Finite-temperature behavior of a relativistic field theory with dynamical symmetry breaking

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Recently, Gross and Neveu have studied a two-dimensional field theory of an N-component fermion in the large-N limit. This theory is asymptotically free and has dynamical spontaneous symmetry breaking. In this paper we study certain finite-temperature properties of this theory, especially those related to the survival of the "condensate," or symmetry breaking. Within the mean-field approximation, we find that the symmetry breaking disappears at a finite temperature T_0, which is of the same order of magnitude as the physical mass of a fermion. However, the mean-field approximation is not good for any finite N. At any nonzero temperature, however small, the system prefers to be in space-dependent field configurations such that the condensate vanishes. The critical temperature is thus zero.

I. INTRODUCTION

There has been some interest in studying finite-temperature properties of systems governed by relativistic quantum field theories. The interest is centered particularly around those theories which exhibit spontaneous symmetry breaking through the acquisition by some field operator of a vacuum expectation value.[1]

Gross and Neveu have investigated an interesting model in two dimensions (one space and one time).[2] The model is an adaptation of the Nambu–Jona-Lasinio model,[3] which in turn was constructed in analogy to superconductivity. It is described by the Lagrangian

$$\mathcal{L} = \sum_{j=1}^{N} [i\bar{\psi}_j \partial\!\!\!/ \psi_j + \tfrac{1}{2} g^2 (\bar{\psi}_j \psi_j)^2] \ . \tag{1.1}$$

In this paper we *assume* that (1.1) is equivalent to

$$\mathcal{L} = \sum_j (i\bar{\psi}_j \partial\!\!\!/ \psi_j - \tfrac{1}{2}\sigma^2 - g\sigma\bar{\psi}_j \psi_j) , \tag{1.2}$$

where σ is a scalar field and the ψ_j are N species of fermions. We shall work with (1.2), and therefore the results might not hold for (1.1) when the assumed equivalence fails.

Gross and Neveu showed that, for $N \to \infty$ and $\lambda \equiv Ng^2$ fixed, the theory has many interesting properties. It is solvable and has a nontrivial S matrix. It is renormalizable and asymptotically free. Spontaneous symmetry breakdown occurs at the one-loop level and the σ field develops a vacuum expectation value.

In this paper we investigate the finite-temperature behavior of systems governed by the Gross-Neveu Lagrangian. In particular, we study whether spontaneous symmetry breaking in such a system, present at $T=0$, survives at finite temperatures, whether there is a phase transition, and if so what the critical temperature is. The discussion and results here will be very simple, owing to the simplicity of the model. We present them purely for their theoretical interest, although there are possible applications of finite-temperature behavior of quantum fields to the real world.[1]

The existence of a nonzero vacuum expectation value $\langle 0|\sigma(x)|0\rangle$ is a property of the Hamiltonian, and has nothing to do with temperature. A general definition of symmetry breaking at any temperature can be given as follows. From the Lagrangian we can obtain the Hamiltonian H. Let the system be confined in a volume L, with periodic boundary condition to secure translational invariance. We also add an external field and define

$$H(h) = H - \int \sigma(x) h \, dx \ . \tag{1.3}$$

Then we say there is symmetry breaking if

$$\langle\sigma\rangle \equiv \lim_{h\to 0}\lim_{L\to\infty} \mathrm{Tr}[\sigma(x)e^{-\beta H(h)}]/\mathrm{Tr}e^{-\beta H(h)} \quad (1.4)$$

is nonzero, where $\beta = 1/T$. Translational invariance implies that $\langle\sigma\rangle$ is independent of x. The vacuum expectation value $\langle 0|\sigma(x)|0\rangle$ is just $\langle\sigma\rangle$ at $T=0$:

$$\langle 0|\sigma|0\rangle = \lim_{h\to 0}\lim_{L\to\infty}\lim_{\beta\to\infty} \mathrm{Tr}[\sigma(x)e^{-\beta H(h)}]/\mathrm{Tr}e^{-\beta H(h)} . \quad (1.5)$$

For our purpose, it is useful to know the probability of $\sigma(x)$ assuming the field configuration given by a classical field $\sigma_c(x)$. In this model the operator $\sigma(x)$ behaves essentially like a classical field, because there is no $\partial\sigma/\partial t$ term in the Lagrangian. This probability is proportional to $\exp[-\beta\Gamma(\sigma_c(x),\beta)]$, where $\Gamma(\sigma_c(x),\beta)$, an effective "thermodynamic potential," is the free energy associated with the field configuration $\sigma_c(x)$:

$$e^{-\beta\Gamma(\sigma_c(x),\beta)} \equiv \mathrm{Tr}_{\sigma_c}(e^{-\beta H}), \quad (1.6)$$

where the trace is taken over all states with $\sigma(x)$ fixed at $\sigma_c(x)$. The "Hamiltonian" to be used in (1.6) is $i\bar{\psi}\gamma_0\partial_0\psi - L$, with L given by (1.2). The object $e^{-\beta\Gamma(\sigma_c(x),\beta)}$ in (1.6) when integrated over $\sigma_c(x)$ will give the correct free energy of the purely fermion system in (1.1). $\Gamma(\sigma_c(x),\beta)$ is to be viewed as a classical potential for the σ field. (For a classical field, whether the kinetic term $\dot{\sigma}^2$ is present or not does not change the partition function by other than a constant.) Γ is *not* quite the finite-temperature equivalent of the "effective action," frequently used to study symmetry breaking in field theories.[4] But the minima of the classical potential $\Gamma(\sigma)$ do indicate whether symmetry breaking takes place or not. Some of these points have been elaborated on in Sec. IV.

In Sec. II we evaluate Γ for the case where $\sigma_c(x) = \sigma_c$ is a constant in space. We find that, for sufficiently low temperatures, $\Gamma(\sigma_c)$ has two symmetrical minima at $\sigma_c = \pm\sigma_M(T)$. At $T=0$, they reduce to the $\pm\sigma_M$ obtained by Gross and Neveu. As T increases, $\sigma_M(T)$ decreases. It vanishes at a temperature T_0. Beyond T_0, $\Gamma(\sigma_c)$ has just one "normal" minimum at $\sigma_c = 0$. If we ignore all but constant field configurations, the result is the mean-field approximation. This turns out to be the leading approximation in the $1/N$ expansion. The symmetry breaking continues with a decreasing $\langle\sigma\rangle = \sigma_M(T)$, until T reaches the critical temperature T_0, where $\langle\sigma\rangle$ vanishes in a second-order phase transition.

This phenomenon is very similar to superconductivity, at least mathematically, with $\sigma_c(x)$ playing the role of the order parameter. Many features of similarity will be discussed in Sec. II. They are not unexpected, given the parentage of the Gross-Neveu model and its obvious similarity to the four-fermion interaction in the BCS theory.

When space-dependent configurations of $\sigma_c(x)$ are not ignored, the results of the mean-field approximation are qualitatively modified. This is the subject of Sec. III. It will be shown that at any temperature, however small, the probability is overwhelming that the system splits into segments, with $\sigma_c(x)$ taking alternating values $\sigma_M(T)$ and $-\sigma_M(T)$. While such segment configurations have higher energies than the uniform configurations with $\sigma_c(x) \approx \sigma_M(T)$, the number of such configurations is sufficiently large (to gain enough entropy) to secure an overwhelming probability. As a result of this segmentation, $\langle\sigma\rangle$ vanishes [i.e., (1.4) vanishes]. Symmetry breaking occurs only at $T=0$ exactly [i.e., (1.5) is not zero]. The critical temperature is not T_0, but 0. Of course, this is just an example of a general rule for systems of one space dimension.[5]

We shall estimate the energy of a segmented configuration by evaluating the energy corresponding to a simple trial function of $\sigma_c(x)$—one which alternates between $+\sigma_M$ and $-\sigma_M$ in sharp steps, or sharp "kinks." The energy per kink will be shown to be proportional to N. For $N\to\infty$, the cost in energy for segmentation is prohibitive. If the infinite-volume limit $L\to\infty$ is taken *after* the limit $N\to\infty$, the segmentation will not take place and the mean-field approximation will be good. More precisely, the size of a segment will be shown to be proportional to $\exp(\alpha M_F N/T)$, where α is a constant of $O(1)$, and M_f is the fermion physical mass.[2] Thus, for sufficiently low temperatures (even when N is not large), the size of segments is very large, and the mean-field results $\sigma_M(T)$ and T_0 are of significance.

At any finite temperature, the alternating kink and "antikink" pairs (i.e., the boundaries separating the $+\sigma_M$ and $-\sigma_M$ segments and vice versa) behave just as sets of particle-antiparticle pairs. Thermodynamic equilibrium requires that they be produced with finite density at any finite temperature, much in the same way as electron-positron pairs must exist in a photon gas at any $T>0$.

The gradient expansion for $\Gamma(\sigma_c(x))$ and the Ginzburg-Landau equation is also briefly dis-

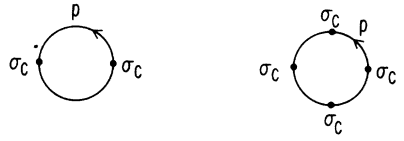

FIG. 1. One-fermion-loop graphs contributing to $\gamma(\sigma_c)$. The black dots represent σ_c, which acts as a constant external field in these graphs.

cussed in Sec. III.

Finally, in Sec. IV we make some remarks about the different kinds of "effective potentials," their interrelationship, and the use of an external field in defining one of them. These remarks are general and not restricted to the Gross-Neveu model.

II. PROBABILITY DISTRIBUTION AND THE MEAN-FIELD APPROXIMATION

A. The potential $\Gamma(\sigma_c(x))$

The probability distribution $e^{-\beta\Gamma}$ as given by (1.6) is the basis for studying statistical properties of the σ field. To find Γ, one fixes $\sigma(x)$ at $\sigma_c(x)$ and computes the free energy of the fermion field. This is to some extent analogous to the Born-Oppenheimer procedure in molecular and solid-state physics. There one fixes the positions of all nuclei and calculates the electronic energy. Do this for all nuclear configurations and the effective potential is obtained.

The Hamiltonian of interest follows from (1.6):

$$H=\int dx\left[\tfrac{1}{2}\sigma^2+\sum_j\left(-i\bar{\psi}_j\gamma^1\frac{\partial}{\partial x}\psi_j+g\sigma\bar{\psi}_j\psi_j\right)\right]. \quad (2.1)$$

Here x will denote the space coordinate only and $\gamma^\mu=(\gamma^0,\gamma^1)$. Note that there is no kinetic-energy term $(\partial\sigma/\partial t)^2$ for the σ field. Thus, the Hamiltonian is diagonal in eigenstates of the operator $\sigma(x)$. The σ field is effectively classical. Γ thus represents the entire Hamiltonian of σ.

For a fixed $\sigma(x)=\sigma_c(x)$, (2.1) is a Hamiltonian for noninteracting fermions in an external field. The free energy Γ can be deduced from the solutions to the Dirac equations. In terms of graphs, we have

$$\Gamma(\sigma_c(x),T)=C-T\sum_n\frac{1}{n!}\int\Gamma_n(x_1,\tau_1,\ldots,x_n,\tau_n)\times\prod_{i=1}^n\sigma_c(x_i)dx_i\,d\tau_i\ , \quad (2.2)$$

where Γ_n is the amputated n-point function for the σ field with one or no fermion loop. The constant C is independent of $\sigma_c(x)$.

Note that graphs in quantum statistical mechanics[6] are identical to those in field theory, except that time variables become purely imaginary $-i\tau$ with $0<\tau<\beta$. In the energy-momentum representation, the energy variables must be even integral multiples of $i\pi T$ for boson lines and odd for fermion lines. For $T\to 0$, the graph rules reduce to the usual Feynman rules with Wick rotations already performed.

B. The mean-field approximation

In practice, the calculation of Γ for an arbitrary $\sigma_c(x)$ is very difficult. Even if this is done, the problem is not over, because, after the probability distribution is known, we still have to perform nontrivial integrals over the $\sigma_c(x)$ space to get average values of interest, such as $\langle\sigma\rangle$.

The calculation is not difficult if $\sigma_c(x)=\sigma_c$, a constant independent of x. If x-dependent configurations are simply ignored, we get the mean-field approximation, which we now study.

Let us define

$$\gamma(\sigma_c)=\Gamma(\sigma_c)/L\ . \quad (2.3)$$

Since we are not interested in the constant C in (2.2), it is sufficient to calculate

$$\begin{aligned}\partial\gamma(\sigma_c)/\partial\sigma_c&=(\delta\Gamma/\delta\sigma_c(x))_{\sigma_c(x)=\sigma_c}\\&=\sigma_c-g\langle\bar{\psi}\psi\rangle\\&=\sigma_c-gNT\sum_\epsilon\int_{-\Lambda}^{\Lambda}\frac{dp}{2\pi}\,\mathrm{Tr}[(\not{p}-g\sigma_c)^{-1}]\ ,\end{aligned} \quad (2.4)$$

where $p=\epsilon\gamma^0-p\gamma^1$, and the sum over ϵ is taken over odd integral multiples of $i\pi T$. The one-loop graphs summed in (2.4) are shown in Fig. 1.

The sum over ϵ in (2.4) can be converted into an integral along the contour shown in Fig. 2:

$$T\sum_\epsilon=-\oint\frac{d\epsilon}{2\pi i}\,\frac{1}{e^{\beta\epsilon}+1}\ . \quad (2.5)$$

The contour can be deformed to pick up the poles at $\pm\epsilon_p$,

$$\epsilon_p\equiv(p^2+g^2\sigma_c{}^2)^{1/2}\ . \quad (2.6)$$

We obtain

$$\partial\gamma/\partial\sigma_c=\sigma_c\left[1-\frac{\lambda}{2\pi}\int_{-\Lambda}^{\Lambda}\frac{dp}{\epsilon_p}\left(1-\frac{2}{e^{\beta\epsilon_p}+1}\right)\right]\ , \quad (2.7)$$

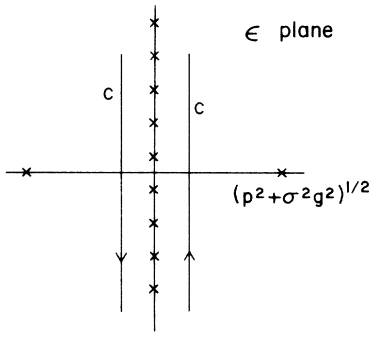

FIG. 2. Summation over the poles at odd integers on the imaginary ϵ axis can be converted to an integral over contour c which can then be distorted to pick up poles at $\pm(p^2+g^2\sigma_c{}^2)^{1/2}$.

where $\lambda \equiv g^2 N$. Note that when $T \neq 0$, there is no infrared divergence in (2.7) even when $\sigma_c = 0$.

The cutoff Λ will disappear after renormalization:

$$\sigma_c \to Z^{1/2}\sigma_c \ , \qquad g \to Z^{-1/2} g \ , \tag{2.8}$$

where Z is furnished by the theory at $T = 0$ by fixing the value of $\partial^2\gamma/\partial\sigma_c{}^2$ at some point. In the large-N limit, it is given by

$$Z = 1 + \frac{\lambda}{2\pi}\left[\ln(4\Lambda^2/g^2\sigma_0{}^2) - 2\right]$$

if

$$(\partial^2\gamma/\partial\sigma_c{}^2)_{T=0,\ \sigma_c=\sigma_0} = 1 \ . \tag{2.9}$$

Substituting (2.8) and (2.9) in (2.7) and setting $\partial\gamma/\partial\sigma_c = 0$, we locate the extrema of γ. There are three of them, $\sigma_c = 0, \pm\sigma_M(T)$ (see Fig. 3). The quantity σ_M satisfies

$$1 = \frac{\lambda}{2\pi Z}\int_{-\Lambda}^{\Lambda}\frac{dp}{\epsilon_p}\tanh\tfrac{1}{2}\beta\epsilon_p \ , \tag{2.10}$$

where ϵ_p is given by (2.6) with $\sigma_c = \sigma_M(T)$. It is easily shown that (i) for $T \to 0$, (2.10) reduces to the Gross-Neveu relation for the minima of their potential; (ii) as T increases, $\sigma_M(T)$ decreases; (iii) at some temperature T_0, $\sigma_M(T_0)$ vanishes, and, for $T > T_0$, (2.10) has no real solution. The temperature T_0 is obtained by setting $\sigma_M(T) = 0$ in (2.10):

$$1 = \frac{\lambda}{2\pi Z}\int_{-\Lambda}^{\Lambda}\frac{dp}{p}\tanh(p/2T_0)$$

$$= \frac{\lambda}{2\pi Z}\,2\ln(1.14\,\Lambda/T_0) \ . \tag{2.11}$$

Using Z from (2.9), we get

$$T_0 = 0.57 M_f \ , \tag{2.12}$$

$$M_f \equiv g\sigma_M(0) \ . \tag{2.13}$$

M_f is the physical fermion mass.[2]

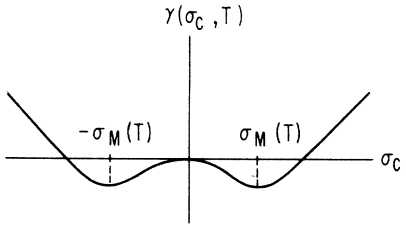

FIG. 3. Shape of the potential $\gamma(\sigma_c, T)$ with minimum at $\pm\sigma_m(T)$ which will approach zero as $T \to T_0$ from below.

In the $L \to \infty$ limit, the probability distribution $\exp(-\beta\Gamma(\sigma_c))$ peaks infinitely sharply at the minima of $\gamma(\sigma_c)$ since $\Gamma(\sigma_c) = L\gamma(\sigma_c)$. The exact symmetry between the two minima $\pm\sigma_M(T)$ can be removed by adding a small external field h, as was done in (1.3). As a result, the peak of the probability distribution at $\sigma_c = +\sigma_M(T)$ dominates and (1.4) gives

$$\langle\sigma\rangle = \sigma_M(T), \quad T < T_0 \ . \tag{2.14}$$

For $T > T_0$, $\langle\sigma\rangle = 0$.

The strong resemblance between these results and those in the BCS theory of superconductivity is evident. The gap $\Delta(T)$ in the BCS theory is akin to $g\sigma_M(T)$ here. Equation (2.10) is identical to the gap equation.[7] So are (2.12) and (2.13) connecting T_0 and M_f identical to the relation between the critical temperature and the zero-temperature gap in BCS theory.

Note that in BCS theory, electron momenta are restricted to the neighborhood of the Fermi surface. As a result, the electron energy depends only on $k - k_F$, where k is the momentum component perpendicular to the Fermi surface, and k_F is the Fermi momentum. The density of one-electron states is nonzero and constant around $k - k_F = 0$. Furthermore, $|k - k_F|$ is restricted to less than the Debye frequency ω_D. In the Gross-Neveu model, the fermion momentum p plays the role of $k - k_F$. There is only one space dimension and the density of states near $p = 0$ is nonzero (and in fact always constant). There is no natural cutoff, however. The theory in terms of bare parameters would diverge. Requiring the renormalized theory to be finite makes the bare coupling constant vanishingly small, and unrenormalized σ_c infinite.

It should also be noted that the condensed σ field here carries no charge and is a real field. There is no superfluidity. This is in contrast to the BCS condensate, which is a complex field of charge $2e$ and does give rise to superfluidity.

C. Finite fermion density

The total number of fermions minus the total number of antifermions

$$\hat{N} = \int dx : \bar{\psi}\gamma_0\psi : \tag{2.15}$$

is a conserved quantity. Let us examine the effect on symmetry breaking of a nonzero fermion density $\hat{N}/L$.

We add a term $-\mu\hat{N}$ to the Hamiltonian where μ is the chemical potential. Mathematically this addition simply changes the free fermion propagator $1/\not{p}$ to $1/(\not{p} + \gamma_0\mu)$. Following previous steps

of calculation, one obtains

$$\partial\gamma/\partial\sigma_c = (\partial\gamma/\partial\sigma_c)_{T=\mu=0} + \frac{\lambda\sigma_c}{\pi}\int_0^\infty \frac{dp}{\epsilon_p}\left(\frac{1}{e^{\beta(\epsilon_p-\mu)}+1} + \frac{1}{e^{\beta(\epsilon_p+\mu)}+1}\right), \tag{2.16}$$

where ϵ_p is given by (2.7). It is a matter of further algebra to obtain the minimum $\sigma_M(T, \mu)$ of γ by solving $\partial\gamma/\partial\sigma_c = 0$. The answer for the case $T = 0$ is particularly simple. Let p_F be the Fermi momentum, i.e.,

$$\epsilon_{p_F} = \mu, \tag{2.17}$$

and let m be defined as $g\sigma_M(0, \mu)$. Then (2.16) gives

$$\frac{m}{M_f} = \left(1 - \frac{2p_F}{M_f}\right)^{1/2}. \tag{2.18}$$

Note that the density of fermions is related to p_F by

$$\hat{N}/L = Np_F/2\pi \tag{2.19}$$

in this case. In view of (2.18), the symmetry breaking is diminished by a nonzero fermion density and vanishes when p_F goes beyond $M_f/2$.

III. FEATURES AT VERY LOW TEMPERATURES

The mean-field approximation discussed above does not adequately describe the physical picture. This is because too little attention is paid to configurations which differ considerably from the uniform configuration $\sigma_c(x) = \text{constant}$. When the effect of nonuniform configurations is taken into account, there is a qualitative modification of the mean-field results. In particular, $\langle\sigma\rangle$ vanishes when $T \neq 0$. This particular modification is peculiar to infinite systems of one dimension, as noted by Landau and Lifshitz.[5] Thus, the critical temperature for this model is actually zero. However, the mean-field results are still meaningful. We shall devote this section to questions related to these facts. Attention will be limited to $T \neq 0$, but $T \ll T_0$.

A. Qualitative aspects

It is very easy to see that $\langle\sigma\rangle$ must be zero for $T \neq 0$. The potential $\Gamma(\sigma_c(x))$ has a minimum when $\sigma_c(x) = \sigma_M$. For the configuration $\sigma(x) = -\sigma_M$, the potential is higher by the amount $2h\sigma_M$ per unit length, where the small field h, introduced in (1.3), is to make $-\sigma_M$ less favorable than σ_M. Now consider a configuration with "kinks" as shown in Fig. 4, where $\sigma_c(x)$ is either σ_M or $-\sigma_M$ except near a kink. Assuming the kinks are sufficiently far apart, each kink contributes some energy ϵ_K above the ground-state energy. If there are n kinks, the energy is $\sim(n\epsilon_K + h\sigma_M L)$, where the last term estimates the gain in energy in the portions with $\sigma_c(x) = -\sigma_M$. It is not affected much when the positions of the kinks are varied. The probability of finding n kinks is then proportional to

$$\frac{1}{n!}\int_0^L dx_1 dx_2 \cdots dx_n e^{-(n\epsilon_K + h\sigma_M L)/T} \approx e^{-[n\epsilon_K + h\sigma_M L - nT(\ln(L/n)+1)]/T}, \tag{3.1}$$

which is strongly peaked as $h \to 0$, at

$$n \approx Le^{-\epsilon_K/T} \tag{3.1'}$$

in the limit of large L. Thus, we conclude that there is a nonzero kink density if $T \neq 0$. As long as ϵ_K is a finite quantity, the internal energy $n\epsilon_K$ required to produce the kinks is more than offset by the gain in entropy $n[\ln(L/n)+1]$ associated with the positions of the kinks. The resulting free energy is lowered by the presence of a finite kink density given by (3.1').

Further, when a finite density of kinks per unit length is present, the value of σ clearly alternates between $+\sigma_M$ and $-\sigma_M$ in alternating segments. As $h \to 0$, the $\sigma = +\sigma_M$ segment is not preferred over the $\sigma = -\sigma_M$ segment. When the locations of the kinks, i.e., the sizes of the segments are integrated over all possible values as in (3.1), it is obvious that the region of $\sigma = +\sigma_M$ will, on the average, have the same weight as those of $\sigma = -\sigma_M$. Thus $\langle\sigma\rangle = 0$, in the limit $h \to 0$.

The reason why this result does not appear in the mean-field approximation is because, by definition, that approximation ignores spatially varying fields. Equivalently, in the $N \to \infty$ limit, the internal energy ϵ_K required to produce a kink becomes infinite, as we show in the following section. Then, the entropy can never offset the internal energy.

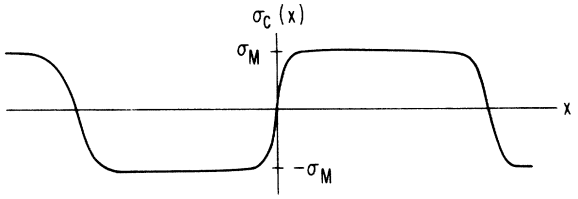

FIG. 4. Typical field configuration with "kinks." The field alternates between $+\sigma_M$ and $-\sigma_M$, which are the minima of the mean-field potential.

The arguments given here are of course heuristic and do not constitute a rigorous proof that $\langle\sigma\rangle = 0$.

The above argument does not apply to systems of more than one dimension. Take the two-dimensional case, for example. A finite but small temperature implies $\sigma_c(x) = \sigma_M$ everywhere except in small regions, "bubbles," where $\sigma_c(x) = -\sigma_M$, scattered over the system. The energy of a bubble is proportional to the circumference, which increases with the area of the bubble. Thus the size of bubbles is limited and is very small when T is very small. Thus $\langle\sigma\rangle \neq 0$ for small enough T. In the one-dimensional case, the "circumference" is just two points, whose "size" does not increase with the size of the region with $\sigma_c = -\sigma_M$.

B. Estimate of energy per kink

For a rough estimate of the energy of a kink, we first evaluate $\Gamma(\sigma_c(x))$ for a simple configuration

$$\sigma_c(x) = (m/g)\,\mathrm{sgn}x\,, \tag{3.2}$$

where m is a positive constant, and then calculate

$$\epsilon_K = \Gamma(\sigma_M\,\mathrm{sgn}x) - \Gamma(\sigma_M) \tag{3.3}$$

by integrating $\partial\Gamma/\partial m$ with respect to m. It is easy to show that

$$m\frac{\partial\Gamma(\sigma_c)}{\partial m} = \int dx(\sigma_c{}^2 + \langle\bar{\psi}(x,0)\psi(x,0)\rangle\sigma_c(x))\,, \tag{3.4}$$

where

$$\langle\bar{\psi}\psi\rangle = N\sum_{p,\lambda}\mathrm{Tr}\psi_{p\lambda}(x)\bar{\psi}_{p\lambda}(x)\theta(\epsilon_p)\,, \tag{3.5}$$

and ϵ_p and $\psi_{p\lambda}(x)$ are eigenvalues and eigenfunctions of the Dirac Hamiltonian:

$$\left(-i\gamma_0\gamma_1\frac{\partial}{\partial x} - m\gamma_0\right)\psi_{p\lambda} = \epsilon_p\psi_{p\lambda}\,. \tag{3.6}$$

For $\epsilon_p > 0$, we find

$$\epsilon_p = (m^2+p^2)^{1/2}\,,$$
$$\psi_{p\lambda,\eta} = [A_\lambda A_\eta\theta(x) + A_{-\lambda}A_{-\eta}\theta(-x)]L^{-1/2}(e^{ipx} - \lambda\eta e^{-ipx})\,,$$
$$\lambda = \pm\,,\quad p>0\,,$$
$$\eta = \pm\,,\quad A_\pm = [(\epsilon_p \pm m)/2\epsilon_p]^{1/2}\,. \tag{3.7}$$

We have used the notation for fermion wave functions

$$\psi = \begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}$$

and the representation $\gamma_0 = \tau_3$, $\gamma_1 = i\tau_2$, where τ_i are Pauli matrices.

Substituting (3.7) in (3.5), we find

$$\langle\bar{\psi}\psi\rangle = mN\int_0^\Lambda\frac{dp}{2\pi}(2/\epsilon_p)(1-\cos 2px)\,\mathrm{sgn}x\,. \tag{3.8}$$

Using (3.4), (3.3), and (3.8), we obtain

$$\epsilon_K = \tfrac{1}{2}NM_f\,. \tag{3.9}$$

Renormalization is easy and needs no explanation. The result (3.9) is for the special kink shape $\sigma_M\,\mathrm{sgn}x$. The shape of the kink can be changed so as to minimize ϵ_K. The minimum would be NM_f times a number somewhat smaller than $\frac{1}{2}$. There is little point in determining this number precisely.[8]

Clearly, for large N, ϵ_K is large. The kink density is proportional to

$$e^{-\epsilon_K/T} \sim e^{-NM_f/2T}\,, \tag{3.10}$$

which is very small. In the $N\to\infty$ limit, it vanishes exponentially. Thus, it will not show up in finite orders of the $1/N$ expansion.

As long as one looks at a finite segment of the system, the probability of finding a kink is vanishingly small. In this sense, the mean-field approximation is not bad.

C. Probability distribution and effective potential for the dc component of $\sigma_c(x)$

Consider the effective potential $\Gamma(\sigma_c(x))$ with $\sigma_c(x) = \sigma$ and the effective potential $A(\bar{\sigma})$ defined by

$$P(\bar{\sigma}) = \left\langle\delta\left(\bar{\sigma} - \frac{1}{L}\int dx\,\sigma(x)\right)\right\rangle$$
$$\propto\int\delta\sigma_c(x)\,e^{-\beta\Gamma(\sigma_c(x))}\delta\left(\bar{\sigma} - \frac{1}{L}\int dx\,\sigma_c(x)\right)$$
$$\equiv e^{-\beta A(\bar{\sigma})}. \tag{3.11}$$

Thus, $A(\bar{\sigma})$ is defined via the probability distribution for the dc component of $\sigma_c(x)$. It is the free energy with one restriction,

$$\frac{1}{L}\int dx\,\sigma_c(x) = \bar{\sigma}\,, \tag{3.12}$$

while $\Gamma(\sigma_c(x) = \bar{\sigma})$ is the free energy with an infinite number of restrictions: $\sigma_c(x) = \bar{\sigma}$ for every point x. What is plotted in Fig. 3 is $\Gamma(\bar{\sigma})/L$ and not $A(\bar{\sigma})/L$.

To determine whether $\langle\sigma\rangle$ is zero, one must find out whether $A(\bar{\sigma})$ has its minimum at a nonzero σ. It is not sufficient to look at $\Gamma(\sigma_c(x) = \bar{\sigma})$.

At $T = 0$, $A(\bar{\sigma})$ is simply the minimum of the potential $\Gamma(\sigma_c(x))$ with any $\sigma_c(x)$ satisfying (3.12). Clearly, for a $\sigma_c(x)$ with one kink, we can make $\bar{\sigma}$ equal to any value between $-\sigma_M$ and σ_M by putting the kink at the right place. Since ϵ_K is finite (for finite N),

$$(A(\bar{\sigma}) - \Gamma(\sigma_c = \sigma_M))/L = \frac{\epsilon_K}{L} = 0 \tag{3.13}$$

in the limit $L\to\infty$, $h\to 0$, for $|\bar{\sigma}| < \sigma_M$. For $|\bar{\sigma}| > \sigma_M$, the uniform $\sigma_c(x) = \bar{\sigma}$ gives the lowest $\Gamma(\sigma_c(x))$, and

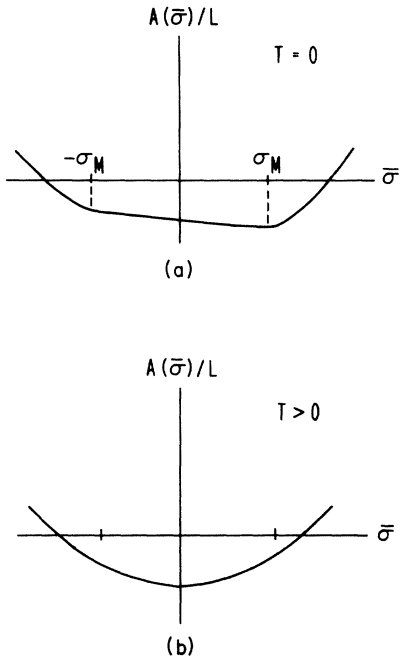

FIG. 5. (a) The potential $A(\bar{\sigma})/L$ at zero temperature, in the presence of a small external field h. (b) The same potential at a small but finite temperature, with h turned off for simplicity.

$A(\bar{\sigma})/L = \Gamma(\sigma_c = \bar{\sigma})/L$. To sum up, $A(\bar{\sigma})/L$ is obtained from Fig. 3 by replacing the portion between $-\sigma_M$ and σ_M by a straight line. For a nonzero h, the line is tilted. [See Fig. 5(a).]

If the temperature is nonzero, configurations of many kinks and satisfying (3.13) become important. They give higher energy but also higher entropy. The net result is a finite density of kinks as described earlier. $A(\bar{\sigma})/L$ then behaves like Fig. 5(b) for a finite but low temperature. It always has a positive curvature. Note that the convexity property

$$\frac{1}{L}A(\tfrac{1}{2}(\bar{\sigma}_1+\bar{\sigma}_2)) \leq \frac{1}{2L}(A(\bar{\sigma}_1)+A(\bar{\sigma}_2)) \tag{3.14}$$

for $L \to \infty$ is a general requirement for thermodynamic stability of homogeneous systems.

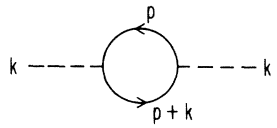

FIG. 6. The coefficient of k^2 in this graph gives the constant a in the Landau-Ginzburg equation (3.15).

D. The Ginzburg-Landau potential

For those $\sigma_c(x)$ which are small and slowly varying, the potential $\Gamma(\sigma_c)$ can be approximated at nonzero T by the Ginzburg-Landau form

$$\Gamma(\sigma_c(x)) \approx \int dx \left[\tfrac{1}{2} a \left(\frac{\partial \sigma_c}{\partial x} \right)^2 + \tfrac{1}{2} b \sigma_c{}^2 + \tfrac{1}{4} c \sigma_c{}^4 \right]. \tag{3.15}$$

This is obtained by expanding $\Gamma(\sigma_c)$ in powers of $\partial\sigma_c/\partial x$ and σ_c, and dropping higher powers. It is useful only when the σ_c of interest are in fact slowly varying and small.

The constants b and c can be obtained by expanding (2.7). The constant a can be obtained by evaluating the k^2 term of the graph in Fig. 6, which is the Fourier transform of

$$\delta^2\Gamma/\delta\sigma_c(x)\delta\sigma_c(0) = -a\frac{\partial^2}{\partial x^2}\delta(x) + \cdots . \tag{3.16}$$

Let us skip the algebra, which is almost identical to Gorkov's derivation of the Ginzburg-Landau equation of superconductivity from BCS theory.[6] The result is

$$\Gamma(\sigma_c) = (\alpha/T^2)\frac{\lambda}{2\pi}\int dx \left[\frac{1}{2}\left(\frac{\partial \sigma_c}{\partial x} \right)^2 + \frac{T^2}{2\alpha}\left(\ln\frac{T}{T_0} \right)\sigma_c{}^2 + \frac{\lambda}{4N}\sigma_c{}^4 \right], \tag{3.17}$$

where

$$\alpha \equiv 7\zeta(3)/8\pi^2,$$

ζ is the Riemann ζ function, and T_0 is defined by (2.23). The extrema of $\Gamma(\sigma_c)$ are determined by the Ginzburg-Landau equation $\delta\Gamma/\delta\sigma_c = 0$:

$$-\frac{\partial^2\sigma_c}{\partial x^2} + \left(\frac{T^2}{\alpha}\ln\frac{T}{T_0} \right)\sigma_c + \frac{\lambda}{N}\sigma_c{}^3 = 0 . \tag{3.18}$$

The statistical mechanics of a one-dimensional classical field described by a Ginzburg-Landau potential such as (3.17) has been studied by Scalapino, Sears, and Ferrell.[9] Their exact calculation shows that $\langle\sigma^2\rangle$ differs from $[\sigma_M(T)]^2$ very little, except in the immediate neighborhood of T_0.

Evidently, the $1/T^2$ factor in front of (3.17) indicates that, for $T \to 0$, the expression will look very different. Since, at $T = 0$, $\sigma_c{}^{-1}$ is the only length parameter (except the one in the logarithm), we expect σ_c to replace T in (3.17). A little algebra shows that, for $T = 0$,

$$\Gamma(\sigma_c) = \int dx \left[\frac{N}{24\pi\sigma_c{}^2}\left(\frac{\partial\sigma_c}{\partial x} \right)^2 + \gamma(\sigma_c) \right], \tag{3.19}$$

plus higher orders in $\sigma_c^{-1}\partial/\partial x$, where $\gamma(\sigma_c)$ is given by (2.7). The applicability of (3.19) is limited to $|\sigma_c|$ sufficiently large. In particular, (3.19) cannot be used to study kink configurations where σ_c passes through zero.

IV. REMARKS ON POTENTIALS

In this section we shall make some trivial but often overlooked remarks concerning the concept of "potential." We shall first draw the distinction between the elementary definition of potential familiar in statistical physics and the definition of "effective action" in recent field-theory literature. Some difficulties associated with the latter will be pointed out.

A. Definitions of potentials

There are different definitions of the term "potential." They are not the same, although they are often mistaken to be.

For simplicity, let us start with a classical scalar field $\phi(x)$ in a d-dimensional Euclidean space and in thermal equilibrium. Ignore the kinetic degrees of freedom. Thus, the Hamiltonian is simply the potential energy. For convenience, the reader can consider the example where the potential energy is

$$\Gamma(\phi(x)) = \int d^d x \left[\tfrac{1}{2}(\nabla\phi)^2 + b\phi^2 + c\phi^4\right]. \tag{4.1}$$

[If the field ϕ interacted with some other field ψ, then the degrees of freedom associated with ψ are summed over and contained in $\Gamma(\phi(x))$ as, for instance, in Sec. II.] The probability distribution for ϕ is

$$P(\phi(x)) \propto e^{-\beta\Gamma(\phi(x))}. \tag{4.2}$$

The potential $\Gamma(\phi(x))$ thus tells us all about the system, in every detail, as far as ϕ dependence is concerned.

Suppose we are only interested in a certain special dynamic variable, for example,

$$\bar{\phi} = L^{-d}\int d^d x\, \phi(x), \tag{4.3}$$

where L^d is the volume of the system. Then we can define a potential $A(\bar{\phi})$:

$$P(\bar{\phi}) \propto \int \delta\varphi(x)\, e^{-\beta\Gamma(\phi(x))}\, \delta(\phi - L^{-d}\int d^d x\, \phi(x)) \equiv e^{-\beta A(\bar{\phi})}. \tag{4.4}$$

The quantity $\bar{\phi}$ is in some sense the "center of mass." The potential $A(\bar{\phi})$ is thus the potential seen by the center of mass.

It is clear that we can define a potential for any set of dynamic variables in a similar way. For example, let

$$\varphi(x) = \sum_{k<\lambda} L^{-d/2} e^{ik\cdot x}\phi_k, \tag{4.5}$$

where

$$\phi_k \equiv L^{-d/2}\int d^d x\, e^{-ik\cdot x}\phi(x), \tag{4.6}$$

and λ is an arbitrary cutoff. The field $\varphi(x)$ is the "long-wavelength part" of $\phi(x)$. The probability distribution for $\varphi(x)$ is

$$P(\varphi) \propto \int \prod_{k>\lambda} d\phi_k\, e^{-\beta\Gamma(\phi)} \equiv e^{-\beta A(\varphi)}. \tag{4.7}$$

$A(\varphi(x))$ is thus the potential seen by $\varphi(x)$. It is something in between the potentials $\Gamma(\phi(x))$ and $A(\bar{\phi})$. Of course, it depends on λ.

These three potentials $\Gamma(\phi(x))$, $A(\varphi(x))$, and $A(\bar{\phi})$ are directly expressing the probability distributions of interest. They are related to the free energy W by

$$\begin{aligned} e^{-\beta W} &= \int \delta\phi(x) e^{-\beta\Gamma(\phi(x))} \\ &= \int \delta\varphi(x) e^{-\beta A(\phi(x))} \equiv \int \prod_{k<\lambda} d\phi_k\, e^{-\beta A(\varphi_k)} \\ &= \int d\bar{\phi}\, e^{-\beta A(\bar{\phi})}. \end{aligned} \tag{4.8}$$

Finally, we come to the potential often appearing in recent field-theory literature.[4] We first define $W(J)$ by

$$e^{-\beta W(J)} = \int \delta\phi \exp\left[-\beta\left(\Gamma - \int d^d x\, J(x)\phi(x)\right)\right]. \tag{4.9}$$

Clearly, $W(J)$ is simply the free energy when an external force $J(x)$ is turned on. The average of $\phi(x)$ at the presence of $J(x)$ is

$$\langle\phi(x)\rangle_J = -\frac{\delta W}{\delta J(x)} \equiv \phi_c(x). \tag{4.10}$$

Then define a potential Ξ as

$$\Xi(\phi_c) = W(J) + \int d^d x\, J(x)\phi_c(x), \tag{4.11}$$

where it is understood that one has solved (4.10) for J in terms of ϕ_c and thereby expresses (4.11) in terms of ϕ_c. We have

$$J(x) = +\frac{\delta\Xi}{\delta\phi_c(x)}. \tag{4.12}$$

If Ξ is known, the free energy $W(0)$ is thus determined as an extremum of Ξ. We can also obtain $\langle\phi(x)\rangle$ as a solution to (4.12).

Note that Ξ does not have a direct connection to the probability distribution of the field ϕ. It does not have a clear physical interpretation in contrast to the potentials $\Gamma(\phi)$, $A(\varphi)$, and $A(\bar{\phi})$.

To begin with, $\Xi(\phi_c)$ may be undefined over some regions in ϕ_c space. For example, let us consider the case $\beta \to \infty$. Then, by (4.9), $W(J)$ is simply the minimum of $\Gamma_J \equiv \Gamma - \int d^d x\, J\phi$. The location of the minimum in ϕ space gives ϕ_c, and Ξ is just $\Gamma(\phi_c)$. One can attempt to adjust J to move the location of the minimum and hence define Ξ for different ϕ_c's. However, adjusting J can never change the curvature of the potential $\Gamma(\phi)$ since $J\phi$ is linear in ϕ. Therefore, if $\delta^2\Gamma/\delta\phi(x)\delta\phi(x')$ is not positive-definite at $\phi_c(x)$, there is no $J(x)$ such that $\Gamma_J \equiv \Gamma - \int J\phi\, d^d x$ is minimum at this ϕ_c. Further, having a positive-definite $\delta^2\Gamma/\delta\phi(x)\delta\phi(x')$ is sufficient only to guarantee that Γ_J can be made *locally* minimum at $\phi(x)$ by taking $J(x) = \delta\Gamma/\delta\phi(x)$. However, a local minimum is not enough. Equation (4.9) demands the absolute minimum. Equations (4.9)–(4.11) make sense only for those ϕ_c where Γ_J has an absolute minimum for some J. For a finite β, the above argument must be modified. It seems evident that, in general, the potential Ξ is defined only for a subset of ϕ_c, although a multisheeted structure for $W(J)$ in J might widen the range of ϕ_c for which $\Xi(\phi_c)$ is defined.

B. Perturbation expansion of the potentials

Clearly, the potential $\Gamma(\phi)$ needs no discussion, if there is only one field ϕ in the problem. It is given as input.

From the definition (4.7), we see that $A(\varphi)$ is the sum of all free-energy graphs for the field $\phi'(x)$ defined by

$$\phi(x) = \phi'(x) + \varphi(x) \,. \tag{4.13}$$

In other words, we substitute (4.13) in $\Gamma(\phi)$ and compute graphs with no external ϕ' line and no internal φ line. Thus all the internal lines have momenta larger than λ. This guarantees that no infrared problem can arise in computing $A(\varphi)$.

For the potential $A(\bar{\phi})$ defined by (4.4), we simply lower the cutoff λ to 0^+. Namely,

$$\phi(x) = \phi'(x) + \bar{\phi} \,. \tag{4.14}$$

One sums all graphs with internal-line momenta not identically zero. In view of our discussion on $A(\bar{\sigma})$ in the previous section, it it clear that one could encounter infrared problems, if there is such a problem, in computing $A(\bar{\phi})$. The kink configurations, for example, are very difficult to account for by perturbation theory.

To compute $\Xi(\phi_c)$ by perturbation theory, one writes $\phi(x) = \phi'(x) + \phi_c(x)$ and sums all one-particle irreducible graphs with ϕ' internal lines. There is no restriction on internal-line momenta, and infrared problems stay.

Note that in counting the power of coupling constants for a given term, one must take into account the order of magnitude of φ, $\bar{\phi}$, or ϕ_c. If, for example, the coefficient b in (4.1) is negative, then the minimum of $\Gamma(\phi)$ will occur at $\phi(x) = \pm(b/c)^{1/2} = O(c^{-1/2})$. This means that we must count φ, $\bar{\phi}$, or ϕ_c as $O(c^{-1/2})$ in computing $A(\varphi)$, $A(\bar{\phi})$, or $\Xi(\phi_c)$, since we are in most cases interested in the minima of $\Gamma(\phi)$.

To the lowest order in c, we have

$$\begin{aligned} A(\varphi) &= \Gamma(\varphi) + U(\varphi) \,, \\ A(\bar{\phi}) &= \Gamma(\bar{\phi}) \,, \\ \Xi(\phi_c) &= \Gamma(\phi_c) \,, \end{aligned} \tag{4.15}$$

when we take φ, $\bar{\phi}$, ϕ_c as $O(c^{-1/2})$. $U(\varphi)$ includes graphs such as Fig. 7. The momenta of the external lines are all less than λ but that of the internal line can be larger than λ. These graphs do not enter for $A(\bar{\phi})$ because the external lines must have zero momentum ($\bar{\phi}$ is a constant) while the internal lines must have nonzero momenta. Equation (4.15) is the so-called "tree approximation." The potentials are of $O(c^{-1})$.

The higher-order terms in the perturbation series are essentially cumulants of ϕ' (which is $\phi - \phi$, $\phi - \bar{\phi}$, or $\phi - \phi_c$). These terms are ordered with ϕ' counted as $O(1)$. This essentially means ϕ' is much smaller than φ or $\bar{\phi}$ or ϕ_c, which is of $O(c^{-1/2})$. For those cases where configurations with large ϕ' [comparable to $O(c^{-1/2})$, for example] are important, the perturbation series may become meaningless as every term becomes of the same order in c. In fact, the kink configurations discussed previously serve as an illustration. There $\Gamma(\phi_c)$, $A(\bar{\sigma})$ play the roles of $\Gamma(\phi)$ and $A(\bar{\phi})$, respectively. Although $\Gamma(\sigma)$ in the GN model is not a simple σ^4 potential, it does imply a perturbation expansion in powers of $1/N$. We have $\sigma_M = O(N^{1/2})$. The $1/N$ expansion is good only if $\sigma' = \sigma - \sigma_M$ is of $O(1)$. The fact that the kink configurations have $\sigma' \sim \sigma_M$ implies the breakdown of the $1/N$ expansion. We showed that kinks are important for configurations with $|\bar{\sigma}| < \sigma_M$, but not

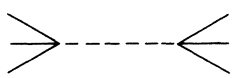

FIG. 7. The tree graph giving $U(\varphi)$ in (4.6). The dashed internal line refers to the same scalar field, but with momenta $k > \lambda$.

for $|\bar{\sigma}| > \sigma_M$. This is why $A(\bar{\sigma})$ is qualitatively different from $\Gamma(\bar{\sigma})$ only for $|\bar{\sigma}| < \sigma_M$. [See Figs. 5(a) and 5(b).]

The discussion so far was restricted to classical fields with no kinetic degrees of freedom. The latter are present in the Hamiltonian usually only through a term $\frac{1}{2}(\pi(x))^2$ where π is the canonical momentum. For a classical field, π and ϕ commute, and integrals for the free energy go over both π_k and ϕ_k for each mode. The integrals over π_k are, however, trivial Gaussians and merely multiply the potentials by temperature-dependent constants. Thus, for instance, the potential in (4.7) becomes

$$e^{-\beta A(\varphi)} = \int \prod_{k>\lambda} d\phi_k \, d\pi_k \exp\left\{-\beta\left[\sum_{k>\lambda} \tfrac{1}{2}\pi_k^2 + \Gamma(\phi)\right]\right\} \tag{4.16}$$

$$= \left(\frac{2\pi}{\beta}\right)^{n/2} \int \prod_{k>\lambda} d\phi_k \, e^{-\beta\Gamma(\phi)} \tag{4.17}$$

and

$$e^{-\beta W} = \int \prod_{k<\lambda} d\phi_k \, d\pi_k \exp\left\{-\beta\left[\tfrac{1}{2}\sum_{k<\lambda} \pi_k^2 + A(\varphi)\right]\right\}, \tag{4.18}$$

where n is the number of modes with $k > \lambda$. For a quantum field, $\pi(x)$ and $\phi(x)$ do not commute. Any one of them, say $\phi(x)$, represents the complete degrees of freedom, and H is not diagonal in $\phi(x)$. Consequently, integrations such as in

$$e^{-\beta A(\varphi)} \equiv \int \prod_{k>\lambda} d\phi_k \times \left\langle \{\phi_k\} \middle| \exp\left\{-\beta\left[\tfrac{1}{2}\sum_{k>\lambda} \pi_k^2 + \Gamma(\phi)\right]\right\} \middle| \{\phi_k\} \right\rangle$$

have to be done by summing the usual quantum statistical diagrams with internal lines of momenta $k > \lambda$.

Finally, note that in the GN model with N fermions, there is no dependence on the σ-boson canonical momentum. Many of the distinctions discussed above disappear in the leading $N \to \infty$ limit. Since no internal (σ') lines exist in the leading term in the $1/N$ expansion,

$$A(\bar{\sigma}) = \Gamma(\bar{\sigma}).$$

Also, the quantum behavior of fermions will affect only the fermion propagators, but do nothing to integrals over boson degrees of freedom.[10]

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