(3) by the vacuum, was varied in a suitable range and was chosen to give the best over-all fit. In particular the scalar-meson masses were found to be very sensitive to the value of δ. The mesonic masses depend effectively on nine independent parameters; these were fixed by a least-squares fit to the 12 masses of the vector, axial-vector, and pseudoscalar mesons.[13] The values of the parameters for $\delta = 0.84$ are

$$\begin{aligned} &\nu_0\mu_0^2 = -4.64 m_\pi^2, \ \lambda\gamma = 8.84 m_\pi^2, \ \nu_1/\nu_0 = 1.06, \\ &m_0 = 0.668 m_\rho, \ g\gamma = 0.703 m_\rho, \ g_0/g = -0.95, \\ &h_1\gamma^2 = -0.658, \ h_2\gamma^2 = -0.058, \ m_A/m_0 = 0.99. \end{aligned} \tag{8}$$

The calculated and experimental values[14] of the masses of the vector, axial-vector, and pseudoscalar mesons are given in Table I. The masses of S_π and S_K are predicted to be 949 MeV and 1025 MeV, respectively. We identify these particles with the resonances $\pi_N(980)$ and $K_N(1080)$. The masses of S_η and S'_η depend on an additional parameter ν_2. Mass values close to the experimental candidates are obtained for small ν_2. For $\nu_2 = (-1.0, 0, 1.0)$, we have $m_{S_\eta} = (1060, 1068, 1083)$ MeV and $m_{S'_\eta} = (560, 694, 794)$ MeV. For the rest of the discussion we assume $\nu_2 = 0$ and identify S_η and S'_η with $S^*(1060)$ and $\epsilon(700)$ resonances.

III. MESONIC COUPLINGS AND DECAYS

Substituting (3), (4), and (4′) in (1) and expressing the resulting Lagrangian in terms of the physical fields, we get an effective Lagrangian from which the various couplings can be read off. In the following we discuss two-body and three-body decay modes of mesons.

A. Two-body decay modes

There are two more parameters, g and h, to be fixed before we proceed to calculate the rates for the two-body decay modes of the mesons. The experimental value of the ratio $\Gamma(K\to\mu\nu)/\Gamma(\pi\to\mu\nu)$ gives

$$\left(\frac{F_K}{F_\pi}\tan\theta_A\right)^2 = 0.775,$$

where θ_A is the Cabibbo angle for the axial-vector current. Using the value $F_K/F_\pi = 1.04$ (see Sec. V) in our model, we have

$$\tan\theta_A = 0.26.$$

Then the rate for $\pi\to\mu\nu$ fixes $F_\pi = 93.3$ MeV. Using this value of F_π (for expression of F_π see Sec. V) and $g\gamma$ fixed earlier, we obtain

$$\frac{g^2}{4\pi} = 1.53.$$

Using the value of g so obtained we can calculate the widths for the $V\to PP$ and $A\to VP$ decays, which are given as functions of h $(= -h_0 m_0^2/g)$ by

$$\begin{aligned} &\Gamma(\rho\to 2\pi) = 84.0(1+2.47h+1.49h^2), \\ &\Gamma(K^*\to K\pi) = 26.7(1+3.07h+2.32h^2), \\ &\Gamma(\phi\to K\bar{K}) = 1.37(1+1.94h+1.77h^2), \\ &\Gamma(A_1\to\rho\pi) = 123.0(1-4.14h+4.20h^2), \\ &\Gamma(K_A\to K^*\pi) = 79.4(1-4.82h+6.11h^2), \\ &\Gamma(E\to K^*\bar{K}+\bar{K}^*K) = 64.7(1-7.67h+14.7h^2). \end{aligned}$$

In Table II these rates are given for some values of h. The best over-all fit is obtained for $h = 0.1$. For this value of h we now calculate the widths for $A\to SP$ decays. $S\to PP$ decay widths which are independent of h and $A\to SP$ decay widths (for $h = 0.1$) are given in Table III. The experimental situation regarding these decays is not clear.

B. Three-body decay modes

We now consider the three-body decays of the axial-vector mesons and the X^0. For the decay

$$A(Q)\to B(q_1)+C(q_2)+D(q_3)$$

let

$$\langle BCD|S|A\rangle = i(2\pi)^4\delta(Q-\textstyle\sum_i q_i)\dfrac{1}{(2\pi)^6} \times \dfrac{m}{(16\omega_1\omega_2\omega_3\omega_Q)^{1/2}}.$$

The width is then given by

$$\Gamma(A\to BCD) = \frac{1}{64\pi^3 M_A} \times \int_{M_2}^{E_{\max}} d\omega_2 \int_{\omega_-}^{\omega_+} d\omega_1 \frac{1}{(2J_A+1)} \sum_{\text{spins}} |m|^2$$

where

$$E_{\max} = \frac{M_A^2+M_2^2-(M_1+M_3)^2}{2M_A}$$

TABLE I. Meson masses in MeV. The values of the nine parameters given in Eq. (8) were obtained by a least-squares fit to these 12 masses. The values in the first row were used in getting the best fit. The second row contains final calculated values (Ref. 14).

	ρ	K^*	ω	ϕ	A_1	K_A	D	E	π	K	η	X^0
Experimental value	765	891.7	783.9	1019.5	1070	1242	1286	1422	140	494	548.8	957.3
Calculated value	776	879	776	1024	1085	1244	1281	1424	140	498	543	958

TABLE II. $V \to PP$ and $A \to VP$ decay widths (in MeV) for some values of h.

h	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	Experimental value [a]
$\rho \to 2\pi$	88.1	92.4	96.7	101.2	105.8	110.5	115.3	120.3	125.3	125 ± 20
$K^* \to K\pi$	29.2	30.9	32.7	34.6	36.5	38.4	40.5	42.5	44.6	50
$\phi \to K\bar{K}$	2.2	2.4	2.6	2.8	2.9	3.1	3.4	3.6	3.8	3.2
$A_1 \to \rho\pi$	112.9	103.4	94.4	85.8	77.6	69.8	62.8	55.6	49.1	95 ± 35
$K_A \to K^*\pi$	71.4	63.9	56.8	50.1	43.9	38.1	32.8	27.9	23.4	40–130
$E \to (K^*\bar{K} + \bar{K}K^*)$	57.6	48.4	40.0	32.5	25.7	19.7	14.5	10.1	6.5	35.5

[a] See Ref. 14.

and $\omega_\pm$ are the two roots of the quadratic equation

$$x\omega^2 + y\omega + z = 0,$$

with x, y, and z being given by

$$x = 4[(\omega_2 - M_A)^2 - \omega_2^2 + M_2^2],$$

$$y = 4t(\omega_2 - M_A),$$

$$z = t^2 + 4M_1^2(\omega_2^2 - M_2^2),$$

and

$$t = M_A^2 - M_3^2 + M_1^2 + M_2^2 - 2M_A\omega_2.$$

The intermediate states which contribute to the various decays are

(1) $A_1 \to 3\pi$: $(\rho\pi)$, $(S_\eta\pi)$, and $(S'_\eta\pi)$;

(2) $K_A \to K\pi\pi$: (ρK), $(K^*\pi)$, $(S_K\pi)$, $(S_\eta K)$, and $(S'_\eta K)$;

(3) $(D, E) \to K\bar{K}\pi$:

$(K^*\bar{K})$, $(\bar{K}^*K)$, $(S_K\bar{K})$, $(\bar{S}_K K)$, and $(S_\pi\pi)$;

(4) $(D, E) \to \eta\pi\pi$: $(S_\pi\pi)$, $(S_\eta\eta)$, and $(S'_\eta\eta)$;

(5) $X^0 \to \eta\pi\pi$: $(S_\pi\pi)$, $(S_\eta\eta)$, and $(S'_\eta\eta)$.

The predicted rates for these decays are given in Table IV. The third column contains the values in a limit in which masses of chiral parameters of Goldstone bosons go to ∞ (see Sec. VIII). In this limit, the contributions of S_π, S_η, and S'_η mesons to these decays drop out. By comparing the two results we see that the scalar-meson contributions to $A \to 3P$ and $X^0 \to \eta\pi\pi$ decays are small.

IV. MESON-MESON SCATTERING

The amplitude $T_{abcd}(s, t, u)$ for the scattering of two pseudoscalar mesons,[15]

$$P_a(q_a) + P_b(q_b) \to P_c(q_c) + P_d(q_d),$$

is defined as

$$\langle P_c P_d | S | P_a P_b \rangle = \delta_{fi} + i(2\pi)^4 \delta(q_c + q_d - q_a - q_b) \times \frac{T(s, t, u)}{(2\pi)^6(16\omega_a\omega_b\omega_c\omega_d)^{1/2}},$$

where the Mandelstam variables s, t, and u are

$$s = -(q_a + q_b)^2,\quad t = -(q_a - q_c)^2,\quad u = -(q_a - q_d)^2.$$

The scattering amplitude F, related to the differential cross section by

$$\frac{d\sigma}{d\Omega} = |F|^2,$$

is given by

$$F(s, t, u) = \frac{1}{8\pi\sqrt{s}}\, T(s, t, u).$$

The scattering length, a_l and the effective range, r_l, for the lth partial wave are given by

$$q^{2l}\,\mathrm{Re}\{[f_l(q^2)]^{-1}\} = a_l^{-1} + \tfrac{1}{2} r_l q^2 + O(q^4),$$

TABLE III. $S \to PP$ and $A \to SP$ decays.

Decay	Width (MeV)
$\pi_N \to \eta\pi$	31.5
$S_K \to K\pi$	94.4
$\epsilon \to \pi\pi$	38.6
$S^* \to \pi\pi$	54.7
$S^* \to K\bar{K}$	26.7
$A_1 \to \epsilon\pi$	28.5
$K_A \to S_K\pi$	33.5
$K_A \to \epsilon K$	4.8
$D \to \pi_N\pi$	8.5
$E \to \pi_N\pi$	44.0
$D \to \epsilon\eta$	6.1
$E \to \epsilon\eta$	17.4

where f_l is the lth partial-wave amplitude defined by

$$F(q^2, \cos\theta) = \sum_{l=0}^{\infty} (2l+1) f_l(q^2) P_l(\cos\theta),$$

and $\vec{q}$ and θ are the center-of-mass momentum and the scattering angle, respectively.

A. π-π scattering

The amplitude $T_{abcd}(s, t, u)$ for the process

$$\pi_a + \pi_b \to \pi_c + \pi_d$$

has the isospin structure

$$T_{abcd}(s, t, u) = \delta_{ab}\delta_{cd} A(s, t, u) + \delta_{ac}\delta_{bd} B(s, t, u) + \delta_{ad}\delta_{bc} C(s, t, u),$$

and the amplitudes T^I ($I = 0, 1, 2$) corresponding to the definite isospin are

$$T^0 = 3A + B + C,$$

$$T^1 = B - C,$$

$$T^2 = B + C.$$

Using the effective couplings obtained earlier we calculate the π-π scattering amplitude in the tree approximation and we get for the s-wave scattering lengths

$$32\pi m_\pi a_s^0 = \frac{7m_\pi^2}{F_\pi^2} + \cdots$$

and

$$32\pi m_\pi a_s^2 = -\frac{2m_\pi^2}{F_\pi^2} + \cdots,$$

where the dots stand for the additional contributions coming from the S_η and S_η' exchanges and the terms written explicitly are the contributions of the contact term and the ρ-meson exchange. If we neglect the contributions of the scalar mesons, which are less than a few percent of the total, our result agrees with the soft-pion calculations of Weinberg.[16] In the case of p-wave scattering we have

$$24\pi m_\pi a_p^1 = F_\pi^{-2} + O(m_\pi^2/m_\rho^2) + \cdots,$$

i.e., the leading term in the expansion of the right-hand side in powers of (m_π/m_ρ) agrees with the soft-pion result. Similar observations have been made by Bhargava[3] in a linear model, but our results are different in two ways: Firstly, the contributions of the scalar mesons to the scattering lengths are small and insensitive to variations in ν_2 in contrast to the results of Ref. 3 where the contributions of the scalar mesons were found to be large and sensitive to the value of the parameter ν_2. The second difference is that we do not have to assume the KSRF (Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin) condition to get current-algebra results as was needed in Ref. 3. The numerical values of the scattering lengths and effective ranges are given in Table V. As, in our model, the scalar-meson contributions are small the predicted values are close to the current-algebra results.

B. K-π scattering

The isospin structure of the amplitude T_{abcd} for the process

$$K_a + \pi_b \to K_c + \pi_d$$

is given by

$$T_{abcd} = \chi_c^\dagger \chi_a \delta_{bd} T_A + \chi_c^\dagger [\tau_d, \tau_b] \chi_a T_B,$$

where χ_a and χ_c are numerical isovectors specifying the isospin of the initial and final kaons, respectively.

The amplitudes for the $I = \frac{3}{2}$ and $I = \frac{1}{2}$ channels are

$$T^{3/2} = T_A - T_B,$$

$$T^{1/2} = T_A + 2T_B.$$

As in the case of π-π scattering we find that the contributions of S_η and S_η' are small and insensitive to variations in ν_2. The numerical values for the s-wave and the p-wave scattering parameters are given in Table VI and are compared with the soft-pion results of Weinberg[16] and the hard-pion calculations of Pond.[19]

TABLE IV. Rates (in MeV) for three-body decay modes of axial-vector mesons and the X^0.

Decay	Width (MeV)	Width in the limit $\lvert\mu_0^2\rvert \to \infty$ [a] (MeV)	Experimental value (MeV)
$A_1 \to 3\pi$	67.5	51.5	125 ± 20
$K_A \to K\pi\pi$	52.4	49.0	40–130
$E \to K\bar{K}\pi$	19.5	18.5	69 ± 4 [b]
$E \to \eta\pi\pi$	30.7	26.5	
$D \to K\bar{K}\pi$	0.24	0.6	33 ± 4 [b]
$D \to \eta\pi\pi$	7.6	7.1	
$X^0 \to \eta\pi\pi$	0.013	0.010	4

[a] See Sec. VIII.

[b] The values quoted are the total widths for the decaying meson and the two modes are the important ones contributing to the total width.

TABLE V. π-π scattering parameters. The s-wave scattering lengths and effective ranges are in units of m_π^{-1}. The p-wave scattering lengths and effective ranges are in units of m_π^{-3} and m_π, respectively.

	a_s^0	r_s^0	a_s^2	r_s^2	a_p^1	r_p^1
Present model	0.16	−7.6	−0.045	64.3	0.035	21.0
Soft-pion results	0.17 ± 0.02	−7.3 ± 0.7	−0.05 ± 0.005	6.0 ± 0.6	0.033 ± 0.003	⋯
$\pi N \to 2\pi N$ [a]	0.19 ± 0.04	⋯	−0.059 ± 0.015	⋯	0.038	⋯
$\pi N \to 2\pi N$ [b]	$0.2^{+0.08}_{-0.1}$	$-5.2^{+1.5}_{-2.0}$	⋯	⋯	⋯	⋯

[a] From high-energy peripheral reaction data (Ref. 17).
[b] From low-energy data (see Ref. 18).

C. K-K scattering

The amplitude T_{abcd} for the K-K scattering

$$K_a + K_b \to K_c + K_d$$

has the form

$$T_{abcd} = \delta_{ac}\,\delta_{bd}\,T_A + \delta_{ad}\,\delta_{bc}\,T_B\,,$$

and the definite isospin amplitudes are

$$T^0 = T_A - T_B\,,$$

$$T^1 = T_A + T_B\,.$$

The s-wave $I=1$ and the p-wave $I=0$ scattering lengths and effective ranges are predicted to be

$$a_s^1 = -0.096 m_\pi^{-1}, \quad r_s^1 = -0.65 m_\pi^{-1},$$

$$a_p^0 = 0.0044 m_\pi^{-3}, \quad r_p^1 = 2.7 m_\pi\,.$$

The s-wave scattering length is close to the value obtained in other effective Lagrangian models.[20] No estimates for the effective ranges and the p-wave scattering length are available.

V. CURRENTS AND MESON-DECAY FORM FACTORS

A. Currents

The vector and axial-vector currents $J_\mu^{V_k}$ and $J_\mu^{A_k}$ can be calculated from the Lagrangian. In terms of 3×3 matrices

$$J_\mu^V = \frac{1}{\sqrt{2}} \sum_{k=1}^{8} J_\mu^{V_k} \lambda_k\,,$$

$$J_\mu^A = \frac{1}{\sqrt{2}} \sum_{k=1}^{8} J_\mu^{A_k} \lambda_k\,.$$

The octet currents are given by[21]

$$J_\mu^V = \frac{1}{\sqrt{2}} (J_\mu^L + J_\mu^R),$$

$$J_\mu^A = \frac{1}{\sqrt{2}} (J_\mu^L - J_\mu^R),$$

where

$$J_\mu^{L,R} = -\frac{m_0^2}{g} X_\mu^{L,R} - \frac{1}{g}\partial_\nu f_{\nu\mu}^{L,R},$$

$$f_{\mu\nu}^L = \hat{X}_{\mu\nu}^L + \tfrac{1}{2}h_1(\hat{X}_{\mu\nu}^L M M^\dagger + M M^\dagger \hat{X}_{\mu\nu}^L) + h_2 M \hat{X}_{\mu\nu}^R M^\dagger + i h_0 (D_\mu M D_\nu M^\dagger - D_\nu M D_\mu M^\dagger),$$

and $$\tag{9}$$

$$f_{\mu\nu}^R = \hat{X}_{\mu\nu}^R + \tfrac{1}{2}h_1(\hat{X}_{\mu\nu}^R M^\dagger M + M^\dagger M \hat{X}_{\mu\nu}^R) + h_2 M^\dagger \hat{X}_{\mu\nu}^L M + i h_0 (D_\mu M^\dagger D_\nu M - D_\nu M^\dagger D_\mu M).$$

In the above expression the currents are written in terms of the original fields X_μ^L, etc., and are now expressed in terms of the physical fields as defined in Sec. II.

TABLE VI. K-π scattering parameters. The s-wave parameters are in units of m_π^{-1}, and the p-wave scattering lengths and effective ranges are in units of m_π^{-3} and m_π, respectively.

	$a_s^{1/2}$	$r_s^{1/2}$	$a_s^{3/2}$	$r_s^{3/2}$	$a_p^{1/2}$	$r_p^{1/2}$	$a_p^{3/2}$	$r_p^{3/2}$
Present model	0.16	−15	−0.058	70	0.014	−33	0.0033	170
Soft-pion calculation [a]	0.16 ± 0.02	−5.0 ± 0.5	−0.078 ± 0.008	17 ± 2	0.014 ± 0.001	⋯	⋯	⋯
Hard-pion calculation [b]	0.115	−12.0	−0.090	14.2	0.0153	−35.5	0.005	−35.0

[a] Reference 16.
[b] The values quoted here are for $F_K/F_\pi = 1.169$ (Ref. 19).

B. Decay constants

The π_{l2} and the K_{l2} decay constants are defined by

$$\langle 0|J_\mu^{A_k}|P^k(q)\rangle = \frac{iq_\mu}{(2\pi)^{3/2}(2\omega_q)^{1/2}} F_{p_k}.$$

Using the above expressions for the currents we get

$$F_\pi = \sqrt{2}\,\alpha Z_\pi^{-1/2},$$

$$F_K = (\alpha+\gamma) Z_K^{-1/2}/\sqrt{2}\,.$$

The decay constant F_{S_K} of the strange scalar meson S_K is defined similarly by

$$\langle 0|J_\mu^{K^*}|S_K(q)\rangle = \frac{-q_\mu}{(2\pi)^{3/2}(2\omega_q)^{1/2}} F_{S_K},$$

and we have

$$F_{S_K} = (\alpha-\gamma) Z_{S_K}^{-1/2}/\sqrt{2}\,.$$

The mesonic masses, the decay constants, and the renormalization constants obey the Glashow-Weinberg relations[5]

$$Z_\pi^{1/2}F_\pi = Z_K^{1/2}F_K + Z_{S_K}^{1/2}F_{S_K},$$

$$Z_\pi^{-1/2}m_\pi^2 F_\pi = Z_K^{-1/2}m_K^2 F_K + Z_{S_K}^{-1/2}m_{S_K}^2 F_{S_K}.$$

For the value 0.84 for the parameter δ we predict

$$\frac{F_K}{F_\pi} = 1.04, \qquad \frac{F_{S_K}}{F_\pi} = -0.14.$$

We do not predict the value of F_π since it was taken as input to fix g as discussed in Sec. III.

For the neutral vector-meson decays into lepton pairs the decay constants are defined by[22]

$$\langle 0|J_\mu^{\text{em}}|V^k(q)\rangle = \frac{\epsilon_\mu}{(2\pi)^{3/2}(2\omega_q)^{1/2}}\frac{m_{V_k}^2}{f_{V_k}}.$$

The f_ρ, f_ω, and f_ϕ are given by

$$f_\rho = gZ_\rho^{1/2},$$

$$f_\omega = 2g_Y/\sin\theta_Y,\quad f_\phi = 2g_Y/\cos\theta_Y,$$

where

$$g_Y = \tfrac{1}{2}\sqrt{3}\,g\left[\left(\frac{m_0}{m_\phi}\cos\theta\right)^2 + \left(\frac{m_0}{m_\omega}\sin\theta\right)^2\right]^{-1/2},$$

$$\tan\theta_Y = (m_\phi/m_\omega)\tan\theta \text{ and } \tan\theta = 2\sqrt{2}\,.$$

The predicted values are

$$\frac{f_\rho^2}{4\pi} = 3.1,\ \frac{f_\omega^2}{4\pi} = 27.5,\ \frac{f_\phi^2}{4\pi} = 24.5,$$

which are to be compared with experimental values[23]

$$\frac{f_\rho^2}{4\pi} = 1.9 \pm 0.19,\ \frac{f_\omega^2}{4\pi} = 14.0 \pm 2.8,\ \frac{f_\phi^2}{4\pi} = 11.0 \pm 0.9$$

obtained from leptonic decay rates and the values[24]

$$\frac{f_\rho^2}{4\pi} = 2.5 - 5.2,\ \frac{f_\omega^2}{4\pi} = 30.4 \pm 7.2,\ \frac{f_\phi^2}{4\pi} = 13.6$$

obtained from the photoproduction data.

The quantities F_π, m_{A_1}, m_ρ and g_ρ $(=gZ_\rho^{1/2})$ are related to each other through the relation

$$F_\pi^2 = \left(\frac{m_\rho^2}{g_\rho}\right)^2\left(\frac{1}{m_\rho^2} - \frac{1}{m_{A_1}^2}\right),$$

which is Weinberg's first sum rule in single-particle approximation.[25] The second sum rule is satisfied only if $h_2 = 0$ is assumed. Also we do not have the KSRF relation[26] $m_\rho^2 = 2F_\pi^2 g_\rho^2$. Numerically, we have

$$m_\rho^2/2F_\pi^2 g_\rho^2 = 0.78.$$

C. K_{l3} form factors

The K_{l3} form factors[27] are defined by

$$\langle \pi^0(p)|\,J_\mu^{K^{*-}}(0)|K^-(k)\rangle = -\frac{1}{(2\pi)^3(4\omega_p\omega_k)^{1/2}} \times [\tfrac{1}{2}(k+p)_\mu f_+(q^2) + \tfrac{1}{2}(k-p)_\mu f_-(q^2)],$$

where $J_\mu^{K^{*-}}$ is the strangeness-changing weak vector current and $q^2 = -(k-p)^2$.

For calculating the form factors we do not use the full current given by Eq. (9) but use only the first term—the term proportional to the X_μ field—which, when expressed in terms of the physical fields, becomes

$$J_\mu^{K^{*-}} = -\frac{m_0^2}{g} Z_{K^*}^{1/2}K_\mu^{*-} + iF_{S_K}\partial_\mu S_K^- - i\,\frac{m_0^2(Z_K Z_\pi)^{1/2}}{4g^2\alpha(\alpha+\gamma)}(K^-\partial_\mu\pi^0 - \pi^0\partial_\mu K^-).$$

The form factors $f_\pm(q^2)$ are calculated and expanded in powers of q^2 as

$$f_\pm(q^2) = f_\pm(0)\left[1 + \lambda_\pm\frac{q^2}{m_\pi^2} + O\left(\frac{q^4}{m_\pi^4}\right)\right].$$

Then we have the following result[5]:

$$f_+(0) = \frac{F_K^2 + F_\pi^2 - F_{S_K}^2}{2F_K F_\pi} = 0.99.$$

The value of $f_+(0)$ is close to 1, the SU(3) symmetry value, in accordance with the Ademollo-Gatto theorem.[28] Other K_{l3} parameters are

$$\xi = f_-(0)/f_+(0) = -0.24,$$

$$\lambda_+ = 0.021,\quad \lambda_- = -0.022.$$

Experimentally,[27] from the Dalitz-plot analysis of

$K_{\mu 3}$ decays we have $\xi = -0.15 \pm 0.5$. The μ polarization data give large negative value: $\xi = -1.45 \pm 0.70$, while the $\Gamma_{\mu 3}/\Gamma_{e3}$ ratio gives $\xi = -0.53 \pm 0.18$. The over-all fit to the three sets of experimental data gives $\xi = -0.85 \pm 0.20$, $\lambda_+ = 0.45 \pm 0.012$. The situation regarding λ_- is not clear and in most of the analyses is assumed to be zero. From the expression

$$R = \frac{0.6457 + 3.8008\lambda_+ + 6.8120\lambda_+{}^2 + 0.1264\xi + 0.4757\xi\lambda_+ + 0.0192\xi^2}{1 + 3.6995\lambda_+ + 5.4777\lambda_+{}^2}$$

for the branching ratio $\Gamma(K \to \pi\mu\nu)/\Gamma(K \to \pi e\nu)$, we get

$$R = 0.640,$$

which is to be compared with the experimental value 0.626 ± 0.19.

Recently it has been suggested that a large negative ξ may be obtained if the Duffin-Kemmer-Petiau (DKP) formalism is used to describe the pseudoscalar mesons.[29] It would be very interesting to check this proposal in a Lagrangian model similar to the present model. However, it is not clear how the Higgs-Kibble mechanism can provide splitting between the vector and axial-vector meson masses since the gauge-invariant kinetic energy term for mesons in the DKP formalism is linear in the gauge fields.

If the full current given by Eq. (9) is used to calculate the form factors we have a small positive value for ξ ($\simeq 0.003$) which is ruled out by the experiments.

VI. ISOSPIN-BREAKING EFFECTS

We now assume that $a_3 \neq 0$ [see Eq. (2)] and that

$$\langle M\rangle_0 = \eta = \frac{1}{\sqrt{2}}(\eta_0\lambda_0 + \eta_8\lambda_8 + \eta_3\lambda_3), \qquad (10)$$

and again perform the polar decomposition:

$$M = e^{i\chi}e^{i\phi}(\eta + \Sigma)e^{i\phi}e^{-i\chi},$$

where now

$$\Sigma = \frac{1}{\sqrt{2}}(\sigma_0\lambda_0 + \sigma_8\lambda_8 + \sigma_3\lambda_3),$$

$$\chi = \begin{bmatrix} 0 & i\sigma_\pi^+/2(\beta-\alpha) & i\sigma_K^+/2(\gamma-\alpha) \\ i\sigma_\pi^-/2(\alpha-\beta) & 0 & i\sigma_K^0/2(\gamma-\beta) \\ i\sigma_K^-/2(\alpha-\gamma) & i\bar{\sigma}_K^0/2(\beta-\gamma) & 0 \end{bmatrix},$$

$$\phi = \frac{1}{\sqrt{2}}\sum_{k=0}^{8} \phi_k\lambda_k/f_k,$$

$$f_k = \begin{cases} (\alpha+\beta), & \text{for } k = 1.2 \\ (\alpha+\gamma), & \text{for } k = 4.5 \\ (\beta+\gamma), & \text{for } k = 6.7 \\ 2\alpha & \text{for } k = 3 \\ (\alpha+\beta+4\gamma)/3, & \text{for } k = 8 \\ 2(\alpha+\beta+\gamma)/3, & \text{for } k = 0. \end{cases}$$

The physical fields are defined and the masses are obtained in the tree approximation as in Sec. II. The form of the consistency conditions, that the VEV's of σ fields are zero, is the same as Eq. (5) except that η is now given by Eq. (10) and A by Eq. (2). Thus we have a relation between η_3 and a_3, and all electromagnetic mass differences and coupling constants are functions of a single parameter which we take to be η_3.

Nonvanishing of η_3 gives rise to the mass differences in various isospin multiplets. To the mass differences so calculated in the tree approximation, the nontadpole contributions must be added. These nontadpole contributions have been calculated by many authors.[30] The nontadpole contribution to Δm_π ($=m_{\pi^+} - m_{\pi^0}$) is close to the experimental value, while that for Δm_K ($=m_{K^0} - m_{K^+}$) is not. In our model, in the tree approximation, Δm_π is of the order $\eta_3{}^2$ and Δm_K is proportional to η_3. The total mass differences are given in Table VII for some values of η_3:

For

$$\delta' \equiv [(\alpha-\beta)/2\gamma]\times 10^3 = -4.0$$

we have, in the tree approximation,

$$m_{\rho^+} - m_{\rho^0} = 2.5 \text{ MeV}, \quad m_{K^{*0}} - m_{K^{*+}} = 4.9 \text{ MeV}.$$

No estimates about the tadpole contributions to these mass differences are available.

In the presence of isospin-violating interactions we can now estimate the rates for $\rho^\pm \to \eta\pi^\pm$ and $\omega \to \pi^+\pi^-$ decays. For $\delta' = -4.0$ we have

$$\Gamma(\rho \to \eta\pi) = 3.8 \text{ keV},$$

which is well below the experimental upper limit of a few MeV. The width for $\omega \to \pi^+\pi^-$, however, turns out to be as large as that for $\rho \to \pi^+\pi^-$. This is because when $\eta_3 \neq 0$ the $\rho-\omega$ mixing is large (mixing angle $= 45°$) and independent of η_3. Therefore, $\Gamma(\omega \to \pi^+\pi^-)$ cannot be made smaller by choosing a smaller value for η_3. This problem is due to the nonet structure of the terms proportional to h_1 and h_2 in the Lagrangian. These terms were essential to have $\omega - \phi$ mixing and a good fit to the spin-one meson masses. One may try to write terms having structure of these terms but with different coupling constants for the octet and the singlet fields, but then the number of parameters becomes too large and the predictive power of the model is considerably reduced.[31]

For the $\eta \to 3\pi$ decays the amplitude m is defined by

$$\langle 3\pi; k_1k_2k_3|S\,|\eta(q)\rangle = i(2\pi)^4\delta(q - \textstyle\sum_{i=1}^3 k_i)$$
$$\times \frac{m}{(2\pi)^6(16\omega_1\omega_2\omega_3\omega_q)^{1/2}}$$

and the width is obtained by integrating the square of the amplitude as discussed in Sec. III.

The slope α of the Dalitz plot for the decay $\eta \to \pi^+\pi^-\pi^0$ is defined by

$$m_{+-0}(y) = m_{+-0}(0)[1 + \alpha y + O(y^2)],$$

where

$$y = \frac{3T_{\pi^0}}{Q}, \quad Q = m_\eta - 2m_{\pi^+} - m_{\pi^0},$$

and T_{π^0} is the kinetic energy of the π^0.

The amplitude for the $\pi^+\pi^-\pi^0$ mode gets contributions from the contact term and the $\rho\pi$, $S_\pi\pi$, $S_\eta\pi$, and $S'_\eta\pi$ intermediate states, while the amplitude for the $3\pi^0$ mode gets contributions from the contact term, the $S_\pi\pi$, $S_\eta\pi$, and $S'_\eta\pi$ intermediate states. If we use the effective vertices, the rates for the $\pi^+\pi^-\pi^0$ and $3\pi^0$ modes, the branching ratio, and the slope parameter are calculated and are given in Table VII along with the electromagnetic mass differences for pseudoscalar mesons for some values of δ'. In the range of values of δ' determined from the pseudoscalar mass differences the rates and slope parameter are lower than the observed values.[14,32]

It has been recently shown[33] that for some simple symmetry-breaking schemes it is not possible to explain the η decays. This conclusion is in agreement with our results for the $(3, 3^*) \oplus (3^*, 3)$ model.

VII. BARYONS

In this section we discuss the interactions of spin-$\frac{1}{2}$ baryons. There are two simple choices for the representation for the baryons: (i) the octet of baryons belonging to the representation $(1, 8) \oplus (8, 1)$, and (ii) a nonet transforming as $(3, 3^*) \oplus (3^*, 3)$. The invariants for the two cases have been written by Gasiorowicz and Geffen.[1] We did not study the $SU(3)\otimes SU(3)$-breaking terms. The choice of invariants was further restricted by keeping the terms which are bilinear in the B's and is at most quadratic in the spin-zero field M.

(i) The $(1, 8) \oplus (8, 1)$ *representation.* The Lagrangian studied was

$$\begin{aligned}
\mathcal{L} = &-\{\bar{B}_+\gamma_\mu D_\mu B_+ + \bar{B}_-\gamma_\mu D_\mu B_-\}\\
&- \alpha_1\{\bar{B}_+\gamma_\mu D_\mu B_+ MM^\dagger + \bar{B}_-\gamma_\mu D_\mu B_- M^\dagger M + \text{H.c.}\}\\
&- \alpha_2\{\bar{B}_+ MM^\dagger\gamma_\mu D_\mu B_+ + \bar{B}_- M^\dagger M\gamma_\mu D_\mu B_- + \text{H.c.}\}\\
&- \alpha_3\{\bar{B}_+ MB_- M^\dagger + \bar{B}_- M^\dagger B_+ M\}\\
&- \alpha_4\{\bar{B}_+\gamma_\mu B_+ MD_\mu M^\dagger + \bar{B}_-\gamma_\mu B_- M^\dagger D_\mu M + \text{H.c.}\}\\
&- \alpha_5\{\bar{B}_+ MD_\mu M^\dagger\gamma_\mu B_+ + \bar{B}_- M^\dagger D_\mu M\gamma_\mu B_- + \text{H.c.}\},
\end{aligned}$$

where

$$B_\pm = \left(\frac{1 \pm \gamma_5}{2}\right)\sum_{k=1}^{8} B_k\lambda_k/\sqrt{2},$$

and the covariant derivatives are

TABLE VII. Variation of the em mass differences (in MeV) of pseudoscalar mesons and $\eta \to 3\pi$ decay parameters with δ'.

δ'	−3.6	−3.8	−4.0	−4.2	−4.4	Experimental value [a]
$(m_{\pi^+} - m_{\pi^0})$ [b]	4.97	5.03	5.08	5.14	5.21	4.604 ± 0.004
$(m_{K^0} - m_{K^+})$ [b]	3.31	3.62	3.94	4.25	4.57	3.94 ± 0.13
$\Gamma(\eta \to \pi^+\pi^-\pi^0)$ [c]	0.285	0.317	0.349	0.391	0.432	0.61 ± 0.16
$\Gamma(\eta \to 3\pi^0)$ [c]	0.252	0.280	0.310	0.342	0.375	0.90 ± 0.21
R	0.88	0.88	0.89	0.87	0.87	1.31 ± 0.11
α	0.10	0.10	0.11	0.12	0.12	−0.50 ± 0.04

[a] Experimental values are from Ref. 14, except for α which is from Ref. 32.
[b] In MeV. The values of nontadpole contributions have been taken from R. H. Socolow, Ref. 30.
[c] In keV.

$$D_\mu B_+ = \partial_\mu B_+ - ig[\hat{X}^L_\mu, B_+] - ig_0 X^{L,0}_\mu(\lambda_0/2)B_+ ,$$
$$D_\mu B_- = \partial_\mu B_- - ig[\hat{X}^L_\mu, B_-] - ig_0 X^{R,0}_\mu(\lambda_0/2)B_- .$$

The equations

$$B_+ = S_L B'_+ S^\dagger_L \exp(i\phi_0\lambda_0/\sqrt{2}f_0) ,$$
$$B_- = S_R B'_- S^\dagger_R \exp(-i\phi_0\lambda_0/\sqrt{2}f_0) \qquad (11)$$

define a new set of baryon fields corresponding to the meson fields defined by Eqs. (3), (4) and (4′).

The three parameters $\alpha_1\gamma^2$, $\alpha_2\gamma^2$, and $\alpha_3\gamma^2$ were fixed by a least-squares fit to the experimental values for the four baryons (N, Σ, Ξ, and Λ). For $\delta = 0.84$ it was found that the baryon renormalization constants become negative.[34] Hence we did not study this case any further.

(ii) $(3, 3^*) \oplus (3^*, 3)$ *representation*. The nonet of baryons

$$B_\pm = \left(\frac{1 \pm \gamma_5}{2}\right) \sum_{k=0}^{8} B_k \lambda_k / \sqrt{2}$$

is chosen to transform as the $(3, 3^*) \oplus (3^*, 3)$ representation. The Lagrangian studied is

$$\begin{aligned}
\mathcal{L} = &-\{\bar{B}_+\gamma_\mu D_\mu B_+ + \bar{B}_-\gamma_\mu D_\mu B_-\} \\
&- \alpha_1\{\bar{B}_+\gamma_\mu D_\mu B_+ M^\dagger M + \bar{B}_-\gamma_\mu D_\mu B_- M M^\dagger + \text{H.c.}\} \\
&- \alpha_2\{\bar{B}_+ M M^\dagger \gamma_\mu D_\mu B_+ + \bar{B}_- M^\dagger M \gamma_\mu D_\mu B_- + \text{H.c.}\} \\
&- \alpha_3 \epsilon_{ijk}\epsilon_{\alpha\beta\gamma}[(\bar{B}_-)_{i\alpha}(B_+)_{j\beta}(M)_{k\gamma} + \text{H.c.}] \\
&- \alpha_4\{\bar{B}_+ M B_- M + \bar{B}_- M^\dagger B_+ M^\dagger\} \\
&- \alpha_5\{\bar{B}_+\gamma_\mu B_+ M^\dagger D_\mu M + \bar{B}_-\gamma_\mu B_- M D_\mu M^\dagger + \text{H.c.}\} \\
&- \alpha_6\{\bar{B}_+ M D_\mu M^\dagger \gamma_\mu B_+ + \bar{B}_- M^\dagger D_\mu M \gamma_\mu B_- + \text{H.c.}\} ,
\end{aligned}$$

where

$$D_\mu B_+ = \partial_\mu B_+ - ig(\hat{X}^L_\mu B_+ - B_+ \hat{X}^R_\mu) - ig_0 X^{L,0}_\mu(\lambda_0/2)B_+ ,$$
$$D_\mu B_- = \partial_\mu B_- - ig(\hat{X}^R_\mu B_- - B_- \hat{X}^L_\mu) - ig_0 X^{R,0}_\mu(\lambda_0/2)B_- .$$

The equations corresponding to Eq. (11) now read

$$B_+ = S_L B'_+ S^\dagger_R \exp(-i\phi_0\lambda_0/\sqrt{2}f_0) ,$$
$$B_- = S_R B'_- S^\dagger_L \exp(i\phi_0\lambda_0/\sqrt{2}f_0) .$$

The five masses of N, Σ, Λ, Ξ, and $Y_0^*(1405)$ have four new parameters $\alpha_1\gamma^2$, $\alpha_2\gamma^2$, $\alpha_3\gamma$, and $\alpha_4\gamma^2$, which were fixed by a least-squares fit to the five masses. The final calculated masses are

$$M_N = 958 \text{ MeV}, \; M_\Lambda = 1103 \text{ MeV}, \; M_{Y_0^*} = 1407 \text{ MeV},$$
$$M_\Sigma = 1186 \text{ MeV}, \; M_\Xi = 1328 \text{ MeV}.$$

The BBP coupling constants are functions of two parameters $\alpha_5\gamma^2$ and $\alpha_6\gamma^2$. No set of values for these parameters was found to give predicted values for the coupling constants close to the experimental values. In Table VIII we give variation of BBP coupling constants with $\alpha_5\gamma^2$ after $\alpha_6\gamma^2$ has been fixed by taking $g_{NN\pi}{}^2/4\pi = 14.5$. We see that the calculated values do not compare well with the experiments.

The scattering lengths and effective ranges for πN, KN, and $\bar{K}N$ scattering processes were calculated. The predicted scattering lengths, except the s-wave πN, do not compare well with the experiments. For example, for $\alpha_5\gamma^2 = -0.4$ we give the results for the scattering parameters. The s-wave scattering lengths and effective ranges are in units of $m_\pi{}^{-1}$, while the p-wave scattering lengths and effective ranges are in units of $m_\pi{}^{-3}$ and m_π, respectively.

TABLE VIII. $g_{BBP}{}^2/4\pi$ for some BBP vertices and $\Gamma(Y_0^* \to \Sigma\pi)$ for some values of $\alpha_5\gamma^2$. $\alpha_6\gamma^2$ is fixed from $g_{NN\pi}{}^2/4\pi = 14.5$.

$\alpha_5\gamma^2$	$N\Lambda K$	$N\Sigma K$	$\Sigma\Sigma\pi$	$\Sigma\Lambda\pi$	$\Gamma(Y_0^* \to \Sigma\pi)$ (MeV)
−1.0	45.9	10.0	73.0	0.75	13.4
−0.8	39.9	5.2	56.9	1.9	34.7
−0.6	34.4	1.9	42.7	3.6	65.5
−0.4	29.3	0.27	30.8	5.8	106.1
−0.2	24.3	0.13	20.7	8.6	156.5
0.0	20.2	1.5	12.6	11.9	216.6
Experimental value [a]	13 ± 3	0 ± 1	11.4 ± 5.0	21.5 ± 7.0	40 ± 10 [b]

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[b] Particle Data Group, Ref. 14.

1. πN scattering lengths and effective ranges.[35]

a. s-wave:

$a_{1/2} - a_{3/2} = 0.237$ $(0.288^{+0.012}_{-0.018})$,

$a_{1/2} + 2a_{3/2} = -0.019$ $(0^{+0.045}_{-0.035})$,

$r_{1/2} = -1.0$, $r_{3/2} = 7.5$.

b. p-wave, $J = \frac{1}{2}$:

$a_{1/2} - a_{3/2} = -0.134$ (-0.045 ± 0.006) ,

$a_{1/2} + 2a_{3/2} = -0.283$ (-0.164 ± 0.008) ,

$r_{1/2} = -7.9$, $r_{3/2} = 30.0$.

c. p-wave, $J = \frac{3}{2}$:

$a_{1/2} - a_{3/2} = -0.153$ (-0.243 ± 0.007) ,

$a_{1/2} + 2a_{3/2} = 0.184$ (0.414 ± 0.021) ,

$r_{1/2} = -77.5$, $r_{3/2} = 18.0$.

2. KN scattering parameters.[36]

a. s-wave:

$a_0 = 0.071$, $r_0 = 14.6$,

$a_1 = -0.41$ (-0.22 ± 0.008), $r_1 = -0.18$.

b. p-wave, $J = \frac{1}{2}$:

$a_0 = 0.011$ (0.039 ± 0.014), $r_0 = -2.9$,

$a_1 = -0.02$ (-0.012 ± 0.002), $r_1 = -0.67$.

c. p-wave, $J = \frac{3}{2}$:

$a_0 = -0.018$, $r_0 = -8.4$,

$a_1 = 0.02$ (-0.012 ± 0.021), $r_1 = 7.3$.

3. $\bar{K}N$ scattering parameters.[37]

a. s-wave:

$a_0 = -1.24$ (-1.2 ± 0.03), $r_0 = -1.08$,

$a_1 = 0.22$ (0.036 ± 0.028), $r_1 = 0.40$.

b. p-wave, $J = \frac{1}{2}$:

$a_0 = -0.085$, $r_0 = -1.35$,

$a_1 = 0.009$ (0.023 ± 0.013) $r_1 = 13.60$.

c. p-wave, $J = \frac{3}{2}$:

$a_0 = 0.015$, $r_0 = 6.30$,

$a_1 = 0.006$ (0.021 ± 0.013), $r_1 = 37.2$.

Most of the scattering lengths do not agree with the experiments. This is mainly due to two reasons: firstly, the *BBP* coupling constants are not predicted correctly, and secondly, due to the fact that some resonances, for example, the decuplet, may give important contributions to some of the amplitudes and have not been taken into account in the present calculation.

When $\eta_3 \neq 0$ the electromagnetic (em) mass differences for baryons can be calculated and are again not in agreement with the experiments. In Table IX we give the em mass differences for some values of δ' where most of the baryonic em mass differences are close to the experimental values. Firstly, we note that $m_n - m_p$ does not agree with the experimental value. Secondly, this range of values of δ' is much lower than the range obtained in Sec. VI obtained from the pseudoscalar em mass differences.

VIII. DISCUSSION

The model gives reasonably good description of low-energy processes involving mesons. But the same model, when extended to include baryons, does not give encouraging results. The isospin-breaking effects are also not described satisfactorily in the sense that a single value of η_3/γ does not explain all the three following results: (i) pseudoscalar-meson em mass differences, (ii) em mass differences of baryons, and (iii) the $\eta \to 3\pi$ decays. Baryonic interactions and the isospin-breaking effects need further study.

We now discuss various aspects of the model.

(i) Values of parameters characterizing the relative strengths of the breaking of $SU(3) \otimes SU(3)$, $SU(3)$, and $SU(2)$ are[38]

$$c = a_8/a_0 = -1.25, \quad d = a_3/a_8 = (0.034 - 0.043) ,$$

$$\xi = \eta_8/\eta_0 = -0.084, \quad \zeta = \eta_3/\eta_8 = (0.039 - 0.050) .$$

TABLE IX. Baryonic em mass differences (in MeV).

δ'	-1.0	-1.1	-1.2	-1.3	-1.4	Experimental value [a]
$m_n - m_p$	-0.65	-0.58	-0.50	-0.43	-0.35	1.293
$m_{\Sigma^-} - m_{\Sigma^0}$	4.04	4.25	4.46	4.67	4.89	4.86 ± 0.07
$m_{\Sigma^0} - m_{\Sigma^+}$	2.04	2.25	2.46	2.68	2.89	3.06 ± 0.20
$m_{\Xi^-} - m_{\Xi^0}$	5.5	5.86	6.24	6.63	7.02	6.6 ± 0.7

[a] See Ref. 14.

The value of c is close to the SU(2)⊗SU(2) value $-\sqrt{2}$ and that for ξ is close to zero, the exact SU(3) value. Thus the Lagrangian is approximately invariant under SU(2)⊗SU(2) symmetry while the vacuum has SU(3) as approximate symmetry. This conclusion is in agreement with results recently obtained in the literature.[39]

The values of the parameters d and ζ characterizing the isospin breaking in the Lagrangian and by the vacuum are near the values obtained by using Ward identities to study the pseudoscalar-meson mass differences which suggest a value ~0.03 for both these parameters.[40] This is also close to the value suggested by Cabibbo and Maiani.[41] The values $d=(0.01-0.014)$ and $c=(0.01-0.016)$ obtained from the baryonic em mass differences are, however, much lower. To explain the η decays one would require a set of values of c and d which are about three times as large as obtained from the em mass differences of pseudoscalar mesons.

(ii) In the limit $|\mu_0{}^2|\to\infty$ and $(\eta-4\nu_1\eta^3-4\nu_2\eta\{\eta^2\})\to 0$, taken in such a way that Eq. (5) is satisfied, masses of the chiral partners of the would-be Goldstone bosons tend to infinity and so do their couplings among themselves. The masses of the spin-one mesons, the pseudoscalar mesons, and S_K (in the case $\eta_3=0$) and their couplings with S_π, S_η, and S'_η remain unchanged. The processes in which the scalar mesons S_π, S_η, and S'_η appear in the final state are not allowed. The contributions of these scalars to the processes, where they appear in the intermediate state only, vanish in the limit.

The three-body decay widths for $A\to 3P$ and $X^0\to\eta\pi\pi$ change slightly and are given in the Table IV. The meson-meson and the meson-baryon scattering parameters do not change significantly in the above limit.

(iii) It has been recently suggested that the Duffin-Kemmer-Petiau (DKP) formalism be used for mesons.[42] It has been shown that some results like K_{l3} form factors, $X^0\to 2\gamma$ decays, etc., can be explained easily with DKP fields for spin-zero mesons. As has been pointed out in Sec. V it is not clear how the Higgs-Kibble mechanism can generate masses for the gauge fields.

(iv) It has been recently shown that it is possible to obtain masses for all gauge fields from spontaneous symmetry breaking only.[43] A model in which all vector-meson masses are generated by the Higgs-Kibble mechanism only will be reported elsewhere.[44]

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APPENDIX

In this appendix we consider the mixing problem in various multiplets. First we consider the mixing of spin-zero and spin-one mesons. When the Lagrangian is written in terms of new fields χ, $\phi,\ldots,$ there is a mixing of type $Y^V_\mu\cdot\partial_\mu\sigma$ and $Y^A_\mu\cdot\partial_\mu\phi$. For the sake of definiteness we consider first the mixing between axial vectors and pseudoscalars and the vector-scalar mixing can be treated in a similar manner.

Let the term giving the A-P mixing be written as

$$Y^{Ai}_\mu\xi_{ij}\partial_\mu\phi_j\ . \tag{A1}$$

To remove this mixing we substitute

$$Y^{Ai}_\mu=\mathcal{Y}^{Ai}_\mu-C^A_{ij}\partial_\mu\phi_j\ .$$

The term (A1) together with the mass term for Y^A_μ $[(-\frac{1}{2})(M_A{}^2)_{ij}Y^{Ai}_\mu Y^{Aj}_\mu]$ becomes

$$\begin{aligned}-\tfrac{1}{2}(M_A{}^2)_{ij}(\mathcal{Y}^{Ai}_\mu-C^A_{ik}\partial_\mu\phi_k)(\mathcal{Y}^{Aj}_\mu-C^A_{jl}\partial_\mu\phi_l)+(\mathcal{Y}^{Ai}_\mu-C^A_{ik}\partial_\mu\phi_k)\xi_{ij}\partial_\mu\phi_j\\ =-\tfrac{1}{2}(M_A{}^2)_{ij}\mathcal{Y}^{Ai}_\mu\mathcal{Y}^{Aj}_\mu+[(M_A{}^2)_{ij}C^A_{jk}+\xi_{ik}]\mathcal{Y}^{Ai}_\mu\partial_\mu\phi_k\\ -\tfrac{1}{2}(M_A{}^2)_{ij}(C^A)_{ik}(C^A)_{jl}\partial_\mu\phi_k\partial_\mu\phi_l+\partial_\mu\phi_k\partial_\mu\phi_jC^A_{ik}\xi_{ij}\ .\end{aligned} \tag{A2}$$

The terms of type $\mathcal{Y}^A_\mu\cdot\partial_\mu\phi$ will be absent if

$$C^A_{ij}=-(M_A{}^{-2})_{ik}\xi_{kj}\ .$$

After the mixing between spin-one and spin-zero fields has been removed, there is mixing between the 8th and 0th (8th, 0th, and 3rd) components in each multiplet when $\eta_3=0$ $(\eta_3\neq 0)$. To illustrate how this mixing is removed and the physical fields are defined we consider the case of pseudoscalar mesons. Other cases can be treated similarly. Let the kinetic energy and the mass term for these fields be written as

$$-\tfrac{1}{2}\partial_\mu\phi^TK\partial_\mu\phi-\tfrac{1}{2}\phi^TM^2\phi\ , \tag{A3}$$

where ϕ is a column with ϕ^8 and ϕ^0 (ϕ^8, ϕ^0, and ϕ^3) as components and K and M^2 are the kinetic energy and mass-squared matrices.

We first diagonalize the matrix K by an orthogonal matrix U_K:

$$U_K K U_K^T = \text{a diagonal matrix, say } K_D ,$$

and define new fields by

$$\phi' = U_K \phi \ . \tag{A4}$$

In terms of the fields

$$\phi'' = K_D^{1/2}\phi' \ , \tag{A5}$$

the expression (A3) becomes

$$-\tfrac{1}{2}\partial_\mu \phi''^T \partial_\mu \phi'' - \tfrac{1}{2}\phi''^T (M_r^{\,2})\phi'' \ , \tag{A6}$$

where $M_r^{\,2}$ is the renormalized mass-squared matrix given by

$$M_r^{\,2} = K_D^{1/2} U_K M^2 U_K^T K_D^{-1/2} \ .$$

$M_r^{\,2}$ is diagonalized by an orthogonal transformation U_M to get the squares of the renormalized masses of the physical pseudoscalar fields defined by

$$P = U_M \phi'' \ . \tag{A7}$$

The original ϕ fields are now easily expressed in terms of physical fields, P, by using Eqs. (A4), (A5), and (A7) to get

$$\phi = U_K^T K_D^{-1/2} U_M^T P \ .$$

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