Proton-proton scattering, the Chou-Yang model, and proton form factors*

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Recent high-energy proton-proton elastic scattering angular distribution data for $0 < -t \leq 5.3$ $(\text{GeV}/c)^2$ are used with the Chou-Yang model to investigate the proton electromagnetic form factors. Comparisons with the measurements of the form factors are given. The use of the proton-proton data in the analysis reveals either the need for measurements at larger momentum transfers or possible inconsistencies when the model is applied at finite energies. Analytical fits to the angular distribution data are also presented.

One of the most interesting results of recent high-energy colliding beam experiments has been the appearance of a rather sharp minimum in the proton-proton (pp) elastic scattering intensity at center-of-mass energies of 45 and 53 GeV.[1] The measurements at 53 GeV extend out to a four-momentum transfer, t, of -5.3 $(\text{GeV}/c)^2$. The appearance of minima in high-energy pp elastic scattering was conjectured and explained some time ago by means of the Chou-Yang model.[2,3] However, recent theoretical analyses predict cross sections which are considerably smaller than the measurements at large values of $-t$.[4]

In the present work we consider the recent pp data and try to extract from them the proton electromagnetic form factors using the Chou-Yang model. A simple fit to the pp elastic scattering data at *small* momentum transfers yields results which are in rather good agreement with the proton form factor data for $q^2 \lesssim 2$ $(\text{GeV}/c)^2$. Even for $2 \lesssim q^2 \lesssim 25$ $(\text{GeV}/c)^2$ the predictions are in good qualitative agreement with the form factor data. However, if we include the pp data at *larger* momentum transfers in the analysis we are led to apparent contradictions of the assumptions of the model when it is applied to these finite-energy collisions.

The elastic scattering differential cross section is written

$$d\sigma/dt = \pi |f(q)|^2 , \tag{1}$$

where $q^2 = -t$. The amplitude $f(q)$ is given in an impact parameter representation by

$$f(q) = \frac{i}{2\pi} \int e^{i\vec{q}\cdot\vec{b}} [1 - S(b)]\, d^2b . \tag{2}$$

The assumptions of the Chou-Yang model imply[2]

$$S(b) = \exp\left[-K \int e^{i\vec{q}\cdot\vec{b}} G^2(q) d^2q \right], \tag{3}$$

where $G(q)$ is related to the density of the proton and K is an arbitrary constant parameter. The function $G^2(q)$ is a real, positive function. $G(q)$ is identified with the electric or magnetic form factor of the proton. Equations (2) and (3) may be solved for G^2 to give

$$G^2(q) = -\frac{1}{2\pi K} \times \int_0^\infty J_0(qb) \ln\left[1 - \frac{1}{i}\int_0^\infty f(q') J_0(q'b) q' dq'\right] b\, db . \tag{4}$$

We may now use Eq. (4) to extract $G^2(q)$ from the pp elastic scattering data. We fit the pp data with an amplitude $f(q)$ given by

$$f(q) = i \int_0^\infty J_0(qb) \left\{1 - \exp\left[(1+i\epsilon) \ln\left(1 - \sum_{i=1}^{n} (c_i/2\alpha_i) e^{-b^2/4\alpha_i}\right)\right]\right\} b\, db , \tag{5}$$

where c_i, α_i, and ϵ are arbitrary parameters. This form has the property that when $\epsilon = 0$, $f(q)$ is a purely imaginary sum of exponentials in t. Furthermore, with this form for $f(q)$, $G^2(q)$ when normalized will not depend upon ϵ.

We first consider the pp data[5] at small momentum transfers $-t \leq 0.4$ $(\text{GeV}/c)^2$. We assume an imaginary amplitude given by a sum of two exponentials ($n = 2$, $\epsilon = 0$). The pp differential cross section at $E_{\text{c.m.}} = 53$ GeV is fitted by

$$d\sigma/dt = 10\pi[0.353604 \exp(15.2934t) + 1.40843 \exp(5.17654t)]^2 \text{ mb}/(\text{GeV}/c)^2. \tag{6}$$

This fit is shown in Fig. 1.[6]

We next use the more complete set of data[1,5] which covers the range $-t \leq 5.3$ (GeV/c)^2. We consider two different fits. The first is obtained from a purely imaginary amplitude ($\epsilon = 0$) given by the sum of three exponentials ($n = 3$), so that

$$d\sigma/dt = 10\pi[1.1862\exp(6.8115t) + 0.48101\exp(3.7617t) - 0.012138\exp(0.97093t)]^2 \text{ mb/(GeV/}c)^2 . \quad (7)$$

This fit is shown by the dashed curve in Fig. 2. The second fit is given by using in Eq. (5) the same values for c_i, α_i ($i = 1, 2, 3$) as those used in Eq. (7) together with $\epsilon = \pm 0.06$. This fit is shown by the solid curve in Fig. 2.[7] Since $G^2(q)$ is independent of ϵ, the two fits in Fig. 2 lead to identical values for $G^2(q)$.

We next use Eq. (4) to obtain $G(q)$ from the pp scattering amplitudes $f(q)$ which we have just used to fit the pp data. In Fig. 3 the dashed curve is the predicted $G(q)$ obtained from the pp amplitude (f) which was used to fit the pp data (see Fig. 1) for $-t \leq 0.4$ (GeV/c)^2. Also shown in Fig. 3 are the measured values[8] of the proton magnetic and electric form factors. The agreement is rather good for $q^2 \lesssim 2$ (GeV/c)^2 and is at least qualitative for $2 \lesssim q^2 \lesssim 25$ (GeV/c)^2.

The predicted values for $G(q)$ obtained from Eq. (4) with the pp amplitude (f) which was used to fit the pp data (see Fig. 2) for the wider range $-t \leq 5.3$ (GeV/c)^2 are given by the solid curve in Fig. 3. This curve differs slightly from the dashed curve for $q^2 \lesssim 1$ (GeV/c)^2. It agrees with the form factor data out to $q^2 \approx 4$ (GeV/c)^2, and perhaps represents an improvement over the dashed curve between $q^2 \approx 1$ and $q^2 \approx 4$ (GeV/c)^2. We thus see that the Chou-Yang model predicts the form factor very well for $q^2 \lesssim 4$ (GeV/c)^2. From $q^2 \approx 4$ to $q^2 \approx 6$ (GeV/c)^2, however, the predicted values of $G(q)$ decrease much too rapidly and quickly become too small. Furthermore, for $q^2 \gtrsim 6.5$ (GeV/c)^2 the predicted values for the quantity $G^2(q)$, given by Eq. (4), are *negative* [at least for $q^2 \leq 25$ (GeV/c)^2]. However, an assumption of the Chou-Yang model is that $G^2(q)$ is positive. Hence it appears that the model in its present form (which was intended for describing the infinite-energy limit and small momentum transfers) should be modified if it is to be applied at, say, $E_{\text{c.m.}} = 53$ GeV and large momentum transfers, or that angular distributions at larger momentum transfers should be measured. These measurements might give rise to significant

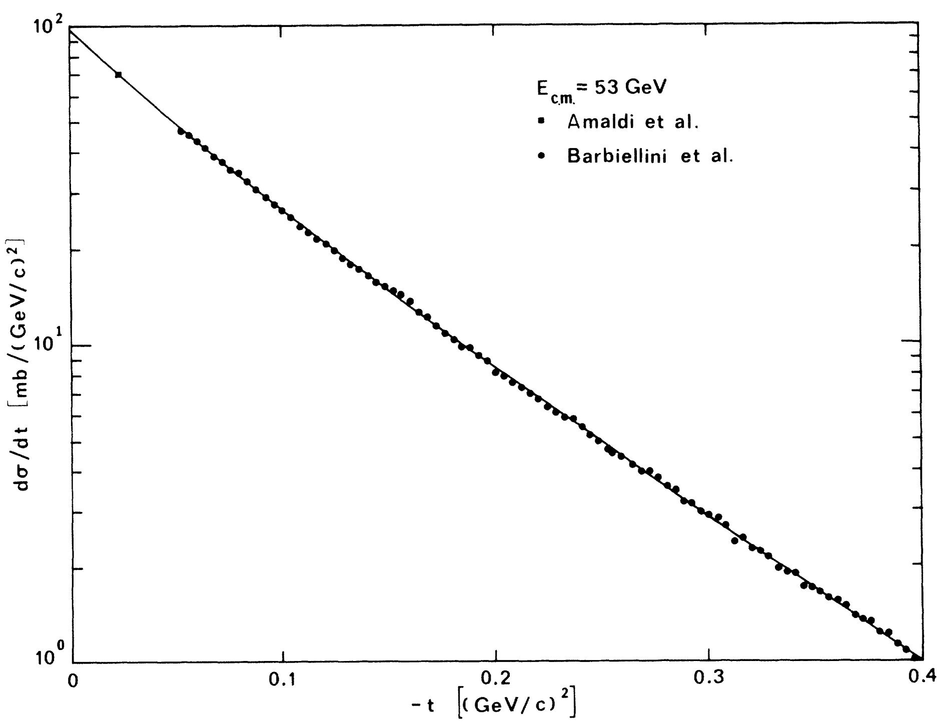

FIG. 1. Fit of Eq. (6) to pp elastic scattering differential cross section data at $E_{\text{c.m.}} = 53$ GeV. The data are from Ref. 5.

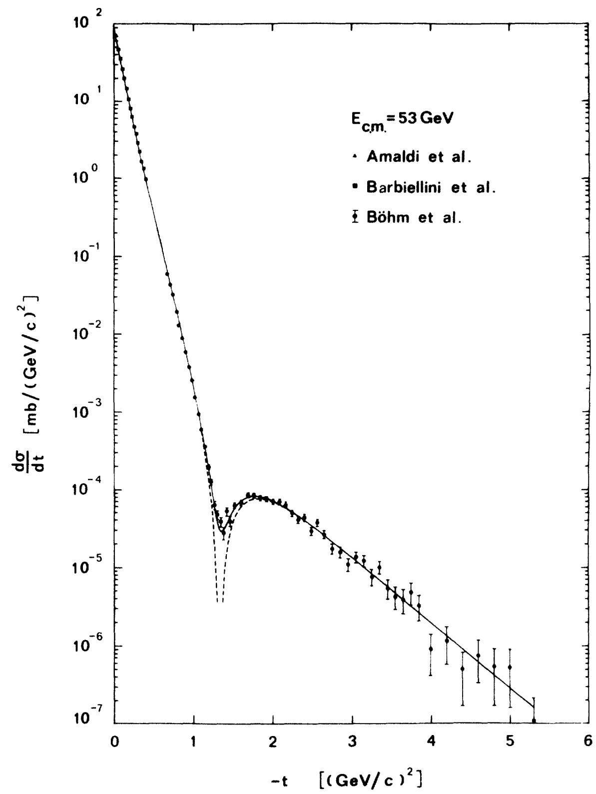

FIG. 2. Fits to pp elastic scattering differential cross section data at $E_{c.m.}$ =53 GeV. The data are from Refs. 1 and 5. The dashed curve is obtained from Eq. (7). The solid curve is obtained from Eq. (5) using the same values for the parameters c_i , α_i as used in Eq. (7) together with $\epsilon = \pm 0.06$.

modifications in $f(q)$ at large q, and the values of $G^2(q)$ at large momentum transfers, deduced from Eq. (4), may yet turn out to be positive (or at least positive over a larger range of q^2) and have a simple relationship to the electromagnetic form factors. For example, if to the three terms in the square brackets in Eq. (7) is added a fourth term, $0.00002\exp(0.02t)$, the fit to the pp elastic scattering data is hardly altered for $-t \leq 5.3$ (GeV/c)^2.

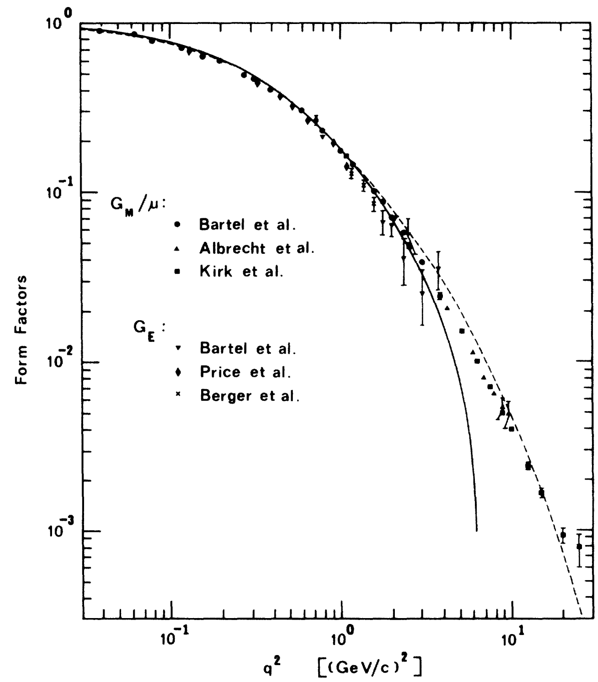

FIG. 3. Predictions of $G^2(q)$ using Eq. (4). The dashed and solid curves result from the scattering amplitudes used in the fits to the pp data shown in Figs. 1 and 2, respectively.

However, this additional term produces a second minimum at $-t \approx 6.7$ (GeV/c)^2. Furthermore, the resulting predicted $G^2(q)$ is positive throughout the range $q^2 \leq 25$ (GeV/c)^2. Unfortunately, however, the qualitative behavior of the resulting $G^2(q)$ is unsatisfactory since $G^2(q)$ becomes almost constant for $6.5 \leq q^2 \leq 25$ (GeV/c)^2, whereas the measured values decrease by a factor of ~10 over this range of q^2 values.

Although we see that the presently available pp data at large momentum transfers do not enable us to predict $G^2(q)$ satisfactorily at large momentum transfers, they do allow us to use the Chou-Yang model to predict the proton form factors quite well for $q^2 \lesssim 4$ (GeV/c)^2.

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[6]This fit yields a value of 97.5 mb/(GeV/c)^2 for $(d\sigma/dt)_{t=0}$ and 43.7 mb for the total cross section.

[7]This fit yields a value of 0.047 for the magnitude of the ratio (ρ) of the real-to-imaginary parts of the pp forward elastic scattering amplitude. This value should be considered an upper limit to the actual value since

a better angular resolution would yield a deeper minimum in the data and hence smaller values for the magnitudes of ϵ and ρ.

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