New systematics in hadron total cross section*

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Analysis of total cross section data from 2 to 200 GeV/c shows striking regularities in the discrepancies between the experimental data and predictions based on two-component duality with exact exchange degeneracy, universality, and SU(3) symmetry. These regularities suggest that the discrepancies are not due to a number of unrelated breaking effects, but are all related and described by a single universal third component which has even-signature isoscalar exchange in the t channel, has exactly the same energy dependence for kaon-nucleon, pion-nucleon, and nucleon-nucleon scattering processes, and scales in the ratio $1:2:\frac{9}{2}$. This new component suggests a common origin for the strangeness dependence of the total cross sections and the deviations from quark-model additivity, and gives a unified description of the following apparently unrelated effects: (1) The $\sigma(\pi N)-\sigma(KN)$ difference, (2) the deviation of $\sigma(\pi N)/\sigma(NN)$ from $\frac{2}{3}$, (3) differences in energy behavior of cross sections, particularly the decrease in $\sigma(pp)$ at low energies, and (4) the differences among $\sigma(K^+p)$, $\sigma(pp)$, and $\sigma(\phi p)$, which are all pure Pomeron in two-component duality.

Hadron total cross section data over the energy range from a few GeV to 200 GeV are consistent with predictions from quark models, Regge pole models, and various symmetry schemes which incorporate universality and exchange degeneracy. However, several apparently unrelated discrepancies exist at the 15% level.[1] The data have been satisfactorily fitted by phenomenologically breaking symmetry, universality, and exchange degeneracy[2] and treating πN, KN, and NN scattering differently with nonuniversal adjustable parameters. This note points out a systematic universality in these discrepancies which suggests that all cross sections should be treated together and described by a simple universal formula. The values of all adjustable parameters should be the same for πN, KN, and NN scattering; the differences should appear only as quark number coefficients [or equivalently as functions of eigenvalues of generators of U(3)×U(3)].

Our approach is to search for striking regularities in the experimental data, *not predicted by theory,* which can motivate further theoretical and experimental work. The regularities found seem too striking to be a numerological accident and have very simple theoretical and experimental implications. The strangeness dependence of the total cross sections seems to be related to the difference between meson-baryon and baryon-baryon cross sections. This relation leads to significant experimental predictions of immediate interest, namely that the differences between nucleon-nucleon and hyperon-nucleon cross sections should be larger by 50% than the predictions of the simple quark model.[1,3]

We first compare experimental cross sections with the simplest theoretical expression obtained with a minimum of adjustable parameters under the assumption of *exact* exchange degeneracy, universality, and SU(3) symmetry. The theoretical input is a Regge component σ_R with intercept $\frac{1}{2}$ and a Pomeron component σ_P having the form $a+b\ln s$. These components are defined universally for the scattering of any hadron H on a proton target with three universal parameters adjusted to fit the data,

$$\sigma_R(Hp)\equiv 1.75(N_{\bar{n}}^H+2N_{\bar{p}}^H)(P_{\rm lab}/20)^{-1/2}\,, \qquad (1a)$$

$$\sigma_P(Hp)\equiv (N_q^H)[13.5+1.25\ln(P_{\rm lab}/20)]/2\,, \qquad (1b)$$

where all cross sections are in millibarns, N_i^H is the number of quarks of type i in the incident hadron H, and N_q^H is the total number of quarks and antiquarks in hadron H. The quark numbers provide a convenient shorthand[4] for expressing conventional requirements of universality, exchange degeneracy, and SU(3). Table I lists the discrepancies between experiment[5] and theory, as defined by

$$\delta\sigma(HP)\equiv\sigma_{\rm tot}(HP)-\sigma_R(HP)-\sigma_P(HP)\,. \qquad (2)$$

We now check whether these differences look like noise or show a clear signal not previously noted. Table I shows such a signal which indicates a new kind of universality. The discrepancies $\delta\sigma(\pi N)$, $\delta\sigma(KN)$, and $\delta\sigma(NN)$ all have exactly the same energy dependence and a magnitude which scales according to a simple quark counting recipe. Conventional phenomenology[2] fits the discrepancies in $\sigma(\pi N)$, $\sigma(KN)$, and $\sigma(NN)$ sepa-

TABLE I. Experimental values of $\delta\sigma$ suitably scaled.

P (GeV/c)	$\delta\sigma\,(K^+p)$ (mb)	$\frac{1}{2}\delta\sigma\,(\pi^-p)$ (mb)	$\frac{2}{9}\delta\sigma\,(pp)$ (mb)	$\delta\sigma\,(K^-p)$ (mb)	$\frac{1}{2}\delta\sigma\,(\pi^+p)$ (mb)	$\frac{2}{9}\delta\sigma\,(\bar{p}p)$ (mb)
2	7.0 ± 0.06	7.1 ± 0.02	6.9 ± 0.01	8.4 ± 0.1	6.5 ± 0.02	10.0 ± 0.01
3	6.1 ± 0.04	6.1 ± 0.01	6.2 ± 0.01	7.2 ± 0.1	6.8 ± 0.01	8.3 ± 0.01
6	5.0 ± 0.1	5.1 ± 0.15	5.0 ± 0.13	5.6 ± 0.3	5.5 ± 0.1	5.6 ± 0.2
8	4.9 ± 0.1	4.8 ± 0.04	4.8 ± 0.13	5.7 ± 0.2	5.2 ± 0.04	5.3 ± 0.2
10	4.7 ± 0.1	4.6 ± 0.04	4.7 ± 0.13	4.9 ± 0.2	4.9 ± 0.04	
12	4.4 ± 0.1	4.5 ± 0.04	4.5 ± 0.13	4.2 ± 0.2	4.7 ± 0.04	4.7 ± 0.2
14	4.3 ± 0.1	4.4 ± 0.04	4.3 ± 0.13	4.3 ± 0.2	4.5 ± 0.04	4.6 ± 0.2
16	3.8 ± 0.1	4.3 ± 0.04	4.2 ± 0.13	4.2 ± 0.4	4.4 ± 0.04	4.4 ± 0.2
18	3.7 ± 0.1	4.2 ± 0.04	4.1 ± 0.13	3.9 ± 0.8	4.3 ± 0.04	4.7 ± 0.8
15	4.2 ± 0.1	4.3 ± 0.06	4.4 ± 0.03	4.4 ± 0.2	4.5 ± 0.04	4.7 ± 0.2
20	3.9 ± 0.2	4.1 ± 0.06	4.2 ± 0.03	4.0 ± 0.1	4.1 ± 0.03	4.3 ± 0.1
25	3.9 ± 0.1	3.9 ± 0.06	4.0 ± 0.03	3.8 ± 0.1	4.0 ± 0.03	4.1 ± 0.1
30	3.7 ± 0.1	3.8 ± 0.06	3.9 ± 0.03	4.0 ± 0.1	3.9 ± 0.03	3.9 ± 0.1
35	3.6 ± 0.1	3.8 ± 0.06	3.8 ± 0.03	3.6 ± 0.1	3.8 ± 0.03	3.9 ± 0.1
40	3.7 ± 0.1	3.7 ± 0.06	3.8 ± 0.03	3.7 ± 0.1	3.7 ± 0.03	3.8 ± 0.1
45	3.4 ± 0.1	3.7 ± 0.06	3.7 ± 0.03	3.7 ± 0.1	3.7 ± 0.03	3.7 ± 0.2
50	3.7 ± 0.1	3.7 ± 0.06	3.7 ± 0.03	3.6 ± 0.1	3.7 ± 0.03	3.5 ± 0.2
55	3.4 ± 0.1	3.8 ± 0.06	3.6 ± 0.03	3.6 ± 0.2	3.7 ± 0.03	
50	3.4 ± 0.1	3.6 ± 0.03	3.6 ± 0.02	3.4 ± 0.1	3.7 ± 0.03	3.6 ± 0.02
100	3.3 ± 0.1	3.4 ± 0.03	3.4 ± 0.01	3.3 ± 0.1	3.5 ± 0.03	3.3 ± 0.02
150	3.3 ± 0.1	3.4 ± 0.03	3.2 ± 0.01	3.3 ± 0.1	3.4 ± 0.03	3.2 ± 0.04
200	3.5 ± 0.1	3.4 ± 0.03	3.2 ± 0.01	3.4 ± 0.1	3.4 ± 0.04	3.2 ± 0.06

rately with symmetry-breaking contributions. Table I suggests a universal additional contribution for all processes rather than different breaking effects in different processes which accidentally add up to give exactly the same energy dependence over an energy range of two orders of magnitude.

For a more detailed analysis we examine the experimental data plotted in Fig. 1, with the baryon-nucleon cross sections multiplied by $\frac{2}{3}$ to exhibit quantities predicted to be equal by the quark model.[3] Also plotted are the linear combinations

$$\sigma(\phi p) \equiv \sigma_{\rm tot}(K^+p) + \sigma_{\rm tot}(K^-p) - \sigma_{\rm tot}(\pi^-p) \tag{3a}$$

and

$$\sigma({\rm Pom}) \equiv \tfrac{3}{2}\sigma_{\rm tot}(K^+p) - \tfrac{1}{3}\sigma_{\rm tot}(pp)\,. \tag{3b}$$

The expression (3a) for $\sigma(\phi p)$ is obtained from the quark model and has been used successfully to describe ϕ photoproduction. The expression (3b) describes the "pure Pomeron" contribution according to the model of Ref. 1. All the cross sections in Fig. 1 are seen to be equal at the 20% level at 200 GeV/c. But more precise analysis shows the discrepancies given in Table I, normally considered as four qualitatively different effects, each fitted with its own individual parameter. There is no strong theoretical reason for or against this procedure, but the experimental data strongly suggest that the following four effects are all related and that a new theoretical approach should be found which relates them.

1. The πN-KN difference. The difference $\sigma_{\rm tot}(\pi^-p) - \sigma_{\rm tot}(K^-p)$ is 5 mb at 2 GeV/c and decreases slowly to about 3.5 mb at 200 GeV/c. This slowly decreasing strangeness-dependent

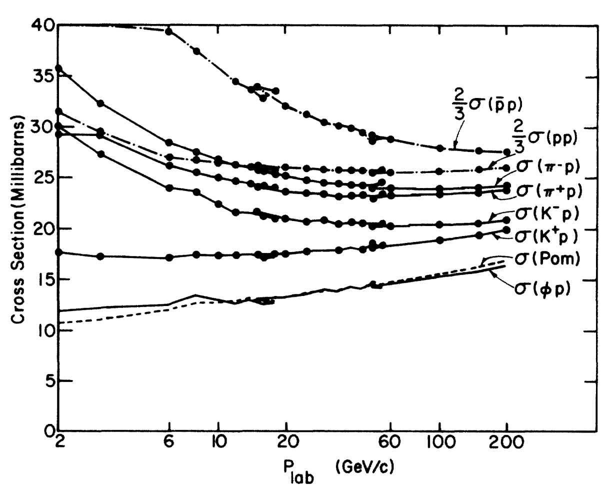

FIG. 1. Experimental total cross section data and the linear combinations (3).

contribution to the total cross section is larger than the contributions from the leading Regge exchanges at Fermilab energies.

2. *The deviation from the Levin-Frankfurt* $\frac{3}{2}$ *rule.* The cross sections $\frac{2}{3}\sigma_{tot}(pp)$ and $\frac{2}{3}\sigma_{tot}(\bar{p}p)$ are not equal to $\sigma(\pi p)$ and $\sigma(Kp)$ as predicted by the quark model[4] but are consistently higher by about 20%. This effect cannot be explained in the additive quark model and indicates the presence of a nonadditive contribution.

3. *The Pomeron contribution.* Duality[6] and exchange degeneracy[4] suggest that there is no contribution from the leading Regge exchanges to $\sigma_{tot}(K^+p)$ and $\sigma_{tot}(pp)$, which are exotic, or to $\sigma_{tot}(\phi p)$ since the ϕ decouples from the leading Reggeons. However, $\sigma_{tot}(K^+p)$, $\sigma_{tot}(pp)$, and the quark-model expression (3a) for $\sigma(\phi p)$ behave very differently as a function of energy between 2 and 200 GeV/c. If one is the Pomeron the other two are not. An additional contribution must be present in addition to the leading Reggeon and the Pomeron.

4. *Different behaviors of rising cross sections.* Rising total cross sections were noticed in the Serpukhov data[5] from 20 to 60 GeV/c and confirmed at higher energies. However, each curve in Fig. 1 shows a different energy behavior. *Striking features not previously noted* are the monotonic rising behavior and the approximate equality of the particular linear combinations of cross sections (3a) and (3b) over the entire energy range from 2 to 200 GeV/c.

We now show that all these apparently unrelated effects seem to have a single common explanation. The new regularities are in isoscalar even-signature exchange normally associated with the Pomeron. For total cross section *differences,* known to be fitted very well by Regge expressions of the form (1a), our analysis adds nothing new. The first three columns of Table I suggest an additional contribution slowly decreasing with energy and coupling to kaons, pions, and nucleons in the ratio $1:2:\frac{9}{2}$. To see this regularity more clearly we remove the well-understood Regge component (1a) from the experimental cross section, and plot the differences $\sigma_{tot}-\sigma_R$ in Fig. 2. The successful removal of the Regge contribution is indicated by the collapse of the particle-antiparticle differences to give three curves independent of charge above 10 GeV/c for $(\frac{2}{3})\sigma(NN)$, $\sigma(\pi N)$, and $\sigma(KN)$.

Two-component duality describes the differences $\sigma_{tot}-\sigma_R$ as Pomeron contributions to the total cross sections. Figure 2 shows that they are not all equal as expected in the Levin-Frankfurt approximation and that the departure from equality shows a striking new regularity, an approximately equal spacing all the way from 2 to 200 GeV/c.

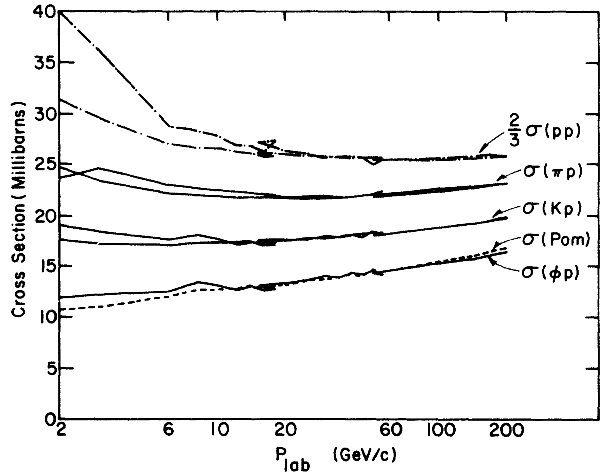

FIG. 2. The differences $\sigma_{tot}-\sigma_R$.

The π-K difference appears to be related to the discrepancy from the Levin-Frankfurt $\frac{3}{2}$ ratio for $\sigma(BB)/\sigma(MB)$, although the former is usually attributed to SU(3) symmetry breaking and the latter completely unrelated to SU(3).

Experimental values of $\delta\sigma$ defined by Eq. (2) are shown in Fig. 3. The three curves representing $\delta\sigma(KN)$, $\delta\sigma(\pi N)$, and $\delta\sigma(NN)$ scale with the same energy dependence and a ratio 1 : 2 : 3. This is emphasized by plots of the same data in Fig. 4 as a universal curve when $\delta\sigma(KN)$, $\delta\sigma(\pi N)$, and $\delta\sigma(NN)$ are multiplied by scaling factors $3:\frac{3}{2}:\frac{2}{3}$.

The spread in the curves at low energies in Figs. 2–4 indicates inadequacies in the prescription (1a) for the Regge exchange component at low energies. We do not attempt to investigate this

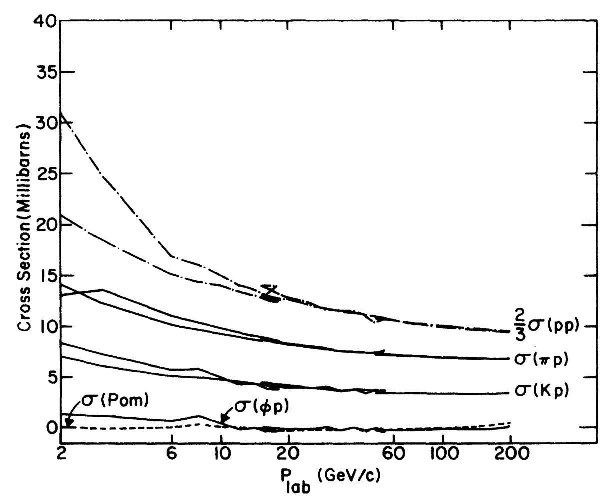

FIG. 3. Experimental values of cross section discrepancies.

point, but note that the scaling of πN, KN, and NN cross sections in Fig. 4 is even more marked when the lowest charge states $\sigma(pp)$, $\sigma(\pi^- p)$, and $\sigma(K^+ p)$ are chosen. These are given in the first three columns in Table I.

Table I and Figs. 3 and 4 present striking evidence for a new component in hadron total cross sections which decreases slowly with energy and satisfies the ratio $1:2:\frac{9}{2}$. One possible dynamical model for such a component[7] has two Pomeron-type contributions, one with $\alpha < 1$ and the other with $\alpha > 1$. We use this model to motivate a universal empirical formula for total cross sections without taking the details too seriously. The residues of the two Pomerons are assumed to be given by well-defined universal quark counting prescriptions except for over-all universal strength factors, and the two strength factors and two intercepts are parameters determined by fits to experiment. This gives the formula

$$\sigma_{tot}(Hp) = (N_q^H)\sigma_1(P_{lab}/20)^{1+\epsilon} + (N_q^H N_{ns}^H)\sigma_2(P_{lab}/20)^{1-\delta} + \sigma_R , \qquad (4)$$

where N_{ns}^H is the total number of nonstrange quarks and antiquarks in hadron H, σ_R is given by Eq. (1a), and σ_1, σ_2, ϵ, and δ are adjustable parameters. We consider only the cross sections $\sigma(pp)$, $\sigma(K^+p)$, and $\sigma(\pi^- p)$ since the others are determined uniquely by these three and σ_R. Table II shows the fit ob-

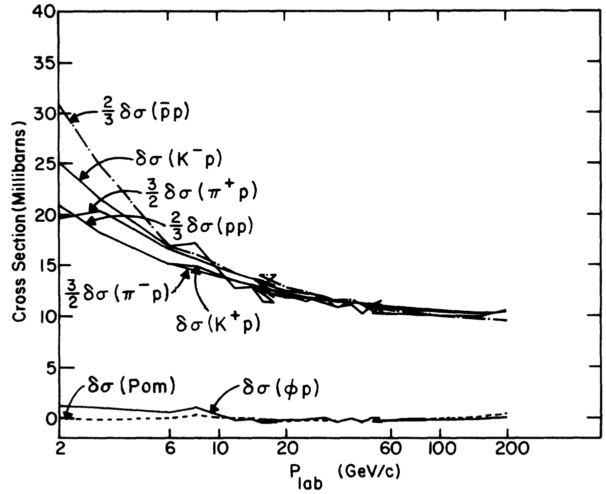

FIG. 4. Values of cross section discrepancies. Quantities plotted are $\frac{2}{3}\delta\sigma(NN)$, $\frac{3}{2}\delta\sigma(\pi N)$, and $\delta\sigma(KN)$.

TABLE II. Comparison with experiment of predictions from the formula (4) with $\sigma_1 = 13$ mb, $\sigma_2 = 4.4$ mb, $\epsilon = 0.13$, $\delta = 0.2$.

P (GeV/c)	σ(pp) (mb) Exp.	σ(pp) Theo.	σ(K⁺p) (mb) Exp.	σ(K⁺p) Theo.	σ(π⁻p) (mb) Exp.	σ(π⁻p) Theo.
2	47.2 ± 0.04	45.8	17.6 ± 0.06	16.6	35.8 ± 0.04	34.7
3	44.4 ± 0.04	44.2	17.2 ± 0.04	16.6	32.4 ± 0.02	32.1
6	40.6 ± 0.6	41.9	17.0 ± 0.1	16.7	28.5 ± 0.3	28.7
8	40.0 ± 0.6	41.1	17.3 ± 0.1	16.8	27.5 ± 0.1	27.6
10	39.9 ± 0.6	40.6	17.3 ± 0.1	16.9	26.8 ± 0.1	26.9
12	39.4 ± 0.6	40.2	17.3 ± 0.1	17	26.3 ± 0.1	26.4
14	39.1 ± 0.6	39.9	17.4 ± 0.1	17.1	25.9 ± 0.1	26.0
16	38.7 ± 0.6	39.6	17.0 ± 0.1	17.2	25.7 ± 0.1	25.7
18	38.7 ± 0.6	39.5	17.1 ± 0.1	17.3	25.4 ± 0.1	25.5
15	39.3 ± 0.1	39.8	17.3 ± 0.1	17.2	25.8 ± 0.1	25.9
20	39.1 ± 0.1	39.3	17.4 ± 0.2	17.4	25.1 ± 0.1	25.3
25	38.8 ± 0.1	39.0	17.7 ± 0.1	17.6	24.8 ± 0.1	24.9
30	38.6 ± 0.1	38.8	17.7 ± 0.1	17.8	24.6 ± 0.1	24.7
35	38.5 ± 0.1	38.7	17.8 ± 0.1	17.9	24.4 ± 0.1	24.5
40	38.5 ± 0.1	38.6	18.0 ± 0.1	18.1	24.3 ± 0.1	24.4
45	38.4 ± 0.1	38.5	17.9 ± 0.1	18.2	24.3 ± 0.1	24.3
50	38.5 ± 0.1	38.5	18.4 ± 0.1	18.3	24.2 ± 0.1	24.2
55	38.4 ± 0.1	38.4	18.2 ± 0.1	18.4	24.5 ± 0.1	24.1
50	38.1 ± 0.1	38.5	18.0 ± 0.1	18.3	24.0 ± 0.1	24.2
100	38.4 ± 0.1	38.4	18.8 ± 0.1	19.2	24.0 ± 0.1	24.0
150	38.6 ± 0.1	38.6	19.3 ± 0.1	19.8	24.1 ± 0.1	24.1
200	38.9 ± 0.1	38.8	19.8 ± 0.1	20.3	24.3 ± 0.1	24.2
500	40.8	40.0		22.1		25.1
1000	42.7	41.5		23.6		26.1
1400	43.2	42.3		24.5		26.8
2000	44.0	43.4		25.4		27.5

tainable with the simple universal four-parameter expression (4) over the very wide energy range from 2 to 200 GeV/c, including CERN ISR data for $\sigma(pp)$ up to 2000 GeV/c equivalent $P_{\rm lab}$. The points at very low energies should be considered only qualitatively, since simple theoretical descriptions are not expected to hold down to 2 GeV. This fit is not presented to support any model but simply to show that regularities must exist in the data to allow a fit to the accuracy of a few percent over a wide energy range with a few universal parameters, rather than by invoking special different mechanisms for πN, KN, and NN channels. The universal formula (4) can be used for any hadron H with no new parameters and predicts hyperon-nucleon cross sections.

The essential new qualitative feature in this approach is independent of the particular parametrization. The second term on the right-hand side of Eq. (4) contains the strangeness dependence and breaks quark-model additivity by a quadratic dependence on the quark number variables. This common origin for the strangeness dependence and additivity breaking explains the regularities of Table I and Figs. 2–4 and predicts a large deviation of the hyperon-nucleon total cross sections from simple quark-model predictions.

This approach directs the attention of theorists and phenomenologists to particular linear combinations of cross sections which have simpler energy behavior and may have simpler theoretical descriptions than the cross sections themselves. The combinations (3) contain contributions only from σ_1 and have the very simple monotonically rising energy behavior shown in Fig. 1. The difference $\sigma(\pi^- p)-\sigma(K^- p)$ has contributions only from σ_2 and also exhibits a simple monotonic energy dependence. Linear combinations which have no contribution from σ_1 or σ_2 and depend only on σ_R have the form

$$A_{\rm Reg}=A(\pi p)-\tfrac{1}{2}A(Kp)-\tfrac{1}{3}A(pp)\,, \qquad (5)$$

where the amplitudes $A(\pi p)$, $A(Kp)$, and $A(pp)$ denote any charge state of the system or any linear combination of amplitudes for different charge states with coefficients whose sum is unity. These have been shown[1] to decrease as $s^{-1/2}$ as expected for a leading Reggeon contribution and to enable the separation of the f-exchange contribution from the Pomeron in isoscalar even-signature amplitudes.

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