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Low-energy Compton scattering: The magnetic polarizability of the nucleon*

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An expression for the magnetic polarizability of the nucleon is derived using gauge and relativistic invariance. Besides the well-known term that contains products of magnetic dipole transition moments it is shown that there exists a second contribution related to third moments of the current-charge density commutator. An estimation of the proton magnetic polarizability leads to a result which is consistent with the available experimental value.

I. INTRODUCTION

Low[1] and Gell-Mann and Goldberger[2] have derived the Compton scattering amplitude on a spin-$\frac{1}{2}$ target up to linear terms in the frequency of the incident photon. The quadratic terms of the amplitude have been investigated in the context of field theory[3-5] and more recently by using only on-mass-shell physical helicity amplitudes.[6] As is well known, in addition to the charge, mass, and anomalous moment, the cross section up to second order in the frequency of the photon is described by two more parameters, α and β, representing the electric and magnetic polarizabilities of the system.

Various authors have estimated the electric polarizabilities of the nucleon, especially the electric polarizability of the neutron.[4,7] More recently a dispersive approach has been used for the electromagnetic polarizabilities.[8]

In this paper we concentrate on the magnetic part. We show that a general closed expression can be obtained for the magnetic polarizability using gauge and relativistic invariance. A first term β_1 which contains products of magnetic dipole transition matrix elements has been derived by Petrun'kin,[5] but the question of whether there are additional contributions or not has been open. We have shown that there is an additional contribution β_2 to the magnetic polarizability which is related to third moments of the current-charge density commutator.

In Sec. II we give a general discussion of the scattering amplitude. Section III is devoted to the calculation of the scattering amplitude to second order, and the full expression of the magnetic polarizability is written down. In Sec. IV we estimate the polarizability of the proton and we obtain a value consistent with recent available data.[8] The results are discussed in Sec. V.

II. THE SCATTERING AMPLITUDE

We write the scattering amplitude of light by the nucleon as

$$S=(4V^2\omega\omega')^{-1/2}\epsilon'_\mu T_{\mu\nu}\epsilon_\nu \ , \tag{1}$$

where[9]

$$T_{\mu\nu}=\int dx\,dy\,e^{-ik'x+iky}\langle\vec{p}'|[T\{J_\mu(x)J_\nu(y)\} - i\rho_{\mu\nu}(x)\delta^4(x-y)]|\vec{p}\rangle \ . \tag{2}$$

Here k and k' are the incident and outgoing photon momenta, ϵ and ϵ' are the corresponding polarizations, and p and p' are the initial and final target momenta. Our metric is defined by $k=(\vec{k}, ik_0)=(\vec{k}, i\omega)$. The term $\rho_{\mu\nu}(x)$ compensates for the noncovariance of the T product and satisfies the identities

$$\rho_{\mu\nu}=\rho_{\nu\mu}, \quad \rho_{0\nu}=0 \ . \tag{3}$$

Covariance of T is ensured by the following equal-time commutation relations for the current operators $J_\mu=(J_i, iJ_0)$:

$$[J_0(x), J_0(y)]\delta(x_0-y_0)=0 \ , \tag{4}$$

$$[J_0(x), J_i(y)]\delta(x_0-y_0)=i\frac{\partial}{\partial x_m}\delta^4(x-y)\rho_{mi}(x) \ . \tag{5}$$

This commutation relation ensures the gauge conditions

$$T_{\mu\nu}k_\nu=k_\mu T_{\mu\nu}=0 \ . \tag{6}$$

In the transverse gauge, $\epsilon'_4=\epsilon_4=0$, $\vec{k}\cdot\vec{\epsilon}=\vec{k}'\cdot\vec{\epsilon}'=0$, the physical amplitude is

$$S=(4V^2\omega\omega')^{-1/2}\epsilon'_i T_{ij}\epsilon_j \ , \tag{7}$$

where, extracting a three-momentum δ function by translation invariance, we obtain

$$T_{ij}=\frac{(2\pi)^3}{V}\,\delta(\vec{p}\,'+\vec{k}\,'-\vec{p}-\vec{k})\left(\int dx\,dy\,e^{-ik'x+iky}\langle p'|[T\{J_i(x)J_j(y)\}-i\rho_{ij}(x)\delta^4(x-y)]|\vec{p}\rangle\right)\ . \tag{8}$$

As we shall see, the additional contribution to the nucleon polarizability is given by second moments of the function $\rho_{ij}(\vec{x})$, which can afterwards be expressed in terms of third moments of the commutator $[J_0(\vec{x}),J_i(0)]$ by means of Eq. (5).

We shall now use a convenient decomposition of the electromagnetic current due to Foldy[10] which is based on the identity

$$\vec{a}e^{i\vec{k}\cdot\vec{y}}=\int_0^1 ds\,\vec{\nabla}(\vec{a}\cdot\vec{y}e^{is\vec{k}\cdot\vec{y}})-i\int_0^1 s\,ds\,e^{is\vec{k}\cdot\vec{y}}(\vec{a}\times\vec{k})\times\vec{y}\ . \tag{9}$$

Contracting this equation with $\vec{J}(y)$, integrating over space coordinates, making a space integration by parts, and using the equation of continuity,

$$\partial_\mu J_\mu=0\ , \tag{10}$$

leads to the following decomposition[10]:

$$\int d\vec{y}J_j(y)e^{i\vec{k}\cdot\vec{y}}=\frac{\partial}{\partial y_0}D_j(\vec{k},y_0)+i\epsilon_{jmr}k_rM_m(\vec{k},y_0)\ , \tag{11}$$

where

$$D_j(\vec{k},y_0)=\int_0^1 ds\int d\vec{y}\,y_jJ_0(y)e^{is\vec{k}\cdot\vec{y}} \tag{12}$$

and

$$M_j(\vec{k},y_0)=\int_0^1 s\,ds\int d\vec{y}(\vec{y}\times\vec{J}(y))_je^{is\vec{k}\cdot\vec{y}}\ . \tag{13}$$

For $\vec{k}=0$, D_j is the electric dipole operator d_j and M_j is the magnetic dipole operator m_j:

$$D_j(\vec{k}=0)=d_j\,,\quad M_j(\vec{k}=0)=m_j\ . \tag{14}$$

It is easy to see that

$$ik_jD_j(\vec{k},y_0)=\int d\vec{y}\,e^{i\vec{k}\cdot\vec{y}}J_0(y)-Q\ , \tag{15}$$

where $Q=\int d\vec{y}J_0(y)$ is the total charge.

Now we substitute Eq. (11) into Eq. (8). After a time integration by parts we can write

$$T_{ij}=\frac{(2\pi)^3}{V}\,\delta(\vec{p}\,'+\vec{k}\,'-\vec{p}-\vec{k})\Big\langle\vec{p}\,'\Big|\Big[i\int dx\,dy_0e^{-ik'x-i\omega y_0}[\omega T\{J_i(x)D_j(\vec{k},y_0)\}+\epsilon_{jmr}k_rT\{J_i(x)M_m(\vec{k},y_0)\}]$$
$$+\int dx\,dy_0e^{-ik'x-i\omega y_0}\delta(x_0-y_0)[J_i(x),D_j(\vec{k},y_0)]-i\int dx\,e^{i(k-k')x}\rho_{ij}(x)\Big]\Big|\vec{p}\Big\rangle\ . \tag{16}$$

Next we use the decomposition (11) in the first two terms on the right-hand side of Eq. (16). One obtains, taking the initial nucleon at rest ($\vec{p}=0$, $p_0=M$),

$$T_{ij}=\frac{(2\pi)^3}{V}\,\delta(\vec{p}\,'+\vec{k}\,'-\vec{k})(T_{ij}^{(1)}+T_{ij}^{(2)})\ , \tag{17}$$

where

$$T_{ij}^{(1)}=\int dx_0dy_0e^{i\omega'x_0-i\omega y_0}\langle\vec{p}\,'|[\omega\omega'T\{D_i(-\vec{k}\,',x_0)D_j(\vec{k},y_0)\}+\omega'\epsilon_{jmr}k_rT\{D_i(-\vec{k}\,',x_0)M_m(\vec{k},y_0)\}$$
$$+\omega\epsilon_{imr}k_r'T\{M_m(-\vec{k}\,',x_0)D_j(\vec{k},y_0)\}+\epsilon_{im'r'}\epsilon_{jmr}k_{r'}'k_rT\{M_{m'}(-\vec{k}\,',x_0)M_m(\vec{k},y_0)\}]|\vec{0}\rangle\ , \tag{18}$$

is the part containing T products and

$$T_{ij}^{(2)}=\int dx_0dy_0e^{i\omega'x_0-i\omega y_0}\delta(x_0-y_0)$$
$$\times\Big\langle\vec{p}\,'\Big|\Big\{\int d\vec{x}e^{-i\vec{k}'\cdot\vec{x}}[J_i(\vec{x},x_0),D_j(\vec{k},y_0)]-i\omega[D_i(-\vec{k}\,',x_0),D_j(\vec{k},y_0)]-i\epsilon_{jmr}k_r[D_i(-\vec{k}\,',x_0),M_m(\vec{k},y_0)]\Big\}\Big|\vec{0}\Big\rangle$$
$$-i\Big\langle\vec{p}\,'\Big|\int dx\,e^{i(k-k')x}\rho_{ij}(x)\Big|\vec{0}\Big\rangle \tag{19}$$

is the part containing equal-time commutators and the function ρ_{ij}.

Using Eqs. (5) and (12), the first equal-time commutator of Eq. (19) gives

$$\int d\vec{x}\, e^{-i\vec{k}'\cdot\vec{x}}[J_i(\vec{x},x_0),D_j(\vec{k},x_0)]=i\int d\vec{x}\, e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}\rho_{ji}(x)+\int d\vec{x}\, e^{-i\vec{k}'\cdot\vec{x}}\int_0^1 s\, ds\, e^{is\vec{k}\cdot\vec{x}}(\vec{k}\cdot\vec{x}\delta_{jm}-x_jk_m)\rho_{mi}(x)\,, \tag{20}$$

where use has been made of the identity

$$\int_0^1 ds(1+is\vec{k}\cdot\vec{y})e^{is\vec{k}\cdot\vec{y}}=e^{i\vec{k}\cdot\vec{y}}\,. \tag{21}$$

Notice that the first term on the right of Eq. (20) will cancel out the last term of Eq. (19).

The second equal-time commutator of Eq. (19) vanishes on account of Eq. (4):

$$[D_i(-\vec{k}',x_0),D_j(\vec{k},x_0)]=0\,. \tag{22}$$

The third equal-time commutator of Eq. (19) gives, with the help of Eqs. (5), (12), and (13),

$$-i\epsilon_{jmr}k_r[D_i(-\vec{k}',x_0),M_m(\vec{k},x_0)]=\int d\vec{x}\int_0^1 ds'\int_0^1 s\, ds\, e^{i(s\vec{k}-s'\vec{k}')\cdot\vec{x}}\rho_{mn}(x)(\delta_{mi}-is'k'_mx_i)(x_jk_n-\vec{k}\cdot\vec{x}\delta_{jn})\,. \tag{23}$$

If we use the identity (21) for the exponential involving $\vec{k}'$ in this equation, the linear term in $\vec{k}'$ will cancel the corresponding one in Eq. (20). If we add Eqs. (20), (22), and (23) only terms at least quadratic in the momenta will survive. After substitution in Eq. (19), we obtain

$$T_{ij}^{(2)}=2\pi i\delta(p_0'+\omega'-M-\omega)$$
$$\times\left\langle\vec{p}'\,\middle|\,\int d\vec{x}\rho_{mn}(\vec{x})\int_0^1 s\,ds\int_0^1 s'ds'e^{i(s\vec{k}-s'\vec{k}')\cdot\vec{x}}[\delta_{mi}\vec{k}'\cdot\vec{x}(x_jk_n-\vec{k}\cdot\vec{x}\delta_{jn})+k'_mx_i(\vec{k}\cdot\vec{x}\delta_{jn}-x_jk_n)]\,\middle|\,\vec{0}\right\rangle. \tag{24}$$

Before we study our decomposition of the amplitude we stop for a few comments: The manipulations leading from Eq. (8) to Eqs. (17), (18), and (19) seem to be highly formal. Actually they are based on the identity (11) and common space and time integrations by parts. We could as well insert a complete set of intermediate states in the T-product term of Eq. (8) and apply our identity for the matrix elements of the currents instead of using them for the current operators themselves. Contracting Eq. (17) with k_j or k_i' and using Eqs. (15) and (11) one can check that the gauge conditions (6) are recovered.

III. CALCULATION TO ORDER ω^2: THE ELECTRIC AND MAGNETIC POLARIZABILITIES

In this section we shall show that the first term $T_{ij}^{(1)}$ in Eq. (17) contains the known results for the polarizabilities[4,5] and the remaining part $T_{ij}^{(2)}$ given in Eq. (24) gives the additional contribution to the magnetic polarizability that we are looking for.

In Eq. (18) we insert a complete set of intermediate states, perform the time integration, and separate out the nucleon intermediate state. The excited-states contribution is certainly of order ω^2, and to that order we can put inside the sum over these intermediate excited states all space momenta equal to zero. One obtains

$$T_{ij}^{(1)}=\frac{2\pi}{i}\,\delta(p_0'+\omega'-M-\omega)[\,a_{ij}^{(N)}+\omega\omega'\alpha\delta_{ij}+(\vec{k}\cdot\vec{k}'\delta_{ij}-k_ik_j')\beta_1]\,, \tag{25}$$

where $a_{ij}^{(N)}$ contains the nucleon contribution,

$$\alpha\delta_{ij}=\sum_n{}'\left[\frac{\langle\vec{0}|d_i|n,\vec{0}\rangle\langle n,\vec{0}|d_j|\vec{0}\rangle}{E_n(\vec{0})-M}+(i\leftrightarrow j)\right], \tag{26}$$

and

$$\beta_1\delta_{ij}=\sum_n{}'\left[\frac{\langle\vec{0}|m_i|n,\vec{0}\rangle\langle n,\vec{0}|m_j|\vec{0}\rangle}{E_n(\vec{0})-M}+(i\leftrightarrow j)\right]. \tag{27}$$

The excited-states contribution coming from the second and third crossed terms of Eq. (18) vanishes, due to the opposite parities of the electric and magnetic dipole moments.

This part of the calculation reproduces the known results.[4,5] The quantity α is the electric polarizability of the nucleon in the conventional sense and the quantity β_1 gives a first contribution to the magnetic polarizability.[5]

In the term $a_{ij}^{(N)}$ one includes[4,5] not only the single-nucleon intermediate state but also the one containing a nucleon pair (zigzag diagrams). The prime in Eqs. (26) and (27) indicates that they contain all but these nucleon intermediate states.

We come now to the part $T_{ij}^{(2)}$ of the scattering amplitude.

From Eq. (24) we see that B_{ij} is at least of order ω^2. To that order the exponentials can be set equal to unity, $\vec{p}\,'=\vec{k}-\vec{k}'$ can be set equal to zero, and one can write

$$T_{ij}^{(2)}=\frac{2\pi}{i}\,\delta(p_0'+\omega'-M-\omega)\tfrac{1}{4}k_m'k_n\left\langle\vec{0}\right|\int d\vec{x}[\rho_{ij}(\vec{x})x_mx_n-\rho_{in}x_jx_m-\rho_{mj}x_ix_n+\rho_{mn}x_ix_j]\left|\vec{0}\right\rangle\ . \tag{28}$$

Consider for instance the first integral of Eq. (28). Since it is symmetric in i,j and m,n its value between spin-$\frac{1}{2}$ states at rest is of the form

$$\left\langle\vec{0}\right|\int d\vec{x}\rho_{ij}(\vec{x})x_mx_n\left|\vec{0}\right\rangle=a\delta_{ij}\delta_{mn}+b(\delta_{in}\delta_{jm}+\delta_{im}\delta_{jn})+c(\epsilon_{irm}\sigma_r\delta_{jn}+\epsilon_{irn}\sigma_r\delta_{jm}+i\leftrightarrow j)\ . \tag{29}$$

Substitution of this form in Eq. (28) leads to

$$T_{ij}^{(2)}=\frac{2\pi}{i}\,\delta(p_0'+\omega'-M-\omega)(\vec{k}\cdot\vec{k}'\delta_{ij}-k_ik_j')\beta_2\ , \tag{30}$$

where

$$\beta_2=\tfrac{1}{2}(a-b)\ . \tag{31}$$

The term proportional to c in Eq. (29), which does not appear in the final result, is actually not present if one invokes time-reversal invariance.

By adding Eqs. (25) and (30) we immediately recognize β_2 as an additional term to the magnetic polarizability.

From Eq. (29) we can express the constants a and b in terms of the second moment of the function ρ_{ij}. From Eq. (31) one obtains

$$\beta_2=\tfrac{1}{12}(\delta_{ij}\delta_{mn}-\delta_{im}\delta_{jn})\left\langle\vec{0}\right|\int d\vec{x}\rho_{ij}x_mx_n\left|\vec{0}\right\rangle\ . \tag{32}$$

It only remains now to express the second moment of ρ_{ij} in terms of the current-charge density commutator.

First of all, applying similar considerations to the case of the moments of the usual currents[11] we can write

$$\left\langle\vec{0}\right|\int d\vec{x}\rho_{ij}x_mx_n\left|\vec{0}\right\rangle=(-i)^2(2\pi)^3\lim_{\vec{Q}\to 0}\frac{\partial}{\partial Q_m}\frac{\partial}{\partial Q_n}\left\langle\frac{\vec{Q}}{2}\right|\rho_{ij}\left|-\frac{\vec{Q}}{2}\right\rangle\ . \tag{33}$$

From Eq. (5) we have

$$\int d\vec{x}\,e^{i\vec{k}\cdot\vec{x}}\left\langle\frac{\vec{Q}}{2}\right|[J_0(\vec{x}),J_i(0)]\left|-\frac{\vec{Q}}{2}\right\rangle=k_j\left\langle\frac{\vec{Q}}{2}\right|\rho_{ji}\left|-\frac{\vec{Q}}{2}\right\rangle\ . \tag{34}$$

Inserting a complete set of intermediate states on the left-hand side of this equation we obtain from Eqs. (32)–(35)

$$\beta_2=\frac{(2\pi)^6}{12}\lim_{\vec{k}\to 0}\lim_{\vec{Q}\to 0}\left(\frac{\partial}{\partial k_i}\nabla_Q^{\,2}-\frac{\partial}{\partial k_n}\frac{\partial}{\partial Q_i}\frac{\partial}{\partial Q_n}\right)\times\sum_n\left[\left\langle\frac{\vec{Q}}{2}\right|J_i\left|n,\vec{k}-\frac{\vec{Q}}{2}\right\rangle\left\langle n,\vec{k}-\frac{\vec{Q}}{2}\right|J_0\left|-\frac{\vec{Q}}{2}\right\rangle-\left\langle\frac{\vec{Q}}{2}\right|J_0\left|n,\frac{\vec{Q}}{2}-\vec{k}\right\rangle\left\langle n,\frac{\vec{Q}}{2}-\vec{k}\right|J_i\left|-\frac{\vec{Q}}{2}\right\rangle\right]. \tag{35}$$

Equation (35) is our main result. It gives the additional contribution that we were looking for.

The total magnetic polarizability is

$$\beta=\beta_1+\beta_2\ , \tag{36}$$

where β_1 is given by Eq. (27) and β_2 is given by Eq. (35).

In the next section we shall estimate β and we shall get a negative contribution from β_2. Of course, β_1 as given in Eq. (28) is a paramagnetic contribution. Although we have not been able to prove in general that β_2 constitutes a diamagnetic contribution, we can expect it to be so. To support this conjecture we resort to the old model of the scattering of light by a nucleon locally coupled to a pseudoscalar-meson field,[1] which in turn will give a qualitative picture of our result. In this case the function ρ_{ij} is given by

$$\rho_{ij}(x)=-2e^2\phi^*(x)\phi(x)\delta_{ij} \tag{37}$$

(we neglect possible difficulties involved in the construction of products of field operators at the same space-time point). In this model, we have from Eq. (32)

$$\beta_2 = -\frac{e^2}{3}\left\langle \vec{0} \right| \int d\vec{r}\,\phi^*\phi(\vec{r}) r^2 \left| \vec{0} \right\rangle . \tag{38}$$

Here we recognize β_2 as diamagnetic. We see that in the model β_2 is given by the mean square radius of the pion cloud distribution.

IV. CALCULATION OF THE PROTON MAGNETIC POLARIZABILITY

In this section we shall estimate the magnetic polarizability of the proton. From Eq. (27) the paramagnetic part is given by

$$\beta_1 = 2\sum_n{}' \frac{|\langle \vec{0}|m_z|n, \vec{0}\rangle|^2}{E_n(\vec{0}) - M} . \tag{39}$$

For the contribution of the first nucleon resonance we have $E_n(\vec{0}) = M^* = 1238$ MeV, and for the magnetic dipole transition moment we use the experimental value[12]

$$\langle p|m_z|N^{*+}\rangle = 1.28 \times \tfrac{2}{3}\sqrt{2}\ \mu_p . \tag{40}$$

With $M = 938$ MeV and $\mu_p = 0.13$ BeV^{-1} it follows that

$$\beta_1 = 12.1 \times 10^{-43}\ \text{cm}^3 . \tag{41}$$

Other contributions are positive but expected to be small due to larger mass differences and expected smaller couplings.

We now saturate the right-hand side of Eq. (35) with the nucleon and $N^*(1238)$. For the proton intermediate state we have

$$\langle p'|J_\mu|p\rangle = \frac{ie}{(2\pi)^3}\,\bar{u}(p')\left[F_1(q^2)\gamma_\mu - \frac{F_2(q^2)}{2M}\sigma_{\mu\nu}q_\nu\right]u(p) , \tag{42}$$

where $q = p' - p$, $F_1(0) = 1$, and $F_2(0) = \lambda$ is the anomalous moment of the proton.

From Eq. (35) we have for the proton intermediate state the diamagnetic contribution

$$\beta_2(p) = -\frac{e^2}{6M}\langle r^2\rangle_{\text{ch}} - \frac{e^2}{8M^3}(1 + 3\lambda + 3\lambda^2) , \tag{43}$$

where

$$\langle r^2\rangle_{\text{ch}} = \langle r_1{}^2\rangle + \frac{3\lambda}{2M^2} \tag{44}$$

is the Sachs mean square charge radius of the proton. It is interesting to note that the first term on the right-hand side of Eq. (43) is exactly the value that one has for spin-zero S-wave atoms where $\langle r^2\rangle$ is the mean square charge radius of the atom.

Using $\lambda = 1.79$ and $\langle r^2\rangle_{\text{ch}}/6 = 2.82$ BeV^{-2} one gets

$$\beta_2(p) = -3.0 \times 10^{-43}\ \text{cm}^3 . \tag{45}$$

For the contribution of N^{*+} we use

$$\langle p'|J_\mu|pN^*\rangle = -\frac{ieC_3}{(2\pi)^3}\,\bar{u}(p')\gamma_5(\gamma_\mu q_\alpha - q\cdot\gamma\delta_{\mu\alpha})u_\mu(p) . \tag{46}$$

One obtains again a diamagnetic contribution,

$$\beta_2(N^{*+}) = -\frac{1}{6M}\left(\frac{eC_3}{M}\right)^2\left(1 + \frac{M^*}{2M} + \frac{5M}{2M^*} + \frac{2M^2}{M^{*2}}\right) . \tag{47}$$

Using[13] $C_3 = 2.14$ BeV^{-1} one gets

$$\beta_2(N^{*+}) = -1.7 \times 10^{-43}\ \text{cm}^3. \tag{48}$$

With Eq. (45) we obtain the diamagnetic value

$$\beta_2 = -4.7 \times 10^{-43}\ \text{cm}^3 . \tag{49}$$

Adding to Eq. (41) we then estimate for the proton magnetic polarizability

$$\beta = 7.4 \times 10^{-43}\ \text{cm}^3 . \tag{50}$$

This value is somewhat larger than the experimental result of Goldanski *et al.*,[14]

$$\beta = (2 \pm 2) \times 10^{-43}\ \text{cm}^3 , \tag{51}$$

but it is consistent with the more recent value quoted by Bernabeu *et al.*,[7] who have reanalyzed the data of Ref. 14 and obtained

$$\beta = [(4 \pm 2) \pm 5] \times 10^{-43}\ \text{cm}^3 , \tag{52}$$

where the first error (±2) is statistical and the second one (±5) is systematic. The authors of Ref. 14, in a dispersive approach, give the theoretical prediction

$$\beta = 10 \times 10^{-43}\ \text{cm}^3 . \tag{53}$$

A better experimental determination of β is needed.

V. DISCUSSION

A closed expression for the magnetic polarizability of the nucleon has been obtained using gauge and relativistic invariance. We have shown that in addition to the well-known paramagnetic term containing products of magnetic dipole transition moments there exists a second term which is related to third moments of the current-charge density commutator.

We have not been able to show that this second term is diamagnetic, but we conjecture it to be so. This conjecture was supported by resorting to the old pseudoscalar-meson theory and also by direct calculation of some of the expected most important contributions, given in Eq. (49). The estimated magnetic polarizability of the proton is consistent with present experimental data, although a better experimental determination of β is needed.

Our approach is quite general and can be applied to targets of arbitrary spin S. The results (27) and (35) also hold for the spin-zero case. For higher-spin states $S \geq 1$ one has to disentangle symmetric terms of the form $\{S_i, S_j\}$.

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