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Goldberger-Treiman relation and chiral-symmetry breaking

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We review the dispersion-theory calculation of the Goldberger-Treiman discrepancy $\Delta \equiv 1 - mg_A/f_\pi g$. Our estimate is still some way from the experimental value. The latter and its SU(3) counterparts can be used to determine the chiral-symmetry-breaking parameter c appearing in the $(3, \bar{3}) + (\bar{3},3)$ Hamiltonian $u_0 + c u_8$. We find $c \approx -1$ in agreement with its determination from phenomenological σ terms.

I. INTRODUCTION AND FORMALISM

The validity or otherwise of the proposed $(3,\bar{3}) + (\bar{3}, 3)$ model[1] for the breaking of the chiral SU(3) $\times$SU(3) symmetry can be tested in various ways. In a previous paper[2] we have compared the predictions of the model against phenomenological σ terms, concluding that the generally large values of these quantities can be reconciled with the model only if the parameter c occurring in the symmetry-breaking Hamiltonian $H' = u_0 + cu_8$ takes the value $c \approx -1$, rather than the value $c \approx -1.25$ derived from the pseudoscalar-meson mass formula.

An alternative means of probing the value of c is provided by the Goldberger-Treiman (GT) relation[3] and its SU(3) counterparts. This is connected to the above determination in two ways. First, the discrepancy between the experimental value $f_\pi \approx 93$ MeV found from $\pi \to \mu\nu$ decay[4] and the theoretical prediction $f_\pi^{\rm GT} = m_N g_A/g \approx 87$ MeV can be reduced slightly if the coupling of the $\sigma(\epsilon)$ particle to pions is large, which, in the σ model, directly follows from the large value[2,5] ($\approx$70 MeV) of the πN σ term $\sigma(\pi N)$. Second, the SU(3) GT discrepancies afford an independent determination of c provided that the hyperon coupling constants are known. In fact, as we shall see, the determination is rather insensitive to SU(3) breaking of these couplings and indeed leads to $c \approx -1$, but cannot credibly be stretched to accommodate the canonical value $c \approx -1.25$.

Both of these aspects of the GT relation have been discussed in the literature before. However, previous evaluations of the GT discrepancy by means of dispersion relations[6,7] are marred by errors and do not use the connection between the σ-meson coupling constants and the πN σ term. Our treatment of the SU(3) relations is in the spirit of the work of Dashen[8] and has the advantage of a now clearer experimental situation as regards the hyperon coupling constants.

The objects of our study will be the GT discrepancies Δ, which for πNN is defined as[6]

$$\Delta_{\pi NN} = 1 - \frac{m g_A}{f_\pi g} \,. \tag{1}$$

Numerically, this discrepancy is found to be

$$\Delta_{\pi NN} = 0.058 \pm 0.013 \,, \tag{2}$$

using the values[9,10] $m \equiv m_N = 938.9$ MeV, $g_A \equiv g_A(0) = 1.25$, $f_\pi = 93.0$ MeV, and[11] $g \equiv g_{\pi NN} = 13.40$.

The formalism for the first aspect of our work has been set up by Pagels.[6] One begins by defining the nucleon-nucleon matrix element of the axial-vector current as[12,13]

$$\langle N(p') | A_\mu^j(x) | N(p) \rangle = i\bar{u}(p')(\tfrac{1}{2}\tau^j)[g_A(q^2)\gamma_\mu + h_A(q^2)q_\mu]\gamma_5 u(p) e^{-iq\cdot x} \,, \tag{3}$$

where $q = p' - p$.

Taking the divergence, one has

$$\langle N(p') | \partial \cdot A^j(0) | N(p) \rangle = D(q^2)\bar{u}(p')\tau^j\gamma_5 u(p) \,, \tag{4}$$

where

$$D(q^2) = m g_A(q^2) + \tfrac{1}{2} q^2 h_A(q^2) \,, \tag{5}$$

which can be separated into a contribution from the pion pole plus the background:

$$D(q^2) = D_{\rm pole}(q^2) + \bar{D}(q^2) \,. \tag{6}$$

Evaluating the pole term at $q^2 = 0$ leads to

$$D(0) = m g_A = f_\pi g + \bar{D}(0) \,. \tag{7}$$

One then assumes an unsubtracted dispersion relation for $\bar{D}(q^2)$:

$$\bar{D}(0) = \frac{1}{\pi}\int_{9\mu^2}^{\infty} {\rm Im}\,\bar{D}(t)\,\frac{dt}{t} \,. \tag{8}$$

The PCAC (partially conserved axial-vector current) relation, $\partial \cdot A = \mu^2 f_\pi \phi$, connects[6] $\overline{D}(q^2)$ to the pionic form factor $K(q^2)$ of the nucleon, leading to

$$\operatorname{Im}\overline{D}(t) = \mu^2 f_\pi g \frac{\operatorname{Im}K(t)}{\mu^2 - t}, \tag{9}$$

where $K(t)$ is normalized to unity at $q^2 \equiv t = \mu^2$.

Thus Eq. (8) can be rewritten as

$$\overline{D}(0) = f_\pi g \frac{\mu^2}{\pi} \int_{9\mu^2}^{\infty} \frac{dt}{t(\mu^2 - t)} \operatorname{Im}K(t), \tag{10}$$

and Eq. (7) as

$$\Delta_{\pi NN} \equiv 1 - \frac{m g_A}{f_\pi g} = \frac{\mu^2}{\pi} \int_{9\mu^2}^{\infty} \frac{dt}{t(t - \mu^2)} \operatorname{Im}K(t). \tag{11}$$

This can be interpreted as a once-subtracted dispersion relation in the pionic form factor of the nucleon, Δ being thus a measure of the variance of this quantity between $q^2 = \mu^2$ and $q^2 = 0$:

$$\Delta_{\pi NN} = K(\mu^2) - K(0). \tag{12}$$

The contributions to the dispersion integral will be examined in Sec. II, where the differences with previous work will be carefully pointed out. The value of the σ term $\sigma(\pi N)$ and the form of the low-energy approximation to the $I_t = 0$ πN amplitude will prove to be important, as discussed in Sec. III.

The second aspect of our work has been discussed briefly by Dashen.[8,14] Here one evaluates the nonpole part of $D(0)$ in Eq. (7) in terms of the structure of the chiral-symmetry breaking Hamiltonian:

$$\overline{D}(0) = \tfrac{1}{2}\langle N | \overline{\partial \cdot A}^{\,3}(0) | N \rangle \tag{13}$$

where

$$i\partial \cdot A^3 = [Q_5^3, H'].$$

In the $(3, \overline{3}) + (\overline{3}, 3)$ model H' takes the form

$$H' = u_0 + c u_8, \tag{14}$$

and the relevant commutators in Eq. (13) are

$$[Q_5^i, u^j] = -i d^{ijk} v_k. \tag{15}$$

Thus, from this point of view

$$\overline{D}(0) = -\frac{\sqrt{2} + c}{2\sqrt{3}} (\overline{v}_\pi)_{NN}, \tag{16}$$

and so

$$\Delta_{\pi NN} = \frac{1}{f_\pi g} \frac{\sqrt{2} + c}{2\sqrt{3}} (\overline{v}_\pi)_{NN}. \tag{17}$$

The SU(3) analogs of this relation can be used to eliminate the unknown quantity $(\overline{v}_\pi)_{NN}$ in favor of hyperon discrepancy functions $\Delta_{KN\Sigma}$, $\Delta_{KN\Lambda}$. This analysis is discussed in Sec. IV, where, in particular, the insensitivity of the derived relation to SU(3) breaking is pointed out.

Following these calculations a discussion of the experimental discrepancy of $\Delta_{\pi NN}$ and the status of the GMOR model is given in Sec. V.

II. SATURATION OF THE DISPERSION RELATION

We now reexamine estimates[6,7] of $\Delta_{\pi NN}$ obtained by saturating (11) with the lowest-lying intermediate states. Barring the existence of a heavy pion or a large 3π enhancement,[15] it has been shown[16] that the contribution from the 3π intermediate state is completely negligible. Furthermore, the high-energy contribution, i.e., the contribution from values of $t > 4m^2$, has been shown[17] to be bounded by 0.01. Thus we can restrict our attention to the resonant $\rho\pi$ and $\sigma\pi$ intermediate states and take the upper limit of integration in Eq. (11) as $4m^2$. The contributions are shown diagrammatically in Fig. 1. In the absence of a complete representation of the right-hand amplitudes $\pi\sigma \to N\overline{N}$ and $\pi\rho \to N\overline{N}$, we can only attempt an estimate thereof using s-, u-, and t-channel pole terms.

We consider first, then, the s- and u-channel nucleon poles, leading to the contributions shown diagrammatically in Figs. 2(a) and 2(b). These poles are the first approximations to a fixed-t dispersion calculation of the right-hand amplitude; in this spirit their couplings are to be taken as constants, as previous authors have done.[6,7]

A. $\rho\pi$ intermediate state

The couplings necessary for this calculation are defined by the following phenomenological Hamiltonians:

$$H_{\pi NN} = g_{\pi NN} \overline{N} \tau_i \gamma_5 N \Pi^i, \tag{18}$$

$$H_{\rho\pi\pi} = g_{\rho\pi\pi} i \epsilon_{ijk} \Pi^{*i} (q_i + q_k)^\mu \rho^j_\mu \Pi^k, \tag{19}$$

$$H_{\rho NN} = g_{\rho NN} \overline{N}(p')(\tfrac{1}{2}\tau_i)[F_1^V \gamma_\mu + F_2^V i\sigma_{\mu\nu}(p' - p)^\nu / 2m] \times \rho^{i,\mu} N(p), \tag{20}$$

where, by vector dominance, F_1^V, F_2^V are respectively the Dirac and Pauli isovector couplings, with the values $F_1^V = 1$, $F_2^V = \kappa_V = 3.7$, and $g_{\rho\pi\pi} \approx g_{\rho NN} \equiv g_\rho$, with $g_\rho^2/4\pi \approx 2.8$, as determined by the width[9] $\Gamma_{\rho\pi\pi} \approx 150$ MeV.

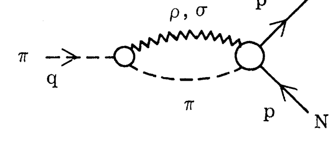

FIG. 1. $\rho\pi$ and $\sigma\pi$ intermediate state contributions to Δ.

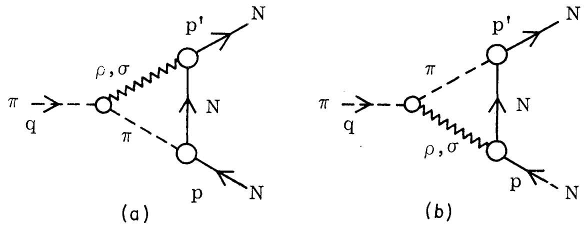

FIG. 2. (a) s-channel nucleon pole diagram, and (b) u-channel nucleon pole diagram.

The contribution to the absorptive part of K is then calculated to be,[18] for $(m_\rho+\mu)^2 \leq t \leq 4m^2$,

$$\operatorname{Im}K^N_{\rho\pi}(t) = \frac{g_\rho{}^2}{4\pi}\left(\frac{Q_\rho}{\sqrt{t}}\,\frac{t-m_\rho{}^2-\mu^2}{m_\rho{}^2} + \frac{\kappa_V Q_\rho{}^2-\mu^2}{P\sqrt{t}}\tan^{-1}A_\rho\right), \tag{21}$$

where

$$Q_\rho = \frac{\{[t-(m_\rho-\mu)^2][t-(m_\rho+\mu)^2]\}^{1/2}}{2\sqrt{t}}, \tag{22}$$

$$P = \tfrac{1}{2}(4m^2-t)^{1/2}, \tag{23}$$

and

$$A_\rho = \frac{4PQ_\rho}{t-m_\rho{}^2-\mu^2}. \tag{24}$$

When inserted into the dispersion integral and evaluated numerically, this gives a contribution to Δ (N for nucleon channels)

$$\Delta^N(\rho\pi) = 0.015, \tag{25}$$

in agreement with the calculation of Braathen.[7]

The inclusion of the pion pole, Fig. 3, leads to a contribution to the absorptive part (s for self-energy)

$$\operatorname{Im}K^s_{\rho\pi}(t) = -\frac{g_\rho{}^2}{4\pi}\,\frac{4Q_\rho{}^3\sqrt{t}}{m_\rho{}^2(t-\mu^2)}, \tag{26}$$

and to Δ

$$\Delta^s(\rho\pi) = -0.0055. \tag{27}$$

B. $\sigma\pi$ intermediate state

In the calculation of the contribution from this intermediate state, we will differ in some important respects from previous treatments. Let us, however, first calculate the absorptive part for a 0^+ σ particle of arbitrary mass m_σ and couplings $g_{\sigma NN}$, $g_{\sigma\pi\pi}$ defined by the phenomenological Hamiltonians

$$H_{\sigma NN} = g_{\sigma NN}\bar{N}N\phi_\sigma, \tag{28}$$

$$H_{\sigma\pi\pi} = g_{\sigma\pi\pi}\Pi^{*i}\delta_{ij}\Pi^j\phi_\sigma. \tag{29}$$

From Figs. 2(a) and 2(b) we obtain an absorptive part

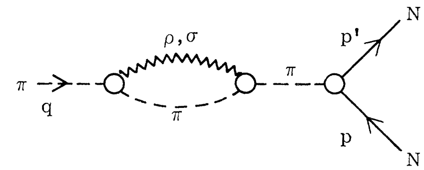

FIG. 3. t-channel pion pole diagram.

$$\operatorname{Im}K^N_{\sigma\pi}(t) = \frac{g_{\sigma\pi\pi}g_{\sigma NN}}{4\pi}\,\frac{m}{t}(t-m_\sigma{}^2+\mu^2)\,\frac{1}{2P\sqrt{t}} \times\tan^{-1}A_\sigma. \tag{30}$$

As discussed in the next section we will use the values of the couplings $g_{\sigma\pi\pi}$, $g_{\sigma NN}$ as given in the σ model,[19] viz.,

$$g_{\sigma NN} = g, \tag{31}$$

$$g_{\sigma\pi\pi} = \frac{g}{2m}(m_\sigma{}^2-\mu^2), \tag{32}$$

and a mass $m_\sigma = 500$ MeV, somewhat lower than the mass of the physical ϵ particle.[9] We choose this value to agree with the pion-nucleon σ term and in recognition of the fact that, as far as the σ particle is concerned, we are not including closed loops, which renormalize the mass upwards.[20]

With these parameters the calculated contribution to Δ is[21]

$$\Delta^N(\sigma\pi) = 0.01. \tag{33}$$

The self-energy diagram, Fig. 3, gives an absorptive part[18]

$$\operatorname{Im}K^s_{\sigma\pi}(t) = -\frac{g_{\sigma\pi\pi}{}^2}{4\pi}\,\frac{t-m_\sigma{}^2+\mu^2}{4t(t-\mu^2)}, \tag{34}$$

which gives a negligible contribution to Δ when inserted in the dispersion integral, Eq. (11).

C. Resonant contributions

So far our discussion has been limited to the inclusion of nucleon poles in the s- and u-channels of the amplitudes $\pi\rho \to N\bar{N}$ and $\pi\sigma \to N\bar{N}$. In the absence of a reliable method of estimating the continuum contribution, we can consider the inclusion of higher-mass resonances in these channels. The structure of the resonance spectrum is such that these resonances do not contribute as much as might have been expected. The reasons for this are as follows:

(i) For the $\rho\pi$ intermediate state, the $\Delta(1236)$ contribution is suppressed (apart from the mass factor) by the $M1$ nature of the $\rho N\Delta$ vertex near resonance.[22] Higher-resonant contributions will be suppressed primarily because of the mass factor, and as an optimistic estimate we may hope for at most

$$\Delta^{\text{proper}}(\rho\pi) \approx 0.02$$

for the proper $\rho\pi$ contribution.

(ii) For the $\sigma\pi$ intermediate state $\Delta(1236)$ is excluded by isospin invariance. Higher resonances such as $N'(1470)$, $N'(1520)$, $N'(1535)$, etc., appear[9] to be rather weakly coupled to the ϵN system. Hence, if we assume that the combined resonance contributions add, it seems that the most we can hope for is

$$\Delta^{\text{proper}}(\sigma\pi) \approx 0.015 .$$

D. Strange-particle intermediate states

The next-highest-mass intermediate states would be the strange counterparts of (A) and (B), viz., $K_V^*\bar{K}$ and $\kappa\bar{K}$, respectively. The couplings of K^* and K can be related by SU(3) to those of ρ and π, giving an overall Clebsch-Gordan factor ≈ 0.4 for the Dirac coupling and 0.2 for the Pauli coupling. Combined with a large reduction factor from higher masses in the dispersion relation, the effect is to make the $K^*\bar{K}$ contribution negligible. Since the $\sigma(\epsilon)$ is predominantly a unitary singlet,[23] the couplings of the κ and the σ cannot be so directly related, but barring an unforeseen enhancement, the mass effect in the dispersion relation should have the same result of also making the $\kappa\bar{K}$ contribution negligible.

To sum up, our calculated contributions to Δ, Eqs. (25), (27), and (33), amount to 0.02. Our value for $\Delta(\rho\pi)$ agrees with that given by Braathen (Ref. 7, Table 5) and is greater than that of Pagels (Ref. 16, footnote 9) because of the inclusion of the Pauli term and the use of a larger value of g_ρ^2. Our value for $\Delta(\sigma\pi)$ is positive and greater than the $\Delta(\epsilon\pi)$ of Braathen[24] because of a different evaluation of the $\sigma\pi$ contribution, as discussed in the next section.

In addition to the calculated contributions we have looked at possible resonance contributions and estimated that they may amount to some 0.01. Finally, there is the high-energy contribution, from states with invariant masses greater than $2m$, which, given the assumptions of Pagels,[6,17] is bounded by 0.01. It is interesting, however, that a reasonable cutoff of 2 GeV leads to a $\rho\pi$ high-energy contribution[7] to Δ of 0.028.

In spite of the increase over previous estimates, with a possible total contribution of, say, 0.035, we are still some way from the present experimental value, Eq. (2). However, the discrepancy is small enough that one may hope for an ultimate reconciliation. It may be that the high-energy bound of Ref. 6 and the assumptions made therein hold the key to the puzzle.

III. THE σ MODEL AND THE πN σ TERM

The parameters of the σ meson used in Sec. II are intimately related to the value of the πN σ term,

$$\sigma(\pi N) \equiv \langle N | [Q_5^\pi, i\partial \cdot A^\pi] | N \rangle . \qquad (35)$$

This can be obtained by an extrapolation of the background (nonpole) forward isotopic-even πN amplitude

$$\bar{F}^{(+)}(\nu, t) = \bar{A}^{(+)} + \nu \bar{B}^{(+)} - g^2/m \qquad (36)$$

to the on-shell subthreshold point[25] $\nu = 0$, $t = 2\mu^2$:

$$\bar{F}^{(+)}(0, 2\mu^2) = \frac{\sigma(\pi N)}{f_\pi^{\,2}} + O(\mu^4), \qquad (37)$$

or to the double off-shell point $\nu = 0$, $t = 0$ with $q^2 = q'^2 = 0$:

$$\bar{F}^{(+)}(0, 0; q^2 = q'^2 = 0) = -\frac{\sigma(\pi N)}{f_\pi^{\,2}} . \qquad (38)$$

Phenomenological analyses using both these methods seem to be converging[2,5,26] on a value of $\sigma(\pi N) \approx 70$ MeV.

The final term in Eq. (36), which arises from the difference between the nucleon pole contributions as evaluated in dispersion theory in contrast to field theory, is crucial to what follows, and has been noted by several authors. In the presence of this term the t-channel "σ dominance" model for $\bar{F}^{(+)}$ reads[27]

$$\bar{F}^{(+)}(\nu, t) \approx \frac{2 g_{\sigma NN} g_{\sigma\pi\pi}}{m_\sigma^{\,2} - t} - \frac{g^2}{m} . \qquad (39)$$

The σ model couplings (31) and (32) then ensure that $\bar{F}^{(+)}(\nu, t)$ vanishes at the on-shell point $\nu = 0$, $t = \mu^2$:

$$F^{(+)}(0, \mu^2) \approx 0 , \qquad (40)$$

which is a good approximation[26] to the required vanishing[28] of the amplitude at the off-shell Adler consistency point $\nu = 0$, $t = \mu^2$, $q^2 = 0$, $q'^2 = \mu^2$. Thus we may have some confidence in the couplings (31) and (32), or at least in their product, which is all that enters into the calculation of $\Delta(\sigma\pi)$.

Parenthetically, we may remark that a similar situation obtains with the isotopic-odd forward scattering amplitude and the ρ couplings. The t-channel "ρ dominance" model[29] for $\bar{F}^{(-)} = \bar{A}^{(-)} + \nu \bar{B}^{(-)}$ implies

$$\nu^{-1} \bar{F}^{(-)}(\nu, t) \approx \frac{g_{\rho\pi\pi} g_{\rho NN}}{m_\rho^{\,2} - t} - \frac{g^2}{2m^2} . \qquad (41)$$

The value of g_ρ we have used in Sec. II, which is in fact that given by the KSRF relation[30]

$$\frac{g_\rho^{\,2}}{m_\rho^{\,2}} = \frac{1}{2 f_\pi^{\,2}} , \qquad (42)$$

is such as to cast (41) into the form of the Adler-Weisberger relation[31] with the GT value for g_A. Once the $O(\mu^2)$ background is taken into account, it can be shown that (41) is in good agreement with the low-energy data for various values of ν and t near threshold.[26]

Returning to the question of the σ couplings, we note that the difference between our evaluation of the $\sigma\pi$ contribution and that of Pagels[6] derives from the extra term in Eq. (39), in the absence of which the values of the πN scattering lengths constrained $g_{\sigma NN}g_{\sigma\pi\pi}$ to be small. Since $g_{\sigma\pi\pi}$ was regarded as fixed by the width $\Gamma_{\sigma\to\pi\pi}$, the net result was to suppress $g_{\sigma NN}$ and hence the triangle graph contributions, leaving only a small (negative) self-energy contribution.[6]

Our attitude is to keep the σ-model relations (31) and (32) for the couplings and then allow the value of the σ term to determine the effective mass to be used in the calculation. The connection is provided by confronting (37) with (39), which gives[32]

$$\sigma(\pi N)\approx g_A{}^2(\mu/m_\sigma)^2 m\,, \tag{43}$$

where, to this order of accuracy, we have used the GT relation itself and have neglected the pion mass relative to the σ mass.

There is some ambiguity at this stage. Since we are using the lowest-order unrenormalized[33] couplings for σ, we should perhaps for consistency take $g_A=1$ in Eq. (43). An input σ term of the order of 70 MeV would then lead to $m_\sigma{}^2\approx 13\mu^2$, or $m_\sigma\approx 500$ MeV. Given this value of m_σ, the σ-model relations then give

$$g_{\sigma\pi\pi}\approx 12\mu\,, \tag{44}$$

while $g_{\sigma NN}$ as always is constrained to have the value g.

In principle (44) can be compared directly with experiment in the form of the σ width.[34,35] Unfortunately, the experimental situation is rather confused, we have the above-mentioned renormalization ambiguity, and the narrow-width approximation is not obviously a very appropriate one.

We wish, however, to make the following observation. According to the narrow-width approximation (NWA) the σ width is given by[34]

$$\Gamma_{\sigma\to\pi\pi}=\frac{3}{2}\,\frac{g_{\sigma\pi\pi}{}^2}{4\pi m_\sigma{}^2}(m_\sigma{}^2-4\mu^2)^{1/2}\,, \tag{45}$$

which, for $m_\sigma{}^2\approx 13\mu^2$, has the value

$$\Gamma_{\sigma\to\pi\pi}\approx 4\mu\approx 550 \text{ MeV} \tag{46}$$

within the range of experiment.[35] Although the use of the NWA is not *a priori* very reasonable for a width of this magnitude, the general feature of this approximation is that a coupling larger than the physical one is needed to obtain agreement with a given physical width. (For example, the $\pi N\Delta$ coupling derived from the Δ width in NWA is 40% greater than the physical coupling.[36]) Thus the neglect in (43) of the factor $g_A{}^2$, the GT approximation, and the neglect of μ^2 compared to $m_\sigma{}^2$, which would have led to a (factor of 2) larger physical coupling constant, to some extent takes account of this effect. We therefore take the attitude of using the coupling constant (44) and at the same time ignoring finite-width effects.

With $\sigma(\pi N)\approx 70$ MeV leading, as we have seen, to $\Gamma_\sigma\approx m_\sigma\approx 500$ MeV, the σ model may be taken as a reasonable approximation to reality. By way of contrast we note that if $\sigma(\pi N)$ were very small, say $\sigma(\pi N)\approx 15$ MeV, then the σ model would lead to $m_\sigma\approx 1100$ MeV, $g_{\sigma\pi\pi}\approx 30\mu$, and $\Gamma_\sigma\approx 2000$ MeV, and would not then be physically relevant.

The difference of our σ contribution from Braathen's ϵ contribution lies in the fact that we have taken $g_{\sigma NN}$ to be fixed by the σ model and determined m_σ from the pion-nucleon σ term, leading to a not unreasonable value for Γ_σ. Braathen, on the other hand, took m_ϵ and Γ_ϵ as input, leading to a much-reduced value for $g_{\epsilon NN}$. Comparison of the resulting Δ's shows the sensitivity of the calculation to such factors.

IV. SU(3) × SU(3) CHIRAL SYMMETRY BREAKING

We now turn to the use of the GT relation to probe the nature of the symmetry-breaking Hamiltonian H'. In particular, if this is assumed to have the $(3,\overline{3})+(\overline{3},3)$ form $H'=u_0+cu_8$, the GT relation provides an independent determination of the value of c.

Equation (17) on its own will not suffice, since we have no way of evaluating $(\overline{v}_\pi)_{NN}$. The method of procedure will therefore be to eliminate this quantity by means of the SU(3) generalizations of Eq. (17). Defining the $K^+p\Lambda$ and $K^+p\Sigma^0$ discrepancies as

$$\Delta_{KN\Lambda}\equiv 1-\frac{\frac{1}{2}(m_\Lambda+m_N)g_A^{KN\Lambda}}{f_K g_{KN\Lambda}} \tag{47}$$

and

$$\Delta_{KN\Sigma}=1-\frac{\frac{1}{2}(m_\Sigma+m_N)g_A^{KN\Sigma}}{f_K g_{KN\Sigma}}\,, \tag{48}$$

we have,[37] as analogs of (17),

$$\Delta_{KN\Lambda}=\frac{1}{f_K g_{KN\Lambda}}\,\frac{\sqrt{2}-\frac{1}{2}c}{2\sqrt{3}}(\overline{v}_K)_{N\Lambda}\,, \tag{49}$$

$$\Delta_{KN\Sigma}=\frac{1}{f_K g_{KN\Sigma}}\,\frac{\sqrt{2}-\frac{1}{2}c}{2\sqrt{3}}(\overline{v}_K)_{N\Sigma}\,. \tag{50}$$

The matrix elements of $\overline{v}$ appearing in (49) and (50) are related in SU(3) according to

$$(\bar{v}_P)_{fi} = 2(\bar{v}_\pi)_{NN}(d_v d_{Pfi} - f_v i f_{Pfi}) , \tag{51}$$

in which we have an unknown f_v ($d_v = 1 - f_v$). This, however, may be eliminated between the two equations, leading to the relation[8,14,38]

$$\frac{\sqrt{2}+c}{\sqrt{2}-\frac{1}{2}c} = \frac{2f_\pi}{f_K}\,\frac{g_{\pi NN}\Delta_{\pi NN}}{-\sqrt{3}\,g_{KN\Lambda}\Delta_{KN\Lambda}+g_{KN\Sigma}\Delta_{KN\Sigma}} . \tag{52}$$

If one further assumes exact kaon PCAC in the operator sense, the left-hand side of (52) becomes $f_\pi m_\pi^2/f_K m_K^2$, in agreement with GMOR. In this case (52) is then equivalent to the sum rules derived by Dashen and Weinstein.[14,8] We prefer, however, to follow the philosophy of Furlan and Paver[14] and regard (52) as the baryon analog of the GMOR meson mass formula from which we can independently derive the value of c.

The procedure we will adopt in analyzing (52) is first of all to assume SU(3) invariance for the baryon couplings to mesons and axial-vector currents,[39] viz.,

$$g_{Pfi} = 2g(d_P d_{Pfi} - f_P i f_{Pfi}) \tag{53}$$

and

$$g_A^{Pfi} = 2g_A(d_A d_{Pfi} - f_A i f_{Pfi}), \tag{54}$$

with $d_P + f_P = d_A + f_A = 1$, and then afterwards to examine the effects of SU(3) breaking.

In the case of exact SU(3), Eq. (52) can be recast in the form

$$-\frac{\sqrt{2}+c}{c} = \tfrac{3}{2}\Delta_{\pi NN}\left[\left(\frac{f_K}{f_\pi}-1\right) - \frac{m g_A}{f_\pi g}\left(\frac{m_\Sigma + m_\Lambda - 2m_N}{4m_N} - f_A\,\frac{m_\Sigma - m_\Lambda}{2m_N}\right)\right]^{-1} . \tag{55}$$

Remarkably, we see that f_P does not appear here, i.e., that the relation is independent[40] of the d/f ratio for the strong-coupling constants once these are assumed to obey SU(3). Moreover, the dependence on the parameter f_A is greatly suppressed by the factor $(m_\Sigma - m_\Lambda)$. The remaining terms in the right-hand denominator are all SU(3)-breaking effects, and are of such a magnitude measured against $\Delta_{\pi NN}$ that the value of c derived from (55) is considerably different from the canonical GMOR value of -1.25.

A single-angle fit to hyperon semileptonic decays gives[39,41]

$$d_A/f_A = 1.7 ,$$

corresponding to $f_A = 0.37$, while, with the aid of the nonrenormalization theorem[42] for K_{l3} decay, f_K/f_π can be determined from the value[43] of $f_K/[f_\pi f_+(0)]$ to be $f_K/f_\pi = 1.24 \pm 0.02$. For $\Delta_{\pi NN}$ we insert the experimental value given in Eq. (2) and obtain

$$c_{\rm GT} = -0.9 \pm 0.1 , \tag{56}$$

in remarkable agreement with the value obtained by an examination of σ terms.[2] The degree of deviation from the GMOR prediction is more fairly measured by the quantity $(\sqrt{2}+c)/(\sqrt{2}-\frac{1}{2}c)$, which, from (56), is

$$\left(\frac{\sqrt{2}+c}{\sqrt{2}-\frac{1}{2}c}\right)_{\rm GT} = 0.28 \pm 0.05 \tag{57}$$

compared with the expected value[1] $m_\pi^2/m_K^2 \approx 0.08$, or[8] $f_\pi m_\pi^2/f_K m_K^2 \approx 0.06$.

We now go back to examine our assumption of SU(3) invariance for the various couplings g_{Pfi}, g_A^{Pfi}.

In the general case (52) can be written as

$$\frac{\sqrt{2}+c}{\sqrt{2}-\frac{1}{2}c} = \frac{\Delta_{\pi NN}}{\dfrac{f_K}{f_\pi}A - \dfrac{m g_A}{f_\pi g}\left[\left(\dfrac{m_\Sigma + m_\Lambda + 2m_N}{4m_N}\right)B - \left(\dfrac{m_\Sigma - m_\Lambda}{2m_N}\right)C\right]} , \tag{58}$$

where

$$A = (-\sqrt{3}\,g_{KN\Lambda} + g_{KN\Sigma})/2g , \tag{59}$$

$$B = (-\sqrt{3}\,g_A^{KN\Lambda} + g_A^{KN\Sigma})/2g_A , \tag{60}$$

and

$$C = -(\sqrt{3}\,g_A^{KN\Lambda} + g_A^{KN\Sigma})/4g_A , \tag{61}$$

which, in the SU(3) limit we have just been discussing, take on the values $A = B = 1$ and $C = f_A$.

The experimental situation for the strong-coupling constants appearing in (59) is as follows.[10] All estimates of $g_{KN\Sigma}^2/4\pi$ have been that this quantity is small, of the order of 2. Modern methods of extracting the coupling constants from KN scattering evaluate the combination $g_Y^2 \equiv (g_{KN\Lambda}^2 + 0.84 g_{KN\Sigma}^2)/4\pi$. In the 1973 compilation[10] the most reliable value was considered to be[44]

$g_Y^2 = 12.2 \pm 2.5$; however, the value $g_Y^2 = 15.2 \pm 2.3$ obtained by the Cutkosky conformal mapping technique[45] was not mentioned. Furthermore, a recent experiment[46] indicates that $g_Y^2 = 19.8 \pm 4.4$ within the zero-range approximation, thus negating the original zero-range estimates[47] of $g_Y^2 \approx 4$. Hence, at the present time, we believe that fair estimates of the coupling constants are

$$g_{KN\Lambda}^2/4\pi = 14 \pm 2\,,\qquad g_{KN\Sigma}^2/4\pi = 2 \pm 1\,, \tag{62}$$

in reasonable agreement with the original evaluation of Kim.[48] Moreover, these values would predict $g_{\pi\Lambda\Sigma}^2/4\pi \approx 9$ by SU(3), compared to the values 10.9 ± 0.3, 12.9 ± 0.8 obtained phenomenologically[49,50] from $\Lambda\pi$ partial-wave dispersion relations. Thus we will certainly believe SU(3) to the extent that it determines the signs of $g_{KN\Lambda}$ and $g_{KN\Sigma}$ and hence deduce from (62) that

$$g_{KN\Lambda} = -13 \pm 1\,,\qquad g_{KN\Sigma} = 5 \pm 1\,. \tag{63}$$

Substituting these values into (59) we find

$$A = 1.0 \pm 0.1\,. \tag{64}$$

The situation with regard to the SU(3)-Cabibbo theory of hyperon semileptonic decays has also improved recently.[41] The experimental values of the axial-vector-to-vector ratio g_1/f_1 in $\Lambda \to pe\bar{\nu}$ appear to be converging, with a mean value of $g_1/f_1 = 0.66 \pm 0.06$. Since the Cabibbo estimate of g_1/f_1 is $(f_A + \frac{1}{3}d_A)g_A \approx 0.70$, using $d_A/f_A \approx 1.9$, which is obtained from the remaining hyperon decays,[41] we can evaluate (60) as

$$B = 1.0 \pm 0.1\,. \tag{65}$$

As previously noted in the symmetry limit, the contribution of C in (58) is greatly suppressed by the factor $(m_\Sigma - m_\Lambda)$, so that we may take $C = f_A$ without serious error.

With these values (58) gives

$$\frac{\sqrt{2} + c}{\sqrt{2} - \frac{1}{2}c} = \frac{0.058 \pm 0.013}{0.21 \pm 0.16}\,. \tag{66}$$

While the error in the denominator of (66) is large, it is clear that the GMOR value of 0.06 to 0.08 for $(\sqrt{2} + c)/(\sqrt{2} - \frac{1}{2}c)$ is still remote. We therefore arrive at the conclusion, bearing out the misgivings expressed in Ref. 8, that, given a symmetry-breaking Hamiltonian of the form $u_0 + cu_8$, the value of c derived from the Goldberger-Treiman relation may not be -1.25, but instead is close to -1, in agreement with the value obtained from phenomenological analyses of σ terms.

V. DISCUSSION AND CONCLUSIONS

We have discussed two aspects of the Goldberger-Treiman relation: the attempt to account for the experimental πN discrepancy $\Delta_{\pi NN}$ by means of dispersion relations, and the use of this quantity and its SU(3) counterparts to test the nature of the chiral-symmetry-breaking Hamiltonian.

The narrowing of the gap between the experimental and theoretical values of $\Delta_{\pi NN}$ arises from two factors: first, a not insignificant contribution from the $\sigma\pi$ intermediate states, ultimately connected to the large value of the $\pi N\,\sigma$ term, and second, a shift in the experimental parameters all in the direction of reducing the experimental value of $\Delta_{\pi NN}$. Thus g_A has increased in time to the currently accepted value[9] 1.25 ± 0.01. The πN coupling constant g has tended to decrease with time, and indeed the most recent and most accurate determination[51] $g = 13.40 \pm 0.08$, which we used in arriving at Eq. (2), is appreciably less than the more generally quoted[10] value $g \approx 13.6$. The nucleon mass[52] has not, of course, varied to any appreciable extent, but the value to be used in Eq. (2) is actually the average of the neutron and proton masses, namely 938.9 MeV. From the π^+ lifetime[9] $\tau = (2.6030 \pm 0.0023) \times 10^{-8}$ sec and using the value[12] $G = (1.026 \pm 0.001) \times 10^{-5} m_p^{-2}$, one can derive

$$f_\pi \cos\theta_A = 90.799 \pm 0.042 \text{ MeV}\,. \tag{67}$$

Given $f_K/f_\pi = 1.24 \pm 0.02$, the ratio of $K_{\mu 2}$ to $\pi_{\mu 2}$ rates implies

$$\cos\theta_A = 0.976 \pm 0.001\,,$$

which leads to the value quoted in Sec. I along with its error:

$$f_\pi = 93.0 \pm 1 \text{ MeV} \tag{68}$$

including the possible 1% radiative corrections to $\pi^+ \to \mu^+\nu$. These are the values we have used in Eq. (2) to obtain $\Delta_{\pi NN}^{\exp}$ together with its error, which, as we have seen, is large enough to make an ultimate agreement between theory and experiment by no means improbable. The change from a few years ago[6] arises from an increased theoretical estimate and a trend in the experimental numbers all in the direction of decreasing $\Delta_{\pi NN}^{\exp}$.

As far as the second aspect is concerned, we have shown in the present paper and in Ref. 2 that two of the measures of SU(2)×SU(2) breaking noted by Li and Pagels,[53] namely,

$$\Delta_{\pi NN} \equiv 1 - \frac{mg_A}{f_\pi g} \approx 0.058 \tag{69}$$

and

$$\frac{\sigma(\pi N)}{m} \approx \frac{70 \text{ MeV}}{939 \text{ MeV}} \approx 0.075\,, \tag{70}$$

can be explained by the same chiral-symmetry-breaking mechanism $H' = u_0 + cu_8$ with $c \approx -1$.

The third measure,

$$\frac{m_\pi^2}{m_K^2} \approx 0.077\,, \tag{71}$$

seems difficult to reconcile with the first two. In the standard GMOR version of SU(3)×SU(3), c is related to the masses by soft pion, kaon, and η PCAC according to

$$c_{\text{GMOR}} \approx -\sqrt{2}\left(\frac{m_K^2 - m_\pi^2}{m_K^2 + \frac{1}{2} m_\pi^2}\right) \approx -1.25\,. \tag{72}$$

The validity of the extrapolations involved in kaon and η PCAC can certainly be questioned, but the same value can be derived either by SU(3) symmetry[54] for the quantities $\langle 0 | v_i | P_i \rangle$ or[55] by SU(3) symmetry of the m_P^2 and only pion PCAC. There thus appears to be a rather fundamental clash between the first two derivations of c and the mass formula, which we have attempted[2] to resolve in terms of some degree of SU(3) breaking of $\langle 0 | v_i | P_i \rangle$ and of the vacuum.[56]

If this resolution is not found convincing [and certainly, as we have shown, the value $c = -1.25$ is in disagreement with the SU(3) GT relation (52), whose derivation is on very firm ground according to the standard approach], the conclusion to be drawn is that the irreducible $(3, \bar{3}) + (\bar{3}, 3)$ representation is not compatible with the data. The most plausible alternative, a reducible representation with a small admixture[57] of (8, 8), has yet to be confronted with all the data in a systematic way.

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