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# Use of inequalities for the experimental test of a general conception of the foundation of microphysics

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It is pointed out that some already-known inequalities (Bell's inequalities) and some new ones presented here can be used to test experimentally the validity of a general conception of the foundations of microphysics. This conception mainly consists in considering sets of propositions (having the structure of lattices but possibly of non-Boolean ones) and in assuming that when a proposition is true on a system $S$ this constitutes an intrinsic property of $S$, which can be neither *imparted* to $S$ nor *withdrawn* from $S$ as long as $S$ is "isolated." It is shown that if experiments of the type of those used to test Bell's inequalities turn out to corroborate the quantum-mechanical predictions, such a result could be used in order to invalidate directly the general conception just described. This is done without reference to the general principles of quantum mechanics. More generally, our derivation does not depend for its validity on assuming the truth of any particular physical theory abstracted by induction from experimental knowledge.

## I. INTRODUCTION

It is a truism that the advent of the modern physical theories—relativity, quantum mechanics, quantum electrodynamics, $S$ matrix theory and so on—has induced us to abandon many familiar intuitive concepts. When we are asked why, our standard—and quite appropriate—answer is that one may well be skeptical about the possibility and usefulness of building up some alternative theoretical framework that would (i) incorporate and use these old concepts and (ii) be as successful as each of our present-day theories in all their respective domains.

On the other hand, a motivated skepticism is far from being equivalent to a disproof. All the successful theories mentioned above are built upon elaborate sets of axioms that are justified *a posteriori*, i.e., by the agreement between some of their consequences and observed facts (and by the absence of discrepancies). But it should be remembered that the appearance of two or more theories using very different basic concepts and yet accounting equally well for a given set of experimental data is *not* quite a rare event in physics. Hence the mere *existence* of the successful theories referred to does not establish that such and such a concept (or general view, or the like) which they reject is indeed to be discarded once and for all as definitely inadequate. For that reason it is quite often asserted that in such a domain we cannot make any absolute statement. Quite frequently, it is even stated as an *obvious* truth that the judgements we can form on these matters are all dependent not only on the facts *but also on the general axioms of the existing theories*.

Still, if not for our *practice* of physical research, at least for our *understanding* of the whole subject we would like to know for certain as many items as we can concerning the adequacy or inadequacy of given concepts or general ideas. In particular, we would be satisfied if we could establish about some given concept or idea not only that it is *useless* at present (i.e., within the framework of the present-day theories), but that it is *false* in that it leads unavoidably to a contradiction with the data.

For that purpose, we stress again that a mere reference to the existing theories is not enough. How then should we proceed? Obviously by trying, as much as possible, to shortcircuit these theories; by trying to compare directly—or as directly as we can—the concept or idea with the experimental facts.

Now if our purpose is really to study *one* concept or *one* particular idea—in isolation, so to speak—then the above program is probably overambitious and cannot in fact be fulfilled. But at least it can be applied, as we show below, to a given *set* of concepts and ideas (assumptions). The result of course is weaker, since when we have shown that this set of concepts and assumptions directly contradicts known facts we can only conclude that one or more of these concepts or assumptions must be rejected, without being able to specify which one. Still, if this set contains only notions and ideas that are *all* deeply ingrained in our minds, even this weaker result is interesting.

In this paper a set of concepts and assumptions is introduced (Sec. II) that is already considered by most experts as not being compatible with quantum mechanics, at least in its most commonly accepted interpretations. The question we are interested in is: Are we *sure* that this set is absolutely unacceptable, i.e., that it will remain so in *any* future theory or interpretation thereof? It is shown that we can answer that question positively, provided only that (a) we consider as significant a few recent experimental results that agree with some quantum-mechanical predictions, and (b) we accept the conjecture according to which such an agreement will *persist* when the experiments alluded to are suitably refined (in technically foreseeable ways).

While the proof of the above statement is the main purpose of this paper, a subsidiary purpose is to supplement Bell's inequalities[1,2,3] with some new ones, and also to show that, when applied to special correlation effects, all these inequalities hold within a theoretical framework that is considerably larger than the one of the hidden-variables theories. In particular, a by-product of our method for deriving such inequalities is to make it clear that they have an *already known* large domain of application, which indeed covers the whole field of what can be called the "classical" probabilities [by this expression we mean the set of all the theories in which elementary events (i) exist and (ii) are all such that several propositions can be formulated that are each necessarily true or false when applied to them]. But it should be stressed that the main purpose of this article is definitely not to establish new inequalities nor indeed to put forward any new physical result. Rather it is to study a new *problem*, which consists in ascertaining directly whether or not a given set of general ideas and concepts is compatible with known facts, independently of *any* formalism.

Some of the views presented here were already put forward— in a provisional form — by the author at the 1972 Trieste conference on the physicists conception of nature.[4] They are reformulated here since they fit naturally with the context.

## II. CONCEPTS AND ASSUMPTIONS

The set of concepts and assumptions that we want to falsify directly—without reference to quantum mechanics—is the following one.

As regards the *concepts* we merely assume that we can use the words *system*, *isolated system*, and *proposition* in the usual way. A silver atom is a system of a given type. A voltmeter and an electron are systems of other types. Provisionally at least, we consider as *isolated* any system lying arbitrarily far from or outside the light cones of all other systems. Propositions are defined operationally.[4] We define a proposition $a$ pertaining to a type $T$ of systems $S$ by specifying the instruments of measurement corresponding to it. We also define the orthogonal complement $a'$ of $a$ by specifying that it corresponds to the same instrumental device as $a$ and that its measurement is said to give the value *yes* whenever that of $a$ gives *no*, and conversely.[5]

These *concepts* cannot be completely separated from the *assumptions* that follow (to some extent, the distinction between assumptions and concepts is artificial in this context).

One of the ideas concerning physical systems that is most deeply ingrained in our general conceptions about nature is that, in some cases at least, some propositions are true about these systems, and that when this is the case, it is so even if nobody is actually going to try to become conscious of the fact. Let us formulate precisely this idea in the following way.

*Assumption 1.* It is meaningful to associate to any proposition $a$ defined on a type $T$ of systems a family $F(a)$ of systems $S$ of the type $T$, $F(a)$ being defined by the two following conditions: (i) The systems $S$ that belong to $F(a)$ are those and only those that are such that if $a$ were measured on $S$ by any method the result *yes* would necessarily be obtained, and (ii) the fact that a given $S$ belongs to $F(a)$ is an intrinsic property of $S$ (i.e., it does not depend on whether or not $S$ will interact with some instrument devised so as to measure $a$).

*Remark 1.* Assumption 1 apparently conflicts with at least some of the conventional interpretations of quantum mechanics. In particular, it seems difficult to reconcile it with some of the views of the Copenhagen school concerning the role of the instruments and the inseparable wholeness they are supposed to constitute with the object. On the other hand, this particular aspect of the Copenhagen interpretation has always remained somewhat controversial, even in the opinion of some physicists who consider themselves as being substantially in agreement with the conception of that school. Indeed, some of the latter physicists seem to have hoped to be able to restore the validity of our assumption 1 by going to a non-Boolean logic,[6] or to a non-Boolean calculus of propositions.[7] One of the points we expect to make in this paper is that such hopes cannot be maintained, and that this is true quite independently of any theory (unless some of the assumptions below are dropped; see Sec. V).

*Remark 2.* The possibility that systems of type $T$ should exist that belong neither to $F(a)$ nor to $F(a')$ is clearly not excluded by assumption 1, nor is even the possibility that some systems should belong to no family of that sort at all. In particular, we do not assume that if $a$ is not true it is false. Indeed, we do not even define a meaning for the latter epithet applied to a proposition bearing on a system.

*Remark 3.* No determinism—neither manifest nor hidden—is postulated.

*Definition 1.* Iff (if and only if) $S$ belongs to $F(a)$, $a$ is said to be *true* on $S$.

*Definition 2.* Let a system $S$ be isolated between times $t_a$ and $t_b$. $a$ is said to be *persistent* on $S$ between $t_a$ and $t_b$ iff the condition that $a$ is true at time $t_1$ entails that it is true at time $t_2$, for any $t_1$ and $t_2$ satisfying

$$t_a < t_1 < t_2 < t_b .$$

*Assumption 2.* Let $t_a < t_1 < t_2 < t_b$, let $S$ be isolated between $t_a$ and $t_b$, and let $a$ be persistent between $t_a$ and $t_b$. Then if $a$ is true at $t_2$, it is also true at $t_1$.

*Remark 4.* Assumption 2 is again one of those that seem to be incompatible with at least some interpretations of quantum mechanics, although this again is controversial. But anyhow it is an assumption that seems quite natural in view of definition 2 and in view of our general opinion that in such matters some kind of time-reversal principle should hold.

*Assumption 3.* Let $C$ be a set of general experimental conditions and let $C'$ be conditions obtained from $C$ by changing only the experimental devices with which $S$ *will* interact *after* time $t$. Then if $C$ entails that $a$ is true on $S$ at time $t$, $C'$ also entails that $a$ is true on $S$ at time $t$.

*Assumption 4.* If $a$ is true on $S$, then it is also true on any system $S+S'$ of which $S$ is a part. Conversely, if $a$ is a proposition defined on systems of the type of $S$, if it bears on $S$, and if it is true on $S+S'$, then it is true on $S$.

## III. CONSEQUENCES

Let us consider the experiment discussed by Bohm,[8] Bell,[1] and others.[9] A spin-zero particle decays at time $t_a$ into two particles $U$ and $V$ of equal spin $S$ by means of a spin-conserving interaction. Let $\{\vec{e}_i\}$ be unit vectors defining directions in space. Let $v_i$ be the proposition "$S^{(V)}(\vec{e}_i) = m$" and let $u_i$ be the proposition "$S^{(U)}(\vec{e}_i) = -m$," where $S^{(W)}(\vec{e}_i)$ is the projection along $\vec{e}_i$ of the spin of particle $W$ ($W = U$ or $V$). Propositions $u_i$ and $v_i$ can be defined by means of suitably oriented Stern-Gerlach devices. It is then apparent that $v_i'$ is the proposition "$S^{(V)}(\vec{e}_i) \neq m$," and similarly for $u_i'$. On the other hand, if we were to measure $u_i$ and $v_i$, in any order, we would always get either two answers *yes* or two answers *no.* This can be considered as a definition of the statement that the composite system $U+V$ has total spin zero (all measurements are assumed here to be "ideal"), and we can consider it as an experimental fact that systems $U+V$ prepared as stated above *do* have spin zero. Combined with assumption 1, the fact that upon measurement of $u_i$ and $v_i$ we would certainly get either two *yeses* or two *noes* implies that if the composite system $U+V$ belongs to $F(u_i)$ it also belongs to $F(v_i)$, and conversely.

Let us consider the case in which, at a time $t_2 > t_a$, $u_i$ is measured on $U$ by means of some instrument $A$. Let us assume first that the result *yes* is obtained. Then, for the reason already mentioned it can be stated with certainty that a measurement of $v_i$ on the corresponding system $V$ would also give the result *yes.* According to assumption 1, $V$ therefore belongs—after time $t_2$—to family $F(v_i)$. Since $v_i$ is a persistent proposition on $V$ from $t = t_a$ to $t = \infty$, assumption 2 then has the consequence that that particular $V$ belongs to $F(u_i)$ also at any time $t_1$ satisfying $t_a < t_1 < t_2$. Assumption 4 then shows that also the composite system $U+V$ of which the considered $V$ is a part belongs to $F(v_i)$. Because of the strict spin correlation established at time $t_a$ it thus also belongs to $F(u_i)$.

Let us now assume that the result of the measurement made on $U$ at time $t_2$ is *no.* Exactly the same argumentation then leads us unavoidably to the conclusion that in that case the composite $U+V$ system belongs at time $t_1$ to families $F(u_i')$ and $F(v_i')$.

Instead of considering one composite system $U+V$ only, let us now consider $N$ such systems, all identically prepared and all of them subjected to a measurement of $u_i$ at $t_2$. For each of them the result of that measurement is necessarily *yes* or *no* so that each of these systems necessarily falls into one of the cases considered above. The previous argument therefore shows that at time $t_1$, under the conditions of the experiment and if assumptions 1, 2, and 4 are correct, the composite systems $U+V$ all belong *either* to $F(u_i)$ and $F(v_i)$ *or* to $F(u_i')$ and $F(v_i')$. If we now also take assumption 3 into account, we must conclude that this situation *would also hold* if the measurement hitherto assumed to be made on $U$ at time $t_2$ were *not* made at all, or were replaced by some other one. But then the same argumentation could be repeated over again with reference to a new pair $u_j, v_j$ of propositions. Hence the conclusion is that in the special case of the decay considered here we have to deal with a situation in which it so happens that any composite $U+V$ system

(i) must belong either to $F(v_i)$ or to $F(v_i')$,

(ii) must belong also to $F(u_i)$ in the first case and to $F(u_i')$ in the second one, and

(iii) belongs as a matter of fact to an infinity of such families at the same time since $\vec{e}_i$ can be chosen in an infinity of ways. We may question these conclusions, but the point is that we may not do this without giving up one or several of the assumptions 1, 2, 3, and 4.

*Remark 1.* This argumentation closely parallels the one developed by Einstein, Podolsky, and Rosen[10] in order to show that quantum mechanics is incomplete. But it is used here with somewhat different assumptions and for a different purpose, since our objective is *not* to test any assumption (e.g., completeness) concerning the axioms of quantum mechanics. As a consequence — in contradistinction with what was the case as regard the article quoted above —the results obtained in this section do not yet constitute a difficulty as regards the assumptions we want to test since no contradiction exists between them and the experimental facts that are used here as reference. In particular, they are fully compatible with the experimental facts usually described under the headings "the spin components along different directions are not simultaneously measurable." Admittedly, the results in question imply for instance that *if* $u_i$ were measured at $t_1$ on some system $U$ the answer *yes would* be obtained. But it asserts nothing about any *actual* sequence of such measurements (concerning which the problem of the perturbation created by the first instrument would have to be taken into account) and, what is even more significant, it does not give us any operational means for effectively sorting out from the statistical ensemble a system $U$ possessing these features. Indeed, under these circumstances it would even seem at first sight that the special character endowed to the considered composite systems $U+V$ by our assumptions has no observable implication whatsoever. If this conclusion were correct, it would reinforce the view that sets of assumptions of this sort are "legitimate but metaphysical." But as we show below, a complete elucidation of the bearing of the Bell-type inequalities must lead us—on the contrary— to give up this view since such inequalities (i) *can* be falsified and (ii) *are* consequences of the results derived directly in the present section from the considered set of assumptions.

*Remark 2.* Some formulations (see, e.g., Ref. 4) introduce the notion of *atomic propositions*. When $a$ is atomic then, if $x$ is a proposition

$$\phi \subseteq x \subseteq a \Rightarrow x=\phi \quad \text{or } x=a\,,$$

it might seem that the results of this section preclude the possibilities of $u_i$ or $v_i$ being atomic on $U$ or $V$, respectively, since the assertion $x=$"$u_i$ and $u_j$" (which was shown to hold on some $U$'s) entails $u_i$ while being different from $\phi$. But the conclusion does not follow since — as pointed out in the foregoing remark — assertion $x$ is not operational and therefore is not a proposition.

On the other hand, this makes clear a point that could be important for the development of the theories gathered under the names of "quantum logic" or "quantum calculus of propositions." This point is that any such theory that implicitly or not makes use of our set of assumptions implicitly contains "built in" significant assertions—such as $x$ above— that are different from propositions.

*Remark 3.* Admittedly, the argumentation of this section implies that in the case in which assumptions 1 to 4 are made, the time evolution of the *particular* systems $U$ and $V$ studied here must be considered as being more "deterministic" than is assumed in the conventional formulation. This does not contradict remark 2 of Sec. II since these supplementary deterministic features emerge here as a *consequence*, true only for *special* systems, not as a *postulate* of *general* validity.

## IV. INEQUALITIES

The semi-positive-definite character of the probabilities (that they cannot be negative) has many consequences—some of which have perhaps not yet been completely exploited; in particular, in conjunction with strict correlation phenomena. Here we derive Bell's inequalities and some generalizations thereof as simple, nay almost trivial, consequences of that semi-positive-definiteness (these inequalities consequently apply for a wide range of physical theories and phenomena, including macroscopic, classical ones).

Through the use of the concepts of measure, conditional probabilities, and so on (and of the corresponding shorthand notations), the following derivations could easily be formulated in concise, abstract terms. However, this would conceal, rather than reveal, their intrinsic simplicity and (what is more important) their corresponding generality. Let us instead use the very simple notion of a number of systems in an ensemble. The number of elements in a statistical ensemble is an inherently non-negative quantity; and the number of elements of the union of two disjoint ensembles is the sum of the numbers of elements of the two constituents. These two trivial but indisputable statements are essentially all we need, and by formulating them in such a concrete manner we hope to show in a convincing way that the basis of the following deduction is extremely difficult to reject.

Let us then consider an ensemble $E$ of system $V$ of a given type $T$. Let $\{v_1 \cdots v_i \cdots v_n\}$ $(n \geq 3)$ be a set of propositions $[\{v_1' \cdots v_i' \cdots v_n'\}$ being the set of their orthogonal complements] defined on systems of type $T$ and such that every element of $E$ belongs for any value of the index $i$ either to $F(v_i)$ or to $F(v_i')$, $F(v_i)$ and $F(v_i')$ being the families of systems defined in assumption 1. In classical physics, ensembles $E$ satisfying such conditions can be constructed in an extremely wide variety of cases (as already mentioned in the Introduction). But even when propositions of a type more general than the classical ones are considered, it may happen (in particular cases) that such ensembles can be considered also. An example is provided by the ensemble $E=E_V$ of the systems $V$ considered in Sec. III. This, as shown in Sec. III, is a consequence of the set of assumptions introduced in Sec. II. Hence the following considerations also apply to $E_V$ as soon as the assumptions 1 to 4 of Sec. II are made, which we assume to be the case.

Let us choose an approach originally used by Wigner[2] in order to deal with the hidden-variables problem: To each element $V$ of $E$ let us associate a sequence $\sigma_1 \cdots \sigma_2 \cdots \sigma_n$ of dichotomic quantities $\sigma_i$ which have the values $+1$ (denoted $+$) if $V$ belongs to $F(v_i)$ and $-1$ (denoted $-$) if $V$ belongs to $F(v_i')$. Let us first consider three $v_i$ only, and then let

$$n(\sigma_1,\sigma_2,\sigma_3)$$

be the number of systems $V$ in $E$ that have the specified values of $\sigma_1,\sigma_2,\sigma_3$. Although we cannot know $n$, it has a well-defined value according to our assumptions (supplemented with the considerations of Sec. III) in all the cases we consider. Moreover, in all these cases

$$\sum_{\sigma_1,\sigma_2,\sigma_3} n(\sigma_1,\sigma_2,\sigma_3)=N,$$

where $N$ is the total number of elements of $E$ $(N\to\infty)$.

Let $M(i,j)$ $(i,j=1,2,3)$ be the mean value on $E$ of the product $\sigma_i\sigma_j$ so that, of course

$$-1\leq M(i,j)\leq 1$$

and

$$M(i,j)=N^{-1}\sum_{\sigma_1,\sigma_2,\sigma_3}\sigma_i\sigma_j\, n(\sigma_1,\sigma_2,\sigma_3). \tag{1}$$

*Proposition 1.*

$$|M(i,j)-M(j,k)|\leq 1-M(k,i) \quad \text{for } i\neq j\neq k. \tag{2}$$

*Proof.* The quantity

$$M(i,j)-M(j,k) = N^{-1}\sum_{\sigma_1,\sigma_2,\sigma_3}\sigma_j(\sigma_i-\sigma_k)\, n(\sigma_1,\sigma_2,\sigma_3) \tag{3}$$

contains no term with $\sigma_i=\sigma_k$, hence has only terms with $\sigma_k=-\sigma_i$, and can therefore be rewritten as

$$M(i,j)-M(j,k)=2N^{-1}\sum_{\sigma_1,\sigma_2,\sigma_3}{}'\,\sigma_j\sigma_i\, n(\sigma_1,\sigma_2,\sigma_3), \tag{4}$$

the symbol $\sum'$ meaning that all the terms in which $\sigma_k=\sigma_i$ must be excluded from the summation, and only these. Similarly,

$$1-M(i,k)=N^{-1}\sum_{\sigma_1,\sigma_2,\sigma_3}(1-\sigma_i\sigma_k)n(\sigma_1,\sigma_2,\sigma_3) \tag{5}$$

also contains no term with $\sigma_k=\sigma_i$ and can be rewritten—with the same convention—as

$$1-M(i,k)=2N^{-1}\sum{}'\, n(\sigma_1,\sigma_2,\sigma_3). \tag{6}$$

In Eqs. (4) and (6) the summations bear on the same terms, but in (6) all these terms are positive whereas in (4) some of them are negative. Hence (2) follows (Q.E.D.).

Since any composite system $U+V$ that belongs to $F(v_i)$ also belongs to $F(u_i)$ as we have shown, $M(i,j)$ can be known experimentally. Indeed

$$M(i,j)=-P(i,j), \tag{7}$$

where $P(i,j)$ is the mean value of the (observable) product of $S^{(U)}_{\vec{e}_i}$ and $S^{(V)}_{\vec{e}_j}$. Equation (2) therefore gives rise to Bell's inequalities[1]:

$$|P(i,j)-P(j,k)|\leq 1+P(k,i). \tag{8}$$

*Proposition 2.*

$$M(12)+M(23)+M(31)\geq -1. \tag{9}$$

*Proof.* The left-hand side—which we designate by $K$—can be written as

$$K=N^{-1}\sum_{\sigma_1,\sigma_2,\sigma_3}(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\, n(\sigma_1,\sigma_2,\sigma_3), \tag{10}$$

$$K=(2N)^{-1}\sum_{\sigma_1,\sigma_2,\sigma_3}[(\sigma_1+\sigma_2+\sigma_3)^2-3]\, n(\sigma_1,\sigma_2,\sigma_3). \tag{11}$$

Since $\sigma_i=\pm 1$, the quantity inside square brackets can only take the values $+6$ (for $\sigma_1=\sigma_2=\sigma_3$) and $-2$ (otherwise). Hence

$$K\geq -N^{-1}\sum_{\sigma_1,\sigma_2,\sigma_3}{}''\, n(\sigma_1,\sigma_2,\sigma_3), \tag{12}$$

where $\sum''$ is a summation extended to all the terms for which not all three $\sigma$'s are equal. Obviously $\sum''\cdots\leq N$, and (9) follows. For the observable quantities $P(i,j)$, (9) gives the new inequality

$$P(12)+P(23)+P(31)\leq 1. \tag{13}$$

*Remark 1.* In the special but important case (used above as an example) in which $U$ and $V$ are equal-spin particles in a state of zero total spin (and in the similar experiments using photons), it can be shown that (13) combined with (8) is equivalent to an inequality derived by Gutkowski and Masotto[11] and relating with one another not the $P$'s but the corresponding numbers of systems (probabilities). On the other hand, the Gutkowski-Masotto inequality is based on the fact that the probabilities of the results $\sigma_i=\pm 1$ are equal. Experiments could probably be imagined in which such an equality would not hold, but for which (9) [or (13)] would still be valid.[12]

*Remark 2.* The effect of the strict spin correlation between $U$ and $V$ is twofold: (i) Together with assumptions 1 to 4, it integrates any ensemble of systems $V$ to the class of those any element of which belongs either to $F(v_i)$ or to $F(v_i')$, and (ii) it has the effect that the quantities $M(i,j)$ become observable, by means of the $P(i,j)$.

*Proposition 3.* Let

$$\begin{aligned} Nx&=n(+,+,+)+n(-,-,-)\leq N,\\ Ny&=n(+,+,-)+n(-,-,+)\leq N,\\ Nz&=n(+,-,-)+n(-,+,+)\leq N,\\ Nt&=n(+,-,+)+n(-,+,-)\leq N. \end{aligned} \tag{14}$$

Then

$$\begin{aligned} A&=\tfrac{1}{2}[1-M(12)]=z+t,\\ B&=\tfrac{1}{2}[1-M(13)]=y+z,\\ C&=\tfrac{1}{2}[1-M(23)]=y+t, \end{aligned} \tag{15}$$

or

$$y=\tfrac{1}{2}(B+C-A),\quad z=\tfrac{1}{2}(A+B-C),$$
$$t=\tfrac{1}{2}(A+C-B).$$

The only inequalities (or equalities) satisfied by $x$, $y$, $z$, and $t$ on these grounds and that can *a priori* be independent from one another are

$$x\geq 0,\quad y\geq 0,\quad z\geq 0,\quad t\geq 0, \tag{16}$$

$$x+y+z+t=1. \tag{17}$$

Hence the only such inequalities that the additive and the semi-positive-definite nature of the entities "numbers of systems" can generate for linear combinations of $A$, $B$, $C$ are those derived from (16) and (17) by substitution. The last three inequalities in (16) give inequalities (2) (Bell's inequalities). Equation (17) and the first inequality in (16) give inequality (9). The first inequality in (16) gives no information. It follows that the inequalities (2) and (9) exhaust the list of the inequalities satisfied by linear combinations of the $M(i,j)$ as a consequence of additivity and semi-positive-definiteness.

*Proposition 4.* Let us consider a fourth unit vector $\vec{e}_4$ and the corresponding proposition $v_4$. Then

$$-2\leq M(12)+M(13)+M(24)-M(34)\leq 2. \tag{18}$$

*Proof.* Let the symbols $\sum'$ and $\sum''$ denote summations over the possible values of the $\sigma$'s from which the terms having, respectively, $\sigma_2=\sigma_3$ and $\sigma_2=-\sigma_3$ are excluded. Let the middle term in (18) be denoted by $B$. With obvious notations $B$ can be written as

$$B=N^{-1}\sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4}[\sigma_1(\sigma_2+\sigma_3)+\sigma_4(\sigma_2-\sigma_3)]\times n(\sigma_1,\sigma_2,\sigma_3,\sigma_4),$$

$$B=N^{-1}\Big[2\sum{}''\sigma_1\sigma_2 n(\sigma_1,\sigma_2,\sigma_3,\sigma_4)+2\sum{}'\sigma_4\sigma_2 n(\sigma_1,\sigma_2,\sigma_3,\sigma_4)\Big].$$

Hence

$$-2N^{-1}\Big[\sum{}''n(\cdots)+\sum{}'n(\cdots)\Big]\leq B\leq 2N^{-1}\Big[\sum{}''n(\cdots)+\sum{}'n(\cdots)\Big],$$

and therefore (since the ensembles $\sum'$ and $\sum''$ are disjoint)

$$-2\leq B\leq 2\quad\text{(Q.E.D.)}.$$

Inequality (18) and the inequalities derived by permuting the symbols give rise to the so-called generalized Bell's inequalities between the $P(i,j)$. These inequalities were first derived within the hidden-variables conception by Clauser, Horne, Shimony, and Holt.[3,13] Within that conception they hold true even if a strict correlation does not hold between $U$ and $V$ in the sense in which this concept is introduced in Sec. III. On the contrary, if we only assume the validity of the set of assumptions listed in Sec. II, these inequalities are only valid in the cases in which strict correlations hold (i.e., in the case of a total spin zero in our example).

More generally, let us now consider the case in which $m$ distinct propositions $u_i$ are taken into account. Let us first consider the case in which $m$ is odd. Then we have the following.

*Proposition 5.*

$$\sum_{i<j} M(i,j) \geq \tfrac{1}{2}(1-m)\,, \quad m \text{ odd} \tag{19}$$

and correspondingly

$$\sum_{i<j} P(i,j) \leq \tfrac{1}{2}(m-1)\,, \quad m \text{ odd}\,. \tag{20}$$

*Proof.* The left-hand side of (19) is

$$(2N)^{-1} \sum_{\sigma_1,\ldots,\sigma_m} [(\sigma_1+\cdots+\sigma_m)^2 - m] \times n(\sigma_1,\ldots,\sigma_m)\,. \tag{21}$$

The smallest term among those written inside the square brackets in (21) has value $1-m$. (19) follows.

When the parity of $m$ is not specified, the inequality obtained by this method is less stringent. It is

$$\sum_{i<j} M(i,j) \geq -\tfrac{1}{2}m\,. \tag{22}$$

The corresponding inequality for the $P(i,j)$ is unlikely to be falsified by experiments made on the $U+V$ systems such as those considered in Sec. III since—contrary to (20)—it is always satisfied by spin-$\frac{1}{2}$ systems obeying quantum mechanics (q.m.). This follows from the fact that $P(i,j)$ can then be written

$$P_{\text{q.m.}}(i,j) = -(\vec{e}_i \cdot \vec{e}_j)$$

so that

$$\sum_{i<j} P_{\text{q.m.}}(i,j) = -2^{-1}[(\vec{e}_1+\cdots+\vec{e}_m)^2 - m] \leq \tfrac{1}{2}m\,, \tag{23}$$

the equality being realized for $\vec{e}_1+\cdots+\vec{e}_m = 0$.

On the other hand, the derivation of inequality (22) can be applied to the more general case in which $M(i,j)$ is the mean value of the product $X_iX_j$ of two random variables. Denoting by $\sigma_i$ the values taken by the $X_i$ we have

$$\sum_{i<j} M(i,j) = N^{-1} \sum_{\sigma_1,\ldots,\sigma_m} \sum_{i<j} \sigma_i\sigma_j\, n(\sigma_1,\ldots,\sigma_m)\,,$$

$$= (2N)^{-1} \sum_{\sigma_1,\ldots,\sigma_m} [(\sigma_1+\cdots+\sigma_m)^2 - (\sigma_1^2+\cdots+\sigma_m^2)]\, n(\sigma_1,\ldots,\sigma_m)\,,$$

and hence

$$\sum_{i<j} M(i,j) \geq -\tfrac{1}{2} \sum_i M(i,i)\,, \quad i,j = 1,\ldots,m\,. \tag{24}$$

In the case in which the $X_i$ are centered and have equal root mean squares, (24) reduces to the inequality

$$\sum_{i<j} r_{i,j} \geq -\tfrac{1}{2}m\,, \quad i,j = 1,\ldots,m \tag{25}$$

between the correlation coefficients

$$r_{i,j} = M(i,j)[M(i,i)M(j,j)]^{-1/2}\,.$$

Inequalities (24) and (25) belong essentially to ordinary probability theory and they should be used as such. Inequalities (2), (9), (18), and (19) can also be used within the same framework. When the $X_i$ can be considered as constituting together a stationary stochastic function of the index $i$, inequality (25) reflects the well-known fact that any correlation function of such a stochastic function is positive-definite.

## V. DISCUSSION

Let us carefully distinguish between, on the one hand, the verifiable predictions of quantum mechanics (which are unambiguous in every case) and, on the other hand, both its formalizations and its conceptual interpretations (which are varied and or controversial). Among the verifiable predictions let us consider in particular the *elementary verifiable quantum predictions*, which we define as the predictions of quantum mechanics that bear on systems composed of a small number of stable particles. If we believe that the elementary verifiable quantum predictions are all correct (even with respect to idealized instruments), then the content of the present article forces upon us (with no commitment to a particular formalism, or to a particular interpretation) the conclusion that the set of the concepts and assumptions listed in Sec. II cannot be kept since such inequalities as (8), (13), and—more generally—(20), that follow from these concepts and assumptions, are violated in some cases by the said verifiable predictions. Such cases include those in which the systems $U$ and $V$ considered in Sec. III are spin-$\frac{1}{2}$ particles and in which the unit vectors $\vec{e}_i$ are chosen in some special ways. This conclusion was shown by Bell[1] as regards inequalities (8). As regards inequalities (13) and (20), it follows for instance from the fact that the symmetrical configuration $\sum_i \vec{e}_i = 0$ corresponds to the equality sign in relation (23) and hence to a violation of (20).

If we do *not* take it for granted that all the elementary verifiable quantum predictions are true, then we must rely upon direct experiments. Hence it must be recalled here that such experiments seem to be quite feasible. In order to test the localized hidden-variables assumption, experiments have indeed already been made[14-17] that can be interpreted as giving significant indications also as regards the present problem. Others are in progress.[18] Admittedly it is technically difficult to build up tests of inequalities (8), (13), or (20) that would be rigorous in that they would rely on no supplementary assumptions. But these difficulties are certainly not considered as insuperable. In this connection it would of course be suitable to vary the tests and this is why we suggest checking the "new" inequalities, such as (13) and (20), also. Independently of that, it should be pointed out that the set of experiments that are suitable for testing the hidden-variables hypothesis does not coincide exactly with the set of experiments that are suitable for testing the set of assumptions under discussion in the present article. For example, the hypothesis that hidden variables exist can be tested (by using the generalized Bell's inequalities already mentioned) even in the case in which the correlation between the spins of $U$ and $V$ is not strict in the sense in which this concept is used in Sec. III above. On the contrary, for testing the validity of the set of assumptions listed in Sec. II the strict correlation effect is essential. This can also be done with photons, as in Refs. 14, 15, and 16. For the reasons alluded to above, the now available results only give indications on that point. Hence, all that can be said at present is that if some of the data reported in Ref. 15, for instance, could be taken at face value (i.e., if we could overlook the fact that their interpretation really requires supplementary assumptions), they would contradict the consequences we have derived from the said set.

It may be noted here that the condition that the spins of $U$ and $V$ should be $\frac{1}{2}$ or that $U$ and $V$ should be photons is by no means necessary for the validity of the considered tests.[19]

As a last remark bearing on experimentation let it be pointed out that when the considered strict correlation takes place between (pseudo-) vectors—as in the example studied above—the experimental devices that serve in testing inequality (13) can be the same as those that are used for testing inequalities (8). This is a consequence of the fact that if the direction of one of the vectors $\vec{e}_i$ is inverted, two of the $P(i,j)$ change sign and (13) becomes identical to one of the inequalities (8). It does *not* mean that (13) is equivalent to (8). For example, in the symmetrical configuration $\vec{e}_1 + \vec{e}_2 + \vec{e}_3 = 0$ inequality (13) is violated by the quantum-mechanical $P(i,j)$ while (8) is not. Nevertheless, the fact just mentioned—together with inequality (13) proved above—has the straightforward consequence that only unoriented directions in space are important. Given three such directions, the question of whether the system of inequalities (8) and (13) is violated or not by the data does not depend on the orientation chosen on any of these lines in order to label as *yes* the response of the instrument oriented along this line.

When photons are used, then of course the inequalities (2), (9), and (19)—i.e., inequalities involving the $M$'s—are to be used instead of those involving the $P$'s; this being true if the conventional definition of the measured quantities, namely

$$M(i,j) = N^{-1}[n(++) + n(--) - n(+-) - n(-+)], \qquad (26)$$

is used, where + means "passes the polarizer," − means "fails to pass," and $n(\sigma_1, \sigma_2)$ are numbers of photon pairs. The transition from (9) to one of the Bell's inequalities (2)—as well as between the latter—then corresponds to exchanging "passes" and "fails to pass" for one of the two photons, that is, to an invariance with respect to rotations of $\frac{1}{2}\pi$ of the polarizers.

For the rest of the discussion let us assume as a working hypothesis that the experimental results have confirmed or will confirm the observable predictions of quantum mechanics, so that the set of concepts and assumptions of Sec. II is falsified. The questions are then: (i) What does this imply as regards the existing approaches of quantum mechanics, and (ii) more generally, by what other assumptions can we replace the set under discussion? On these two questions we formulate here but a few remarks.

*(i) Question of the approaches to quantum mechanics.* As regards this point, the main interest of the present analysis is probably that it discriminates between several interpretations of conventional quantum mechanics and that it questions some of them. In particular, it discriminates between the Copenhagen formulations and at least some versions of what could be called for short the axiomatical-logical formulations. As

mentioned in the Introduction, the conclusions we have reached are in no disagreement whatsoever with the Copenhagen interpretation, simply because this interpretation does *not* postulate the entire set of assumptions that has been falsified above. Indeed that interpretation discards assumption 1, since (as Bohr in particular has repeatedly stressed) according to it, microsystems do not have any properties *of their own* (which means properties that would be independent of the experimental arrangement, *including* the apparatus with which these microsystems *will* be observed).

On the other hand, some of the axiomatical-logical formulations *do* involve assumptions that are strictly equivalent to our assumption 1. For the sake of definiteness let us, for example, consider the approach of Jauch and Piron.[7] These authors first define "yes-no experiments" and decide to "say that the yes-no experiment $\alpha$ is *true* if a measurement of $\alpha$ will give the result yes with certainty." Then they accept it as an empirical fact that certain pairs $\alpha, \beta$ of such experiments have the property

$$\alpha \text{ true} \Rightarrow \beta \text{ true}, \tag{27}$$

which they write $\alpha < \beta$. They call "equivalent" two yes-no experiments $\alpha_1$, $\alpha_2$ having the properties that

$$\alpha_1 < \alpha_2 \text{ and } \alpha_2 < \alpha_1 . \tag{28}$$

Next, they denote by $a = \{\alpha\}$ the class of all such yes-no experiments and call $a$ a *proposition.* Finally the quoted authors observe that $a$ is true if any (and therefore all) of the $\alpha \in a$ are true and decide that "if the proposition $a$ is true we shall call it a *property* of the system."[7]

Now, the question is: When we introduce in this way the notion of propositions defined on systems, can we avoid making assumption 1 of Sec. II? It seems that the answer is no. Since the fact that $a$ is true is considered as a property of certain systems, we may call $F(a)$ a family of such systems; and then the fact that a given system $S$ belongs to $F(a)$ is (this is a tautology) an intrinsic property of $S$, independent of whether $S$ will be observed by such and such an instrument. The only conceivable doubt we might have would be in connection with the definition of truth. As we have just seen, the quoted authors use the future tense for defining the truth of a yes-no experiment $\alpha$ (they write "*will* give the result yes"), whereas in our assumption 1 the conditional (*would*) was used. If this use of the future were to imply that $\alpha$ is true only in the cases in which an instrument for measuring $\alpha$ is actually set up (and ready for the measurement), then we would again have to do with a theory in which, as in Bohr's conception, the microsystems have no properties independent from the complete experimental environment; we would thus avoid making assumption 1. Unfortunately, if we understand the use of the future instead of the conditional in such a restrictive way, (i) we come in conflict with the sentence quoted above that $a$ (and therefore $\alpha$ also) is "a property of the system," and (ii) we get into difficulties in giving a meaning to relation (27): If the statement "$\alpha$ true" has a meaning only when a complete experimental device is present that is designed so as to measure $\alpha$ in the future, then how can this statement imply "$\beta$ true," an assertion which is related to some other, quite different experimental arrangement?

As a result of this discussion we are tempted to believe that the very method by means of which the quoted authors introduce the concept of a proposition makes it impossible for them to avoid making, effectively, assumption 1. On the other hand, we also believe that this same method is entirely in the spirit of the general axiomatical-logical approach to the foundations of quantum mechanics and that, far from being unduly specific, it has—on the contrary—the great merit of making explicit what was implicit before it. In particular we believe that the said axiomatical-logical approach is inherently based on the idea that, somehow, even microsystems *have* properties of their own, albeit these properties are described by propositions not obeying the usual Boolean logic. But if so, then it is the validity of this entire approach which is questioned by the present analysis, unless it could be shown that it does not postulate implicitly the validity of assumptions 2, 3, and 4, or unless the very existence of strict correlations is doubted, see, e.g., Jauch in Ref. 9.

*(ii) Question of the alternative assumptions.* Quite independently of the whole controversy that is still continuing on the foundations of quantum mechanics it follows from the present analysis and from the experimental results[14,15] (if taken at their face value) that we must abandon one at least of the concepts and assumptions of Sec. II and, moreover, that we must do so even in a simple case, in which $S$ is a *stable* particle and $a$ is a (so-called) constant of the motion whenever $S$ is isolated.

If the usual notion of a system is kept, it seems rather artificial to give up assumption 4 only. Analyses of the Einstein, Podolsky, and Rosen (EPR) problem that seem to proceed along this line are occasionally put forward, but as a rule they implicitly deny some other assumptions of the set also.

As regards assumption 3, there exist some subtle ways of violating the principle it refers to while keeping all the other ones and producing no *observable* effects of the future on the past. It is well known that deterministic theories can be found that violate none of the predictions of quantum mechanics; they are of the contextualistic[20] nonseparable variety. But their nonseparability (i) leads to no observable violation of the principle of finite-velocity propagation of signals and (ii) can be accounted for as a consequence of a retroactive effect; in the phenomenon described in Sec. III this effect would consist of a retroactive influence of the measurement made on $U$ at time $t_2$ on the parameters describing the state of the system $U+V$ at time $t_a$. Up to now, however, the scientific community seems to be reluctant as regards the idea that the future could act upon the past, even in a way that is not directly observable.

A real *violation* of assumption 3 is therefore unlikely. But should we not consider—as a conceivable solution—the possibility that assumption 3 (and perhaps also assumption 2), while being correct, simply would not *apply* in the context of Sec. III where it was used? Conceptually, this indeed is possible. To understand why, let us consider again the argumentation of that Sec. III, but let us observe that all the instruments of measurement presently available are such that in practice they must be completely fixed long before time $t_a$, also as regards the direction $\vec{e}_i$ along which they measure the spin. Hence it is conceivable that they might influence necessarily the decaying object even *before* $t_a$, through forces—or more generally "influences" or "interactions"—of an unknown type but that would not violate the law of finite propagation of signals. Admittedly, it is also conceivable that these unknown influences would operate precisely in such a way as to restore the quantum-mechanical correlation. However, if (in accordance with the quantum-mechanical result) it is believed that the latter correlation law does not depend on the distance between $U$ and $V$, then the explanation presently considered implies that somehow the said unknown influences do not vanish when the distance increases without limits. Again, this sounds like a most unlikely assumption and like one that, if true, might entail important restrictions to the use of the very notion of an *isolated* system. Moreover, it should also be noticed that an explanation of this type, if it were taken seriously, would imply the idea that the quantum principles are relative to the presently available apparatus. It seems that in some cases they *must* fail if it is assumed that the elementary, verifiable quantum predictions would apply even if hypothetical "versatile" instruments were used. By "versatile" we mean instruments that could be fixed along $\vec{e}_i$ *after* $t_a$: There is no known principle of physics that excludes *a priori* the possibility of building such instruments.

The idea that assumption 2 should be violated is somewhat less unattractive than those considered up to this point. At any rate it does not sound completely unfamiliar to many of the theorists who have studied the foundations of quantum mechanics. But it partakes of similar difficulties. And the main question is: Should it be abandoned *alone*? If it is, then a strange kind of irreversibility is thereby introduced in the fundamental laws of physics. Hence it may seem more natural to abandon also assumption 1.

Finally, there are two main possible substitutions for the set of assumptions 1 and 2. One of them introduces the idea of a nonseparability existing between the microsystem and the experimental arrangement (including the instruments with which the system will *later* interact). This seems to have been Bohr's view and the essence of his answer to the EPR criticism. Along with many satisfactory aspects, such a view has the well-known but nevertheless surprising feature of expressing the laws of the microworld by using approximate classical concepts referring essentially to *our* experience of the macroworld. Moreover, it also violates the general principle lying behind assumption 3. The other possible substitution to assumptions 1 and 2 offers a way of avoiding this: It introduces a nonseparability between microsystems that have once interacted.[21] That type of nonseparability is closely parallel to the nonseparability of the quantum-mechanical wave function. It is the (sometimes implicit) common feature of two or three otherwise different descriptions: the one that introduces hidden variables in a deterministic[22,23] but contextualistic and nonseparable theory, the one that makes consciousness an active agent,[24-26] and finally—if it can be proved that it does not reduce to one of the latter two—the description that makes objective the entire wave function of the universe.[27,28]

## VI. CONCLUSION

For the sake of convenience let us call the following principle the *principle of separability*, as formulated by Einstein: If $S_1$ and $S_2$ are two systems that have interacted in the past but are now arbitrarily distant, "the real, factual situation of system $S_1$ does not depend on what is done with system $S_2$ which is spatially separated from the former."[29] This principle can be somewhat widened—or understood in a broad sense—so as to mean: No finite influences can propagate arbitrarily far away. Alternatively, it can be given the more restrictive sense that there are no space-like propagations of influences. The result of the foregoing analysis can then be summarized as follows: (a) If the principle of separability is assumed to hold in its broad sense, experimental tests of the set Σ of concepts and assumptions listed in Sec. II are in principle available. (b) Their results are predicted to be negative by conventional quantum mechanics, and there are indeed preliminary experimental indications supporting these predictions. (c) Quite independently of the assumption made in (a), Σ seems to be incompatible with all the "verifiable" predictions of quantum mechanics if these are extended to measurements with "versatile" instruments. Finally, (d) the most likely substitutes to the set Σ would imply that the principle of separability is *false*, even in its restrictive sense. Hence this principle can hardly be expected to hold except if there exist *either* large-distance violations of the elementary quantum predictions *or* necessarily operating, finite influences propagating arbitrarily far (in which case it can hold only in the restrictive sense, of course).[30]

Superficially such a conclusion seems neither surprising nor new. After all, the nonlocality—in the general case—of the many-particle wave functions is quite obvious. It finds its best illustration in the Pauli principle (which also questions the possibility of individualizing systems in the way separability would have it). Nay, even classical physics admits the existence of correlations between spatially separated events and hence of sudden changes of the probability distributions, induced by distant measurements. On the other hand, the very easiness with which we find these apparent counterexamples to the separability principle should make us doubtful about their real validity as such. Obviously, none of the facts we have just listed were unknown to Einstein. That this latter author could nevertheless give credence to the separability principle should therefore induce us to try to be as critical in the use of our conceptual frameworks as we are accustomed to be in the use of our mathematical formalism. Now as soon as we decide to make such an effort we discover that of course Einstein was quite right. In his times, separability could be *questioned*, but could not be *disproved*. For example, the fact alluded to above that distant correlations can take place even between spatially separated events (when influenced by some common anterior one) has, in fact, nothing to do with the principle of separability, which refers, as just recalled, not to our knowledge but to "the real factual situation." More generally, the nonlocality of the many-particle wave function cannot be used straightaway as an argument against separability. For that purpose it must indeed be associated with an *interpretation* of that wave function, and this leads to quite a long chain of arguments that hinges on the validity of the general axioms of quantum mechanics and the meaning we give to them, and that has led to long and subtle controversies.

Since in its principle our whole analysis is completely independent from quantum mechanics, its conclusion against separability is free from such inconveniences. Its main defect is that the experimental results which should normally constitute its firm basis are recent, incomplete, and still somewhat controversial. We may, however, be confident that such experimental ambiguities will be resolved very soon.

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