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Electron-laser pulse scattering*

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We present a formalism for the calculation of scattering amplitudes for electron-laser pulse processes. Our results make use of standard Feynman-Dyson S-matrix techniques and the Lehmann-Symanzik-Zimmermann reduction formalism. The laser pulse is described quantum mechanically as a coherent-state wave packet. The electron is represented by wave-packet solutions of the Dirac equation in the case of scattering boundary conditions, or by Volkov wave packets when we desire nonscattering boundary conditions. We demonstrate the existence of new, nonlinear corrections of a quantum-mechanical nature for the process of stimulated Compton scattering.

I. INTRODUCTION

The problem of the interaction of an electron with a laser pulse is a difficult one, as evidenced by the large number of authors who have written about the subject.[1]

These authors fall roughly into three groups: The first group[2] (classical) calculated the electron orbit in the laser pulse and evaluated cross sections using the methods of classical electrodynamics—the electron was treated nonrelativistically or relativistically, whereas the laser pulse was taken to be a monochromatic plane wave of infinite extent or a damped monochromatic plane wave. The second group[3] (semiclassical) computed scattering amplitudes via time-dependent perturbation theory—the electron was described by the Volkov[4] wave function and the laser pulse by a plane wave. The third group[5] (quantum field-theoretic) used covariant perturbation theory (conventional or null-plane) and the adiabatic switch-off[6] technique to compute scattering amplitudes. As applied to the calculation of the stimulated Compton scattering amplitude,[1-3,5] the methods of these groups of authors gave different results.

The conflicting classical results were due to the imposition of initial conditions on the electron while simultaneously demanding that the laser pulse be described as a monochromatic plane wave of infinite extent. Eberly and Sleeper,[7] however, calculated an electron orbit in the presence of a wave-packet laser pulse.

As Fried, Baker, and Korff[8] pointed out, much of the disagreement in the semiclassical results could be traced to the use of Volkov[4] wave functions to compute scattering amplitudes for experimental configurations where the laser pulse and electron are asymptotically noninteracting. Thus, we see that the disagreements arising in the classical and semiclassical results are related.

Much of the disagreement in the field-theoretic case can be traced to the imposition of disparate boundary conditions also. Fried and Eberly[5] invoked the adiabatic hypothesis[6] (incorrectly in this author's opinion) in their calculations while at the same time assuming a monochromatic plane-wave field for laser pulse. They found no intensity-dependent frequency shift (denoted hereafter by IDFS) in their calculations. Subsequently, Eberly and Reiss[9] were able to show that a class of diagrams was omitted by Fried and Eberly,[5] which, when properly accounted for, would give rise to an IDFS.

Dawson and Fried[5] treated the electron-laser pulse (henceforth denoted by ELP) process of stimulated Compton scattering using a model where a neutral scalar particle (the electron) interacts with a scalar, massless external field (the laser pulse). The electron was described by Volkov-type[4] solutions, while the laser pulse was given by a square pulse shape and also by the more realistic Lorentzian shape. They found that the frequency profile of the scattered photon depended not only on the intensity of the external field, but on the laser pulse shape as well. Neville and Rohrlich[5] used the unconventional null-plane formulation of quantum electrodynamics and obtained an IDFS and a shape-dependent frequency profile using Volkov[4] wave packets and a monochromatic square pulse (pulse and electron were asymptotically noninteracting). Their method, however, introduced singular electromagnetic field intensities.

In this paper we will present a general formalism for calculating scattering amplitudes for ELP processes. Our results make use of the standard Feynman-Dyson S-matrix techniques[10] and the LSZ formalism,[11,12] although the Dirac interaction[13] picture is replaced by the Furry representation.[14] The laser pulse is described quantum mechanically as a coherent-state[15] wave packet; the electron, for scattering boundary conditions (i.e., the experimental setup is such that the electron and laser pulse are asymptotically noninteracting), is described by wave-packet solutions to the Dirac equation. For nonscattering boundary conditions (i.e., we imagine the experimental situation when, for all practical purposes, the electron and laser pulse are never decoupled), the electron is represented by Volkov[4] wave packets. Our method is more general and much easier to utilize than previous quantum field-theoretic methods, in that it may be applied to situations where the incident laser pulse has an arbitrary shape, where we may or may not have scattering boundary conditions, and where the incident laser pulse is modified significantly through its interaction with the electron.

As a bonus, we also demonstrate the existence of new, purely quantum-mechanical, nonlinear corrections present in any ELP scattering process.

In Sec. II we introduce our model for the ELP interaction using appropriate modifications of a method due to Zwanziger.[16]

In Sec. III we define and calculate S-matrix elements in the Heisenberg picture using the reduction technique.[11,12] We make use of Zwanziger's fundamental formula for the reduction of coherent states.[16] We also obtain scattering amplitudes where vacuum expectation values of the various operators are to be computed in the Furry picture.[14]

In Sec. IV we apply our methods to the specific process of stimulated Compton scattering[1-3,5] and explicitly demonstrate the new, nonlinear quantum-mechanical corrections.

In Sec. V we present a nonrigorous argument for the existence of an IDFS in any ELP process.

II. ELP SCATTERING MODEL

Our model for the ELP interaction is conceptually very simple. The initial state consists of an electron with four-momentum p, an arbitrary number of photons $\{k_i(\mu_i)\}$ with four-momentum k_i, and polarization μ_i, and a laser pulse intensity is determined by the classical field

$$\alpha_\mu^{\text{in}}(x)=\int\Delta(x-y)j_\mu^{\text{in}}(y)dy\,,$$

where

$$\Delta(x)=\Delta^{\text{ret}}(x)-\Delta^{\text{adv}}(x)\,,$$

$$\Delta(x)=\frac{i}{(2\pi)^3}\int(e^{-ikx}-e^{ikx})\frac{d^3k}{2\omega}\,,$$

and

$$\Box\Delta^{\text{ret}}(x)=\Box\Delta^{\text{adv}}(x)=\delta(x)\,.$$

Here $k^\mu=(k^0,\vec{k})=(\omega,\vec{k})=(|k|,\vec{k})$.[17] The final state consists of an electron with four-momentum p', an arbitrary number of photons $\{k_f'(\mu_f')\}$ with momentum k_f' and polarization μ_f', and a laser pulse with intensity determined by the classical radiation field

$$\alpha_\mu^{\text{out}}(x)=\int\Delta(x-y)j_\mu^{\text{out}}(y)dy\,,$$

where

$$j^{\text{out}\,\nu}(x)\equiv j^{\text{in}\nu}(x)+j^\nu(x)\,. \tag{2.1}$$

We see that it is $j(x)$ that can cause the intensity of the out-laser pulse to be different from that of the in-laser pulse.

Our Lagrangian is

$$\mathcal{L}(x)=\mathcal{L}_{\text{Fermi}}(x)+\mathcal{L}_{\text{Dirac}}(x)+\mathcal{L}'(x)\,,$$

where

$$\begin{aligned}\mathcal{L}_{\text{Fermi}}&=-\tfrac{1}{2}(\partial^\mu A^\nu)(\partial_\mu A_\nu)\,,\\ \mathcal{L}_{\text{Dirac}}&=-\tfrac{1}{2}\bar\psi(-i\gamma^\mu\partial_\mu+m)\psi\\ &\quad-\tfrac{1}{2}(i\partial_\mu\bar\psi\gamma^\mu+m\bar\psi)\psi\,,\\ \mathcal{L}'(x)&=-J^\mu(x)A_\mu(x)\,.\end{aligned} \tag{2.2}$$

$J^\mu(x)$ is the quantum-mechanical current, $J^\mu=-\frac{1}{2}e[\bar\psi\gamma^\mu,\psi]$, and $A^\mu(x)$ is the interpolating electromagnetic field operator. No mass counterterm is present in (2.2) as radiative corrections due to the quantized fields are expected to be unobservably small for ELP interactions.[18,19]

From (2.2) we obtain

$$\Box A^\mu=J^\mu\,,$$

$$\vec{D}\psi=e\gamma^\mu A_\mu\psi\,,$$

$$\bar\psi\overleftarrow{D}=e\bar\psi\gamma^\mu A_\mu\,,$$

where

$$\vec{D}\equiv(-i\gamma\cdot\vec\partial+m),\quad\overleftarrow{D}\equiv(i\gamma\cdot\overleftarrow\partial+m)\,.$$

The canonical commutation relations for the in and out fields are

$$[A_\mu^{\text{in}}(x),A_\nu^{\text{in}}(y)]=[A_\mu^{\text{out}}(x),A_\nu^{\text{out}}(y)]=ig_{\mu\nu}\Delta(x-y)\,.$$

We also have the usual expansions for the in and out fields:

$$A_\mu^{\text{out(in)}}(x)=(2\pi)^{-3/2}\int\frac{d^3k}{2\omega}\,[a_\mu^{\text{out(in)}}(k)e^{-ikx}+a_\mu^{\dagger\text{out(in)}}(k)e^{ikx}]\,, \tag{2.3}$$

where

$$[a_\mu^{\text{out(in)}}(k),a_\nu^{\dagger\text{out(in)}}(k')]=-2k^0g_{\mu\nu}\delta(\vec{k}-\vec{k}')\,.$$

We can set up the usual Yang-Feldman formalism[20] relating the asymptotic quantum fields:

$$A_\mu(x) = A_\mu^{\text{in}}(x) + \int \Delta^{\text{ret}}(x-y)J_\mu(y)dy\,,$$
$$A_\mu(x) = A_\mu^{\text{out}}(x) + \int \Delta^{\text{adv}}(x-y)J_\mu(y)dy\,, \tag{2.4}$$
$$\Box A_\mu^{\text{in}}(x) = \Box A_\mu^{\text{out}}(x) = 0\,.$$

We have similar relations for the asymptotic classical fields:

$$\Box\alpha_\mu(x) \equiv j_\mu(x)\,,$$
$$\alpha_\mu(x) = \alpha_\mu^{\text{in}}(x) + \int \Delta^{\text{ret}}(x-y)j_\mu(y)dy\,,$$
$$\alpha_\mu(x) = \alpha_\mu^{\text{out}}(x) + \int \Delta^{\text{adv}}(x-y)j_\mu(y)dy\,, \tag{2.5}$$
$$\Box\alpha_\mu^{\text{in}}(x) = \Box\alpha_\mu^{\text{out}}(x) = 0\,.$$

$\alpha_\mu(x)$ is the interpolating classical field.

We relate the classical and quantum asymptotic fields in the following manner[16,21]:

$$s^{\text{out}}(j^{\text{out}})A_\mu^{\text{out}}s^{\dagger\text{out}}(j^{\text{out}}) = A^{\text{out}} - \alpha^{\text{out}} \equiv A_f^{\text{out}}\,, \tag{2.6}$$
$$s^{\text{out}}(j^{\text{out}})s^{\dagger\text{out}}(j^{\text{out}}) = s^{\dagger\text{out}}(j^{\text{out}})s^{\text{out}}(j^{\text{out}}) = 1\,, \tag{2.7}$$

and similarly for out $\to$ in. From (2.4) and (2.5) it follows that

$$\Box A_f^{\text{out}} = \Box A_f^{\text{in}} = 0\,.$$

If s exists at all it is of the form

$$s^{\text{out}}(j^{\text{out}}) = T\left[\exp\left(-i\int j^{\text{out}}(x')\cdot A^{\text{out}}(x')dx'\right)\right].$$

Our basis states are

$$|p,\{k_i\},\alpha^{\text{in}}\rangle^{\text{in}} = s^{\text{in}}(j^{\text{in}})\,|p,\{k_i\}\rangle^{\text{in}}\,, \tag{2.8}$$
$$|p,\{k_f'\},\alpha^{\text{out}}\rangle = s^{\text{out}}(j^{\text{out}})\,|p',\{k_f'\}\rangle^{\text{out}}\,, \tag{2.9}$$
$$|p,\{k\},\alpha\rangle = |p\rangle \otimes |\{k\},\alpha\rangle\,, \tag{2.10}$$

where polarization indices have been suppressed.

We have immediately that states $|\alpha^{\text{in}}\rangle^{\text{in}}$ and $|\alpha^{\text{out}}\rangle^{\text{out}}$ are coherent[15,16]:

$$[A_\mu^{\text{in}}(x)]^{(-)}|\alpha^{\text{in}}\rangle^{\text{in}} = [\alpha_\mu^{\text{in}}(x)]^{(-)}|\alpha^{\text{in}}\rangle^{\text{in}}\,,$$

and similarly for in $\to$ out.

The basis states $|p,\{k\},\alpha\rangle$ have the same inner product as the usual Fock states if the incoming and outgoing laser pulses are the same. However, when they are different, that is, when $\alpha^{\text{in}} \neq \alpha^{\text{out}}$, the inner product ${}^{\text{out}}\langle\alpha^{\text{out}}|\alpha^{\text{in}}\rangle^{\text{in}}$ will depend upon $j(x)$.

A sufficient condition that $s(j)$ exist is that

$$|\tilde{j}(k)|^2 \to 0$$

faster than

$$|\vec{k}|^{-(2+\gamma)}, \quad \gamma > 0$$

where

$$j_\mu(x) = \int \tilde{j}_\mu(k)e^{-ikx}dk\,.$$

This condition can be obtained by normal ordering s,

$$s(j) = C(j):s(j):\,, \tag{2.11}$$

and then counting powers of k in $C(j)$.[22] It turns out that

$$C(j) = \exp\left[\tfrac{1}{4}\int j_\mu(x)\Delta_F(x-y)j^\mu(y)dx\,dy\right],$$

where

$$\Delta_F(x-y) = \frac{2i}{(2\pi)^4}\int dp\,\frac{e^{-ip(x-y)}}{p^2+i\epsilon}\,.$$

III. CALCULATION OF SCATTERING AMPLITUDES

Now that our basis states have been defined and field equations given, we may proceed with the calculation of S-matrix elements. For the case where we have only the laser pulse and an electron present in the initial state, the S-matrix element is given by

$$S_{fi} = {}^{\text{out}}\langle\alpha^{\text{out}},\{k_f'\},p'\,|p,\alpha^{\text{in}}\rangle^{\text{in}}\,.$$

Here, in keeping with the Lehmann-Symanzik-Zimmermann (LSZ) formalism,[11,12] the electron and laser pulse do not interact in the remote past or future and are therefore represented by wave-packet states. Thus, we assume scattering boundary conditions for the ELP interaction.

Upon reducing out the electrons in the usual manner we obtain

$$S_{fi} = (i)^2\int dx\,dy\,\bar{u}_{p'}^{(s')}(x)\vec{D}_x\,{}^{\text{out}}\langle\alpha^{\text{out}},\{k_f'\}|\,T(\psi(x)\bar{\psi}(y))\,|\alpha^{\text{in}}\rangle^{\text{in}}\overleftarrow{D}_y u_p^{(s)}(y)\,,$$

where $u_p^{(s)}(y)$ is a wave-packet solution of the free Dirac equation. Using Eqs. (2.1), (2.8), (2.9), and (2.10), we obtain

$$V \equiv {}^{\text{out}}\langle\alpha^{\text{out}},\{k_f'\}|\,T(\psi(x)\bar{\psi}(y))\,|\alpha^{\text{in}}\rangle^{\text{in}}$$
$$= {}^{\text{out}}\langle\{k_f'\}|\,s^{\dagger\,\text{out}}(j^{\text{in}}+j)T(\psi(x)\bar{\psi}(y))s^{\text{in}}(j^{\text{in}})\,|0\rangle^{\text{in}}\,.$$

Now

$$s(j_1+j_2) = e^{i\phi}s(j_1)s(j_2)\,,$$

where ϕ is real and j_1, j_2 are arbitrary.[15] It is also true that

$$T\left[\exp\left(-i\int j\cdot A^{\mathrm{out\,(in)}}dx\right)\right] = e^{i\phi'}\exp\left(-i\int j\cdot A^{\mathrm{out\,(in)}}dx\right),$$

where

$$\phi' = \tfrac{1}{4}\int j_\mu(x)j^\mu(x')\epsilon(x-x')\Delta(x-x')dx\,dx',$$

and ϕ' is real. That is, time ordering of s contributes only a phase factor.[23]

At this point, we must use the fundamental reduction formula for coherent states[16]

$$\exp\left(i\int j_1\cdot A^{\mathrm{out}\,(-)}dx\right)T(\psi\bar\psi)\exp\left(-i\int j_1\cdot A^{\mathrm{in}(-)}dx\right) = T\left(\psi\bar\psi\exp\left(-i\int J\cdot\alpha_1^{(+)}dx\right)\right), \tag{3.1}$$

which may be rewritten in the form

$$\exp\left(i\int A^{\mathrm{out}}\overleftrightarrow{\partial}_0\alpha_1^{(+)}d^3x\right)T(\psi\bar\psi)\exp\left(-i\int A^{\mathrm{in}}\overleftrightarrow{\partial}_0\alpha_1^{(+)}d^3x\right) = T\left(\psi\bar\psi\exp\left(-i\int J\cdot\alpha_1^{(+)}dx\right)\right),$$

where

$$\alpha_1^\mu(x) = \int\Delta(x-y)j_1^\mu(y)dy,$$

and

$$\alpha_1^\mu(x) = \begin{cases}\alpha^{\mathrm{in}}(x), & j_1 = j^{\mathrm{in}}\\ \alpha^{\mathrm{out}}(x), & j_1 = j^{\mathrm{out}}\\ \alpha'(x)\equiv\alpha^{\mathrm{out}}-\alpha^{\mathrm{in}}, & j_1 = j\,.\end{cases} \tag{3.2}$$

The identity also holds if $\alpha_1^{(+)}\rightleftarrows\alpha_1^{(-)}$ and $\alpha_1^{(+)}\to\alpha_1$.

Using the reduction formula we obtain

$$V\equiv {}^{\mathrm{out}}\!\left\langle 0\left|\{k_f'\}s^{\dagger\mathrm{out}}(j)T\left(\psi\bar\psi\exp\left(-i\int J\cdot\alpha^{\mathrm{in}}dx\right)\right)\right|0\right\rangle^{\mathrm{in}}.$$

From Eqs. (2.11), (3.1), and (3.2), we find

$$V\equiv C^*(j)\;{}^{\mathrm{out}}\!\left\langle 0\left|\prod_i a_f^{\mu_i'}(k_i')T\left(\psi\bar\psi\exp\left(-i\int J\cdot(\alpha^{\mathrm{in}}+\alpha'^{(+)})dx\right)\right)\right|0\right\rangle^{\mathrm{in}},$$

where the operators $a_f^{\mu_i'}(k_i')$ are defined through Eqs. (2.3), (2.6), and (2.9). We changed the index "f" of the photons to "i" to avoid confusion.

Using the reduction formula[16]

$${}^{\mathrm{out}}\langle 0|a^{\nu_i}(k_i')T(\psi\bar\psi J)|0\rangle^{\mathrm{in}} = i\int d^3x_i f_{k_i'}^*(x_i)\overleftrightarrow{\Box}_{x_i}\,{}^{\mathrm{out}}\langle 0|T(A^{\nu_i}(x_i)\psi\bar\psi J)|0\rangle^{\mathrm{in}},\quad f_k(x)\equiv(2\pi)^{-3/2}e^{-ikx}$$

and Eqs. (2.3) and (2.6), and upon dropping disconnected photon momenta, we obtain

$${}^{\mathrm{out}}\!\left\langle 0\left|\prod_{i=1}^n a_f^{\nu_i}(k_i')T(\psi\bar\psi J)\right|0\right\rangle^{\mathrm{in}} = (i)^n\int\prod_{i=1}^n d^3x_i f_{k_i'}^*(x_i)\overleftrightarrow{\Box}_{x_i}\,{}^{\mathrm{out}}\langle 0|T([A^{\nu_1}(x_1)-\alpha^{\nu_1}(x_1)]\cdots[A^{\nu_n}(x_n)-\alpha^{\nu_n}(x_n)]\psi\bar\psi J)|0\rangle^{\mathrm{in}}.$$

Thus, the scattering amplitude is

$$\begin{aligned}S_{fi} &= {}^{\mathrm{out}}\langle\alpha^{\mathrm{out}},\{k_i'\},p'|p,\alpha^{\mathrm{in}}\rangle^{\mathrm{in}}\\ &= (i)^{n+2}C^*(j)\int\bar u_{p'}^{(s')}(x)\prod_{i=1}^n d^3x_i f_{k_i'}^*(x_i)\overrightarrow{D}_x\overleftrightarrow{\Box}_{x_i}\\ &\quad\times{}^{\mathrm{out}}\!\left\langle 0\left|T\left([A^{\nu_i}(x_i)-\alpha^{\nu_i}(x_i)]\psi(x)\bar\psi(y)\exp\left(-i\int J\cdot(\alpha^{\mathrm{in}}+\alpha'^{(+)})dx'\right)\right)\right|0\right\rangle^{\mathrm{in}}\\ &\quad\times\overleftarrow{D}_y u_p^{(s)}(y)dx\,dy\,.\end{aligned} \tag{3.3}$$

If we make the polarization four-vector explicit, we obtain

$$S_{fi} = (i)^{n+2}C^*(j)\int \bar{u}_p^{(s')}(x)\prod_{i=1}^{n} dx_i g_{\lambda_i' \lambda_i'} f^*_{k_i'}(x_i)\epsilon_{\mu i}(k_i', \lambda_i')\vec{D}_x \vec{\Box}_{x_i} W \vec{D}_y u_p^{(s)}(y)dx\,dy\,,$$

where

$$W \equiv {}^{\text{out}}\Big\langle 0\Big| T\Big(\big[A^{\mu_i}(x_i) - \alpha^{\mu_i}(x_i)\big]\psi(x)\bar{\psi}(y)\exp\Big(-i\int J\cdot(\alpha^{\text{in}} + \alpha'^{(+)})dx'\Big)\Big)\Big|0\Big\rangle^{\text{in}}\,.$$

The presence of the factors $[A^{\mu_i}(x_i) - \alpha^{\mu_i}(x_i)]$ indicates that the use of the Furry picture[14] may be judicious. We therefore define a new interpolating field

$$A' \equiv A - \alpha\,.$$

Thus we obtain

$$\Box A' = J - j\,.$$

Equations (2.2), in terms of A', give rise to the interaction Lagrangian

$$\mathcal{L}' = -(J-j)\cdot(A' + \alpha) = -\mathcal{H}'\,.$$

We note that the term $j_\mu(x)\alpha^\mu(x)$ may be neglected.[24]

The relationship between the Heisenberg representation and the Furry picture is given by the following equations[14]:

$$i\frac{\delta U[\sigma]}{\delta\sigma(x)} = \mathcal{H}_I'(x)U[\sigma, \sigma_0]\,,$$

$$O_I[\sigma] = U[\sigma, \sigma_0]\,O\,U^\dagger[\sigma, \sigma_0]\,,$$

$$|\psi_I[\sigma]\rangle = U[\sigma, \sigma_0]\,|\psi[\sigma, \sigma_0]\rangle\,,$$

$$i\frac{\delta V[\sigma]}{\delta\sigma(x)} = J_I(x)\cdot\alpha(x)V[\sigma]\,,$$

$$O_F[\sigma] = V^{-1}[\sigma]O_I[\sigma]V[\sigma]\,,$$

$$|\psi_F[\sigma]\rangle = V^{-1}[\sigma]\,|\psi_I[\sigma]\rangle\,,$$

where σ and σ_0 are spacelike surfaces, σ_0 fixed. The subscript I denotes the interaction picture, while F denotes the Furry picture. No subscript denotes the Heisenberg representation. O is an arbitrary operator while $|\psi[\sigma_0]\rangle$ is a state vector of the physical system.

From these equations we may obtain the field equations for $A_\mu'(x)$ and $\psi(x)$ in the Furry picture[14]:

$$\Box A_F' = \Box A_I' = 0\,,$$

$$\vec{D}\psi_F = e\gamma\cdot\alpha\psi_F\,,$$

$$A_F' = V^{-1}A_I'V = A_I'\,,$$

$$\psi_F = V^{-1}\psi_I V\,.$$

The S matrix is

$$S = T\Big(\exp\Big(-i\int (J_F - j)\cdot A_F'\,dx\Big)\Big)\,.$$

Dropping the prime on A_F', we obtain for W the expression

$$W = \frac{\langle 0|T(\cdots A_F^{\mu_i}(x_i)\cdots\psi_F(x)\bar{\psi}_F(y)\exp[-i\int(J_F - j)\cdot(A_F + \alpha^{\text{in}} + \alpha'^{(+)})dx])|0\rangle}{\langle 0|S|0\rangle}\,,$$

where $|0\rangle$ is the Furry vacuum. We note that $J_p(x) \equiv \langle 0|J_F(x)|0\rangle$ does not vanish for an arbitrary field $\alpha_\mu(x)$.[14,25-27] In fact this term gives use to all the vacuum polarization phenomena one encounters and contains a logarithmically divergent part which effects a charge renormalization.[14,25-27] This means that in our expression for W one must acknowledge that $J_F^\mu(x)$ is not a normally ordered operator, and therefore it is necessary to include any Wick contractions of $J_F^\mu(x)$ that might occur in the expansion of a time-ordered product for a particular process.[28]

However, it is well known that the polarization induced in the vacuum due to a plane wave whose field strength tensor is $F_{\mu\nu}(\tau) = f_{\mu\nu}(\tau)$, where $\tau = \eta^\mu x_\mu$, $\eta^\mu\eta_\mu = 0$, $\eta^\mu f_{\mu\nu} = 0$, F is an arbitrary function of τ, and $f_{\mu\nu}$ is a constant tensor, is zero.[26] We shall therefore consider for simplicity $\alpha_\mu^{\text{in}}(x)$ to be a plane-wave packet. This then implies that the vacuum polarization of our ELP process is due only to the potential

$$\alpha_p^\mu(x) \equiv (\alpha^\mu(x) - \alpha^{\text{in}\,\mu}(x))$$

$$= \int \Delta^{\text{ret}}(x-y)j^\mu(y)dy\,.$$

If we assume that the intense laser pulse incident on one or more electrons will not be appreciably changed by the interaction, then we can consider that $j_\mu(x) \approx 0$ and $\alpha^{\text{in}} \approx \alpha^{\text{out}}$. If we further assume that $\Box j_\mu \approx 0$, then $J_p(x)$ can also be neglected.[2?] The expression for W becomes

$$W=\frac{\langle 0|T(\cdots A_F^{\mu_i}(x_i)\cdots\psi_F(x)\bar\psi_F(y)\exp[-i\int J_F\cdot(A_F+\alpha^{\rm in})dx'])|0\rangle}{\langle 0|T(\exp[-i\int J_F\cdot A_F dx'])|0\rangle}, \tag{3.4}$$

where now $J_F^\mu(x)$ can be considered a normally ordered operator.

As usual the denominator $\langle 0|S|0\rangle$ represents the vacuum Feynman graphs and eliminates all disconnected graphs in the expansion of W.[14,29,30]

We note that the more realistic case where $j_\mu\neq 0$ and $\alpha_\mu^{\rm in}\neq\alpha_\mu^{\rm out}$ can also be handled by our formalism.

IV. STIMULATED COMPTON SCATTERING

We now apply the results of the previous sections to the process of stimulated Compton scattering (we assume $j_\mu\approx 0$, i.e., pulse depletion is neglected) where the scattering amplitude is given by

$$S_{fi}={}^{\rm out}\langle\alpha^{\rm out},k'(\lambda'),e'|e,\alpha^{\rm in}\rangle^{\rm in}.$$

For W, we obtain

$$W=\frac{\langle 0|T(A_F^\nu(z)\psi_F(x)\bar\psi(y)\exp\{-i\int J_F(u)\cdot[A_F(u)+\alpha^{\rm in}(u)]du\})|0\rangle}{\langle 0|S|0\rangle}.$$

The zeroth-order term vanishes. The first-order term gives (connected graphs only)

$$W(1)=(ie)\int du[-\tfrac12 g^{\nu\rho}D_F(z-u)][-\tfrac12 S_F^{(e)}(x,u)]\,\gamma_\rho[-\tfrac12 S_F^{(e)}(u,y)], \tag{4.1}$$

where

$$\langle 0|T(A_F^\mu(x)A_F^\nu(x'))|0\rangle=-\tfrac12 D_F(x-x')g^{\mu\nu},$$

$$\langle 0|T(\psi_F(x)\bar\psi_F(y))|0\rangle=-\tfrac12 S_F^{(e)}(x,y).$$

Here

$$\vec D_x S_F^{(e)}(x,y)=2i\delta(x-y)+e\gamma^\mu\alpha_\mu^{\rm in}(x)S_F^{(e)}(x,y), \tag{4.2}$$

$$S_F^{(e)}(x,y)\overleftarrow{D}_y=-2i\delta(x-y)+eS_F^{(e)}(x,y)\gamma^\mu\alpha_\mu^{\rm in}(y), \tag{4.3}$$

$$\Box_x D_F(x-x')=-2i\delta(x-x'), \tag{4.4}$$

$$S_{fi}(1)=(i)^3\int dx\,dy\,dz\,g_{\lambda'\lambda'}f_{k'}^*(z)\epsilon_\nu(k',\lambda')\bar u_{p'}^{(s')}(x)[\vec D_x\vec\Box_z W(1)\overleftarrow{D}_y]u_p^{(s)}(y), \tag{4.5}$$

where

$$f_{k'}(z)=(2\pi)^{-3/2}e^{-ik'z}.$$

We obtain

$$\begin{aligned}\vec D_x\vec\Box_z W(1)\overleftarrow{D}_y=\;&\delta(x-z)(-e\gamma^\nu)\delta(z-y)+\tfrac12 i(\gamma\cdot[-e\alpha^{\rm in}(x)]S_F^{(e)}(x,y)[-e\gamma^\nu]\,\delta(z-y))\\&+(-\tfrac12 i)([-e\gamma^\nu]S_F^{(e)}(z,y)\gamma\cdot[-e\alpha^{\rm in}(y)]\delta(x-z))\\&+\tfrac14(\gamma\cdot[-e\alpha^{\rm in}(x)]S_F^{(e)}(x,z)[-e\gamma^\nu]S_F^{(e)}(z,y)\gamma\cdot[-e\alpha^{\rm in}(y)]).\end{aligned}$$

The first term does not contribute because the electrons (in and out) and the scattered photon are on their respective mass shells.

We can elucidate the structure of the remaining terms by utilizing the Fourier transform of $S_F^{(e)}(x,y)$ and $\alpha^{\rm in}(x)$:

$$S^{(e)}(x,y)=\int dp_1dp_2e^{-ip_1x}e^{ip_2y}S_F^{(e)}(p_1,p_2),$$

$$\begin{aligned}\alpha_\mu^{\rm in}(x)&=\int dq\,\alpha_\mu^{\rm in}(q)\,e^{-iqx}\\&=\int dq\sum_\lambda\alpha^{\rm in}(q,\lambda)\epsilon_\mu(q,\lambda)e^{-iqx}.\end{aligned}$$

We find that the second term in Eq. (4.1) contributes to

$$S_{fi}(1) = S_{fi}^{(1)}(1) + S_{fi}^{(2)}(1) + S_{fi}^{(3)}(1) + S_{fi}^{(4)}(1), \quad S_{fi}^{(1)}(1) = 0 \tag{4.6}$$

the expression

$$S_{fi}^{(2)}(1) = \tfrac{1}{2}(2\pi)^{7/2} e^2 g_{\lambda'\lambda'} \bar{u}(p', s') M^{(2)} u(p, s),$$

where $\bar{u}$ and u are Dirac spinors such that

$$U_p^{(s)}(x) = f_p(x) u(p, s)$$

and

$$M^{(2)} = \sum_{\lambda} \int \gamma \cdot \epsilon(q, \lambda) \alpha^{\text{in}}(q, \lambda) S_F^{(e)}(p' - q, p - k') \gamma \cdot \epsilon(k', \lambda') dq .$$

From the third term, we obtain

$$S_{fi}^{(3)}(1) = -\tfrac{1}{2}(2\pi)^{7/2} e^2 g_{\lambda'\lambda'} \bar{u}(p, s) M^{(3)} u(p, s),$$

where

$$M^{(3)} = \sum_{\lambda} \int dq\, \gamma \cdot \epsilon(k', \lambda') S_F^{(e)}(p' + k', p + q) \gamma \cdot \epsilon(q, \lambda) \alpha^{\text{in}}(q, \lambda).$$

Similarly, for the fourth term, we obtain

$$S_{fi}^{(4)}(1) = \tfrac{1}{4} i (2\pi)^{15/2} e^3 g_{\lambda'\lambda'} \bar{u}(p', s') M^{(4)} u(p, s),$$

where

$$M^{(4)} = \sum_{\lambda\lambda_1} \int \gamma \cdot \epsilon(q, \lambda) \alpha^{\text{in}}(q, \lambda) S_F^{(e)}(p' - q, p_1) \gamma \cdot \epsilon(k', \lambda') S_F^{(e)}(p_1 + k', p + q_1) \gamma \cdot \epsilon(q_1, \lambda_1) \alpha^{\text{in}}(q_1, \lambda_1) dp_1 dq\, dq_1 .$$

If we write the Furry picture propagator in terms of free propagators using the expression[31]

$$\begin{aligned} S_F^{(e)}(p_1, p_2) = {} & S_F(p_1)\delta(p_1 - p_2) + [(2\pi)^4/(-2i)] S_F(p_1) e\gamma \cdot \alpha^{\text{in}}(p_1 - p_2) S_F(p_2) \\ & + [(2\pi)^4/(-2i)]^2 S_F(p_1) \left[\int dq\, dq_1 e\gamma \cdot \alpha^{\text{in}}(q) S_F^{(e)}(p_1 - q, p_2 + q_1) e\gamma \cdot \alpha^{\text{in}}(q_1) \right] S_F(p_2), \end{aligned} \tag{4.7}$$

where

$$S_F(p) = \frac{-2i}{(2\pi)^4} \frac{1}{\gamma \cdot p - m},$$

we immediately see that $M^{(2)}$ represents the sum of all Feynman diagrams where the scattered photon k' is emitted "first" (see Fig. 1), $M^{(3)}$ represents the sum of all the diagrams where the scattered photon is emitted "last" (see Fig. 2), $M^{(4)}$ represents the sum of all diagrams where the scattered photon is emitted "somewhere in between" (see Fig. 3).[32]

Thus $S_{fi}(1)$ is the total, nonlinear amplitude for stimulated Compton scattering, assuming that $j_\mu(x) \approx 0$ and $J_p(x) \approx 0$ and ignoring the fact that the laser pulse is actually a Bose-Einstein system. Substituting Eq. (4.7) into Eq. (4.6) and considering terms of order e^2 only, we obtain what is essentially the Klein-Nishina scattering amplitude.[33] The terms in $S_{fi}(1)$ of order higher than e^2 give rise to nonlinear effects. Upon computing S_{fi} to higher order in the Furry picture, that is, $S_{fi}(2)$, $S_{fi}(3), \ldots$, we find new nonlinear corrections not previously given in the literature. It appears that these new corrections originate in our use of a full quantum-mechanical description of the

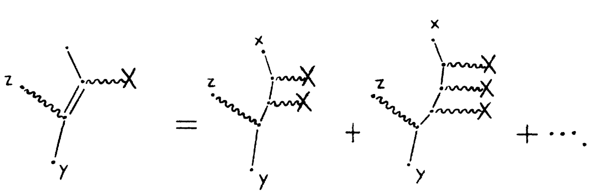

FIG. 1. Diagram in the Furry picutre which represents the sum of all Feynman diagrams where the scattered photon k' is emitted "first."

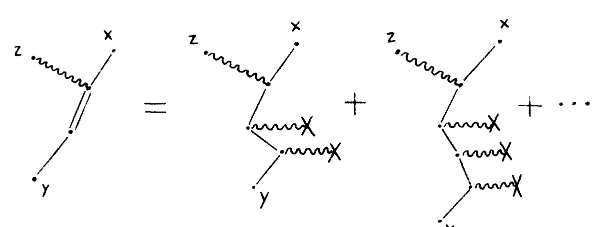

FIG. 2. Diagram in the Furry picture which represents the sum of all Feynman diagrams where the scattered photon k' is emitted "last."

laser pulse as a Bose-Einstein system via the coherent-state formalism. They involve the dimensionless parameter (eA/m) (A characterizes the amplitude of the laser pulse), which is of the order of 10^{-2} for present high-intensity optical lasers. As an example of the new nonlinear corrections, we illustrate the computation of $S_{fi}(2)$. Considering connected graphs only,

$$W(2)=(-i)^2(-e)^2\int du_1du_2(-\tfrac{1}{2})^4D_F(z-u_1)[S_F^{(e)}(x,u_1)\gamma^\nu S_F^{(e)}(u_1,u_2)\gamma\cdot\alpha^{\text{in}}(u_2)S_F^{(e)}(u_2,y)$$
$$+S_F^{(e)}(x,u_2)\gamma\cdot\alpha^{\text{in}}(u_2)S_F^{(e)}(u_2,u_1)\gamma^\nu S_F^{(e)}(u_1,y)]$$

and

$$\vec{D}_x\vec{\Box}_zW(2)\vec{D}_y=-\tfrac{1}{2}ie\delta(x-z)\gamma^\nu S_F^{(e)}(z,y)[-e\gamma\cdot\alpha^{\text{in}}(y)]-\tfrac{1}{2}ie\delta(z-y)[-e\gamma\cdot\alpha^{\text{in}}(x)]S_F^{(e)}(x,z)\gamma^\nu$$
$$+\tfrac{1}{4}e[-e\gamma\cdot\alpha^{\text{in}}(x)]S_F^{(e)}(x,z)\gamma^\nu S_F^{(e)}(z,y)[-e\gamma\cdot\alpha^{\text{in}}(y)]$$
$$-\tfrac{1}{4}e[-e\gamma\cdot\alpha^{\text{in}}(x)]S_F^{(e)}(x,z)\gamma^\nu S_F^{(e)}(x,y)[-e\gamma\cdot\alpha^{\text{in}}(y)]$$
$$-\tfrac{1}{4}e\delta(x-z)\int du_2\gamma^\nu S_F^{(e)}(z,u_2)[-e\gamma\cdot\alpha^{\text{in}}(u_2)]S_F^{(e)}(u_2,y)[-e\gamma\cdot\alpha^{\text{in}}(y)]$$
$$-\tfrac{1}{4}e\delta(z-y)\int du_2[-e\gamma\cdot\alpha^{\text{in}}(x)]S_F^{(e)}(x,u_2)[-e\gamma\cdot\alpha^{\text{in}}(u_2)]S_F^{(e)}(u_2,z)\gamma^\nu$$
$$-\tfrac{1}{8}ie\int du_2[-e\gamma\cdot\alpha^{\text{in}}(x)]S_F^{(e)}(x,z)\gamma^\nu S_F^{(e)}(z,u_2)[-e\gamma\cdot\alpha^{\text{in}}(u_2)]S_F^{(e)}(u_2,y)[-e\gamma\cdot\alpha^{\text{in}}(y)]$$
$$-\tfrac{1}{8}ie\int du_2[-e\gamma\cdot\alpha^{\text{in}}(x)]S_F^{(e)}(x,u_2)[-e\gamma\cdot\alpha^{\text{in}}(u_2)]S_F^{(e)}(u_2,z)\gamma^\nu S_F^{(e)}(u_2,y)[-e\gamma\cdot\alpha^{\text{in}}(y)]\,.$$

Thus,

$$S_{fi}(2)=(i)^3\bar{u}(p',s')\int dx\,dy\,dz\,f^*_{p'}(x)f_p(y)g_{\lambda'\lambda'}f^*_{k'}(z)\epsilon_\nu(k',\lambda')[\vec{D}_x\vec{\Box}_zW(2)\vec{D}_y]u(p,s)\,.$$

Further evaluation of $S_{fi}(2)$ is not very illuminating.

It is interesting to note that for the spinor electron, with nonscattering boundary conditions (i.e., the electron and laser pulse are never decoupled) and using Volkov wave packets, the amplitude for stimulated Compton scattering can be obtained by making the following replacements in Eq. (4.5):

$$\vec{D}_x\to[\vec{D}_x-e\gamma\cdot\alpha^{\text{in}}(x)]\equiv\vec{D}_{Vx}\,,\tag{4.8}$$

$$\vec{D}_y\to[\vec{D}_y-e\gamma\cdot\alpha^{\text{in}}(x)]\equiv\vec{D}_{Vy}\,.\tag{4.9}$$

In that case, we find that

$$S_{fi}(1)=(i)^3\int dx\,dy\,dz\,\bar{U}^{V(s')}_{p'}(x)[g_{\lambda'\lambda'}f_{k'}(z)\epsilon_\nu(k',\lambda')]$$
$$\times\vec{D}_{Vx}\vec{\Box}_zW(1)\vec{D}_{Vy}U^{V(s)}_p(y)\,,\tag{4.10}$$

where U^V and $\bar{U}^V$ now satisfy the equations

$$\bar{U}^{V(s')}_{p'}(x)\vec{D}_{Vx}=0\,,\tag{4.11}$$

$$\vec{D}_{Vy}U^{V(s)}_p(y)=0\,.\tag{4.12}$$

Using Eqs. (4.1)–(4.4) and the replacements, we obtain

$$S_{fi}(1)=\frac{ie}{(2\pi)^{3/2}}\int g_{\lambda'\lambda'}e^{ik'x}\,\bar{U}^{V(s')}_{p'}(x)\gamma\cdot\epsilon(k',\lambda')$$
$$\times U^{V(s)}_p(x)dx\,.\tag{4.13}$$

Presumably, this is the stimulated Compton scattering amplitude for the experimental configuration where one observes the scattered photon k' while the laser pulse and electron are still interacting.

It should be noted that because the ELP interaction is intrinsically nonlinear, one cannot construct the scattering amplitude for an arbitrary pulse in terms of scattering amplitudes using monochromatic fields. One must, in general,

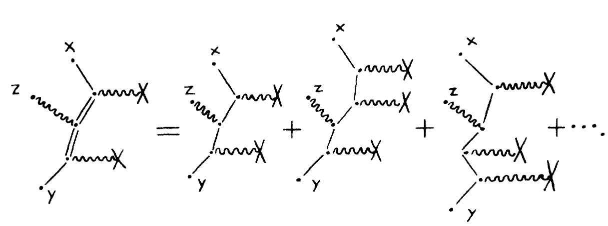

FIG. 3. Diagram in the Furry picture which represents the sum of all Feynman diagrams where the scattered photon k' is emitted "somewhere in between."

construct amplitudes using normalizable three-dimensional wave packet for the laser pulse.[34]

Our method for computing ELP scattering amplitudes uses only the familiar Feynman-Dyson S-matrix theory and the LSZ formalism and thus is much easier to utilize than previous field-theoretic techniques—although first the Furry propagator must be known or approximated and then nontrivial integrations be performed for any calculation. It should also be noted that the extension of our method to situations where an arbitrary number of electrons and/or positrons are in the "in" or the "out" state is straightforward. One simply applies the reduction technique as many times as needed.

V. IDFS

Although the intent of this paper is to present a quantum field-theoretic formalism for handling ELP processes, we comment briefly on the subject of the existence of the IDFS for stimulated Compton scattering. For scattering or nonscattering boundary conditions, the amplitude depends on $W(1)$ and therefore on the propagator for the electron in the laser pulse. Because of the intimate relationship between the poles of the propagator and the IDFS, to make a statement about existence of the IDFS is tantamount to making a statement about the pole behavior of the propagator $S_F^{(e)}$. For the monochromatic plane-wave case, the work of a number of authors[35] has shown that the electron will experience a mass shift $\delta m^2 \propto A^2$ (A is the plane-wave amplitude) given by the poles of the propagator. More generally, Dittrich[36] has shown that for any plane-wave field, the electron will experience a mass shift $\delta m^2 = \delta m^2(\alpha^\mu(x))$. Thus, one can reasonably conclude—at least for the plane-wave case—that the IDFS exists.

For the more realistic situation where the laser pulse is a normalizable three-dimensional wave packet, not much is known about the propagator $S_F^{(e)}$. However, one expects the pole behavior of the propagator to differ considerably from that of a free-particle propagator. Indirect evidence attesting to this is the quantum field-theoretic results of Neville and Rohrlich[37] and Dawson and Fried[34] and the semiclassical results of Reiss,[38,39] where wave-packet descriptions were used for the electron and laser pulse (Neville and Rohrlich used a monochromatic square pulse).

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