Effective-Lagrangian formulation of generalized vector dominance. II

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As in a preceding paper we generalize the Lagrangian of Lee and Zumino to include several mutually interacting vector mesons. The treatment is more general in the sense that *all* possible interactions between the vector mesons, compatible with the field-current proportionality relations, are now discussed. It is moreover demonstrated that also the fields corresponding to the *physical* vector mesons satisfy a field-current proportionality relation of exactly the same form. Comparison of the different schemes and their implications for the magnetic moments of the vector mesons are discussed.

I. INTRODUCTION

In a preceding paper,[1] hereafter denoted by I, we generalized the Lagrangian of Lee and Zumino[2] to include several mutually interacting vector mesons. The mutual interactions were chosen so as to guarantee a current-fields proportionality relation analogous to the original current-field identity of Kroll, Lee, and Zumino.[3] It was our purpose in that paper to demonstrate that this goal can be achieved even with nontrivial mixings of the vector mesons, and we did not care so much about the most general mixing scheme. Thus we considered a simplified situation where g — the trilinear coupling constant — was identical for the fields ρ and ρ'. This led to *two* possible coupling schemes, namely the Lagrangian (I; 3.3) as well as the Lagrangian (I; 3.3b). ["(I; 3.3)" denotes Eq. (3.3) of paper I.]

In Sec. II we shall show that when the trilinear coupling constants g and g' for ρ and ρ' respectively are taken to be unequal, only scheme (I; 3.3) is possible. The current-fields proportionality relation now reads

$$j_\mu^{\mathrm{em}} = e\left(\frac{m^2}{g}\rho_\mu^3 + \frac{m'^2}{g'}\rho_\mu'^3\right). \tag{1.1}$$

In Sec. III it will be shown that when expressed in terms of the *physical* fields ρ^a, ρ^b — i.e., the two fields which diagonalize the quadratic part of the Lagrangian — the electromagnetic current is given through

$$j_\mu^{\mathrm{em}} = e\left(\frac{m_a{}^2}{g_a}\rho_\mu^{a,3} + \frac{m_b{}^2}{g_b}\rho_\mu^{b,3}\right), \tag{1.2}$$

with m_a, m_b the masses of ρ^a, ρ^b and with

$$\frac{1}{g_a} = \frac{a}{g} + \frac{b}{g'}, \qquad \frac{1}{g_b} = \frac{c}{g} + \frac{d}{g'}, \tag{1.3}$$

where a, b, c, d are the matrix elements of the transformation between the physical fields ρ^a, ρ^b and the original ρ and ρ',

$$\vec{\rho}_\mu^{\,a} = a\vec{\rho}_\mu + b\vec{\rho}_\mu{}', \qquad \vec{\rho}_\mu^{\,b} = c\vec{\rho}_\mu + d\vec{\rho}_\mu{}'. \tag{1.4}$$

In Sec. IV a mixing in the quadratic mass terms of the Lagrangian, i.e., a "mass mixing scheme," will be considered. It will also be shown here that it is possible to arrange things so as to obtain (1.2)–(1.4). However, in contrast to the mixing in the trilinear terms, it will now turn out that the magnetic moments of the charged ρ^a and ρ^b are *not* affected by the mixing and are always exactly 2.

In Sec. V we shall show that the generalized-vector-meson-dominance scheme of Fujikawa and of the Berkeley group[4] is in fact identical to our mass mixing scheme of Sec. IV.

II. EFFECTIVE-LAGRANGIAN FORMULATION OF GENERALIZED VECTOR DOMINANCE – THE TWO-MESON CASE

This section follows closely Sec. III of I, with the sole difference that now the trilinear terms in the Lagrangian have different coupling constants, i.e.,

$$\begin{aligned}\mathcal{L}_V = &-\tfrac{1}{4}(\vec{G}_{\mu\nu})^2 + \tfrac{1}{2}m^2(\vec{\rho}_\mu)^2 + \tfrac{1}{2}g\vec{G}_{\mu\nu}\cdot(\vec{\rho}_\mu\times\vec{\rho}_\nu) + \tfrac{1}{2}\beta\vec{G}_{\mu\nu}\cdot(\vec{\rho}_\mu{}'\times\vec{\rho}_\nu{}') \\ &-\tfrac{1}{4}(\vec{G}_{\mu\nu}{}')^2 + \tfrac{1}{2}m'^2(\vec{\rho}_\mu{}')^2 + \tfrac{1}{2}g'\vec{G}_{\mu\nu}{}'\cdot(\vec{\rho}_\mu{}'\times\vec{\rho}_\nu{}') + \tfrac{1}{2}\beta'\vec{G}_{\mu\nu}{}'\cdot(\vec{\rho}_\mu\times\vec{\rho}_\nu) - \tfrac{1}{2}\alpha\vec{G}_{\mu\nu}\cdot\vec{G}_{\mu\nu}{}' + F(\vec{\rho}_\mu, \vec{\rho}_\mu{}').\end{aligned} \tag{2.1}$$

As in I we now construct

$$\vec{S}_\mu = \frac{\partial \mathcal{L}}{\partial \vec{\rho}'_{\nu,\mu}} \times \vec{\rho}_\nu = [-\vec{G}_{\mu\nu} - \alpha \vec{G}'_{\mu\nu} + g(\vec{\rho}_\mu \times \vec{\rho}_\nu) + \beta(\vec{\rho}'_\mu \times \vec{\rho}'_\nu)] \times \vec{\rho}_\nu ,$$
$$\vec{S}'_\mu = \frac{\partial \mathcal{L}}{\partial \vec{\rho}'_{\nu,\mu}} \times \vec{\rho}'_\nu = [-\vec{G}'_{\mu\nu} - \alpha \vec{G}_{\mu\nu} + g'(\vec{\rho}'_\mu \times \vec{\rho}'_\nu) + \beta'(\vec{\rho}_\mu \times \vec{\rho}_\nu)] \times \vec{\rho}'_\nu . \tag{2.2}$$

Isospin invariance of (2.1) implies

$$\partial_\mu(\vec{S}_\mu + \vec{S}'_\mu) = 0 . \tag{2.3}$$

The equations of motion implied by (2.1) are

$$\partial_\nu\left(\frac{\partial \mathcal{L}_V}{\partial \vec{\rho}_{\mu,\nu}}\right) = m^2 \vec{\rho}_\mu - g(\vec{G}_{\mu\nu} \times \vec{\rho}_\nu) - \beta'(\vec{G}'_{\mu\nu} \times \vec{\rho}_\nu) + \frac{\partial F}{\partial \vec{\rho}_\mu} ,$$
$$\partial_\nu\left(\frac{\partial \mathcal{L}_V}{\partial \vec{\rho}'_{\mu,\nu}}\right) = m'^2 \vec{\rho}'_\mu - g'(\vec{G}'_{\mu\nu} \times \vec{\rho}'_\nu) - \beta(\vec{G}_{\mu\nu} \times \vec{\rho}'_\nu) + \frac{\partial F}{\partial \vec{\rho}'_\mu} . \tag{2.4}$$

As before, since $\partial \mathcal{L}_V / \partial \vec{\rho}_{\nu,\mu}$ and $\partial \mathcal{L}_V / \partial \vec{\rho}'_{\nu,\mu}$ are antisymmetric in μ and ν one has $\partial_\mu \partial_\nu \partial \mathcal{L} / \partial \vec{\rho}_{\nu,\mu} = \partial_\mu \partial_\nu \partial \mathcal{L} / \partial \vec{\rho}'_{\nu,\mu} = 0$ and so

$$0 = \partial_\mu\left(\frac{m^2}{g}\vec{\rho}_\mu + \frac{m'^2}{g'}\vec{\rho}'_\mu\right) - \partial_\mu\left[(\vec{G}_{\mu\nu} \times \vec{\rho}_\nu) + \frac{\beta'}{g}(\vec{G}'_{\mu\nu} \times \vec{\rho}_\nu) + (\vec{G}'_{\mu\nu} \times \vec{\rho}'_\nu) + \frac{\beta}{g'}(\vec{G}_{\mu\nu} \times \vec{\rho}'_\nu)\right] + \partial_\mu\left(\frac{1}{g}\frac{\partial F}{\partial \vec{\rho}_\mu} + \frac{1}{g'}\frac{\partial F}{\partial \vec{\rho}'_\mu}\right) . \tag{2.5}$$

To utilize (2.2) and (2.3) as in I one has to take

$$\frac{\beta'}{g} = \frac{\beta}{g'} = \alpha ; \tag{2.6}$$

then one has, as before, the condition on F

$$\frac{1}{g}\frac{\partial F}{\partial \vec{\rho}_\mu} + \frac{1}{g'}\frac{\partial F}{\partial \vec{\rho}'_\mu} = g(\vec{\rho}_\mu \times \vec{\rho}_\nu) \times \vec{\rho}_\nu + \alpha g'(\vec{\rho}'_\mu \times \vec{\rho}'_\nu) \times \vec{\rho}_\nu + \alpha g(\vec{\rho}_\mu \times \vec{\rho}_\nu) \times \vec{\rho}'_\nu + g'(\vec{\rho}'_\mu \times \vec{\rho}'_\nu) \times \vec{\rho}'_\nu , \tag{2.7}$$

which yields

$$F = -\tfrac{1}{4} g^2 (\vec{\rho}_\mu \times \vec{\rho}_\nu)^2 - \tfrac{1}{2}\alpha g g'(\vec{\rho}_\mu \times \vec{\rho}_\nu) \cdot (\vec{\rho}'_\mu \times \vec{\rho}'_\nu) - \tfrac{1}{4} g'^2 (\vec{\rho}'_\mu \times \vec{\rho}'_\nu)^2 . \tag{2.8}$$

Recall that condition (2.7) originated in our requirement to have the field-current identity (1.1). With a conserved electromagnetic current $\partial_\mu j_\mu^{\mathrm{em}} = 0$, one needs

$$\partial_\mu\left(\frac{m^2}{g}\vec{\rho}_\mu + \frac{m'^2}{g'}\vec{\rho}'_\mu\right) = 0 , \tag{2.9}$$

which together with (2.5) gives (2.7).

The electromagnetic interaction is again introduced through the replacement

$$\rho^3_\mu \to \rho^3_\mu + \frac{e}{g} A_\mu ,$$
$$\rho'^3_\mu \to \rho'^3_\mu + \frac{e}{g'} A_\mu \tag{2.10}$$

in all terms of (2.1) except the mass terms. The proof of the gauge invariance of the resulting Lagrangian as well as of the field-current relation (1.1) follows the same pattern as in I and will therefore not be reproduced here.

The Lagrangian (2.1) was the generalization of (I; 3.3) to the case of unequal g and g'. Let us now turn to the analogous generalization of (I; 3.3b); this yields

$$\mathcal{L}_V = -\tfrac{1}{4}(1+\alpha)(\vec{G}_{\mu\nu})^2 + \tfrac{1}{2} m^2 (\vec{\rho}_\mu)^2 + \tfrac{1}{2} g \vec{G}_{\mu\nu} \cdot (\vec{\rho}_\mu \times \vec{\rho}_\nu) + \tfrac{1}{2}\beta \vec{G}_{\mu\nu} \cdot [(\vec{\rho}_\mu \times \vec{\rho}'_\nu) - (\vec{\rho}'_\mu \times \vec{\rho}_\nu)]$$
$$- \tfrac{1}{4}(1+\alpha)(\vec{G}'_{\mu\nu})^2 + \tfrac{1}{2} m'^2 (\vec{\rho}'_\mu)^2 + \tfrac{1}{2} g' \vec{G}'_{\mu\nu} \cdot (\vec{\rho}'_\mu \times \vec{\rho}'_\nu) + \tfrac{1}{2}\beta' \vec{G}'_{\mu\nu} \cdot [(\vec{\rho}'_\mu \times \vec{\rho}_\nu) - (\vec{\rho}_\mu \times \vec{\rho}'_\nu)] - \tfrac{1}{2}\alpha \vec{G}_{\mu\nu} \cdot \vec{G}'_{\mu\nu} + F(\vec{\rho}_\mu, \vec{\rho}'_\mu) . \tag{2.1'}$$

As before we construct

$$\vec{S}_\mu = \{-(1+\alpha)\vec{G}_{\mu\nu} - \alpha \vec{G}'_{\mu\nu} + g(\vec{\rho}_\mu \times \vec{\rho}_\nu) + \beta[(\vec{\rho}_\mu \times \vec{\rho}'_\nu) - (\vec{\rho}'_\mu \times \vec{\rho}_\nu)]\} \times \vec{\rho}_\nu ,$$
$$\vec{S}'_\mu = \{-(1+\alpha)\vec{G}'_{\mu\nu} - \alpha \vec{G}_{\mu\nu} + g'(\vec{\rho}'_\mu \times \vec{\rho}'_\nu) + \beta'[(\vec{\rho}'_\mu \times \vec{\rho}_\nu) - (\vec{\rho}_\mu \times \vec{\rho}'_\nu)]\} \times \vec{\rho}'_\nu . \tag{2.2'}$$

Isospin invariance of (2.1′) implies

$$\partial_\mu(\vec{S}_\mu + \vec{S}'_\mu) = 0 . \tag{2.3'}$$

The equations of motion implied by (2.1′) are

$$\partial_\nu\left(\frac{\partial\mathfrak{L}_V}{\partial\vec{\rho}_{\mu,\nu}}\right)=m^2\vec{\rho}_\mu-g(\vec{G}_{\mu\nu}\times\vec{\rho}_\nu)-\beta(\vec{G}_{\mu\nu}\times\vec{\rho}'_\nu)-\beta'(\vec{G}'_{\mu\nu}\times\vec{\rho}'_\nu)+\frac{\partial F}{\partial\vec{\rho}_\mu}\,,$$
$$\partial_\nu\left(\frac{\partial\mathfrak{L}_V}{\partial\vec{\rho}'_{\mu,\nu}}\right)=m'^2\vec{\rho}'_\mu-g'(\vec{G}'_{\mu\nu}\times\vec{\rho}'_\nu)-\beta'(\vec{G}'_{\mu\nu}\times\vec{\rho}_\nu)-\beta(\vec{G}_{\mu\nu}\times\vec{\rho}_\nu)+\frac{\partial F}{\partial\vec{\rho}'_\mu}\,, \tag{2.4'}$$

leading to

$$0=\partial_\mu\left(\frac{m^2}{g}\vec{\rho}_\mu+\frac{m'^2}{g'}\vec{\rho}'_\mu\right)-\partial_\mu\left[(\vec{G}_{\mu\nu}\times\vec{\rho}_\nu)+\frac{\beta}{g}(\vec{G}_{\mu\nu}\times\vec{\rho}'_\nu)+\frac{\beta'}{g}(\vec{G}'_{\mu\nu}\times\vec{\rho}'_\nu)\right]$$
$$-\partial_\mu\left[(\vec{G}'_{\mu\nu}\times\vec{\rho}'_\nu)+\frac{\beta'}{g'}(\vec{G}'_{\mu\nu}\times\vec{\rho}_\nu)+\frac{\beta}{g'}(\vec{G}_{\mu\nu}\times\vec{\rho}_\nu)\right]+\partial_\mu\left(\frac{1}{g}\frac{\partial F}{\partial\vec{\rho}_\mu}+\frac{1}{g'}\frac{\partial F}{\partial\vec{\rho}'_\mu}\right). \tag{2.5'}$$

To guarantee (2.8) by the usual technique, i.e., by substitution of (2.3′) into (2.5′), one has to require $\beta'/g=\beta/g'=\beta/g=\beta'/g'=\alpha$, in other words, $\beta'=\beta=\alpha g=\alpha g'$, which implies $g=g'$. We conclude that, as already claimed, the Lagrangian (2.1′) with $g\neq g'$ *cannot* lead to a field-current identity and is therefore not suitable for a formulation of generalized vector dominance. However, it was this Lagrangian which led in I to $\mu_\rho>2$ for the lightest vector meson, in contrast to the claim in Ref. 5. Can it now happen that our Lagrangian (2.1), because of the greater freedom $g\neq g'$, leads to the same result? In view of the arguments of Shtokhamer and Singer it is of interest to demonstrate that this indeed is the case.

For this purpose it is again sufficient to consider the special case where $m=m'$. The analog of (I; 3.29) obtainable from (3.4) is

$$\mathfrak{L}_{VV\gamma}=eA_\mu\{(\rho^1_\nu\overleftrightarrow{\partial}_\mu\rho^2_\nu)+\alpha[(\rho'^1_\nu\overleftrightarrow{\partial}_\mu\rho^2_\nu)+(\rho^1_\nu\overleftrightarrow{\partial}_\mu\rho'^2_\nu)]+(\rho'^1_\nu\overleftrightarrow{\partial}_\mu\rho'^2_\nu)\}$$
$$+eF_{\mu\nu}\{(2+\alpha g/g')\rho^1_\mu\rho^2_\nu+\alpha(\rho'^1_\mu\rho^2_\nu+\rho^1_\mu\rho'^2_\nu)+(2+\alpha g'/g)\rho'^1_\mu\rho'^2_\nu\}\,. \tag{2.11}$$

The transformation (I; 3.19) diagonalizes the quadratic piece of the Lagrangian (2.1) just as in I. Expressing (2.11) in terms of $\vec{\rho}_a$ and $\vec{\rho}_b$ through application of (I; 3.19) leads to the generalization of (I; 3.30) for $g\neq g'$

$$\mathfrak{L}_{VV\gamma}=eA_\mu[(\rho^{a,1}_\nu\overleftrightarrow{\partial}_\mu\rho^{a,2}_\nu)+(\rho^{b,1}_\nu\overleftrightarrow{\partial}_\mu\rho^{b,2}_\nu)]$$
$$+eF_{\mu\nu}\left\{\left[2+\alpha+\frac{\alpha}{2}\left(\frac{g}{g'}+\frac{g'}{g}\right)\right](1+\alpha)^{-1}\rho^{a,1}_\mu\rho^{a,2}_\mu\right.$$
$$\left.+\frac{\alpha}{2}\left(\frac{g'}{g}-\frac{g}{g'}\right)[(1+\alpha)(1-\alpha)]^{1/2}(\rho^{a,1}_\mu\rho^{b,2}_\nu+\rho^{b,1}_\mu\rho^{a,2}_\nu)+\left[2-\alpha+\frac{\alpha}{2}\left(\frac{g}{g'}+\frac{g'}{g}\right)\right](1-\alpha)^{-1}\rho^{b,1}_\mu\rho^{b,2}_\nu\right\}. \tag{2.12}$$

As in I, $m_a<m_b$ for $\alpha>0$, i.e., ρ is the lighter vector meson. Since $g/g'+g'/g\geq 2$ it follows that

$$\mu(\rho^a)=\left[2+\alpha+\frac{\alpha}{2}\left(\frac{g}{g'}+\frac{g'}{g}\right)\right](1+\alpha)^{-1}\geq 2\,,$$

which is what we set out to prove.

III. FIELD-CURRENT IDENTITY IN TERMS OF PHYSICAL VECTOR FIELDS

The fields ρ and ρ' entering (1.1) are not the physical fields since the quadratic part of the Lagrangian (2.1) is not diagonal but contains the mixing term $\alpha\vec{G}_{\mu\nu}\cdot\vec{G}'_{\mu\nu}$. The Lagrangian (2.1), (2.8) can be written in the form

$$\mathfrak{L}_V=-\tfrac{1}{4}(\vec{G}_{\mu\nu}-g\vec{h}_{\mu\nu})^2-\tfrac{1}{2}(\vec{G}_{\mu\nu}-g\vec{h}_{\mu\nu})\cdot(\vec{G}'_{\mu\nu}-g'\vec{h}'_{\mu\nu})$$
$$-\tfrac{1}{4}(\vec{G}'_{\mu\nu}-g'\vec{h}'_{\mu\nu})^2+\tfrac{1}{2}m^2\vec{\rho}_\mu{}^2$$
$$+\tfrac{1}{2}m'^2\vec{\rho}'^2_\mu+\vec{\rho}_\mu\cdot\vec{J}_\mu+\vec{\rho}'_\mu\cdot\vec{J}'_\mu\,, \tag{3.1}$$

with $\vec{h}_{\mu\nu}\equiv\vec{\rho}_\mu\times\vec{\rho}_\nu$, $\vec{h}'_{\mu\nu}\equiv\vec{\rho}'_\mu\times\vec{\rho}'_\nu$ and with $\vec{J}_\mu,\vec{J}'_\mu$ the external sources of $\vec{\rho}_\mu,\vec{\rho}'_\mu$.

$\vec{J}_\mu$ and $\vec{J}'_\mu$ are obviously not functions of ρ and ρ', in contrast to $\vec{h}_{\mu\nu}$, $\vec{h}'_{\mu\nu}$. Note, however, that $h^3_{\mu\nu}$ does *not* depend on ρ^3_μ but only on ρ^1_μ, ρ^2_ν. Therefore the Lagrangian for the third isospin component is

$$\mathfrak{L}^3_V=-\tfrac{1}{4}(G_{\mu\nu})^2-\tfrac{1}{2}\alpha G_{\mu\nu}G'_{\mu\nu}-\tfrac{1}{4}(G'_{\mu\nu})^2+\tfrac{1}{2}m^2(\rho_\mu)^2$$
$$+\tfrac{1}{2}m'^2(\rho'_\mu)^2+\tfrac{1}{2}G_{\mu\nu}T_{\mu\nu}+\tfrac{1}{2}G'_{\mu\nu}T'_{\mu\nu}+\rho_\mu J_\mu+\rho'_\mu J'_\mu\,, \tag{3.2}$$

with

$$T_{\mu\nu}=gh_{\mu\nu}+\alpha g'h'_{\mu\nu}\,,$$
$$T'_{\mu\nu}=g'h'_{\mu\nu}+\alpha gh_{\mu\nu}\,. \tag{3.3}$$

The isovector indices of the fields $T_{\mu\nu}$, J_μ, etc. were omitted. The index-free symbols always stand for the third component of the corresponding isovector.

The substitution of (2.10) in (3.2) and the addition of the free Maxwell Lagrangian yield the following total Lagrangian:

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}{}^2 + \mathcal{L}_V^3 + \frac{e}{g}[-\tfrac{1}{2}F_{\mu\nu}(G_{\mu\nu} - \alpha G'_{\mu\nu}) + F_{\mu\nu}T_{\mu\nu} + A_\mu J_\mu] + \frac{e}{g'}[-\tfrac{1}{2}F_{\mu\nu}(G'_{\mu\nu} - \alpha G_{\mu\nu}) + F_{\mu\nu}T'_{\mu\nu} + A_\mu J'_\mu]. \tag{3.4}$$

The field-current identity is derived, as one recalls from I, through comparison of the equations of motion for ρ^3_μ, ρ'^3_μ derived from (3.2) and the ones for A_μ derived from (3.4). Note that $T_{\mu\nu}$, $T'_{\mu\nu}$ do not depend on the third components of $\vec{\rho}_\mu$ and $\vec{\rho}'_\mu$ and are from the point of view of the equations of motion for ρ^3_μ, ρ'^3_μ on the same footing as J_μ and J'_μ, i.e., they may be considered, for the time being, also as external sources.

Now if

$$\vec{\rho}_a = a\vec{\rho} + b\vec{\rho}', \quad \vec{\rho}_b = c\vec{\rho} + d\vec{\rho}' \tag{3.5}$$

is the transformation that diagonalizes the quadratic part of $\mathcal{L}_V$ then in particular

$$\rho_a = a\rho + b\rho', \quad \rho_b = c\rho + d\rho' \tag{3.5'}$$

diagonalizes the quadratic part of $\mathcal{L}_V^3$.

Define also

$$\frac{1}{g_a} \equiv \frac{a}{g} + \frac{b}{g'}, \quad \frac{1}{g_b} \equiv \frac{c}{g} + \frac{d}{g'} \tag{3.6}$$

so that $1/g$ and $1/g'$ transform like ρ and ρ', respectively. Then the transformation (3.5′) which diagonalizes the kinetic energy (and mass) term in (3.2) *also* diagonalizes the FG, FG' terms in (3.4) to yield the piece

$$-\tfrac{1}{2}F_{\mu\nu}\left(\frac{1}{g_a}G^a_{\mu\nu} + \frac{1}{g_b}G^b_{\mu\nu}\right).$$

Also because of (3.5′),

$$\begin{aligned}\rho J + \rho' J' &\equiv \rho_a J_a + \rho_b J_b \\ &= (a\rho + b\rho')J_a + (c\rho + d\rho')J_b \\ &= \rho(aJ_a + cJ_b) + \rho'(bJ_a + dJ_b).\end{aligned}$$

Hence

$$J = aJ_a + cJ_b, \quad J' = bJ_a + dJ_b \tag{3.7}$$

From this follows that

$$\begin{aligned}\frac{e}{g}AJ + \frac{e}{g'}AJ' &= \frac{e}{g}A(aJ_a + cJ_b) + \frac{e}{g'}(bJ_a + dJ_b) \\ &= eA\left[J^a\left(\frac{a}{g} + \frac{b}{g'}\right) + J^b\left(\frac{c}{g} + \frac{d}{g'}\right)\right] \\ &= eA\left(\frac{J_a}{g_a} + \frac{J_b}{g_b}\right).\end{aligned} \tag{3.8}$$

Similarly

$$GT + G'T' = G_aT_a + G_bT_b \tag{3.9}$$

implies

$$\frac{e}{g}FT + \frac{e}{g'}FT' = eF\left(\frac{T_a}{g_a} + \frac{T_b}{g_b}\right).$$

To summarize: Equations (3.2) and (3.4) go, after transformation into the physical vector fields, over into

$$\mathcal{L}_V^3 = -\tfrac{1}{4}(G^a_{\mu\nu})^2 - \tfrac{1}{4}(G^b_{\mu\nu})^2 + \tfrac{1}{2}m_a{}^2(\rho^a_\mu)^2 + \tfrac{1}{2}m_b{}^2(\rho^b_\mu)^2 + G^a_{\mu\nu}T^a_{\mu\nu} + G^b_{\mu\nu}T^b_{\mu\nu} + \rho^a_\mu J^a_\mu + \rho^b_\mu J^b_\mu, \tag{3.2'}$$

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}{}^2 + \mathcal{L}_V^3 + \frac{e}{g_a}(-\tfrac{1}{2}F_{\mu\nu}G^a_{\mu\nu} + F_{\mu\nu}T^a_{\mu\nu} + A_\mu J^a_\mu) + \frac{e}{g_b}(-\tfrac{1}{2}F_{\mu\nu}G^b_{\mu\nu} + F_{\mu\nu}T^b_{\mu\nu} + A_\mu J^b_\mu). \tag{3.4'}$$

The equations of motion for the vector-meson fields following from (3.2′) are

$$\begin{aligned}\partial_\nu(G^a_{\nu\mu} - T^a_{\nu\mu}) + m_a{}^2\rho^a_\mu &= -J^a_\mu, \\ \partial_\nu(G^b_{\nu\mu} - T^b_{\nu\mu}) + m_b{}^2\rho^b_\mu &= -J^b_\mu.\end{aligned} \tag{3.10}$$

The equation of motion for the electromagnetic field following (3.4′) is

$$\partial_\nu F_{\nu\mu} + e\partial_\nu\left[\frac{1}{g_a}(G^{\nu\mu}_a - T^{\nu\mu}_a) + \frac{1}{g_b}(G^{\nu\mu}_b - T^{\nu\mu}_b)\right] = -e\left(\frac{J^a_\mu}{g_a} + \frac{J^b_\mu}{g_b}\right). \tag{3.11}$$

Combining (3.10) and (3.11) finally yields

$$\partial_\nu F_{\nu\mu} = J^{\text{em}}_\mu = e\left(\frac{m_a{}^2}{g_a}\rho^a_\mu + \frac{m_b{}^2}{g_b}\rho^b_\mu\right), \tag{3.12}$$

which is exactly what we wished to show.

IV. THE MASS MIXING MODEL

In Sec. II a model was presented where the mixing between the vector mesons occurred in the trilinear part of the Lagrangian. Another possibility is to introduce a mixing in the quadratic mass term, i.e., to consider, in place of (2.1), a Lagrangian

$$\mathcal{L}_V = -\tfrac{1}{4}(\vec{G}_{\mu\nu} - g\vec{h}_{\mu\nu})^2 - \tfrac{1}{4}(\vec{G}'_{\mu\nu} - g'\vec{h}'_{\mu\nu})^2 + \tfrac{1}{2}m^2(\vec{\rho}_\mu)^2 + \mu^2\vec{\rho}_\mu\cdot\vec{\rho}'_\mu + \tfrac{1}{2}m'^2(\vec{\rho}'_\mu)^2 + \vec{\rho}_\mu\cdot\vec{J}_\mu + \vec{\rho}'_\mu\cdot\vec{J}'_\mu. \tag{4.1}$$

To discuss the introduction of the electromagnetic interaction, the field-current identity, and the gauge invariance, one can proceed as before, i.e., substitute (2.10) into all terms of (4.1) except the mass terms. However, for our purpose here it is more suitable to introduce the substitution (2.10) just in the opposite way, i.e., *only* into the mass terms. That these two procedures are equivalent is well known and quite easy to see. It is demonstrated, for the one-meson case, in Ref. 6.

The total Lagrangian is now

$$\mathcal{L} = -\tfrac{1}{4}(\vec{G}_{\mu\nu} - g\vec{h}_{\mu\nu})^2 - \tfrac{1}{4}(\vec{G}'_{\mu\nu} - g'\vec{h}'_{\mu\nu})^2 + \tfrac{1}{2}m^2\left(\vec{\rho}_\mu - \frac{e}{g}\vec{n}A_\mu\right)^2 + \mu^2\left(\vec{\rho}_\mu - \frac{e}{g}\vec{n}A_\mu\right)\cdot\left(\vec{\rho}'_\mu - \frac{e}{g'}\vec{n}A_\mu\right) + \tfrac{1}{2}m'^2\left(\vec{\rho}'_\mu - \frac{e}{g'}\vec{n}A_\mu\right)^2 + \vec{\rho}_\mu\cdot\vec{J}_\mu + \vec{\rho}'_\mu\cdot\vec{J}'_\mu - \tfrac{1}{4}F_{\mu\nu}{}^2\,, \tag{4.2}$$

with $n_i = \delta_{i3}$.

The demonstration of the *physical* field-current identity (1.2) is now much simpler than in the former case of trilinear mixing. One has only to demonstrate that the transformation (1.4) which diagonalizes the quadratic meson piece in (4.2) together with the ansatz (1.3) also diagonalizes the quadratic ρ^3-A piece in (4.2). To show this note that the ρ^3-A piece is

$$eA_\mu\left(\frac{m^2}{g}\rho_\mu + \frac{\mu^2}{g'}\rho_\mu + \frac{\mu^2}{g}\rho'_\mu + \frac{m'^2}{g'}\rho'_\mu\right). \tag{4.3}$$

The quantity inside the brackets has the same transformation properties (recall that $1/g$ and $1/g'$ transform as ρ and ρ') as the mass piece

$$\tfrac{1}{2}(m^2\rho_\mu{}^2 + 2\mu^2\rho_\mu\rho'_\mu + m'^2\rho'^2_\mu) \tag{4.4}$$

and so diagonalizes simultaneously with it to yield

$$eA_\mu\left(\frac{m_a{}^2}{g_a}\rho^a_\mu + \frac{m_b{}^2}{g_b}\rho^b_\mu\right). \tag{4.5}$$

This now establishes straightforwardly Eq. (1.2).

To demonstrate that the magnetic moments of ρ^a and ρ^b in the mass mixing model are exactly 2 note that (1.4) is now a pure rotation since the (quadratic) kinetic energy part in (4.2) is now diagonal *with equal coefficients*, in contrast to the situation in Sec. III where the diagonal mass term had unequal coefficients m^2 and m'^2. So we can write

$$\vec{\rho} = \cos\theta\vec{\rho}_a + \sin\theta\vec{\rho}_b\,, \quad \vec{\rho}_a = \cos\theta\vec{\rho} - \sin\theta\vec{\rho}'\,,$$
$$\vec{\rho}' = -\sin\theta\vec{\rho}_a + \cos\theta\vec{\rho}_b\,, \quad \vec{\rho}_b = \sin\theta\vec{\rho} + \cos\theta\vec{\rho}'\,, \tag{4.6}$$

with identical formulas for $1/g$, $1/g'$, $1/g_a$, and $1/g_b$.

The trilinear part of (4.2) when written out explicitly is

$$\mathcal{L}_{\rm tr} = gG^3_{\mu\nu}(\rho^1_\mu\rho^2_\nu - \rho^2_\mu\rho^1_\nu) + g'G'^3_{\mu\nu}(\rho'^1_\mu\rho'^2_\nu - \rho'^2_\mu\rho'^1_\nu) + g\rho^3_\mu(G^2_{\mu\nu}\rho^1_\nu - G^1_{\mu\nu}\rho^2_\nu) + g'\rho'^3_\mu(G'^2_{\mu\nu}\rho'^1_\nu - G'^1_{\mu\nu}\rho'^2_\nu)\,. \tag{4.7}$$

Let us for simplicity treat only the "purely magnetic" part of (4.7), namely the term

$$\mathcal{L}_{{\rm tr},M} = gG^3_{\mu\nu}\rho^1_\mu\rho^2_\nu + g'G'^3_{\mu\nu}\rho'^1_\mu\rho'^2_\nu\,. \tag{4.8}$$

Expressing ρ^3, ρ'^3 in terms of $\rho^{a,3}$, $\rho^{b,3}$ yields

$$\begin{aligned}\mathcal{L}_{{\rm tr},M} &= g(\cos\theta G^{a,3}_{\mu\nu} + \sin\theta G^{b,3}_{\mu\nu})\rho^1_\mu\rho^2_\nu + g'(-\sin\theta G^{a,3}_{\mu\nu} + \cos\theta G^{b,3}_{\mu\nu})\rho'^1_\mu\rho'^2_\nu \\ &= G^{a,3}_{\mu\nu}(g\cos\theta\rho^1_\mu\rho^2_\nu - g'\sin\theta\rho'^1_\mu\rho'^2_\nu) + G^{b,3}_{\mu\nu}(g\sin\theta\rho^1_\mu\rho^2_\nu + g'\cos\theta\rho'^1_\mu\rho'^2_\nu)\,.\end{aligned} \tag{4.9}$$

Utilizing (1.2) and (4.9) gives for the electromagnetic interaction of ρ^a and ρ^b the following magnetic part:

$$\begin{aligned}\mathcal{L}_{{\rm em},M} &= \frac{eF_{\mu\nu}}{g_a}(g\cos\theta\rho^1_\mu\rho^2_\nu - g'\sin\theta\rho'^1_\mu\rho'^2_\nu) + \frac{eF_{\mu\nu}}{g_b}(g\sin\theta\rho^1_\mu\rho^2_\nu + g'\cos\theta\rho'^1_\mu\rho'^2_\nu) \\ &= eF_{\mu\nu}\left[\rho^1_\mu\rho^2_\nu g\left(\frac{\cos\theta}{g_a} + \frac{\sin\theta}{g_b}\right) + \rho'^1_\mu\rho'^2_\nu g'\left(-\frac{\sin\theta}{g_a} + \frac{\cos\theta}{g_b}\right)\right] \\ &= eF_{\mu\nu}(\rho^1_\mu\rho^2_\nu + \rho'^1_\mu\rho'^2_\nu) \\ &= eF_{\mu\nu}(\rho^{a,1}_\mu\rho^{a,2}_\nu + \rho^{b,1}_\mu\rho^{b,2}_\nu)\,.\end{aligned} \tag{4.10}$$

In proving (4.10) we have used the transformation equations for g_a and g_b and in the last step we have also used the fact that (4.6) is an orthogonal transformation.

Similarly the other terms in (4.7) lead to diagonal $VV\gamma$ terms. The term we have just treated contributes to the magnetic interaction the amount $\left(\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right)$. The $g\rho^3_\mu$, $g'\rho'^3_\mu$ terms contribute $\left(\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right)$ to the electric charge interaction and, after partial integration (see the identical treatment in I), also make a *further* $\left(\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right)$ contribution to the magnetic moment, which thereby gives the *total* value $\left(\begin{smallmatrix}2&0\\0&2\end{smallmatrix}\right)$.

V. DISCUSSION

The formulation of generalized vector dominance, and the original formulation of vector dominance due to Lee and Zumino, are based on Yang-Mills Lagrangians with additional mass terms which spoil their renormalizability. One can attempt a formulation of vector dominance and of generalized vector dominance in renormalizable Yang-Mills theories whose mass terms are obtained through the Higgs-Kibble mechanism. This has been done within the framework of the M scheme of the Berkeley group.[4] It is of interest to compare the results of this scheme with our effective-Lagrangian prescriptions. For this purpose note that the relevant part of the Lagrangian in Ref. 4 is given by

$$\mathcal{L} = -\tfrac{1}{4}(\vec{G}_{\mu\nu} + g\vec{h}_{\mu\nu})^2 - \tfrac{1}{4}(\vec{G}'_{\mu\nu} + g'\vec{h}'_{\mu\nu})^2 + \tfrac{1}{2}v^2(g\vec{\tau}\cdot\vec{\rho}_\mu - g'\vec{\tau}\cdot\vec{\rho}'_\mu)^2 + \tfrac{1}{2}v'^2(g'\vec{\tau}\cdot\vec{\rho}_\mu - eA_\mu\tau_3)^2 - \tfrac{1}{4}F_{\mu\nu}{}^2\,, \tag{5.1}$$

where v and v' are the vacuum expectation values of the Higgs scalars M and M'.

Equation (5.1) can be rewritten in the form

$$\mathcal{L} = -\tfrac{1}{4}(\vec{G}_{\mu\nu} + g\vec{h}_{\mu\nu})^2 - \tfrac{1}{4}(\vec{G}'_{\mu\nu} + g'\vec{h}'_{\mu\nu})^2 + \tfrac{1}{2}v^2\left[g\vec{\tau}\cdot\left(\vec{\rho}_\mu - \vec{n}\frac{e}{g}A_\mu\right) - g'\vec{\tau}\cdot\left(\vec{\rho}'_\mu - \vec{n}\frac{e}{g'}A_\mu\right)\right]^2 + \tfrac{1}{2}v'^2\left[g'\vec{\tau}\cdot\left(\vec{\rho}'_\mu - \vec{n}\frac{e}{g'}A_\mu\right)\right]^2 + \tfrac{1}{4}F_{\mu\nu}{}^2\,, \tag{5.2}$$

which on comparison with (4.2) shows the identity with the mass mixing scheme of Sec. IV.

The relation between the two prescriptions, as far as vector dominance is concerned, is given through

$$m^2 = v^2g^2\,,\quad \mu^2 = -v^2gg'\,,\quad m'^2 = (v^2 + v'^2)g'^2\,. \tag{5.3}$$

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