# Amplitudes for $\pi^- p \rightarrow K^0\Lambda^0$ and $\pi^- p \rightarrow K^0\Sigma^0$ at 5 GeV/$c$ *

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Using our data on $d\sigma/dt$ and polarization in $\pi^- p \rightarrow K^0\Lambda^0$ and $\pi^- p \rightarrow K^0\Sigma^0$ at 5 GeV/$c$, and assuming SU(3) octet dominance of the baryon couplings in $s$-channel helicity amplitudes, we determine amplitudes for these reactions up to an undetermined over-all phase. We calculate the spin rotation parameters $\hat{A}$ and $\hat{R}$ and compare our predictions with those from phenomenological models. Assuming a Regge phase for the helicity-flip amplitude, we find that the imaginary part of the nonflip amplitude for these reactions shows peripheral behavior but the real part does not.

The study of the associated production reactions $\pi N \rightarrow K\Lambda$ and $\pi N \rightarrow K\Sigma$ can yield important information about exchange mechanisms and absorptive effects. The amplitude structure for these reactions is particularly simple in a Regge description, since only one pair of opposite-signature Regge exchanges is allowed. The predictions of exchange-degenerate models, even with conventional absorptive corrections, are in contradiction with experiment.[1,2] Accordingly, it seems important to obtain the amplitude structure of these reactions directly. Several amplitude analyses of hypercharge-exchange reactions have been published[3-5]; however, they share the feature of assuming a theoretical model, then fitting the data to obtain the parameters of that model.

In the absence of $R$ and $A$ measurements the amplitudes cannot be determined from experimental data alone, so that some theoretical input is required. We show in this paper that simply the assumptions of pure $I=\frac{1}{2}$ exchange (which can be checked experimentally) and SU(3) octet baryon couplings in the $s$-channel helicity amplitudes, in conjunction with measured $d\sigma/dt$ and polarizations, are sufficient to determine the amplitudes up to an over-all phase. We have calculated the amplitudes at 5 GeV/$c$, using our recent data on $\pi^- p \rightarrow K^0\Lambda^0$ and $\pi^- p \rightarrow K^0\Sigma^0$ (see Refs. 6, 7) as well as older polarization data from $\pi^+ p \rightarrow K^+\Sigma^+$.[8] In addition to having high statistical quality, the two cross sections were measured simultaneously in the same experimental apparatus, thus greatly reducing possible errors resulting from normalization discrepancies.

The validity of $I=\frac{1}{2}$ exchange dominance is most thoroughly demonstrated by recent measurements[9] of the double-charge-exchange reaction $\pi^- p \rightarrow K^+\Sigma^-$ giving cross sections three orders of magnitude lower than those where $I=\frac{1}{2}$ exchange is allowed. However, we can also check the prediction

$$\frac{d\sigma}{dt}(\pi^- p \rightarrow K^0\Sigma^0) = \frac{1}{2}\cdot\frac{d\sigma}{dt}(\pi^+ p \rightarrow K^+\Sigma^+)\,;$$

Fig. 1 illustrates the striking agreement of the prediction with the data.[6,8,10] We will accordingly make use of another consequence of pure $I=\frac{1}{2}$ exchange, namely the equality of the polarizations, $P(\pi^- p \rightarrow K^0\Sigma^0) = P(\pi^+ p \rightarrow K^+\Sigma^+)$.

In order to explain the mirror symmetry of the polarizations in $\pi^- p \rightarrow K^0\Lambda^0$ and $\pi^+ p \rightarrow K^+\Sigma^+$, Martin, Michael, and Phillips have given[11] an SU(3) relation between $s$-channel helicity amplitudes based on octet exchange dominance, which we can write for the $\Lambda^0$ and $\Sigma^0$ as

$$\frac{H_{++}^{\Lambda}}{2x_+ + 1} = \frac{H_{++}^{\Sigma}}{\sqrt{3}\,(2x_+ - 1)} \equiv f_+\,, \tag{1}$$

where $f_+$ is a reduced helicity-nonflip amplitude,

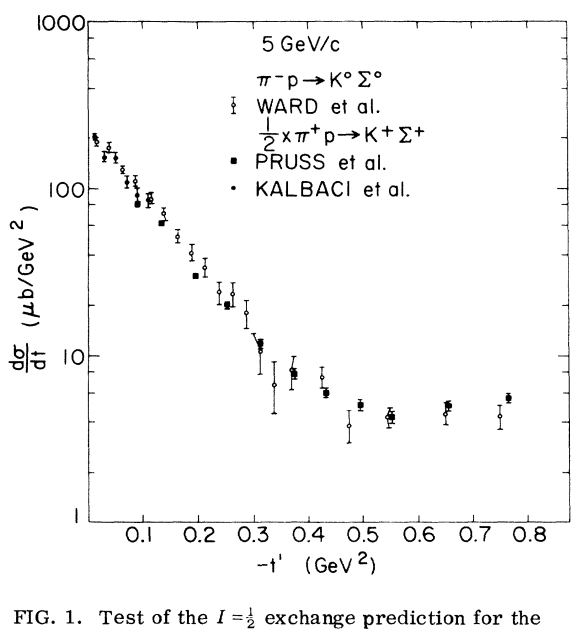


FIG. 1. Test of the $I=\frac{1}{2}$ exchange prediction for the $\pi p \rightarrow K\Sigma$ cross sections.

and the real parameter $x_+$ is $F_+/(F_+ + D_+)$, where $F$ and $D$ refer to the SU(3) couplings for exchange amplitudes of definite $G$ parity. A similar relation holds for the helicity-flip amplitudes $H_{+-}$ in terms of a reduced amplitude $f_-$ and the parameter $x_-$. (The values of $x_+$ and $x_-$ can be different in general, but we expect them to be independent of $s$ and $t$.) Relation (1) holds only if the $F/D$ ratios for vector and tensor exchange are equal, as suggested by Rosner *et al.*[12] We will make an experimental check of this condition.

Relation (1) leads to the following expressions for the cross sections and polarizations for $\pi^- p \to K^0\Lambda^0$:

$$\frac{d\sigma_\Lambda}{dt} = (2x_+ + 1)^2 |f_+|^2 + (2x_- + 1)^2 |f_-|^2 , \tag{2}$$

$$P_\Lambda \frac{d\sigma_\Lambda}{dt} = 2(2x_+ + 1)(2x_- + 1)|f_+||f_-|\sin\phi_{+-} , \tag{3}$$

and to similar expressions for $\pi^- p \to K^0\Sigma^0$; $\phi_{+-}$ is the phase of $f_-$ relative to $f_+$.

The value of $x_+$ can be determined from the vanishing of the spin-flip amplitude at 0°, through the ratio

$$R_z \equiv \frac{\frac{d\sigma_\Sigma}{dt}(0°)}{\frac{d\sigma_\Lambda}{dt}(0°)} = \frac{3(2x_+ - 1)^2}{(2x_+ + 1)^2} . \tag{4}$$

$R_z$ has been determined at 3, 4, 5, and 6 GeV/$c$ from the data of Ref. 6 and is shown in Fig. 2(a); its value is constant with energy, with a weighted average value of $0.708 \pm 0.014$. In turn, $x_-$ can be determined from the ratio

$$R_p \equiv \frac{P_\Sigma \frac{d\sigma_\Sigma}{dt}}{P_\Lambda \frac{d\sigma_\Lambda}{dt}} = \frac{3(2x_+ - 1)(2x_- - 1)}{(2x_+ + 1)(2x_- + 1)} . \tag{5}$$

$R_p$ is plotted as a function of $t'$ in Fig. 2(b) using the data at 5 GeV/$c$; within the errors it is independent of $t'$, with a value $-0.304 \pm 0.067$. [In the determination of $x_-$, we have set the unphysically large measured values of $\Lambda^0$ polarization beyond $t' = -0.84$ GeV$^2$ equal to $-1.0$, and increased the errors accordingly.] From these results we obtain $x_+ = 1.445 \pm 0.007$ and $x_- = 0.327 \pm 0.031$, in good agreement with earlier results.[11]

The constancy of $R_z$ with respect to $s$ and $R_p$ with respect to $t$ is consistent with the assumption of the SU(3) relation (1) between $s$-channel helicity amplitudes, and within the errors with the equality of the $F/D$ ratios for vector and tensor exchange.

Using the values obtained for $x_+$ and $x_-$, we can solve for $|f_+|$, $|f_-|$, and $\sin\phi_{+-}$. Values for $\sin\phi_{+-}$ were determined using the $\Lambda^0$ and $\Sigma^+$ polarization data separately. The two sets of values agreed within statistics, so they were averaged to give final results for $\sin\phi_{+-}$. The results are given in Table I and Fig. 3. The quoted errors result from the statistical errors in the data and the errors on $x_+$ and $x_-$. The values of $|f_-|$ near $t' = 0$ have large errors, but presumably $|f_-|$ varies as $\sqrt{-t'}$ as $t' \to 0$.

We can now calculate the spin rotation parameters $\hat{R}$ and $\hat{A}$, defined as

$$\hat{R}\frac{d\sigma}{dt} = 2\,\mathrm{Re}(H_{++}H^*_{+-}) ,$$

$$\hat{A}\frac{d\sigma}{dt} = |H_{++}|^2 - |H_{+-}|^2 .$$

Our predictions are shown in Table I and Fig. 3. Where the values of $\sin\phi_{+-}$ were determined to be greater than 1.0 owing to experimental errors, in the subsequent analysis we set $\sin\phi_{+-}$ equal to 1.0 and increased the errors accordingly. There is an ambiguity in the sign of $\hat{R}$; we have selected the sign which would be given by models with the following properties near $t' = 0$: a flip amplitude having a Regge phase $e^{-i\pi\alpha}$, and a sign for the imaginary part of the nonflip amplitude in agreement with finite-energy sum rule calculations.[5]

We have indicated in Fig. 4 the predictions for the parameters $\hat{R}$ and $\hat{A}$ from analyses based on the dual absorption model (DAM),[4] exchange-degenerate Regge-pole exchange with absorptive corrections,[1] and the strong-cut Regge absorption model (SCRAM).[13] The analysis of Barger and Martin,[3] which assumes an unmodified exchange-degenerate Regge-pole spin-flip amplitude, gives results very similar to the Regge model of Ref. 1. As Berger and Fox have pointed out,[13] the complicated zero structure of the amplitudes in some models, such as SCRAM and DAM, requires con-

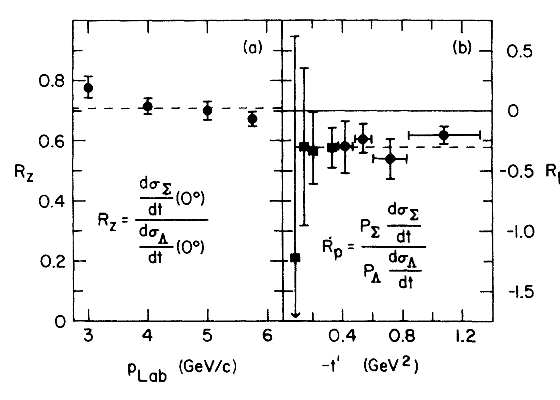


FIG. 2. The ratios $R_z$ and $R_p$. The SU(3) relation assumed in the text predicts that these should be constant with $s$ and $t$.

TABLE I. Input values and results of the analysis.

Input values

| $-t'$ (GeV$^2$) | $\frac{d\sigma(\Lambda)}{dt}$ ($\mu$b/GeV$^2$) | $\frac{d\sigma(\Sigma)}{dt}$ ($\mu$b/GeV$^2$) | $P(\Lambda)$ | $P(\Sigma)$ |
|---|---|---|---|---|
| 0.075 | 186.4 ± 5.2 | 122.1 ± 4.8 | 0.07 ± 0.10 | −0.13 ± 0.09 |
| 0.135 | 114.2 ± 4.0 | 71.5 ± 3.7 | 0.09 ± 0.12 | −0.04 ± 0.07 |
| 0.195 | 70.1 ± 3.2 | 40.1 ± 2.9 | 0.22 ± 0.15 | −0.12 ± 0.10 |
| 0.255 | 44.4 ± 2.3 | 22.7 ± 2.4 | 0.48 ± 0.19 | 0.00 ± 0.12 |
| 0.315 | 31.3 ± 2.3 | 11.2 ± 1.2 | −0.59 ± 0.25 | 0.49 ± 0.16 |
| 0.405 | 19.5 ± 1.2 | 7.3 ± 0.9 | −0.40 ± 0.22 | 0.33 ± 0.16 |
| 0.525 | 11.9 ± 0.7 | 4.2 ± 0.5 | −1.07 ± 0.26 | 0.53 ± 0.20 |
| 0.645 | 8.3 ± 0.7 | 4.5 ± 0.6 | −1.13 ± 0.36 | 0.58 ± 0.20 |
| 0.765 | 8.1 ± 0.7 | 4.1 ± 0.6 | −0.50 ± 0.39 | 0.68 ± 0.18 |
| 0.885 | 7.3 ± 0.6 | 4.1 ± 0.6 | −1.33 ± 0.43 | 0.78 ± 0.19 |

Results

| $-t'$ (GeV$^2$) | $\|f_+\|$ ($\sqrt{\mu b}$/GeV) | $\|f_-\|$ ($\sqrt{\mu b}$/GeV) | $\sin\phi_{+-}$ | $\hat{R}_\Lambda$ | $\hat{A}_\Lambda$ | $\hat{R}_\Sigma$ | $\hat{A}_\Sigma$ |
|---|---|---|---|---|---|---|---|
| 0.075 | 3.35 ± 0.09 | 2.50 ± 0.80 | 0.18 ± 0.16 | −0.58 ± 0.15 | 0.82 ± 0.11 | 0.26 ± 0.10 | 0.96 ± 0.08 |
| 0.135 | 2.55 ± 0.09 | 2.43 ± 0.64 | 0.13 ± 0.13 | −0.69 ± 0.14 | 0.72 ± 0.14 | 0.34 ± 0.11 | 0.94 ± 0.11 |
| 0.195 | 1.88 ± 0.09 | 2.46 ± 0.52 | 0.26 ± 0.14 | −0.82 ± 0.10 | 0.53 ± 0.18 | 0.44 ± 0.12 | 0.89 ± 0.15 |
| 0.255 | 1.39 ± 0.11 | 2.35 ± 0.46 | 0.26 ± 0.15 | −0.91 ± 0.08 | 0.32 ± 0.23 | 0.54 ± 0.14 | 0.83 ± 0.23 |
| 0.315 | 0.90 ± 0.13 | 2.63 ± 0.37 | −0.58 ± 0.17 | −0.80 ± 0.10 | −0.21 ± 0.24 | 0.68 ± 0.19 | 0.56 ± 0.37 |
| 0.405 | 0.74 ± 0.09 | 2.03 ± 0.26 | −0.41 ± 0.16 | −0.90 ± 0.07 | −0.16 ± 0.21 | 0.73 ± 0.18 | 0.60 ± 0.32 |
| 0.525 | 0.55 ± 0.07 | 1.63 ± 0.20 | −0.87 ± 0.19 | −0.49 ± 0.17 | −0.23 ± 0.20 | 0.42 ± 0.18 | 0.55 ± 0.33 |
| 0.645 | 0.63 ± 0.06 | 0.93 ± 0.28 | −1.25 ± 0.36 | 0.00 ± 0.34 | 0.43 ± 0.30 | 0.00 ± 0.18 | 0.86 ± 0.28 |
| 0.765 | 0.59 ± 0.06 | 1.02 ± 0.26 | −0.77 ± 0.33 | −0.60 ± 0.22 | 0.30 ± 0.31 | 0.36 ± 0.16 | 0.82 ± 0.30 |
| 0.885 | 0.60 ± 0.05 | 0.82 ± 0.26 | −1.57 ± 0.47 | 0.00 ± 0.42 | 0.49 ± 0.29 | 0.00 ± 0.23 | 0.88 ± 0.26 |

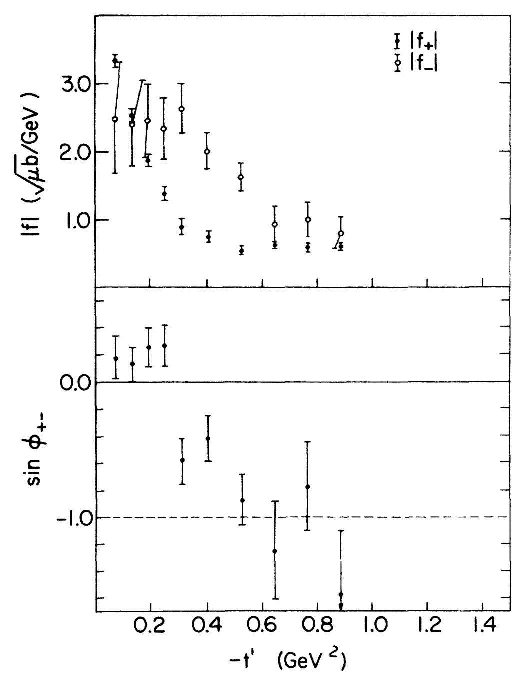


FIG. 3. The moduli and relative phase of the reduced amplitudes.

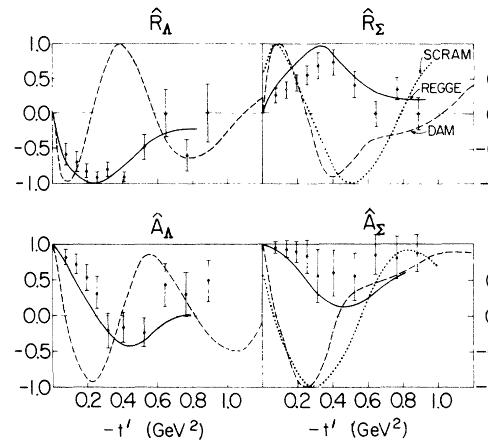


FIG. 4. Predicted values for the spin rotation parameters $\hat{R}$ and $\hat{A}$. Solid line: Regge model (Ref. 1). Dashed line: dual absorption model (Ref. 4). Dotted line: strong-cut Regge absorption model (Ref. 13). Points with error bars: this analysis.

siderable structure in these observables. Our analysis indicates that such structure is incompatible with the SU(3) relation (1) between $s$-channel helicity amplitudes. This conflict with SU(3) is not required by the Regge pole model. Thus, measurements of $\hat{R}$ and $\hat{A}$ for the associated production reactions would necessarily discredit either (a) SCRAM and DAM or (b) the SU(3) relations between the $s$-channel helicity amplitudes, or both. On the other hand, if such $\hat{R}$ and $\hat{A}$ measurements agree with the predictions presented here, they will be unable to distinguish between all dynamical models consistent with the SU(3) relations. Tests of such models could presumably be made only in combination with measurements of the line-reversed processes $\bar{K}N \to \pi\Lambda, \Sigma$.

Another possibility is that the SU(3) relation (1) holds for the $t$-channel helicity amplitudes rather than for the $s$-channel ones. In this case all of the foregoing results would still hold, except that the predicted values of $\hat{R}$ and $\hat{A}$ would be related by a rotation to those we have presented. The linear relation between the two sets of predictions is unique and can be determined from the $s$-$t$ crossing relations.

Finally, assuming a Regge phase $\exp(-i\pi\alpha)$ for the helicity-flip amplitude, where $\alpha(t) = 0.35 + 0.82t$ is the straight-line trajectory through the $K^*(892)$ and $K^{**}(1420)$ masses, we can separate the real and imaginary parts of each helicity amplitude. The resulting amplitude structure for $\pi^- p \to K^0\Lambda^0$ is shown in Fig. 5. The imaginary part of $H^\Lambda_{++}$ has the shape of a $J_0$ Bessel function, which can be associated with a peripheral interaction. Re $H^\Lambda_{++}$ does not show this behavior. $H^\Lambda_{+-}$ shows simply the assumed Regge behavior.

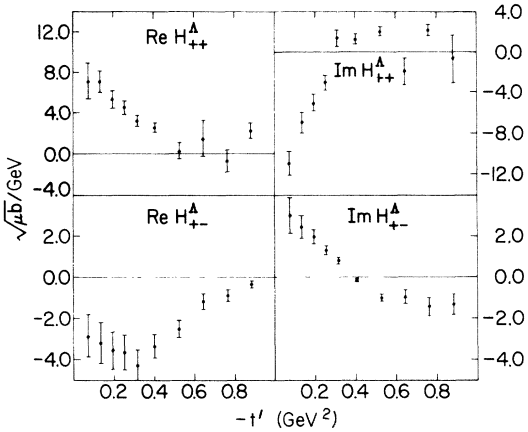


FIG. 5. Real and imaginary parts of the $\pi^- p \to K^0\Lambda^0$ $s$-channel helicity amplitudes, assuming a Regge phase for the helicity-flip amplitude.

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