PHYSICAL REVIEW D VOLUME 11, NUMBER 7 1 APRIL 1975

Inclusive production on nuclei at large transverse momentum*

Jon Pumplin and Ed Yen†

Department of Physics, Michigan State University, East Lansing, Michigan 48824

(Received 29 October 1974)

We show that inclusive production data on nuclear targets at large transverse momentum can be interpreted according to the general features of the parton model. The data are inconsistent with "cascade" models, in which produced particles are assumed to interact independently with the nucleons in the nucleus. We propose an interpretation of the $A^{1.1}$–$A^{1.3}$ behavior which has been observed at $s = 560$ GeV^2.

I. PREDICTIONS

Single-particle inclusive spectra have been measured on nuclear targets, up through Fermi National Accelerator Laboratory energies.[1] Cosmic ray data, which necessarily involve nuclear targets, offer information up to much higher energy, although of admittedly lower quality. It is therefore of interest to consider how such data should be interpreted.

In this paper, we will show that the large-transverse-momentum (p_t) region, defined formally by $s \to \infty$ at fixed $x_\perp \equiv 2p_t/\sqrt{s}$, can be interpreted on the basis of parton-model ideas. We further show that the data are inconsistent with a "cascade" model,[2] in which products of an interaction with one nucleon in the nucleus are assumed to interact as free particles with subsequent nucleons.

We begin by presenting our point of view, and deriving some consequences of it. We will then compare these consequences with experiment. In the parton model, the colliding particles are imagined to be made of quasifree constituents called partons. If the small Fermi momenta are neglected, the parton wave function of the nucleus is the same as that of the proton and neutron, scaled up by their numbers Z and $(A-Z)$. Large-transverse-momentum particles are produced by collisions between "hard" = "leading" = "nonwee" partons, i.e., those with momentum $\propto \sqrt{s}$ in the center-of-mass system. For such collisions, rescattering (or absorptive) effects are negligible, since these effects only change the distribution of *wee* partons. The requirement of a hard collision implies that there is very little coherence involved in the sum over amplitudes which correspond to scattering from individual nucleons. It is therefore correct to add cross sections rather than amplitudes, just as one does in deriving the Bjorken scaling limit from the parton model of deep-inelastic electron scattering. The large p_t^2 in our problem plays the same role as large q^2 in the latter one. Finally, the hard interactions which are responsible for large-transverse-momentum secondaries are extremely rare. It is therefore unlikely that more than one such interaction will take place in a single pp or even p-nucleus collision.

The above considerations lead directly to the prediction

$$\rho_{pA,c} = Z\rho_{pp,c} + (A-Z)\rho_{pn,c} \ , \tag{1}$$

where $\rho_{pA,c}$ is the inclusive cross section $E(d\sigma/d^3p)_{pA\to c+\text{anything}}$ and $c = \pi^\pm, K^\pm, p, \bar{p}, n$, etc., at large transverse momentum. These quantities are to be compared at equal energies and angles in the nucleon-nucleon center-of-mass frame. Equation (1) is our basic result. We proceed to discuss several predictions which follow from it, and which can easily be tested experimentally. In Sec. III, we will discuss a modification of Eq. (1) which appears to be necessary in the light of recent data.

To the extent that the inclusive spectrum at large p_t is the same on protons and neutrons, or to the extent that the ratio of neutrons to protons in the nucleus is constant, Eq. (1) leads to *prediction (i): $E(d\sigma/d^3p)_{pA\to c+\text{anything}}$ is proportional to A*. Note that we predict no shadow effect. The nucleons at the center of a large nucleus are no less effective in producing large-p_t secondaries than the ones at the surface. Hence the cross section varies like $\sim A$ rather than $\sim A^{2/3}$. This comes about because the interactions which correspond to shadowing in small-momentum-transfer processes—e.g., two-body Reggeon exchange—only serve to change the distribution of the *wee* partons.

According to Eq. (1), we further obtain *prediction (ii): The shape of the transverse-momentum spectrum is independent of A*, and also *prediction (iii): The particle production ratios, $p:\pi^+:\pi^-:K^+:K^-:\bar{p}$, are independent of A*. These predictions follow from the idea that the hard interactions responsible for large-p_t particles are rare, so that multiscattering processes are unimportant.

Since the large-p_t processes involve nonwee partons, we expect the cross sections to depend on the quantum numbers of the incident particles. Thus for example we do not expect particle ratios such as π^-/π^+ or $\bar{p}/p$ to approach unity in the limit $s \to \infty$, $x_\perp = 2p_t/\sqrt{s}$ finite, in either pp or pA scattering. In fact, we expect *prediction (iv): The particle ratios at finite* $x_\perp$, $s\to\infty$, $x \to 0$ *should be similar to the ratios observed in the fragmentation region of finite* x, $s \to \infty$, $x_\perp \to 0$. This prediction differs, for example, from what would be expected on the basis of a statistical model for large p_t, where the large number of particles produced (mostly at small p_t) would wash out the significance of the incident quantum numbers.

II. COMPARISON WITH EXPERIMENT-I

To test prediction (i), we compare measurements of $pp \to \pi^0 +$anything[3] with $pW \to \frac{1}{2}(\pi^+ + \pi^-)$ +anything,[1] at approximately the same energy ($\sqrt{s} \simeq 23$ GeV) and angle ($\simeq 90°$) in the nucleon-nucleon center-of-mass frame. The average of π^+ and π^- is assumed to approximate the π^0 distribution, which has not been measured. The similarity between the π^+ and π^- distributions (their ratio is about 1.2 ± 0.1 for the data points we use) suggests that this assumption is reasonable, and also that we are allowed to neglect the difference between neutron and proton target particles in this region. Table I shows the ratio r of these cross sections, which is to be compared with $A = 184$ according to the parton model. The agreement is rather good. In view of the normalization error of ~50%, which is quoted for the experiments[1,3] but not included in Table I, r is consistent with being equal to A. Beyond this basic observation, one must not look too closely at Table I because of the possible normalization errors and further possible systematic effects which may be present due to deviations of the ratio of $(\pi^+ + \pi^-)/2\pi^0$ from unity[4] and due to the fact that the tungsten data were actually measured at fixed angle in the laboratory, rather than at 90° in the nucleon-nucleon center-of-mass frame.

If one imagined absorptive effects to be important, one would expect $r = A_{\text{eff}}$, where $A_{\text{eff}} \simeq A$ at small A and $A_{\text{eff}} \simeq A^{2/3}$ at very large A. For the tungsten data, Cronin *et al.*[1] have estimated A_{eff} to be given by the inelastic cross section:

$$A_{\text{eff}} = \sigma_{pW}^{\text{inel}}/\sigma_{pp} = 1635 \text{ mb}/40 \text{ mb} = 40.6 \ .$$

In fact, they divide by this factor in reporting their cross section. Table I shows that the ratio r is in strong disagreement with this value, and supports (within rather large errors) the parton-model prediction of no shadow effect at large p_t. To estimate the equivalent p-nucleon cross section, it would therefore be better to divide by A rather than A_{eff}. At smaller p_t, the A dependence becomes weaker than A; e.g., at $p_t = 0.8$ GeV/c the same experiments yield $r \simeq 109$. This value, which is closer to A_{eff}, makes sense in terms of the parton model, since wee partons become important at small p_t.

If, on the other hand, one imagined multiple scattering or cascade effects to be important, one would expect r to be much larger than A, since the p_t distribution would involve a two-dimensional random walk, which would increase the cross section for large A at large p_t. To estimate this effect, we represent the transverse-momentum distribution for a single interaction by the large-p_t fit given in Ref. 3 for $E(d\sigma/d^3p)_{pp,\pi^0}$, with a modification to provide a reasonable description at small p_t as well:

$$\rho = c_1 \exp(-6p_t) + \frac{c_2 \exp(-1.11 p_t)}{c_3 + p_t^{8.24}} \ , \tag{2}$$

where $c_1 = 1.2 \times 10^{-25}$ cm^2, $c_2 = 1.5 \times 10^{-26}$ cm^2, and $c_3 = 100$, in units where GeV = 1. We then calculate the distribution on tungsten, assuming equal probabilities for single and double scattering:

$$\rho_{pW,\pi^0}(\vec{p}_t) = 184 \left[\rho(\vec{p}_t) + \frac{\int \rho(\frac{1}{2}\vec{p}_t + \vec{q})\rho(\frac{1}{2}\vec{p}_t - \vec{q}) d^2q}{\int \rho(q) d^2 q}\right] . \tag{3}$$

This estimate assumes that all interactions have essentially the same p_t distribution. Otherwise, it is a very conservative estimate of the amount of multiscattering: For example, if independent interactions are assumed, the average number of inelastic collisions on a nucleus of uniform density and radius R is easily shown to be $A\sigma^{\text{inel}}/\pi R^2$, which equals 3.5 if the inelastic cross section on a single nucleon is 30 mb. In a cascade model, the multiplicity of particles which are capable of in-

TABLE I. Inclusive cross sections $E\, d\sigma/d^3p$ on tungsten and hydrogen targets, at $\sqrt{s} \simeq 23$ GeV and $\theta \simeq 90°$ in the nucleon-nucleon center-of-mass frame, in units of cm^2/GeV^2.

p_t (GeV/c)	$pW \to \frac{1}{2}(\pi^+ + \pi^-)X$	$pp \to \pi^0 X$	Ratio
2.29	$(1.56 \pm 0.11) \times 10^{-28}$	$(1.31 \pm 0.14) \times 10^{-30}$	119 ± 15
3.05	$(8.86 \pm 0.63) \times 10^{-30}$	$(5.32 \pm 0.57) \times 10^{-32}$	116 ± 22
3.81	$(6.57 \pm 0.49) \times 10^{-31}$	$(3.66 \pm 0.45) \times 10^{-33}$	179 ± 26
4.58	$(4.55 \pm 0.33) \times 10^{-32}$	$(3.41 \pm 0.48) \times 10^{-34}$	133 ± 22

teracting is >1, so the average number of interactions is still larger. The predictions of this single-plus-double-scattering calculation, in the region of $2 \lesssim p_t \lesssim 5$ GeV/c, is that r should be equal to about $5\times 184 = 920$. Such a prediction would conflict strongly with the data in Table I.

Since a shadow effect would make the cross section $\propto A^n$ with $n<1$, and multiscattering would give $n>1$, it is of course conceivable that a theory which combines these effects could fit the data.

The approximate constancy of r in Table I agrees with prediction (ii), that the shape of the p_t distribution is similar for proton and nuclear targets. In cascade or multiscattering models, one expects the p_t distribution at small p_t to be much broader on a large nucleus than on a single nucleon because of the random walk in p_t space. In the region $2<p_t<5$ GeV/c, however, single scattering and double scattering are rather similar in shape. Therefore, the agreement of prediction (ii) with experiment in this region, which is shown in Table I, does not show, by itself, that double scattering is small. That result follows, however, from the agreement of the other predictions.

The result stated in the preceding paragraph, that the double-scattering convolution

$$D(p) = \int \rho(\tfrac{1}{2}\vec{p} + \vec{q})\rho(\tfrac{1}{2}\vec{p} - \vec{q})d^2q \qquad (4)$$

is similar in shape, at fairly large p, to the input single scattering $\rho(p)$, may be verified by numerical integration, using forms such as (2) which represent the data. The result may also be understood as follows: $\rho(p)$ can be fitted well by a form which falls asymptotically as a power, and if $\rho(p)\sim p^{-n}$ as $p\to\infty$ then $D(p)\sim p^{-n}$ as well. When $\rho(p)$ falls as a power, the dominant regions in the integral (4) are $q\simeq \pm\frac{1}{2}p$, i.e., one scattering is hard and the other soft.

Prediction (iii) states that the ratios K/π, p/π, and $\bar{p}/\pi$ should be independent of A. It is now known experimentally that these ratios vary like $A^{0.1-0.2}$. This observation and a possible explanation for it will be discussed in Sec. III. For the present, we remark that according to a cascade model these ratios would, to the contrary, *fall* rapidly with A. This is because secondary particles, which have lower energy than the incident beam, are relatively ineffective in producing particles such as K and $\bar{p}$, whose invariant cross sections rise with energy.

Lastly, we wish to test prediction (iv), that the particle ratios at fixed $x_\perp$ should be similar to the ratios at fixed x. We first note that the ratios π^-/π^+ and K^-/K^+ deviate considerably from unity at large p_t, while they are close to unity at small p_t. Presumably, this will also be true of $\bar{p}/p$ at extremely high energies, where the scaling limit is approached for fixed p_t and $s\to\infty$. To examine prediction (iv) in more detail, we consider the energy dependence of the particle ratios. For small x and fixed p_t, the x dependences of ρ_{pp,K^+} and ρ_{pp,π^+} are similar.[5] In the same region, $\rho_{pp,p}$ increases with x while ρ_{pp,π^+} decreases. ρ_{pp,K^-} decreases faster than x with ρ_{pp,π^-}, and likewise $\rho_{pp,\bar{p}}$ decreases faster than ρ_{pp,π^-}. Consequently, when s increases at fixed $x_\perp$, i.e., at large p_t, we predict that K^+/π^+ should be about constant, p/π^+ should decrease, and K^-/π^- and $\bar{p}/\pi^-$ should both increase. These qualitative behaviors are observed in the tungsten data,[1] as shown in Table II.

We have shown that the cascade model disagrees with the data in several respects. In one last regard, let us consider the variation of the mean multiplicity in a p-nucleus collision, $\langle n\rangle_{pA}$, with the number of nucleons A. We have seen that if one were to parameterize $\rho_{pA,c}\propto A^\lambda$, then $\lambda\simeq 1$ at large p_t and $\lambda<1$ for small p_t near $p_\parallel^* \simeq 0$. We would expect in fact that $\lambda\lesssim 1$ in all regions. If this is found to be true, then the inclusive sum rule

$$\int E\left(\frac{d\sigma}{d^3p}\right)\left(\frac{d^3p}{E}\right) = \langle n\rangle\sigma^{\text{inel}} \qquad (5)$$

will require $\langle n\rangle_{pA}/\langle n\rangle_{pp} < A\sigma_{pp}/\sigma_{pA}$. Since experimentally $\sigma_{pA}^{\text{inel}}/\sigma_{pp}^{\text{inel}} \simeq A^{0.75}$, this would imply $\langle n\rangle_{pA}/\langle n\rangle_{pp} < A^{0.25}$. Such a slow growth of the multiplicity with A, if substantiated by direct measurements or through the sum rule, would constitute further evidence against a cascade picture, which naturally produces a rapid growth of $\langle n\rangle$ with A.

When $s\to\infty$ at fixed p_t, the Mueller-Regge analysis suggests $r\to A_{\text{eff}}$. According to the sum rule (2), this would mean that the increase in multiplicity with A would have to come entirely

TABLE II. Particle production ratios on tungsten as a function of laboratory momentum at $p_t = 2.29$ (4.58) GeV/c.

	$P_{\text{lab}} = 200$ GeV/c	$P_{\text{lab}} = 300$ GeV/c
K^+/π^+	0.434 ± 0.021 (0.536 ± 0.033)	0.421 ± 0.018 (0.531 ± 0.022)
p/π^+	1.020 ± 0.011 (1.11 ± 0.03)	0.871 ± 0.009 (0.742 ± 0.015)
K^-/π^-	0.204 ± 0.010 (0.170 ± 0.014)	0.231 ± 0.010 (0.205 ± 0.008)
$\bar{p}/\pi^-$	0.093 ± 0.003 (0.033 ± 0.003)	0.124 ± 0.002 (0.043 ± 0.002)

from the large-p_t region. This will be easily testable when multiplicity data on several nuclei become available.

III. COMPARISON WITH EXPERIMENT-II

After the preceding part of this paper was completed, measurements of single-particle spectra on Be ($A=9$) and Ti ($A=48$) became available.[6] These measurements allow a much more precise test of our theory, and are less susceptible to systematic errors because the same detection apparatus is used with each target element. For π^+ and π^- production, the data agree with our theory qualitatively, but they disagree with it in detail, in a manner which could be attributed to the presence of a small amount of double scattering. For $K^\pm$ production, and especially for p and $\bar{p}$ production, the discrepancies are larger. We will discuss these cases separately.

The measured A dependence is $E(d\sigma/d^3p) \propto A^n$ with $n \simeq 1.1$. To first approximation, this is close to our prediction of $A^{1.0}$. It is far, for instance, from the "shadowing" prediction $A_{\text{eff}} \simeq A^{2/3}$. To interpret the deviation from $A^{1.0}$, let us assume that there is a small amount of double scattering. At a given impact parameter, the probability of a single interaction is proportional to the thickness of the target nucleus, while the probability of two interactions is proportional to the square of that thickness. The cross section is an integral over impact parameters, and hence contains an extra factor of radius squared. It is therefore proportional to $R^3 \propto A^{1.0}$ for single scattering (S), and to $R^4 \propto A^{4/3}$ for double scattering (D). *We are therefore led to fit* $E(d\sigma/d^3p)_{pA\to\pi^\pm}$ *with the form* $S(1+D/S) = C_1A(1+C_2A^{1/3})$, rather than with the *ad hoc* form A^n. We find that $C_2 = 0.14$ fits the data at $p_t > 3.8$ GeV/c just as well as $A^{1.1}$ does. According to the calculation of Sec. II, equal *probabilities* for single and double scattering would make $D/S \simeq 4$ in the p_t region of interest. Hence the ratio of double-scattering to single-scattering probabilities implied by the data is about $0.035A^{1/3}$, which amounts to 0.07 for $A=9$, 0.13 for $A=48$, and 0.20 for $A=184$. The amount of double scattering required to fit the A dependence is thus quite small. It corresponds to an effective interaction cross section of 5 mb (see Appendix). Our basic theory, in which there is no double scattering, may therefore be not far from the truth, even though the numerical corrections to it, $C_2A^{1/3}$ (or $\simeq A^{0.1}$) are not so small.

The cross section for *proton* production varies about like $A^{1.3}$.[6] This does not fit with our picture of single scattering together with perhaps a small amount of double scattering. It is rather as if double scattering were dominant. A possible explanation is that the basic mechanism which produces large-p_t protons is a collective effect involving more than one parton from the target. This is indeed the case if the relevant partons are valence quarks with baryon number $\frac{1}{3}$. It can also happen in parton interchange models.[7] If we assume for example that large-p_t protons are formed from three quarks—one from the beam and two from the nucleus or the other way around—the resulting A dependence must be $A^{1.0}+A^{4/3}$, which is similar to $A^{1.25}$ for the nuclei measured and is close to the experimental result. The observation that *pion* production is relatively close to $A^{1.0}$ can be understood from this point of view, since pions can be formed from a single quark from the target together with one from the beam particle. If the partons indeed have quark quantum numbers, then *kaons* can also be produced from two partons. Because of their strangeness quantum number, however, they require at least one quark which is not a valance quark from the beam or target. It is therefore natural to expect multiparton processes to be more important for production of kaons than for pions. This is in fact the observation: The A dependence of K production falls between that for π and that for p.

We conclude that parton-model concepts are helpful in understanding the large-p_t production data on nuclear targets. In particular, "cascade" effects and shadowing do not occur. The observed A dependence indicates that two or more partons from the target may participate in the large-p_t production of p and $\bar{p}$. In general, the nuclear A dependence will provide a useful test for specific models of large-transverse-momentum production.

APPENDIX

Let the nucleus have a uniform density distribution, with radius $R=R_0A^{1/3}$ where $R_0 = 1.1$ F. If the interaction cross section for some system on a single nucleon is σ, then the cross section for n interactions in the nucleus is

$$\sigma_n = 2\pi\int_0^R b\,db\,e^{-y}y^n/n!\ ,$$

$$y = (R^2-b^2)^{1/2}\times 3\sigma/2\pi R_0^{\,3}\ .$$

To second oreder in σ, we have $\sigma_1 = \sigma A - (9\sigma^2/8\pi R_0^{\,2})A^{4/3}$ and $\sigma_2 = (9\sigma^2/16\pi R_0^{\,2})A^{4/3}$. If we assume, as in the text, that the cross-section ratio of double scattering to single scattering is 4:1 if their probabilities are equal, we obtain

$$\sigma(A) \propto \sigma_1 + 4\sigma_2 = \sigma A[1+(9\sigma/8\pi R_0^{\,2})A^{1/3}]\ .$$

Hence the observed dependence $E\,d\sigma/d^3p \propto A(1+0.14A^{1/3})$ for pions at large p_t corresponds to an effective interaction cross section of $\sigma = 4.7$ mb. This rather small value indicates that multiscattering is not very important in pion production. The value would change very little if we kept the higher powers of σ in fitting the A dependence of $E\,d\sigma/d^3p$.

---

*Work supported by National Science Foundation.

†Present address: Department of Physics, National Tsing Hua University, Hsinchu, Taiwan, Republic of China.

[1]J. Cronin *et al.*, Phys. Rev. Lett. 31, 1426 (1973).

[2]P. Fishbane and J. Trefil, Phys. Rev. Lett. 31, 734 (1973); A. Dar and J. Vary, Phys. Rev. D 6, 2412 (1972); K. Zalewski, Acta Phys. Pol. B4, 973 (1973).

[3]F. Büsser *et al.*, Phys. Lett. 46B, 471 (1973).

[4]A. Wu Chao and Chen Ning Yang, Phys. Rev. D 9, 2505 (1974).

[5]E. Yen, Phys. Rev. D 10, 836 (1974).

[6]J. Cronin *et al.*, papers submitted to the XVII International Conference on High Energy Physics, London, 1974 (unpublished).

[7]J. F. Gunion, S. J. Brodsky, and R. Blankenbecler, Phys. Lett. 42B, 461 (1972); S. J. Brodsky and G. R. Farrar, Phys. Rev. Lett. 31, 1153 (1973); R. Blankenbecler, lecture notes distributed at the SLAC Summer Institute, Stanford, 1974 (unpublished).