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Nonvanishing neutrino mass and the process $\gamma\gamma \rightarrow \nu\bar{\nu}$

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Within the framework of the local $V - A$ theory, I find that the experimental uncertainty in the μ-neutrino mass is not sufficiently great to allow the process $\gamma\gamma \rightarrow \nu\bar{\nu}$ to be an important neutrino energy-loss mechanism in stellar objects.

I. INTRODUCTION

Some time ago, Pontecorvo[1] and Chiu and Morrison[2] suggested the process $\gamma\gamma \rightarrow \nu\bar{\nu}$ as a potentially important energy-loss mechanism during certain stages of stellar evolution. Their first expectations were for rates comparable to what are now considered the dominant neutrino energy-loss mechanisms[3-8]

$\gamma e \rightarrow \nu\bar{\nu}$,

$e\bar{e} \rightarrow \nu\bar{\nu}$,

plasmon $\rightarrow \nu\bar{\nu}$.

However, Gell-Mann[9] has noted that the $\gamma\gamma \rightarrow \nu\bar{\nu}$ cross section actually vanishes in the local $V - A$ theory to first order in the Fermi coupling constant. Rosenberg[10] has obtained a nonvanishing cross section by replacing one of the real photons by a virtual photon supplied by a nuclear Coulomb field, but the effect is small.

A nonvanishing cross section also results from a nonlocal weak interaction, and a number of authors have attempted to calculate this process in the intermediate vector-boson theory. Most of these calculations[11-14] cast away several divergent Feynman diagrams, producing non-gauge-invariant and/or cutoff-dependent results that are, moreover, not in agreement with Gell-Mann's theorem in the local limit. Levine[15] has put together all of the highly divergent diagrams and has obtained a finite cutoff-independent result. The effect is once again very small. As pointed out in that paper, a neutral current does not contribute to this process as it does in other neutrino energy-loss mechanisms.[16] In the language of gauge theories,[17] Levine's calculation was performed in the unitary gauge, and in view of the need to argue away the Adler anomaly,[18] it is comforting to know that Gell-Mann's theorem is also obeyed in the local limit when the calculation is performed in the renormalizable gauge.[19]

The Gell-Mann theorem depends crucially on the vanishing of the neutrino mass. In this paper I shall, within the framework of an otherwise unmodified local $V - A$ theory, consider the possibility of a neutrino mass as large as its experimental upper limit.[20]

II. CALCULATION

This calculation is based on the Fierz-rearranged local $V - A$ weak interaction Lagrangian,

$$L_W(x) = \frac{G}{\sqrt{2}} i\bar{\nu}_j(x)\gamma_\beta(1+\gamma_5)\nu_j(x)[V^j_\beta(x) + A^j_\beta(x)],$$

$$\begin{pmatrix} V^j_\beta(x) \\ A^j_\beta(x) \end{pmatrix} = i\bar{l}_j(x)\gamma_\beta \begin{pmatrix} 1 \\ \gamma_5 \end{pmatrix} l_j(x) \quad (1)$$

and the usual electromagnetic interaction

$$L_{\text{em}}(x) = ie\bar{l}_j(x)\gamma_\lambda l_j(x)A_\lambda(x) . \quad (2)$$

Here $Gm_p{}^2 = 10^{-5}$, $e^2/4\pi = (137)^{-1}$, and $j =$ electron, muon. Henceforth, the lepton subscript, j, will be omitted. The matrix element for the process $\gamma\gamma \rightarrow \nu\bar{\nu}$ can be written to first order in G as

$$M = \frac{Ge^2}{\sqrt{2}} \bar{\nu}^{(+)}\gamma_\beta(1+\gamma_5)\nu^{(-)}\langle 0|V_\beta(0) + A_\beta(0)|\gamma\gamma\rangle, \quad (3)$$

where ν^+ and ν^- are positive- and negative-energy neutrino spinors, respectively, and normalized to $\bar{\nu}(\vec{p},E,h)\nu(\vec{p},E,h) = 1$. Since $V_\beta(x)$ is odd under charge conjugation,

$$\langle 0|V_\beta(0)|\gamma\gamma\rangle = 0 . \quad (4)$$

Also, spin $1 \rightarrow \gamma\gamma$ is forbidden by gauge invariance and Bose statistics, so that $\langle 0|\vec{A}(0)|\gamma\gamma\rangle = 0$ in the barycentric frame, the Lorentz-covariant statement of which is

$$\langle 0|A_\beta(0)|\gamma\gamma\rangle = Q_\beta D/S ,$$

$$D = Q \cdot \langle 0|A(0)|\gamma\gamma\rangle,$$

$$Q = q + \bar{q} = p + \bar{p},$$

$$S = -Q^2 . \quad (5)$$

q and $\bar{q}$ are the photon momenta and p and $\bar{p}$ are the neutrino momenta. Equations (3)–(5) along with the Dirac equation for the neutrino spinors

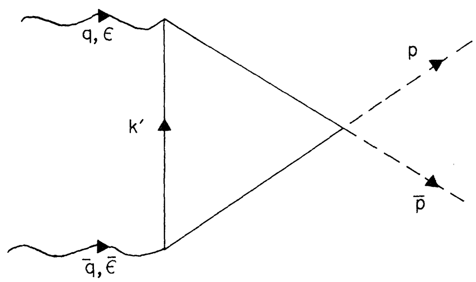

FIG. 1. Feynman diagram for $\gamma\gamma \rightarrow \nu\bar{\nu}$ to lowest order in G and e. Wavy lines are photons, solid lines electrons, and dashed lines neutrinos. Momenta and polarization vectors are indicated.

yield

$$M=\sqrt{2}\,Ge^2 m_\nu \bar{\nu}^{(+)}(1+\gamma_5)\nu^{(-)}D/S\,. \qquad (6)$$

To lowest order in e, D is associated with the Feynman diagram of Fig. 1 and given by

$$D=\int \frac{d^4k'}{(2\pi)^4}\,\frac{Q_\beta N_{\beta\sigma\rho}\epsilon_\sigma \bar{\epsilon}_\rho}{(k'^2+m^2-i\epsilon)(A^2+m^2-i\epsilon)(B^2+m^2-i\epsilon)}$$

$$+\gamma_1 \leftrightarrow \gamma_2\,,$$

$$N_{\beta\sigma\rho}=\mathrm{Tr}\,\gamma_5\gamma_\beta(A\!\!\!/+im)\gamma_\sigma(k\!\!\!/'+im)\,\gamma_\rho(B\!\!\!/+im)\,,$$

$$A=k'+q\,,$$

$$B=k'-\bar{q} \qquad (7)$$

where m is the electron or muon mass. The integral appears to be logarithmically divergent, but by using the relation

$$Q\!\!\!/=(A\!\!\!/-im)-(B\!\!\!/+im)+2im$$

and evaluating some traces it can be written as the obviously convergent integral

$$D=m^2\epsilon_\sigma\bar{\epsilon}_\rho q_\alpha \bar{q}_\beta \epsilon_{\alpha\sigma\rho\beta} I\,,$$

$$I=16\int \frac{d^4k'}{(2\pi)^4}\{(k'^2+m^2)[(k'+q^2+m^2]$$

$$\times[(k'-\bar{q})^2+m^2]\}^{-1}\,. \qquad (8)$$

By means of the well-known formula

$$\frac{1}{\beta+i\epsilon}=\frac{1}{i}\int_0^\infty e^{i(\beta+i\epsilon)z}dz$$

I find with $r^2=z_1+z_2+z_3$ that

$$I=\frac{1}{\pi^2}\int \frac{dz_1dz_2dz_3}{r^2}\exp\left[ir\left(m^2+S\frac{z_1z_2}{z^2}\right)\right]. \qquad (9)$$

For $S \ll m^2$, this is easily evaluated as

$$I=i/2\pi^2\,. \qquad (10)$$

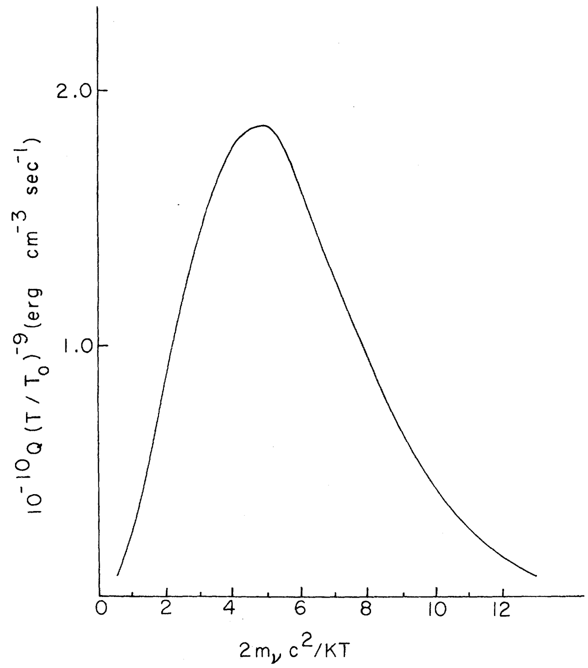

FIG. 2. Energy-loss rate as a function of neutrino rest energy. The experimental upper limit on η at $T=10^9$ °K is $\eta=3$.

The cross section is given by

$$\sigma=\frac{m_\nu^2}{8q_0\bar{q}_0}\,\frac{1}{(2\pi)^2}$$

$$\times\int \frac{d^3p}{p_0}\,\frac{d^3\bar{p}}{\bar{p}_0}\,\tfrac{1}{4}\sum_h |M|^2\delta^4(p+\bar{p}-q-\bar{q})\,, \qquad (11)$$

where $\sum_h$ is a sum over photon and neutrino helicities. I readily find that

$$\sigma=\sigma_0\left(\frac{m_\nu}{1\text{ MeV}}\right)^2\left(\frac{S-4m_\nu^2}{S}\right)^{1/2},$$

$$\sigma_0=\frac{1}{2\pi}(Gm_p^2)^2\left(\frac{e^2}{4\pi}\right)^2\left(\frac{1\text{ MeV}}{m_p}\right)^2 m_p^{-2}$$

$$\cong 0.4\times 10^{-48}\text{ cm}^2\,. \qquad (12)$$

Taking $m_\nu=1$ MeV for muons and $S=(3\text{ MeV})^2$, a typical S value at 10^{10} degrees Kelvin, yields a cross section of 3×10^{-49} cm^2. This is much larger than the cross section produced by the IVB (intermediate vector boson) nonlocality[15] and is at least comparable to that produced by a nuclear Coulomb field.[10] It is therefore of some interest to calculate the contribution of this process to the stellar energy-loss rate.

The energy loss per unit volume per unit time, Q, is given by

$$Q = 4 \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} 2\sigma(\omega_1+\omega_2)[\exp(\omega_1/KT)-1]^{-1} \times [\exp(\omega_2/KT)-1]^{-1}. \tag{13}$$

Changing integration variables to

$$x = (\omega_1+\omega_2)/2m_\nu, \quad y = (\omega_1-\omega_2)/2m_\nu, \quad z = \sqrt{S}/2m_\nu, \tag{14}$$

and performing the z integration yields

$$Q = (0.5\times 10^7 \text{ erg cm}^{-3}\text{ sec}^{-1})(T/T_0)^9 R,$$

$$T_0 = 10^9\ {}^\circ\text{K}$$

$$R = \eta^9 \int_1^\infty dx \int_0^{(x^2-1)^{1/2}} dy\, x(x^2-y^2)[e^{\eta(x+y)/2}-1]^{-1}[e^{\eta(x-y)/2}-1]^{-1} \times \left\{(x^2-y^2)^{1/2}(x^2-y^2-1)^{1/2} - \ln\left[1+\left(\frac{x^2-y^2-1}{x^2-y^2}\right)^{1/2}\right]\right\}, \quad \eta = 2m_\nu/KT. \tag{15}$$

Numerical integration of R (Ref. 21) yields the neutrino mass dependence of the loss rate shown in Fig. 2. The peak value of $Q/(T/T_0)^9$ is therefore about 2×10^{10} erg/cm^3 sec. While substantially greater than the rate produced by the IVB nonlocality, this is still two orders of magnitude below the principal conventional mechanisms at a temperature of 10^9 °K.

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[20]The present limits are $\nu_e = 60$ eV, $\nu_\mu = 1.2$ MeV, ignoring the 8-eV figure employing cosmological evidence.

[21]The crude estimate, $R \cong \eta^9 e^{-\eta}$, comes within an order of magnitude of the numerical result.