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Scattering amplitudes for inverse bremsstrahlung and pair production in a laser pulse

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The scattering amplitudes for the processes of inverse bremsstrahlung and pair production in a laser pulse are given for scattering and nonscattering boundary conditions.

In a previous paper,[1] one of the authors (M.D.S.) derived quantum field theoretically formal scattering amplitudes for electron-laser pulse (ELP) interactions where the initial state consisted of a single electron and a laser pulse, and the final state consisted of a single electron, an arbitrary number of photons, and the laser pulse which may be different from the incident pulse. We mentioned that extension to cases where there are several electrons and/or positrons in the "in" or the "out" state is straightforward. One simply applies the reduction technique as many times as needed.[2] The amplitudes in paper I were derived for either scattering boundary conditions (SBC) or nonscattering boundary conditions (NSBC). We have SBC when the initial- and final-state electron and laser pulse are noninteracting. For the experimental configuration where the electron and laser pulse can be assumed to interact even in the remote past or future, we have NSBC.

In this paper we present formal results for scattering amplitudes for the ELP processes of inverse bremsstrahlung and pair production using SBC and NSBC. We simplify calculations by assuming that the laser pulse is essentially unchanged by the interaction and that vacuum polarization effects are unimportant. However, the case where the final-state laser pulse is very different from the incident pulse can also be handled by the methods presented in paper I.

INVERSE BREMSSTRAHLUNG

The scattering amplitude for inverse bremsstrahlung with NSBC is given by the equation

$$S_{fi} = {}^{\text{out}}\langle \alpha^{\text{out}}, p' | p, \alpha^{\text{in}}\rangle^{\text{in}} . \tag{1}$$

Here $\alpha^{\text{in}}(x) \approx \alpha^{\text{out}}(x) = \alpha(x)$ is the laser pulse represented as a plane-wave, coherent-state[3] wave packet; p and p' are, respectively, the initial and final electron momenta. The expression for S_{fi} becomes

$$S_{fi} = (i)^2 \int dx\, dy\, \overline{U}^V_{p'}(x) \vec{D}_{Vx} W \overleftarrow{D}_{Vy} U^V_p(y) , \tag{2}$$

where $\overline{U}^V_{p'}(x)$ and $U^V_p(y)$ are Volkov[4] wave packets which satisfy the equations

$$\begin{aligned} &\vec{D}_{Vx} U^V_{p'}(x) = 0 = \overline{U}^V_p(y) \overleftarrow{D}_{Vy} , \\ &\vec{D}_{Vx} \psi \equiv (-i\gamma^\mu \partial_\mu + m - e\gamma^\mu \alpha_\mu(x)) \psi , \\ &\overline{\psi} \overleftarrow{D}_{Vy} \equiv i\partial_\mu \overline{\psi}\gamma^\mu + m\overline{\psi} - e\overline{\psi}\gamma^\mu \alpha_\mu . \end{aligned} \tag{3}$$

W is given by Eq. (I.3.4), i.e.,

$$W = \frac{\langle 0 | T(\psi_F(x)\overline{\psi}_F(y) \exp[-i\int J_F \cdot (A_F + \alpha) dx']) | 0\rangle}{\langle 0 | T(\exp[-i\int J_F \cdot A_F dx']) | 0\rangle} . \tag{4}$$

In lowest order, we obtain for W the expression

$$W(0) = -\tfrac{1}{2} S_F^{(e)}(x, y) , \tag{5}$$

the propagator for an electron moving in the laser pulse, and the Coulomb potential given by the equation

$$\beta_\mu(x) = \frac{Ze}{4\pi|\vec{x}|} \delta_{\mu 0} . \tag{6}$$

Using Eqs. (I.4.2) and (I.4.3) with $\alpha(x)$ replaced by $\alpha(x) + \beta(x)$, we obtain

$$\begin{aligned} S_{fi}(0) = &-i \int dx\, dy\, \overline{U}^V_{p'}(x) \vec{D}_{Vx} \delta(x-y) U^V_p(y) \\ &+ ie \int dx\, \overline{U}^V_{p'}(x) \gamma \cdot \beta(x) U^V_p(x) \\ &+ \tfrac{1}{2} e^2 \int dx\, dy\, \overline{U}^V_{p'}(x) \gamma \cdot \beta(x) S_F^{(e)}(x, y) \\ &\times \gamma \cdot \beta(y) U^V_p(y) . \end{aligned} \tag{7}$$

The first term in Eq. (7) is the forward scattering amplitude and will be dropped. If we want the scattering amplitude only to first order in the Coulomb potential, then we drop the third term in Eq. (7) also. Thus, the scattering amplitude for inverse bremsstrahlung with NSBC to first order in the Coulomb potential is

$$S_{fi}(0) = ie \int dx\, \overline{U}^V_{p'}(x) \gamma \cdot \beta(x) U^V_p(x) . \tag{8}$$

This amplitude agrees with results published previously using time-dependent perturbation[5] theory.

For the inverse bremsstrahlung scattering amplitude with SBC, we proceed exactly as before except that now $\overline{U}_p'(x)$ and $U_p(y)$ are free-particle wave packets and $\vec{D}_x$ and $\vec{D}_y$ are the ordinary Dirac operators; $W(0)$ and $S_F^{(e)}(x, y)$ are still given formally by Eq. (5). We find for the scattering amplitude the expression

$$S_{fi}(0) = i \int dx\, dy\, \overline{U}_{p'}(x)\delta(x-y)\vec{D}_y U_p(y) - e\int dx\, \overline{U}_{p'}(x)\gamma\cdot[\alpha^{\text{in}}(x)+\beta(x)]\,U_p(x) + \tfrac{1}{2}e^2\int dx\, dy\, \overline{U}_{p'}(x)\gamma\cdot[\alpha^{\text{in}}(x)+\beta(x)]\,S_F^{(e)}(x,y)\gamma\cdot[\alpha^{\text{in}}(y)+\beta(y)]\,U_p(y)\,. \tag{9}$$

We can expand the propagator $S_F^{(e)}(x, y)$ in terms of the propagator $S_F^{\alpha}(x, y)$ which describes the motion of the electron in the laser pulse alone:

$$S_F^{(e)}(x,y) = S_F^{\alpha}(x,y) + \int S_F^{\alpha}(x,z_1)(-\tfrac{1}{2}ie)\gamma\cdot\beta(z_1)S_F^{\alpha}(z_1,y)dz_1 + \int S_F^{\alpha}(x,z_1)(-\tfrac{1}{2}ie)\gamma\cdot\beta(z_1)S_F^{\alpha}(z_1,z_2)(-\tfrac{1}{2}ie)\gamma\cdot\beta(z_2)S_F^{\alpha}(z_2,y)dz_1dz_2 + \cdots\,. \tag{10}$$

Dropping the forward scattering term and to first order in the Coulomb potential, we have

$$S_{fi}(0) = -e\int dx\, \overline{U}_{p'}(x)\gamma\cdot[\alpha^{\text{in}}(x)+\beta(x)]\,U_p(x) + \tfrac{1}{2}e^2\int dx\, dy\, \overline{U}_{p'}(x)\Big[\gamma\cdot\alpha^{\text{in}}(x)S_F^{\alpha}(x,y)\gamma\cdot\alpha^{\text{in}}(y) + \gamma\cdot\alpha^{\text{in}}(x)S_F^{\alpha}(x,y)\gamma\cdot\beta(y) + \gamma\cdot\beta(x)S_F^{\alpha}(x,y)\gamma\cdot\alpha^{\text{in}}(y) + (-\tfrac{1}{2}ie)\gamma\cdot\alpha^{\text{in}}(x)\int dz_1 S_F^{\alpha}(x,z_1)\gamma\cdot\beta(z_1)S_F^{\alpha}(z_1,y)\gamma\cdot\alpha^{\text{in}}(y)\Big]U_p(y)\,. \tag{11}$$

That Eq. (11) is plausible is easily verified by setting $\alpha(x)=0$. Then the amplitude reduces to

$$S_{fi}(0) = -e\int dx\, \overline{U}_{p'}(x)\gamma\cdot\beta(x)U_p(x) = -\overline{u}(p', s')\left[\frac{Ze^2}{(2\pi)^2}\,\frac{\delta(p_0'-p_0)\gamma^0}{|\vec{p}\,'-\vec{p}\,|^2}\right]u(p,s)\,. \tag{12}$$

This is precisely what one expects to obtain as the scattering amplitude of an electron interacting once with the Coulomb potential.

PAIR PRODUCTION

The pair-production amplitude with NSBC is given by

$$S_{fi} = {}^{\text{out}}\langle \alpha^{\text{out}}\,\overline{q}\,', p' \,|\, \alpha^{\text{in}}\rangle^{\text{out}}\,. \tag{13}$$

Here, $\overline{q}\,'$ is the four-momentum of a positron. With appropriate modifications to Eqs. (I.3.3) and (I.3.4) and to first order in the Coulomb potential, we obtain

$$S_{fi}(0) = ie\int dx\, \overline{U}^{V}_{p'}(x)\gamma\cdot\beta(x)V^{V}_{\overline{q}'}(x)\,, \tag{14}$$

where $V^{V}_{\overline{q}'}(x)$ is a negative-frequency Volkov solution to the Dirac equation in the presence of the laser pulse $\alpha(x)$. This amplitude agrees with results published previously using time-dependent perturbation theory.[6]

If we assume SBC, we find

$$S_{fi}(0)=e\bar{u}(p',s')\int dx\, f^*_{p'}(x)\gamma\cdot[\alpha^{\rm in}(x)+\beta(x)]f^*_{\bar{q}'}(x)v(\bar{q}',s'')$$
$$+\tfrac{1}{2}e^2\bar{u}(p',s')\int dx\,dy\, f^*_{p'}(x)\Big[\gamma\cdot\alpha^{\rm in}(x)S_F^\alpha(x,y)\gamma\cdot\alpha^{\rm in}(x)+\gamma\cdot\alpha^{\rm in}(x)S_F^\alpha(x,y)\gamma\cdot\beta(y)+\gamma\cdot\beta(y)S_F^\alpha(x,y)\gamma\cdot\alpha^{\rm in}(y)$$
$$+(-\tfrac{1}{2}ie)\gamma\cdot\alpha^{\rm in}(x)\int dz_1 S_F^\alpha(x,z_1)\gamma\cdot\beta(z_1)S_F^\alpha(z_1,y)\gamma\cdot\alpha^{\rm in}(y)\Big]f^*_{\bar{q}'}(y)v(\bar{q}',s''). \quad (15)$$

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