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Asymptotic behavior of form factors in a class of field theories*

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It is shown that for all renormalizable field theories, excluding gluon and gauge theories, the asymptotic behaviors of the on-mass-shell form factors are related to the short-distance behaviors of the theories. In particular, if the Gell-Mann–Low eigenvalue conditions are satisfied, the asymptotic behavior of the on-mass-shell electromagnetic form factor is $(m^2/-q^2)^\gamma$, where γ is the anomalous dimension of the proton field. We also show that the form factor of the axial-vector current has the same asymptotic behavior as that of the vector current.

I. INTRODUCTION

The asymptotic behavior of the electromagnetic form factor is one of the outstanding problems in high-energy physics. The experiments[1] in the last decade showed a rapidly decreasing elastic form factor[2] of the proton of the form $(-q^2)^{-\gamma}$, with γ equal to or slightly larger than 2. The data can be fitted adequately by the dipole formula[3]

$$\left[\frac{1}{1+(-q^2)/0.71\ \mathrm{GeV}^2}\right]^2$$

over a very wide range of spacelike q^2. The success of the simple power-law fit leads naturally to the question of whether there are any deeper physical implications of this behavior. Many attempts have been made to understand this. They are summarized very nicely in a paper by Appelquist and Primack.[4] Within the general framework of renormalizable field theory, the problem of the form factors has been studied using approximate integral equations,[5] a renormalization group,[6] leading-logarithmic approximation,[4] cluster expansion,[7] and the Callan-Symanzik equation.[8] The conclusions obtained so far using these approaches are unsatisfactory in one way or another.

The purpose of this paper is to study the problem of the asymptotic behaviors of the on-mass-shell form factors, again using the Callan-Symanzik equation.[9] We show that in all renormalizable field theories, excluding gluon and gauge theories, the asymptotic behaviors of the on-mass-shell form factors are controlled by the short-distance behaviors of the theories. In particular, if the Gell-Mann–Low eigenvalue conditions[6] have solutions in these theories, the asymptotic behaviors can be described by powers of $(-q^2)$. Here the exponents are related to the anomalous dimensions[10] of the fields. Our conclusion is in agreement with the long-cherished hope that asymptotic behaviors of form factors would reveal the deeper structure of hadrons.

We recall that in the usual applications of the Callan-Symanzik equation, the mass-insertion terms are often dropped by letting all external momenta go to the asymptotic Euclidean region and by appealing to Weinberg's theorem.[11] There are, however, many physically interesting cases in which only some subset of momenta goes to infinity and the others remain fixed. These cases include all on-mass-shell amplitudes as well as partially on-mass-shell amplitudes as in lepton-nucleon experiments. Therefore, it would be desirable if we could say something about mass-insertion terms so that the Callan-Synamzik equation would become solvable and useful. Indeed the Callan-Symanzik equation with mass-insertion term has been employed successfully in the discussion of the low-energy theorem for $\pi^0\to 2\gamma$ (Ref. 12) and lepton-nucleon experiments,[13] as well as the inclusive annihilation process in ϕ^4 field theory.[14] There is yet another case: elastic form factors, where the Callan-Symanzik equation can yield useful results.

This paper is organized as follows: In Sec. II we consider the elastic electromagnetic form factor of the proton in the pseudoscalar theory of a proton and a neutral pion. As illustrations, we show how to handle the mass-insertion terms in the Callan-Symanzik equation for some typical low-order diagrams. We also indicate why the asymptotic behaviors of the form factors in gauge theories and gluon theory are not related to the short-distance behaviors of the theories. We present the general treatment of mass-insertion terms in Sec. III using the Bethe-Salpeter equation. We show that the mass-insertion terms can be neglected in the asymptotic limit of $-q^2 \gg p^2=p'^2=m^2$. The Callan-Symanzik equation for the elastic form factor is then solved in this limit. It is found that the asymptotic behavior is related to the short-distance behavior of the theory. In particular, if the Gell-Mann–Low eigenvalue conditions are satisfied, the asymptotic behavior of the form factor is $(-q^2)^{-\gamma}$, where γ is the anomalous dimension of proton field. In Sec. IV, the form factors of the axial-vector current as well as the vector current are studied in the σ model.[15] We show that in this model the axial-vector and vector currents will have the same asymptotic behaviors, in agreement with the available experimental data. In the last section we summarize the results we obtained and compare with the previous work of Landau *et al.*,[5] Bogoliubov *et al.*,[6] Appelquist and Primack,[4] and others.

II. LOW-ORDER RESULTS

For the sake of simplicity, we consider the asymptotic behavior of the electromagnetic form factor of the proton in a simple model. The model consists of a proton field and a neutral-pion field only and is described by the renormalizable pseudoscalar γ_5 interaction:

$$\mathcal{L}_I = -ig_0\bar\psi\gamma_5\psi\pi - \lambda_0(\pi)^4\,. \tag{2.1}$$

The electromagnetic interaction is treated to the lowest order. For illustration, let us begin with the lowest nontrivial calculation of the electromagnetic form factor of the proton. It is given by

$$\begin{aligned}\Gamma^\mu &= \int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2-\mu^2}g_0\gamma_5\frac{i}{\not p'-\not k-m}\gamma^\mu\frac{i}{\not p-\not k-m}g_0\gamma_5\\ &= -ig_0^2\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-\mu^2}\gamma_5\frac{\not p'-\not k+m}{(p'-k)^2-m^2}\gamma^\mu\\ &\qquad\times\frac{\not p-\not k+m}{(p-k)^2-m^2}\gamma_5\,,\end{aligned} \tag{2.2}$$

which corresponds diagrammatically to Fig. 1. Using the fact that $p^2=p'^2=m^2$ and $(\not p-m)u(p) =u(p')(\not p-m)=0$, we can rewrite the integral as

$$ig_0^2\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-\mu^2}\frac{\not k\gamma^\mu\not k}{(-2p'k+k^2)(-2pk+k^2)}\,. \tag{2.3}$$

It is very easy to see that when $\mu=0$ and $m=0$, the integral in (2.3) has no infrared divergence at all because of the presence of two $\not k$'s in the numerator. The result of the calculation[16] is

$$\begin{aligned}\Gamma^\mu &= \gamma^\mu\left(-\frac{g_0^2}{32\pi^2}\right)\ln\left(\frac{-q^2}{\Lambda^2}\right)\\ &\quad+O\left(\frac{\mu^2}{-q^2},\frac{m^2}{-q^2},\frac{\mu^2}{\Lambda^2},\frac{m^2}{\Lambda^2}\right).\end{aligned} \tag{2.4}$$

In this example the dominant contribution is from the ultraviolet region.

We recall that the Callan-Symanzik equation is a useful tool for investigating the asymptotic behaviors in renormalizable field theories. We are interested here in the proper vertex function of the electromagnetic current, which we denote by $\tilde\Gamma^\mu$. The Callan-Symanzik equation for $\tilde\Gamma^\mu$ is

$$\left(m\frac{\partial}{\partial m}+\mu\frac{\partial}{\partial\mu}+\beta_1\frac{\partial}{\partial g}+\beta_2\frac{\partial}{\partial\lambda}-2\gamma\right)\tilde\Gamma^\mu=\Delta\tilde\Gamma^\mu\,, \tag{2.5}$$

where m is the mass of the proton, μ is the mass of the pion, the tilde is used to denote renormalized quantities, $\Delta\tilde\Gamma^\mu$ is the mass-insertion term, and

$$\begin{aligned}\beta_1 &= \left(m\frac{d}{dm}+\mu\frac{d}{d\mu}\right)g\,,\\ \beta_2 &= \left(m\frac{d}{dm}+\mu\frac{d}{d\mu}\right)\lambda\,,\\ 2\gamma &= Z_1^{-1}\left(m\frac{d}{dm}+\mu\frac{d}{d\mu}\right)Z_1\\ &= Z_2^{-1}\left(m\frac{d}{dm}+\mu\frac{d}{d\mu}\right)Z_2\,.\end{aligned} \tag{2.6}$$

Here β_1, β_2, and γ are all cutoff-independent, and Z_2 is the wave-function renormalization constant for the proton field. It is understood that, in taking derivatives in Eq. (2.6), the bare coupling constants g_0, λ_0 as well as the cutoff Λ are held fixed. In deriving Eq. (2.5) we have made use of the fact that $Z_1=Z_2$ (Ward-Takahashi identity[17]).

Our interest is the behavior of the on-mass-shell vertex $\tilde\Gamma^\mu$ in the limit of $-q^2\gg p^2=p'^2=m^2$. As in other applications of the Callan-Symanzik equation, we have to know $\Delta\tilde\Gamma^\mu$ in order to obtain useful information on $\tilde\Gamma^\mu$. The analysis of the large-q^2 behavior of $\Delta\tilde\Gamma^\mu(q^2,p^2=p'^2=m^2)$ is complicated by the fact that, in the limit of $m=\mu=0$, we have the situation of exceptional momenta[18] and therefore Weinberg's theorem is no longer applicable. Instead of presenting the general method of handling the mass-insertion term $\Delta\tilde\Gamma^\mu$,

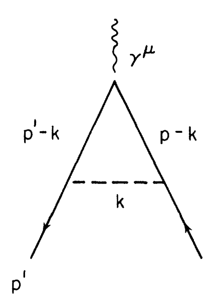

FIG. 1. Second-order diagram contributing to the proton electromagnetic form factor in neutral pseudoscalar γ_5 theory.

we restrict ourselves in this section to the discussion of $\Delta\tilde{\Gamma}^\mu$ for some low-order diagrams.

For the Feynman diagram in Fig. 1, it is easy to work out the expression for $\Delta\tilde{\Gamma}^\mu$. One finds that

$$\Delta\tilde{\Gamma}^\mu = O\left(\frac{\mu^2}{-q^2}, \frac{m^2}{-q^2}\right).$$

The precise expression for $\Delta\tilde{\Gamma}^\mu$ is not important to us. Therefore we find that in this example

$$\Delta\tilde{\Gamma}^\mu = O\left(\frac{\mu^2}{-q^2}, \frac{m^2}{-q^2}\right)\tilde{\Gamma}^\mu, \tag{2.7}$$

i.e., $\Delta\tilde{\Gamma}^\mu$ is of the order of $\mu^2/(-q^2)$, $m^2/(-q^2)$ (up to logarithms) as compared to $\tilde{\Gamma}^\mu$ and therefore can be neglected in the limit of $-q^2 \gg m^2$, μ^2.

For comparison, let us consider the lowest nontrivial expression for $\tilde{\Gamma}^\mu$ in the neutral-gluon model. The second-order expression for Γ^μ is given by[19]

$$\int \frac{d^4k}{(2\pi)^4}\frac{-i}{k^2-\mu^2}(-ig_0\gamma^\alpha)\frac{i}{\not{p}'-\not{k}-m}\gamma^\mu\frac{i}{\not{p}-\not{k}-m}(-ig_0\gamma_\alpha) = -ig_0^2\int\frac{d^4k}{(2\pi)^4}\frac{\gamma^\alpha(\not{p}'-\not{k}+m)\gamma^\mu(\not{p}-\not{k}+m)\gamma_\alpha}{(k^2-\mu^2)(-2p'k+k^2)(-2pk+k^2)}. \tag{2.8}$$

Here, in contrast with the previous case of Fig. 1, the infrared contribution is the dominant one. This can be seen easily by noting that as $\mu\to 0$, the k integration around the origin is logarithmically divergent. By commuting $(\not{p}'-\not{k}+m)$ with γ^α and $(\not{p}-\not{k}+m)$ with γ_α, one finds that the contribution from the infrared region is[19]

$$ig_0^2(2q^2)\gamma^\mu\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-\mu^2)(-2p'k+k^2)(-2pk+k^2)} + \text{less important terms}$$

$$= -\frac{g_0^2}{16\pi^2}\gamma^\mu \ln^2\left(-\frac{q^2}{\mu^2}\right) + \text{nonleading terms}, \tag{2.9}$$

while the ultraviolet contribution is of the order of $\ln(-q^2/\Lambda^2)$.

One can also work out the expression for $\Delta\tilde{\Gamma}^\mu$ in the gluon model. One finds that

$$\Delta\tilde{\Gamma}^\mu = O(1)\tilde{\Gamma}^\mu.$$

In other words, $\Delta\tilde{\Gamma}^\mu$ is of the same order as $\tilde{\Gamma}^\mu$ (up to logarithmic factors). The usefulness of the Callan-Symanzik equation in this case is limited.

The important difference between the second-order expressions of Γ^μ in pseudoscalar γ_5 theory and gluon theory is that the factors $(\not{p}-\not{k})/(-2pk+k^2)$, $(\not{p}'-\not{k})/(-2p'k+k^2)$ behave effectively as $-\not{k}/(-2pk+k^2)$, $-\not{k}/(-2p'k+k^2)$ in pseudoscalar γ_5 theory, while they behave as $\not{p}/(-2pk+k^2)$, $\not{p}'/(-2p'k+k^2)$ for small k in the gluon model.

We can use a similar argument to show that the asymptotic behavior of the on-mass-shell Γ^μ is not related to the short-distance behavior in gauge theories.

Let us return to pseudoscalar γ_5 theory. The next diagram we want to discuss is the one with coupling-constant renormalization subdiagrams. The diagram under discussion is shown in Fig. 2. The vertex function Γ^μ is given by

$$\Gamma^\mu = \int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-\mu^2}\left(\int\frac{d^4k_2}{(2\pi)^4}\frac{i}{k_2^2-\mu^2}g_0\gamma_5\frac{i}{\not{p}'-\not{k}_2-m}g_0\gamma_5\frac{i}{\not{p}'-\not{k}-\not{k}_2-m}g_0\gamma_5\right)$$

$$\times\frac{i}{\not{p}'-\not{k}-m}\gamma^\mu\frac{i}{\not{p}-\not{k}-m}\left(\int\frac{d^4k_1}{(2\pi)^4}\frac{i}{k_1^2-\mu^2}g_0\gamma_5\frac{i}{\not{p}-\not{k}_1-\not{k}-m}g_0\gamma_5\frac{i}{\not{p}-\not{k}_1-m}g_0\gamma_5\right). \tag{2.10}$$

Just as before, we have to worry about the effects of $(\not{p}'-k+m)/(-2p'k+k^2)$ and $(\not{p}-\not{k}+m)/(-2pk+k^2)$ in the k integration. At first sight it looks as though $\not{p}'$ and $\not{p}$ in the numerator would cause troubles when $\mu\to 0$. Fortunately this is not the case. The expression

$$\int\frac{d^4k_1}{(2\pi)^4}\frac{i}{k_1^2-\mu^2}g_0\gamma_5\frac{i}{\not{p}-\not{k}_1-\not{k}-m}g_0\gamma_5\frac{i}{\not{p}-\not{k}_1-m}g_0\gamma_5 \tag{2.11}$$

is nothing but the familiar vertex correction for

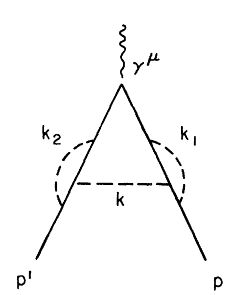

FIG. 2. A sixth-order diagram.

the pseudoscalar coupling constant. It behaves as a constant times γ_5 as $k\to 0$. Therefore we can move $\not p$ in $(\not p-\not k+m)/(-2pk+k^2)$ through the expression (2.11) when $k\to 0$. Exactly the same reason can be used for $\not p'$ in $(\not p'-\not k+m)/(-2p'k+k^2)$. With these in mind, one finds that the k integration in Eq. (2.10) is free from infrared divergences when $\mu\to 0$.

It is also easy to see that the contribution to Γ^μ from the ultraviolet region of the k integration has a nice zero-mass limit. The contribution can be written as

$$\text{polynomial in }\ln\left(-\frac{q^2}{\Lambda^2}\right)+O\left(\frac{\mu^2}{-q^2},\frac{m^2}{-q^2}\right).$$

The contribution to $\Delta\Gamma^\mu$ from the ultraviolet region of the k integration is, therefore, $O(\mu^2/(-q^2), m^2/(-q^2))$. Combining these two contributions, we find

$$\Delta\tilde\Gamma^\mu=O\left(\frac{\mu^2}{-q^2},\frac{m^2}{-q^2}\right)\tilde\Gamma^\mu,$$

or, equivalently,[20] that in the limit $m\to 0$, $\mu\to 0$

$$\Gamma^\mu(q^2,m^2,\mu^2,g_0,\lambda_0,\Lambda)\to\Gamma^\mu(q^2,0,0,g_0,\lambda_0,\Lambda)$$

exists.

The last example that we want to discuss here is represented diagrammatically in Fig. 3. We can write Γ^μ as

$$\Gamma^\mu_{da}=\int\frac{d^4k}{(2\pi)^4}\,{}_c\!\left(\frac{i}{\not p'-\not k-m}\gamma^\mu\frac{i}{\not p-\not k-m}\right)_b K_{dc;ba}, \tag{2.12}$$

where

$$\begin{aligned}K_{dc;ba}=\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\int\frac{d^4k_3}{(2\pi)^4}\int\frac{d^4k_4}{(2\pi)^4}\delta^{(4)}(k-k_3-k_4)\frac{i}{k_1^2-\mu^2}\,\frac{i}{k_2^2-\mu^2}\,\frac{i}{k_3^2-\mu^2}\,\frac{i}{k_4^2-\mu^2}\\ \times{}_b\!\left(g_0\gamma_5\frac{i}{\not p-\not k_1-\not k_3-\not k_4-m}g_0\gamma_5\frac{i}{\not p-\not k_1-\not k_4-m}g_0\gamma_5\frac{i}{\not p-\not k_1-m}g_0\gamma_5\right)_a\\ \times{}_d\!\left(g_0\gamma_5\frac{i}{\not p'-\not k_2-m}g_0\gamma_5\frac{i}{\not p'-\not k_4-\not k_2-m}g_0\gamma_5\frac{i}{\not p'-\not k_3-\not k_4-\not k_2-m}g_0\gamma_5\right)_c\end{aligned} \tag{2.13}$$

and is represented diagrammatically in Fig. 4.

The detailed analysis of the diagram in Fig. 3 is a long and tedious one. It is best treated by the general method in the next section. We are content here with pointing out possible difficulties and complications in the analysis.

We would like to know whether the unrenormalized vertex function $\Gamma^\mu(q^2,m^2,\mu^2,g_0,\lambda_0,\Lambda)$ corresponding to Fig. 3 has a nice zero-mass limit. One contribution to K is from the region where k_3 and k_4 are of the order of k. This contribution is of the order of $1/k^2$. At first sight, one assumes that this would cause infrared divergence in Γ^μ because $(\not p'-\not k)/(-2p'k+k^2)\sim 1/k$, $(\not p-\not k)/(-2pk+k^2)\sim 1/k$ as $k\to 0$. In the two previous examples, we have shown that we can move $\not p$ and $\not p'$ outside to operate on the spinors so that effectively $(\not p-\not k)/(-2pk+k^2)\sim\not k/(-2pk+k^2)$ and $(\not p'-\not k)/(-2pk+k^2)\sim-\not k/(-2p'k+k^2)$. Here we also have to show that, in fact, the $\not p$ and $\not p'$ in the numerators of $(\not p-\not k)/(-2pk+k^2)$, $(\not p'-\not k)/(-2p'k+k^2)$ will not cause infrared divergence in Γ^μ. We postpone this until the next section.

In the next section, we will present a general argument that, indeed, $\Gamma^\mu(q^2,m^2,\mu^2,g_0,\lambda_0,\Lambda)$ has a nice zero-mass limit, i.e., $\Gamma^\mu(q^2,0,0,g_0,\lambda_0\Lambda)$ exists. In other words, the mass-insertion term

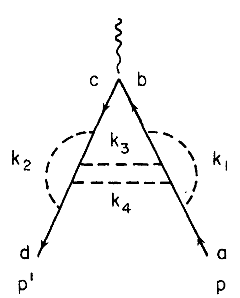

FIG. 3. A eighth-order diagram.

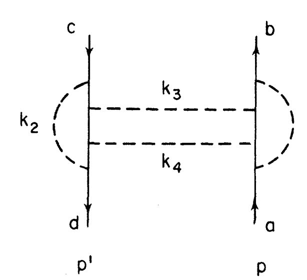

FIG. 4. Diagram for the kernel K in Eq. (2.13).

$\Delta\tilde{\Gamma}^\mu$ is negligible as compared to $\tilde{\Gamma}^\mu$ in the asymptotic limit $-q^2 \gg m^2, \mu^2$.

III. ASYMPTOTIC BEHAVIOR OF ELECTROMAGNETIC FORM FACTOR

The Callan-Symanzik equation for the vertex function $\tilde{\Gamma}^\mu$ was introduced in the previous section. We have emphasized the importance of knowing the behavior of the mass-insertion term in determining the asymptotic behavior of the vertex function $\tilde{\Gamma}^\mu$. Several examples were discussed in Sec. II. In two cases where the analysis can be easily carried out, we find that $\Delta\tilde{\Gamma}^\mu = O(\mu^2/(-q^2), m^2/(-q^2))\tilde{\Gamma}^\mu$, i.e., the mass insertion term is negligible as compared to $\tilde{\Gamma}^\mu$ in the asymptotic region. It is easy to see that the difficulties of analyzing $\Delta\tilde{\Gamma}^\mu$ increase as the order of the diagram increases. Any attempts to study $\Delta\tilde{\Gamma}^\mu$ order by order in perturbation would be a hopeless task. We need some better way to study $\Delta\tilde{\Gamma}^\mu$. Fortunately, for the vertex function Γ^μ we have a well-known iterative equation: the Bethe-Salpeter equation

$$\Gamma^\mu = \gamma^\mu - \int \Gamma^\mu S_F' S_F' K \,. \tag{3.1}$$

Diagrammatically, this is represented in Fig. 5. In Eq. (3.1) all quantities are unrenormalized quantities and therefore depend on the cutoff Λ; K is the two-particle-irreducible kernel in the q channel.

In this section we discuss the behavior of $\Delta\tilde{\Gamma}^\mu$ in the asymptotic limit using the Bethe-Salpeter equation [Eq. (3.1)] as our starting point. We will show that

$$\Delta\tilde{\Gamma}^\mu = O\left(\frac{\mu^2}{-q^2}, \frac{m^2}{-q^2}\right)\tilde{\Gamma}^\mu, \tag{3.2}$$

or, equivalently, that in the limit $m \to 0$, $\mu \to 0$

$$\Gamma^\mu(q^2, m^2, \mu^2, g_0, \lambda_0, \Lambda) \to \Gamma^\mu(q^2, 0, 0, g_0, \lambda_0, \Lambda) \tag{3.3}$$

exists, in the pseudoscalar γ_5 theory described by Eq. (2.1).

The Bethe-Salpeter equation tells us that, to the nth order in the coupling constant, the vertex function Γ^μ is given by

$$(\Gamma^\mu)^{(n)} = -\int (\Gamma^\mu)^{(n_1)}(S_F')^{(n_2)}(S_F')^{(n_3)}K^{(n_4)}, \tag{3.4}$$

where the superscripts n, n_1, n_2, n_3, n_4 refer to the order in perturbation and $n = n_1 + n_2 + n_3 + n_4$. Here n_2 and n_3 are all larger than or equal to zero and n_4 is larger than or equal to 2. Therefore we only need $(\Gamma^\mu)^{(n_1)}$ for $n_1 = 0$ to $n_1 = n-2$ in order to determine Γ^μ to nth order.

Our aim is to prove that Eqs. (3.2) and (3.3) are valid in each order of perturbation in neutral pseudoscalar γ_5 theory. The proof is based on mathematical induction. We assume that Eq. (3.3) is true to order $n = N-2$. We then use the Bethe-Salpeter equation [Eqs. (3.1), (3.4)] to establish that Eq. (3.3) is true for $n = N$.

First let us restrict ourselves to diagrams without self-energy corrections. For such diagrams, we can replace the full propagators S_F' and Δ_F' by the bare propagators S_F and Δ_F. In determining $(\Gamma^\mu)^{(N)}$ through Eq. (3.4), one has to know some general properties of the two-particle-irreducible kernel K. We recall that K involves many internal integrations. For convenience, let us separate each pion line into two parts. We call the region of pion momentum around the origin $k \sim 0$ soft; otherwise it is called hard.[21]

The contributions from internal integrations of K can be divided into two classes.

Class 1: All pion lines from the p line to p' line are soft. This class can be further divided into two subclasses.

(1a): There is no hard integration along p and p' lines. In this subclass, all pion lines are soft inside K. For the pion line closest to $u(p)$ and $u(p')$, respectively, we can replace $(\not{p} - \not{k}_1)/(-2pk_1 + k_1^2)$ by $-\not{k}_1/(-2pk_1 + k_1^2)$ and $(\not{p}' - \not{k}_2)/(-2p'k_2 + k_2^2)$ by $-\not{k}_2/(-2p'k_2 + k_2^2)$, where both k_1 and k_2 are of the order of k. Therefore the k integration in Γ^μ will be infrared-divergence-free.

(1b): There are hard integrations along the p line and the p' line. In this subclass, we follow the p line and the p' line separately. Consider first the p line (Fig. 6). As we have mentioned before,

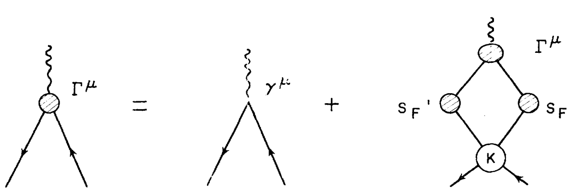

FIG. 5. Bethe-Salpeter equation for the vertex function Γ^μ.

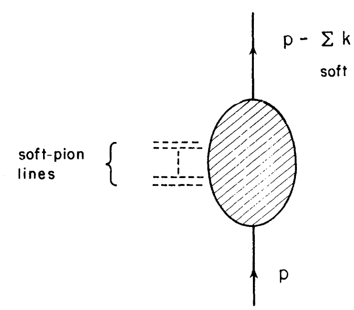

FIG. 6. Diagram with p line and soft-pion lines.

we have to check whether the $\not p$ and $\not p'$ in the numerators of $(\not p-\not k)/(-2pk+k^2)$, $(\not p'-\not k)/(-2pk+k^2)$ can cause infrared divergences in Γ^μ.

The integrand that we have to worry about is of the form

$$\not p\gamma_5\frac{1}{\not p-\sum\not k_i}\gamma_5\cdots\gamma_5\frac{1}{\not p-\not k_1}\gamma_5 \times(\text{product of hard-pion propagators})\,. \tag{3.5}$$

This has to be integrated over hard regions. Since the integrand transforms as either a scalar or a pseudoscalar [depending on the number of γ_5's in the integrand in (3.5)], the result of the hard integration is

$$A1+B\gamma_5+C_\mu\gamma^\mu+D_\mu\gamma^\mu\gamma_5+E_{\mu\nu}\sigma^{\mu\nu}\,. \tag{3.6}$$

Here A, B, C_μ, D_μ, and $E_{\mu\nu}$ are all constructed out of p and soft k's (and are at least linear in p^μ) times a function of a, where a is used to distinguish between soft and hard pions.[21] Since $p^2=0$, the only terms among A, B, C_μ, D_μ, and $E_{\mu\nu}$ that may approach a constant as $k\to 0$ are C_μ and D_μ. They may be proportional to p_μ. Fortunately, the terms $C_\mu\gamma^\mu$ and $D_\mu\gamma^\mu\gamma_5$ will not contribute to the on-mass-shell vertex Γ^μ because $\not p u(p)=0$. The final result is therefore at least linear in k. The same conclusion can be reached for the p' line.

From the usual dimensional consideration, the kernel K is of the order of the inverse square of the mass. Thus,

$$\dim(a^{-r_1}\,\text{soft contribution})=-2 \tag{3.7}$$

or $r_1+r_2=2$, where r_2 is related to the dimension of the soft contribution. As we have mentioned before, we restrict ourselves to Γ^μ with no self-energy subdiagrams (we postpone the discussion for Γ^μ with self-energy subdiagrams until later). Since $r_1\geq 0$ for any diagrams of K without self-energy subdiagrams, we conclude that K is at most $O(1/k^2)$ as $k\to 0$. When we put this information into the Bethe-Salpeter equation, together with the fact that $\not p$ and $\not p'$ in $(\not p-\not k)/(-2pk+k^2)$ and $(\not p'-\not k)/(-2p'k+k^2)$ do not contribute in the infrared region, we find that the contribution of this class to $(\Gamma^\mu)^{(N)}(q^2,m^2,\mu^2,g_0,\lambda_0,\Lambda)$ has a nice zero-mass limit, i.e., $(\Gamma^\mu)^{(N)}(q^2,0,0,g_0,\lambda_0,\Lambda)$ exists.

Class 2: Some pions from the p line to the p' line are hard. In this case, when we perform hard integrations, we find that the result is

$$\frac{1}{a^{r_1}}f\left(\frac{pp'}{a^2}\right)\quad\text{as } k\to 0\,. \tag{3.8}$$

For this class of contributions, it is easy to see that r_1 is always greater than or equal to 2. Thus,

$$\dim[a^{-r_1}f(pp'/a^2)\text{ soft contributions}]=-2 \tag{3.9}$$

implies that the contributions from this class to K behave like $O(1)$ when $k\to 0$. When this kernel K is substituted into the Bethe-Salpeter equation, we find that there is no divergence in the k integration around the origin. The ultraviolet contribution in the k integration also causes no problem, for there are no infrared divergences from the internal integrations inside K because of the two-particle irreducibility of the kernel K.

Combining classes 1 and 2, we find that for diagrams without self-energy subdiagrams the vertex function $(\Gamma^\mu)^{(N)}$ has a nice zero-mass limit.

It is a simple task to include diagrams of Γ^μ with self-energy-correction subdiagrams. All we have to do is first sum over all such self-energy-correction subdiagrams. Instead of the bare propagators Δ_F and S_F, we have to use the full propagators

$$\begin{aligned}\Delta_F'&=[k^2-\pi(k^2)]^{-1}\,,\\ S_F'&=[\not k-\Sigma(\not k)]^{-1}\,.\end{aligned} \tag{3.10}$$

Since $\pi(k^2)=O(k^2)$ and $\Sigma(\not k)=O(k)$, all of our previous conclusions remain the same even for Γ^μ with self-energy-correction subdiagrams. We also know that the lowest-order result $(\Gamma^\mu)^{(0)}=\gamma^\mu$ has a nice zero-mass limit. This completes our inductive proof that Eqs. (3.2) and (3.3) are true, in each order in perturbation theory, in the pseudoscalar γ_5 theory containing a neutral pion and a proton.

With the aid of this new information [Eqs. (3.2), (3.3)], we can neglect $\Delta\tilde\Gamma^\mu$ as compared to $\tilde\Gamma^\mu$ in the asymptotic limit $-q^2\gg m^2$, μ^2. The asymptotic expression of $\tilde\Gamma^\mu$ satisfies the partial differential equation

$$\left(m\frac{\partial}{\partial m}+\mu\frac{\partial}{\partial\mu}+\beta_1\frac{\partial}{\partial g}+\beta_2\frac{\partial}{\partial\lambda}-2\gamma\right)\tilde\Gamma^\mu(\text{asy})=0\,. \tag{3.11}$$

It is convenient to express $\tilde\Gamma^\mu$ in terms of form factors F_1 and F_2,[22]

$$\tilde\Gamma^\mu=\gamma^\mu F_1+i\sigma^{\mu\nu}q_\nu(\kappa/2m)F_2\,. \tag{3.12}$$

Since $F_2=O(\mu^2/(-q^2),m^2/(-q^2))F_1$ in each order in perturbation theory, the form factor F_2 is negligible as compared to F_1. Equation (3.11) determines the asymptotic expression for the form factor F_1. One finds that

$$\begin{aligned}F_1&=F_1\left(-\frac{q^2}{m^2},\frac{\mu}{m}\right)\\ &=f_1\left(\bar g(t),\bar\lambda(t),\frac{\mu}{m}\right)\exp\left[-2\int_0^t dt'\gamma(\bar g',\bar\lambda')\right],\end{aligned} \tag{3.13}$$

where $t=\frac{1}{2}\ln(-q^2/m^2)$ and

$$\frac{d\bar{g}(t)}{dt}=\beta_1(\bar{g}(t),\bar{\lambda}(t)),$$
$$\frac{d\bar{\lambda}(t)}{dt}=\beta_2(\bar{g}(t),\bar{\lambda}(t)),\tag{3.14}$$

with the initial condition $g(t)=g$, $\lambda(t)=\lambda$ at $t=0$. It is easy to see that the asymptotic behavior of the form factor F_1 is related to the short-distance behavior of the neutral pseudoscalar γ_5 theory.

In the special case that β_1 and β_2 have zeros at $g=g_f$, $\lambda=\lambda_f$ and $g\to g_f$, $\lambda\to\lambda_f$ as $t\to\infty$, the asymptotic behavior of the form factor is[23]

$$F_1=c_1(m^2/-q^2)^{\gamma_0},\tag{3.15}$$

where $\gamma_0=\gamma(g_f,\lambda_f)$ is the anomalous dimension of the proton field. Our method here tells us nothing about F_2 because in every order in perturbation theory F_2 is of the same order as the neglected mass-insertion term $\Delta\tilde{\Gamma}^\mu$ in the asymptotic limit.

We have shown in this section that is possible to obtain power-law behavior for the asymptotic behavior of the electromagnetic form factor in the neutral pseudoscalar γ_5 theory. The determination of the exponent γ_0 is, however, a very complicated dynamical problem.

IV. FORM FACTOR OF THE AXIAL-VECTOR CURRENT

Up to now, we have restricted ourselves to the electromagnetic form factor. However, it is very easy to see that the same technique can be used to study the asymptotic behavior of other on-mass-shell form factors. One such form factor is the form factor of the axial-vector current. Since the vector current and the axial-vector currents play similar roles in hadron physics, both are measurable quantities in lepton-hadron scattering. We will study the asymptotic behavior of both form factors together in this section.

We assume as usual that the axial-vector current is partially conserved (PCAC assumption). One model which incorporates both PCAC and current algebra is the well-known renormalizable σ model.[15] For the sake of simplicity, we consider here only the truncated version[24] of the σ model which contains a proton, a neutral pion, and a neutral scalar meson. The problem of renormalization of the σ model has been discussed by Lee,[25] Gervais and Lee,[26] and Symanzik.[27] We refer to them for details. The unrenormalized Lagrangian is

$$\mathcal{L}=\bar{\psi}_0 i\not{\partial}\psi_0+\tfrac{1}{2}[(\partial\pi_0)^2+(\partial\sigma_0)^2]-\tfrac{1}{2}\mu_0{}^2(\pi_0{}^2+\sigma_0{}^2)$$
$$-g_0\bar{\psi}_0(\sigma_0+i\pi_0\gamma_5)\psi_0-\tfrac{1}{4}\lambda_0{}^2(\sigma_0{}^2+\pi_0{}^2)^2-c_0\sigma_0\,.\tag{4.1}$$

The model after renormalization is characterized by the coupling constants g,λ, the proton mass m, and the pion mass μ. The Green's functions are renormalized according to

$$\tilde{\Gamma}=Z_\Gamma\Gamma\,.\tag{4.2}$$

The Callan-Symanzik equation can be derived as follows. Let us vary the proton mass m and pion mass μ:

$$\left(m\frac{d}{dm}+\mu\frac{d}{d\mu}\right)\tilde{\Gamma}=\left(m\frac{\partial}{\partial m}+\mu\frac{\partial}{\partial\mu}+\beta_1\frac{\partial}{\partial g}+\beta_2\frac{\partial}{\partial\lambda}\right)\tilde{\Gamma}$$
$$=Z_\Gamma\Gamma Z_\Gamma{}^{-1}\left(m\frac{d}{dm}+\mu\frac{d}{d\mu}\right)Z_\Gamma$$
$$+Z_\Gamma\left(m\frac{d}{dm}+\mu\frac{d}{d\mu}\right)\Gamma\,.\tag{4.3}$$

This can be rewritten as

$$\left(m\frac{\partial}{\partial m}+\mu\frac{\partial}{\partial\mu}+\beta_1\frac{\partial}{\partial g}+\beta_2\frac{\partial}{\partial\lambda}-\gamma_\Gamma\right)\tilde{\Gamma}=\Delta\tilde{\Gamma},\tag{4.4}$$

where

$$\beta_1=\left(m\frac{d}{dm}+\mu\frac{d}{d\mu}\right)g,$$
$$\beta_2=\left(m\frac{d}{dm}+\mu\frac{d}{d\mu}\right)\lambda,$$
$$\gamma_\Gamma=Z_\Gamma{}^{-1}\left(m\frac{d}{dm}+\mu\frac{d}{d\mu}\right)Z_\Gamma,\tag{4.5}$$
$$\Delta\tilde{\Gamma}=Z_\Gamma\left(m\frac{d}{dm}+\mu\frac{d}{d\mu}\right)\Gamma\,.$$

It is understood that in taking total derivatives with respect to m and μ, the bare coupling constants g_0,λ_0 as well as the cutoff Λ are fixed.

Let us concentrate on the proper vertex functions of the vector current and the axial-vector currents. In the σ model, the PCAC condition is

$$\partial_\mu J_5^\mu=c_0\pi_0\,.\tag{4.6}$$

Preparata and Weisberger[28] have shown that in a theory with the PCAC condition the vertex function of the axial-vector current is multiplicatively renormalizable. Furthermore, the cutoff-dependent factor needed to renormalize Γ_5^μ is Z_2, i.e.,

$$\tilde{\Gamma}_5^\mu=Z_2\Gamma_5^\mu\,.\tag{4.7}$$

The Callan-Symanzik equations for the vector and axial-vector currents are

$$\left(m\frac{\partial}{\partial m}+\mu\frac{\partial}{\partial\mu}+\beta_1\frac{\partial}{\partial g}+\beta_2\frac{\partial}{\partial\lambda}-2\gamma\right)\tilde{\Gamma}^\mu=\Delta\tilde{\Gamma}^\mu,$$
$$\left(m\frac{\partial}{\partial m}+\mu\frac{\partial}{\partial\mu}+\beta_1\frac{\partial}{\partial g}+\beta_2\frac{\partial}{\partial\lambda}-2\gamma\right)\tilde{\Gamma}_5^\mu=\Delta\tilde{\Gamma}_5^\mu.\tag{4.8}$$

We wish to show that

$$\Delta\tilde{\Gamma}^{\mu} = O\left(\frac{\mu^2}{-q^2}, \frac{m^2}{-q^2}\right)\tilde{\Gamma}^{\mu},$$
$$\Delta\tilde{\Gamma}_5^{\mu} = O\left(\frac{\mu^2}{-q^2}, \frac{m^2}{-q^2}\right)\tilde{\Gamma}_5^{\mu}, \tag{4.9}$$

or, equivalently, that

$$\Gamma^{\mu}(q^2, m^2, \mu^2, g_0, \lambda_0, \Lambda) \text{ and } \Gamma_5^{\mu}(q^2, m^2, \mu^2, g_0, \lambda_0, \Lambda)$$
$$\text{exist in the limit of } m \to 0,\ \mu \to 0\,. \tag{4.10}$$

From the work of Gervais and Lee[26] one learns that $m = 0$, $\mu = 0$ implies $c_0 = 0$ and $\langle\sigma_0\rangle = v_0 = 0$. Therefore the problem reduces to showing the existence of Γ^{μ}, Γ_5^{μ} in the zero-mass symmetric theory. In other words, the only interaction terms that we have to take into account are

$$-g_0\bar{\psi}_0(\sigma_0 + i\pi_0\gamma_5)\psi_0 - \tfrac{1}{4}\lambda_0^{\,2}(\sigma_0^{\,2} + \pi_0^{\,2})^2.$$

We do not have to worry about tadpole terms or interactions cubic in the meson fields.

The argument we used in Sec. III for the neutral pseudoscalar γ_5 theory can be readily adapted to our σ model. The same conclusion can be reached as before as long as the meson-fermion vertex is either a scalar or a pseudoscalar interaction. Fortunately, this is the case for the σ model.

Therefore we conclude that Eqs. (4.9) and (4.10) are valid. The asymptotic behaviors of $\tilde{\Gamma}^{\mu}$ and $\tilde{\Gamma}_5^{\mu}$ satisfy the partial differential equations

$$\left(m\frac{\partial}{\partial m} + \mu\frac{\partial}{\partial \mu} + \beta_1\frac{\partial}{\partial g} + \beta_2\frac{\partial}{\partial \lambda} - 2\gamma\right)\tilde{\Gamma}^{\mu}(\text{asy}) = 0\,,$$
$$\left(m\frac{\partial}{\partial m} + \mu\frac{\partial}{\partial \mu} + \beta_1\frac{\partial}{\partial g} + \beta_2\frac{\partial}{\partial \lambda} - 2\gamma\right)\tilde{\Gamma}_5^{\mu}(\text{asy}) = 0\,. \tag{4.11}$$

It is worth emphasizing that the asymptotic forms of $\tilde{\Gamma}^{\mu}$ and $\tilde{\Gamma}_5^{\mu}$ satisfy the same partial differential equation. The on-mass-shell vertices $\tilde{\Gamma}^{\mu}$, $\tilde{\Gamma}_5^{\mu}$ can be expressed in terms of the form factors:

$$\tilde{\Gamma}^{\mu} = \gamma^{\mu}F_1 + i\sigma^{\mu\nu}q_{\nu}(\kappa/2m)F_2\,,$$
$$\tilde{\Gamma}_5^{\mu} = \gamma^{\mu}\gamma_5 F_1^A + (q^{\mu}/m)F^P\,, \tag{4.12}$$

where F^P is the induced pseudoscalar term. [The axial-vector current considered in this paper is the first-class current. Therefore the term $i\sigma^{\mu\nu}\gamma_5(q_\nu/m)F_2^A$ will not be present in Eq. (4.12).[29]] Since $F_2 = O(\mu^2/-q^2, m^2/-q^2)F_1$ and $F^P = O(\mu^2/-q^2, m^2/-q^2)F_1^A$, in each order in perturbation, they are negligible as compared to F_1 and F_1^A in the asymptotic limit. Equation (4.11) determines F_1, F_1^A in this limit. One finds that

$$F_1 = F_1\left(\frac{-q^2}{m^2}, \frac{\mu}{m}\right) = f_1\left(\bar{g}(t), \bar{\lambda}(t), \frac{\mu}{m}\right)\exp\left[-2\int_0^t dt'\gamma(\bar{g}', \bar{\lambda}')\right]$$

and

$$F_1^A = F_1^A\left(\frac{-q^2}{m^2}, \frac{\mu}{m}\right) = f_1^A\left(\bar{g}(t), \bar{\lambda}(t), \frac{\mu}{m}\right)\exp\left[-2\int_0^t dt'\gamma(\bar{g}', \bar{\lambda}')\right], \tag{4.13}$$

where

$$t = \tfrac{1}{2}\ln(-q^2/m^2)\,,$$
$$\frac{d\bar{g}}{dt} = \beta_1(\bar{g}, \bar{\lambda})\,, \tag{4.14}$$
$$\frac{d\bar{\lambda}}{dt} = \beta_2(\bar{g}, \bar{\lambda}) \quad \text{with } \bar{g} = g,\ \bar{\lambda} = \lambda \text{ when } t = 0.$$

In the case that the Gell-Mann–Low eigenvalue conditions are satisfied,

$$\beta_1(g_f, \lambda_f) = 0\,, \quad \beta_2(g_f, \lambda_f) = 0$$

and

$$g \to g_f\,, \quad \lambda \to \lambda_f \quad \text{as } t \to \infty\,,$$

the asymptotic behaviors of F_1 and F_1^A are[23]

$$F_1 \sim c_1(m^2/-q^2)^{\gamma_0}\,,$$
$$F_1^A \sim c_2(m^2/-q^2)^{\gamma_0}\,, \tag{4.15}$$

where $\gamma_0 \equiv \gamma(g_f, \lambda_f)$ is the anomalous dimension of the proton field, and c_1, c_2 are functions of μ/m. We conclude that the on-mass-shell axial-vector form factor has the same asymptotic behavior as the vector form factor. The experimental data from high-energy lepton-hadron scatterings seem to be consistent with this result.

V. SUMMARY AND DISCUSSION

In this section we want to summarize the results we obtained and make comparison with previous work.

We have shown that in the renormalizable pseudoscalar field theory with a proton and a neutral pion the asymptotic behavior of the on-mass-shell electromagnetic form factor is governed by the short-distance behavior of the theory. In particular, if the strong interaction has a nontrivial fixed point at g_f and λ_f, the asymptotic behavior is $(m^2/-q^2)^{\gamma(g_f, \lambda_f)}$.

We have also shown that in the truncated σ model with a proton, a neutral pion, and neutral scalar meson, the asymptotic behavior of the vector-current and axial-vector-current form factors is governed by the short-distance behavior of the theory. They will have the same asymptotic behavior if the truncated σ model has a fixed point. Experimental data obtained so far seem to support this result.

Although we restrict our discussion to the neutral pseudoscalar field theory and the truncated σ model, it is easy to convince ourselves that the arguments and discussions can be readily extended to any renormalizable field theory (with or without internal symmetry) with Yukawa-type interactions and quartic interactions. These include all renormalizable field theories except gluon theories and gauge theories. We leave the generalizations to the interested readers.

It would be useful to compare our results with the results of previous work on the asymptotic behavior of form factors in field theory, especially the work of Landau *et al.*,[5] Bogoliubov *et al.*,[6] and Appelquist and Primack.[4] We recall that Landau and collaborators obtained the asymptotic behavior of the form factor, in neutral pseudoscalar theory, when all the squared momenta are asymptotic, by solving a set of approximate integral equations. The result is

$$\tilde{\Gamma}^\mu = \gamma^\mu \left[1 - \frac{10g^2}{32\pi^2}\ln\left(\frac{-k^2}{m^2}\right)\right]^{1/10}.$$

The same result was obtained by Bogoliubov and collaborators using the renormalization group. The more interesting case of the asymptotic behavior of the on-mass-shell form factor was obtained by Appelquist and Primack using the leading-logarithmic approximation. Their result is exactly the same as that of Landau *et al.*[5] and Bogoliubov *et al.*[6] There is one very serious difficulty in their result. For large spacelike q^2, the vertex function $\tilde{\Gamma}^\mu$ becomes complex, which violates unitary and analyticity. Therefore the result they obtained is not physically meaningful. Appelquist and Primack tried to include the term of next to leading order in the logarithm. The similar unreasonable properties of the result persisted.

It is shown in this paper that for general renormalizable field theories, excluding gluon and gauge theories, the asymptotic behaviors of the on-mass-shell form factors can be treated by the renormalization group (or the Callan-Symanzik equation). The renormalization-group equation and the Callan-Symanzik equation provide us with a powerful method for summing up all logarithmic contributions (leading as well as nonleading). We find that for this class of field theories the asymptotic behaviors of the form factors are governed by the short-distance behaviors of the theories. The asymptotic behaviors of the form factors have the right unitarity and analyticity properties.

Recently, Marques[8] applied the Callan-Symanzik equation technique to the asymptotic behavior of form factors. He shows that $\Delta\tilde{\Gamma}^\mu$ can be neglected for some restricted set of diagrams (ladder diagrams and fourth-order diagrams). No general discussion for $\Delta\tilde{\Gamma}^\mu$ was given. His results in pseudoscalar γ_5 theories did not go beyond those of Appelquist and Primack.

Our last comment is that if all three squared momenta are large the vertex functions will satisfy the same equation as the on-mass-shell vertex functions for the class of theories we discussed in this paper. The asymptotic behavior, in the case that the Gell-Mann–Low eigenvalue conditions are satisfied, will be $[m^2/(-q^2)]^{\gamma_0} f(p^2/q^2, p'^2/q^2)$.

Added note. In this paper we have not touched upon the problem of form factors of bound states. Much work has been done in this direction, including the recent work of Appelquist and Poggio.[30]

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