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Multiperipheral description of clustering in charged-particle production in the central plateau*

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A two-channel Chew-Pignotti–type model is constructed which describes the two-charged-particle correlations and the rapidity-gap distribution for the nondiffractive component of the hadronic cross section at high energies.

I. INTRODUCTION

Although it is generally conceded that a multiperipheral or Mueller-Regge model describes particle production in hadron-hadron collisions at high energy, no detailed model has yet been proposed to fit the data.[1] Given the equivalence of the models[2] one should expect one model (i.e., one set of parameters) which would fit the gross properties of both inclusive and exclusive data. There are many obstacles to overcome before this can be done, although the outline of the solution of each problem is known. The main problems are (1) the Pomeron or large rapidity-gap problem, (2) isotopic spin, (3) end couplings and baryon exchange, and (4) thresholds in multiperipheral phase space. In the next section these problems will be discussed to explain the limits of application of the present model; however, we now state that we are considering a model of charged-particle production in the central region by the nondiffractive or short-ranged component of the cross section.

The question which is considered is: Can a simple generalization of a Chew-Pignotti (CP) model[3] explain the high particle density and the rather strong two-particle correlation (both magnitude and correlation length)? That is, can such a model handle clustering in a way which is consistent with the data? Also the experimental rapidity-gap distribution from the 200-GeV bubble chamber data has recently become available and is a sensitive test of whether any model is handling clustering correctly. We will compare these data to the model.

We will show that a two-channel generalization of the CP model can describe adequately all the presently known clustering effects seen in the central region (except whatever actual production of resonances exists). Before proceeding with this model we want to consider quite precisely where it fits, in the hope for a general model of the Mueller-Regge or multiperipheral type to explain all the data.

II. THE CONTEXT OF THIS MODEL

It is now well established that the Pomeron problem can be separated from the remaining multiperipheral model.[4] The two-component concept,[5] diffractive + short-ranged components, is just this separation. The short-ranged or nondiffractive component, which is about $\frac{2}{3}$ of the total cross section, is the zeroth-order term in an expansion in the triple Pomeron coupling and contains none of the self-consistency problems this coupling causes for the total cross section. In the remainder of this paper we will consider only the short-ranged component of the cross section, its multiplicity, inclusive distributions, etc. This also implies that all the inclusive distributions should be normalized by the nondiffractive cross section instead of the elastic or total cross sections as is now the practice.

At present it is not known whether isotopic spin is more of an inconvenience or a real problem. In an examination of a large collection of isospin multiperipheral models Coulter and Snider[6] showed that certain features of the data which are particularly sensitive to the details of isospin in the models could be understood, but still other long-standing failures of all isospin models remained. The most severe of these is the failure of all models to date to yield a pair of degenerate $I=0$ and 1 trajectories below the leading $I=0$ trajectory which all models produce. The other problems (e.g., difficulty obtaining a positive correlation between neutral and charged particles) probably are related to this failure to obtain degenerate $I=0$ and 1 secondary trajectories. The more reasonable (in other respects) models always seem to yield the leading $I=1$ trajectory between the leading and second $I=0$ trajectories.[6,7] Without this degeneracy there is little chance of fitting correlations, etc., which depend on secondary trajectories. We will avoid this problem in this paper by only considering charged-particle data. Because we sum over neutrals and ignore the sign of the charge, isotopic spin is not a useful concept.

End couplings, baryon exchange, and the approach to scaling will be neglected here by our only proposing a model for the central plateau. In the central region mostly pions are produced, and for pion production it is believed that the effects of thresholds (or hard-core repulsion in the language of the Feynman gas analogy) are not very important; hence we neglect them. At the end of the paper we will return to this effect and find that it is surprisingly small.

III. THE MOTIVATION OF THE MODEL

As mentioned in the first section the original Chew-Pignotti model does not contain clustering. Hence it has no correlations among the secondaries. However, the need for some clustering effect is more profound than this; it already exists when one considers only the single-particle density. Assuming that the nondiffractive cross section is nearly constant, CP found

$$\rho \approx 1-\bar{\beta}\,, \tag{1}$$

where ρ is the density of produced particles in rapidity space and $\bar{\beta}$ determines the asymptotic energy dependence of the exclusive (nondiffractive) cross sections (modulo $\ln s$) by

$$\sigma^n(s) \sim s^{\bar{\beta}-2}\,. \tag{2}$$

A similar result holds for the charged particles produced and the *prong* cross section[6]

$$\rho_{\text{ch}} \approx 1-\bar{\beta}_{\text{ch}}\,, \tag{3}$$

$$\sigma^{n_{\text{ch}}}(s) \sim s^{\beta_{\text{ch}}-2}\,. \tag{4}$$

In the CP (single-channel) model $\bar{\beta}_{\text{ch}}$ and β_{ch} would be identical; we introduce different symbols here to be able to discuss each independently. We know today that ρ_{ch} is approximately 2.0 (remember, this is the density for the nondiffractive part only), but the asymptotic decrease of the prong cross section is probably in the range[6]

$$0.0 < \beta_{\text{ch}} < 0.2\,. \tag{5}$$

That β_{ch} is in this range is reasonable when one considers the known Regge trajectories with intercepts near 0.5 ($\beta = 2\alpha - 1$). (Since neutrals have been summed over, $[\beta + \frac{1}{3}(1-\beta)] \geq \beta_{\text{ch}} \geq \beta$.[6]) The fact that $\bar{\beta}_{\text{ch}}$ in Eq. (3) and β_{ch} above differ so greatly means that there must be strong clustering of the particles in rapidity. For instance, if we used a model with an average of two charged particles per cluster and with clusters produced independently in a multi-Regge amplitude with $J = 0.5$ trajectories exchanged we could fit both the large particle density and the slow decrease with energy of the prong cross sections, because Eq. (3) would now read

$$\rho_{\text{ch}} \approx 2(1-\bar{\beta}_{\text{ch}})\,. \tag{6}$$

However, an approximately equivalent description, by duality, is to use a two-channel generalization of the CP model. Then $\bar{\beta}_{\text{ch}}$ is some sort of weighted average of the two β's and β_{ch} in Eq. (4) is the higher of them. This two-channel description has the advantage over the resonance description of producing two output Regge trajectories and hence the exponential-type short-ranged correlations predicted by Mueller and apparently found in the data.

Thus we are led to consider a two-channel CP-type model which has one input singularity at $\beta = 0.1$ (to fit the energy dependence of the prong cross sections) and one lower. It must produce one output pole at 1.0 (for constant nondiffractive cross section) and one at 0.5 (to obtain the correct correlation length).[8] Beyond this we want the charged-particle density and normalization of the two-charged-particle correlation to be correct. Finally we will check the entire description against the experimental rapidity-gap distribution.

The two-charged-particle correlation

$$C^{\text{ch}}(y_1 y_2) = \frac{1}{\sigma}\frac{d^2\sigma}{dy_1 dy_2} - \frac{1}{\sigma}\frac{d\sigma}{dy_1}\frac{1}{\sigma}\frac{d\sigma}{dy_2} \tag{7}$$

has for its integral f_2^{ch}, where

$$f_2^{\text{ch}} = \langle n_{\text{ch}}(n_{\text{ch}}-1)\rangle - \langle n_{\text{ch}}\rangle^2\,. \tag{8}$$

Since we are considering the above quantities for the nondiffractive part of the data $C^{\text{ch}}(y_1 y_2)$ will asymptotically only be a function of $(y_1 - y_2)$ and will be given by

$$C^{\text{ch}}(y_1 y_2) \sim C_0 e^{|y_1-y_2|/L}\,, \tag{9}$$

where L, the correlation length, is

$$L \equiv \frac{1}{\Delta\alpha} = 2 \tag{10}$$

if we fix the two output poles at 1.0 and 0.5.

Then

$$f_2^{\text{ch}} = \int_0^Y dy_1 \int_0^Y dy_2\, C^{\text{ch}}(y_1 y_2) = 2LC_0 Y\,, \tag{11}$$

where $Y = \ln s$ is the total available rapidity interval. Of course we expect constant and $s^{-1/4}$ correction but the above gives the leading part ($\ln s$) correctly. From the above it appears that we should be able to determine C_0 experimentally from either the two-particle correlation function or from the multiplicity distributions. However, two considerations make either of these determinations difficult; we are not at asymptotic energies and most of the data are for all inelastic events instead of just nondiffractive. The separation of diffractive and nondiffractive cross sections is, at best, difficult. One estimate given in Appendix A leads to

$$C_0 = 1.3 \pm 0.9 \tag{12}$$

and

$$f_2^{\text{ch}} = (5.2 \pm 3.6) Y + \text{const} \ , \tag{13}$$

which shows the large imprecision.

IV. THE CALCULATION

We name the multiperipheral channels 1 and 2 and the Mueller-Regge channels + and −. The multiperipheral couplings and propagators are

$$G = \begin{pmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{pmatrix}$$

and (14)

$$F = \begin{pmatrix} 1/(J-\beta_1) & 0 \\ 0 & 1/(J-\beta_2) \end{pmatrix} .$$

We introduce the fugacity for charged particles, w, and solve for the output poles by finding the J value where

$$\text{Det}[1 - wGF(J)] = 0 . \tag{15}$$

This yields

$$\alpha_\pm(w) = \frac{\bar{\beta}_1(w) + \bar{\beta}_2(w)}{2} \pm \frac{\Delta\alpha(w)}{2} \ , \tag{16}$$

where

$$\bar{\beta}_i(w) = \beta_i + w g_{ii} \tag{17}$$

and

$$\Delta\alpha(w) = [(\bar{\beta}_1(w) - \bar{\beta}_2(w))^2 + 4w^2 g_{12}{}^2]^{1/2} \ . \tag{18}$$

The physical output poles are α_+ and α_- evaluated at $w = 1$; as explained earlier these will be set equal to 1.0 and 0.5, respectively.

The asymptotic form of the multiplicity moments, f_i^{ch}, where

$$f_1^{\text{ch}} = \langle n_{\text{ch}} \rangle \sim \rho_{\text{ch}} Y \tag{19}$$

and

$$f_2^{\text{ch}} = \langle n_{\text{ch}}(n_{\text{ch}} - 1) \rangle - \langle n_{\text{ch}} \rangle^2 \tag{20}$$

are found by differentiating α_+ with respect to w. That is,

$$\frac{f_i^{\text{ch}}}{Y} \sim \left. \frac{\partial^i \alpha_+(w)}{\partial w^i} \right|_{\omega=1} \ . \tag{21}$$

This yields

$$\frac{f_1^{\text{ch}}}{Y} = \frac{g_{11} + g_{22}}{2} + \frac{(\bar{\beta}_1 - \bar{\beta}_2)(g_{11} - g_{22}) + 4 g_{12}{}^2}{2(\alpha_+ - \alpha_-)} + O(1/Y) \ , \tag{22}$$

$$\frac{f_2^{\text{ch}}}{Y} = \frac{2 g_{12}{}^2 (\bar{\beta}_1 - \bar{\beta}_2)^2}{(\alpha_+ - \alpha_-)^3} + O(1/Y) . \tag{23}$$

We note that at first glance the diagonal elements of G do not affect f_2^{ch}, but the off-diagonal element does in a direct manner. Hence to have a large f_2^{ch} we need a large g_{12}. But for α_+ and α_- fixed g_{12} is limited to [see Eqs. (16) and (18)]

$$g_{12} \leq \tfrac{1}{2}(\alpha_+ - \alpha_-) . \tag{24}$$

To obtain the maximum values (and incidentally, simply the results) we take

$$\bar{\beta}_1 = \bar{\beta}_2 = \frac{\alpha_+ + \alpha_-}{2} = 0.75 \tag{25}$$

and fix

$$g_{12} = \frac{\alpha_+ - \alpha_-}{2} = 0.25 . \tag{26}$$

This also fixes $g_{11} = 0.65$ from Eq. (17). The only undetermined parameter is g_{22} (or β_2) and the f's are given by

$$\frac{f_1^{\text{ch}}}{Y} = 0.575 + \frac{g_{22}}{2} \tag{27}$$

and

$$\frac{f_2^{\text{ch}}}{Y} = (\beta_1 - \beta_2)^2$$

$$= (g_{22} - 0.65)^2 . \tag{28}$$

Thus we see clearly that (with the previously imposed constraints) a large value of g_{22} is required. If we take $g_{22} = 2.85$ we can obtain

$$\frac{f_1^{\text{ch}}}{Y} = 2.0 \tag{29}$$

and

$$\frac{f_2^{\text{ch}}}{Y} = 4.8 \ , \tag{30}$$

which is quite close to our estimates of the experimental numbers. This means that the lower input singularity is quite low,

$$\beta_2 = \bar{\beta}_2 - g_{22} = -2.10 . \tag{31}$$

If we look for a real Regge pole to produce such a singularity it would have to have an intercept near $J = -0.55$, perhaps a daughter of the $J = +0.5$ trajectories. Instead this is probably some average of various low-lying singularities.

Notice from Eqs. (27) and (28) that although we can comfortably fit the data, they border on being impossible to fit. If as the present imprecise experimental values of f_1^{ch} and f_2^{ch} *for the nondiffractive cross section* get more precise we find that the slope of f_2^{ch} vs Y becomes greater than 5.0 while ρ_{ch} remains at 2.0 or less, then this type of model is in trouble. That is, there is a limit to how large the two-particle correlation may be in this model and the present data indicate that this limit is essentially reached.

V. THE RAPIDITY-GAP DISTRIBUTION

By the rapidity-gap distribution we mean the distribution in rapidity *difference* between *neighboring* final-state particles (when ordered in rapidity). Each $2-n$ event contributes to the distribution $n-1$ times. This distribution is independent of the two-particle correlations since it receives contributions only from rapidity differences between *neighboring* particles. If we assume that a multiperipheral model is correct, this distribution is probably as close to directly measuring the kernel as any measurement one can imagine. For a single-channel model it is just the kernel, and for a multichannel model it is, of course, determined by the kernel.

Since we are only considering charged particles we use instead the rapidity interval between two consecutive charged particles independently of whether there are intervening neutral particles. Because we are interested in the distribution in the central region we neglect the leading charged particle on each end in the rapidity space. This means we have no contribution from the two-prong events and the n-prong events contribute $n-3$ times. Since at FNAL energies almost all diffraction is single diffraction this cut practically eliminates the contribution from the rapidity interval across which the Pomeron connects. The other intervals in a diffractive event are expected to be typical of nondiffractive events so that we should obtain the same distribution as for the nondiffractive or short-ranged component of the cross section.

In Appendix B we show that the rapidity-gap distribution asymptotically can be written in the form

$$\frac{dn}{dr}(r) = A_1 e^{-(\alpha_+ - \beta_1)r} + A_2 e^{-(\alpha_+ - \beta_2)r}, \tag{32}$$

where A_i are given in Appendix B.

Notice that since β_2 is so low we expect a sharp decrease for small r,

$$\frac{dn}{dr} \sim e^{-3.1r} . \tag{33}$$

Using the values chosen previously, namely $\alpha_+ = 1.0$, $\alpha_- = 0.50$, $\bar{\beta}_1 = \bar{\beta}_2 = 0.75$, $\beta_1 = 0.10$, and $\beta_2 = -2.10$, gives

$$\frac{dn}{dr} = 0.20\, e^{-0.9r} + 2.40\, e^{-3.1r} . \tag{34}$$

In Fig. 1 this prediction is compared to the experimental distribution obtained from the 205-GeV/c data on particle production in pp collisions.[9] The experimental distribution is expected to have attained its asymptotic value only over a range of r from $r=0$ to some finite value, which from the data appears to be 2.5 or 3.0. Thus our asymptotic distribution should agree only over this range, which it appears to do reasonably well. In particular the rapid decay at low r is found in the data and it also appears to have another part which decreases more slowly.

The continuation of the sharp exponent rise down to very small r is surprising. Hard-core repulsion (multiperipheral phase-space cutoff) is expected to cause dn/dr to have a dip at $r=0$.

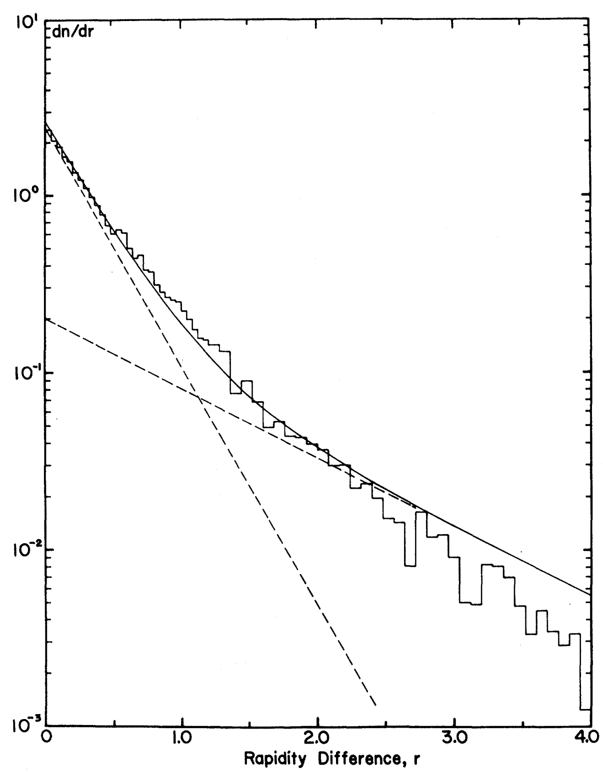

FIG. 1. The rapidity-gap distribution for charged particles in the central region. The experimental distribution is from the 205-GeV/c bubble chamber data on particle production in pp collisions. The bin size is first 0.04; then for $r > 1.2$ it is 0.08. The theoretical curve is from Eq. (34) in the text. The dashed lines show the contribution of the two terms individually.

There appears no indication of this for r down to 0.04. (Finite experimental resolution could fill in the dip at $r=0$, but it would not produce a sharp peak.) This lack of repulsion, if true, indicates that complex poles probably are not important in the nondiffractive cross section.[10]

Before concluding we must point out what may be an important failing of this model. By using Eq. (21) to obtain the asymptotic form of $f_2^{\rm ch}$ we find that asymptotically $f_3^{\rm ch} = -3f_2^{\rm ch}$, which disagrees with the experimental $f_3^{\rm ch}$ at present energies.[11] However, we must remember that this prediction is for the nondiffractive component only, while the data are strongly affected by the diffractive part. Also, each successive $f_i^{\rm ch}$ seems to need higher energy to approach its asymptotic form, so it may be too early to reach any conclusion about the asymptotic form of $f_3^{\rm ch}$.

Except for this possible source of trouble we have a model which appears to describe correctly the clustering effects in the charged-particle production cross sections, both inclusive and exclusive.

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APPENDIX A: ESTIMATING THE EXPERIMENTAL VALUE OF C_0

The easiest way to find C_0 is to find $C^{\rm ch}(y_1=y_2=0)$ directly in the two-charged-particle correlation results. For instance, Singer *et al.* give[12]

$$C_{\rm ch\,ch}(y_1=y_2=0)=1.38\pm 0.16 \tag{A1}$$

at 205 GeV/c.

Perhaps a better estimate may be obtained by carefully trying to correct for the various "perturbing effects." If we use the CERN ISR data[13] and try to fit $R_{\rm ch\,ch}$,

$$R(y_1y_2)=\frac{C(y_1y_2)}{\rho(y_1)\rho(y_2)}\ , \tag{A2}$$

to an $e^{-0.5(y_1-y_2)}$ form we get a higher value for $R_{\rm ch\,ch}(00)$ than experimentally found, perhaps 0.7 instead of 0.6. Thus we take

$$R_{\rm ch\,ch}(00)=0.7\pm 0.1\,. \tag{A3}$$

However, this still contains the long-ranged correlations due to diffraction. If we assume that these are charge independent and noting that R_{--} is essentially constant at 0.3 ± 0.1 we subtract this value we get for the short-ranged part alone

$$R_{\rm ch\,ch}(00)=0.4\pm 0.2\,. \tag{A4}$$

For the entire inelastic cross section $\rho_{\rm ch}(0)=1.66$,[12] but we estimate that for the nondiffractive part

$$\rho_{\rm ch}(0)=1.8\pm 0.2\,. \tag{A5}$$

This yields

$$C_0=1.3\pm 0.9 \tag{A6}$$

and hence, using Eq. (11),

$$f_2^{\rm ch}=(5.2\pm 3.6)Y\,, \tag{A7}$$

a very imprecise value and not very different from taking the experimental result directly.

If we try to get $f_2^{\rm ch}$ from the multiplicity distributions we are in even worse trouble because of diffraction. An attempt at a linear fit to f_2^- vs Y gives a range of slopes of perhaps 1.4 ± 0.5.[11]

From charge conservation we have[14]

$$f_2^{\rm ch}=Q+2f_1^-+4f_2^-\,, \tag{A8}$$

where Q is constant. Taking $\rho_-=(0.9\pm 0.1)$, which is the slope of f_1^- vs Y, we get for the linear part of $f_2^{\rm ch}$

$$\frac{f_2^{\rm ch}}{Y}\sim 2\rho_- +\frac{4f_2^-}{Y}=7.4\pm 2.2\,. \tag{A9}$$

However, the presence of diffraction makes f_2^- increase like Y^2 even if $f_{2\rm nondiff}^-=0$. Some have argued on the basis of duality that $f_{2\rm nondiff}^-$ is indeed zero, and the entire increase we see in f_2^- is due to diffraction.[1,14] This would mean

$$\frac{f_2^{\rm ch}}{Y}\sim 2\rho_- =1.8\pm 0.2\,. \tag{A10}$$

Hence, we must conclude that the presence of the diffraction makes it almost impossible to get $f_2^{\rm ch}$ *for the nondiffractive component* from the unseparated multiplicity distributions, and we thus return to our estimates from the two-particle correlations. Notice that it would be quite useful to separate the cross sections over a wide energy range into diffractive and nondiffractive components using a unique prescription.

APPENDIX B: THE RAPIDITY-GAP DISTRIBUTION

If $A_n(y_1, y_2, \ldots, y_n)$ is the *probability* of producing n charged particles with rapidities $y_1 < y_2 < \cdots < y_n$ then our rapidity-gap distribution is

$$\langle n-3\rangle \frac{dn}{dr}=\sum_{n=4}^{\infty}\sum_{i=2}^{n-2}\int dy_1\int dy_2\cdots\int dy_i\int dy_{i+2}\cdots\int dy_n A_n(y_1\cdots y_i, y_i+r, y_{i+2},\ldots,y_n)\,. \tag{B1}$$

In our multiperipheral model we have (since we are not interested in the end particles we fix them)

$$A_n=\delta(y_1)D\hat{F}(y_2-y_1)G\hat{F}(y_3-y_2)\cdots\hat{F}(y_n-y_{n-1})\times D\delta(Y-y_n)\,, \tag{B2}$$

where $\hat{F}$ and G are matrices. To simplify the algebra we transform to the J plane. Let

$$\langle n-3\rangle \frac{dn}{dr}(r,Y)=\hat{h}(r,Y)\,. \tag{B3}$$

Then from the normalization of dn/dr we have

$$\frac{dn}{dr}(r,Y)=\frac{\hat{h}(r,Y)}{\int_0^Y dr\,\hat{h}(r,Y)}\,. \tag{B4}$$

We now define the double Laplace transform of $\hat{h}(r,Y)$ with $(j-1)$ the conjugate variable to r and $(J-1)$ the conjugate to $Y-r$ [one can also say "$(J-1)$ conjugate to Y and $(j-J)$ the conjugate to r"]:

$$\hat{h}(r,Y)=\frac{1}{(2\pi i)^2}\int_{c-i\infty}^{c+i\infty}dJ\int_{c-i\infty}^{c+i\infty}dj\,e^{(J-1)(Y-r)+(j-1)r}\times h(j,J)\,. \tag{B5}$$

Then

$$h(j,J)=\int_0^\infty dY\int_0^\infty dr\,\theta(Y-r)e^{-(J-1)(Y-r)-(j-1)r}\times\hat{h}(r,Y)\,. \tag{B6}$$

Notice that $h(J,J)$ is just the (single) Laplace transform of $\int_0^Y dr\,\hat{h}(r,Y)$ with $(J-1)$ the conjugate variable to Y. The transform of an n-prong cross section is just

$$\sigma^{n_{\rm ch}}(J)=D^TF(J)[GF(J)]^{n-2}D\,, \tag{B7}$$

where $F(J)$ is just the Laplace transform of $\hat{F}$. As explained in Pinsky, Snider, and Thomas[2] an integrated n-charged-particle inclusive cross section (Laplace transformed) is given by[15]

$$P_n(J)=\Delta^T\Phi(J)[\Gamma\Phi(J)]^n\Delta\,, \tag{B8}$$

where

$$\Delta=S^TD\,, \tag{B9}$$

$$\Gamma=S^TGS\,, \tag{B10}$$

and

$$\Phi(J)=S^T[F(J)^{-1}-G]^{-1}S\,. \tag{B11}$$

The explicit form of $\Phi(J)$, the Mueller-Regge propagator, is

$$\Phi(J)=\begin{pmatrix}1/(J-\alpha_+) & 0\\ 0 & 1/(J-\alpha_-)\end{pmatrix}\,. \tag{B12}$$

Here S is the J-independent orthogonal matrix which transforms from the multiperipheral to the Mueller-Regge picture. It is not difficult to generalize when one uses the multiple Laplace transforms introduced above that the unintegrated exclusive cross sections may be written in the form of Eq. (B7) except that the F's each have different arguments (which are the conjugate variables to rapidity *differences*). A similar statement holds for the unintegrated inclusive cross sections. For instance, the double transform of the single-particle density with j_1 the conjugate to y and j_2 the conjugate to $Y-y$ is

$$\rho(j_1,j_2)=\Delta^T\Phi(j_1)\Gamma\Phi(j_2)\Delta\,. \tag{B13}$$

In a similar manner we see that the quantity we want, $h(j,J)$, is

$$\begin{aligned}h(j,J)&=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}D^TF(J)[GF(J)]^nGF(j)GF(J)[GF(J)]^mD\\&=D^T[F(J)^{-1}-G]^{-1}GF(j)G[F(J)^{-1}-G]^{-1}D\\&=\Delta^T\Phi(J)S^TGF(j)GS\Phi(J)\Delta\\&=\Delta^T\Phi(J)\Gamma S^TF(j)S\Gamma\Phi(J)\Delta\,.\end{aligned} \tag{B14}$$

Notice that

$$h(J,J)=\Delta^T\Phi(J)\Gamma\Phi(J)\Delta-\Delta^T\Phi(J)\Delta+D^TF(J)D$$

so that its transform is asymptotically $\langle n-3\rangle\sigma^{nd}$, as expected (see Ref. 15).

If we only want the asymptotic (in Y) contribution we keep only the leading (+) pole in the Φ's which gives

$$h(j,J)_{\rm asym}=\Delta_+\Phi_{++}(J)[S^TGF(j)GS]_{++}(J)\Delta_+\,. \tag{B15}$$

Transforming back and dividing by the asymptotic integrated $\hat{h}$ yields

$$\frac{dn}{dr}(r)_{\rm asym}=\frac{e^{-(\alpha_+-1)r}[S^TG\hat{F}(r)GS]_{++}}{\Gamma_{++}}\,. \tag{B16}$$

Thus to obtain an equation for the rapidity-gap distribution we are going to need the matrix S, so we might as well also find the equivalent elements of the Mueller-Regge description of the model.

For completeness we will obtain S for the more general case than when $\bar{\beta}_1=\bar{\beta}_2$. S has the form

$$S=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\quad, \tag{B17}$$

where

$$\cos\theta=\left(\frac{\alpha_+-\bar{\beta}_2}{\Delta\alpha}\right)^{1/2}=\left(\frac{\bar{\beta}_1-\alpha_-}{\Delta\alpha}\right)^{1/2} \tag{B18}$$

and

$$\sin\theta=\left(\frac{\alpha_+-\bar{\beta}_1}{\Delta\alpha}\right)^{1/2}=\left(\frac{\bar{\beta}_2-\alpha_-}{\Delta\alpha}\right)^{1/2}. \tag{B19}$$

A useful identity is

$$\sin\theta\cos\theta=\frac{g_{12}}{\Delta\alpha}\ . \tag{B20}$$

We obtain the elements of Γ,

$$\Gamma=\begin{pmatrix}\gamma_{++} & \gamma_{+-}\\ \gamma_{-+} & \gamma_{--}\end{pmatrix}\quad,$$

by using the above S in Eq. (B10) to obtain

$$\gamma_{++}=\frac{1}{\Delta\alpha}[g_{11}(\alpha_+-\bar{\beta}_2)+2(\alpha_+-\bar{\beta}_2)(\alpha_+-\bar{\beta}_1)+g_{22}(\alpha_+-\bar{\beta}_1)]\ , \tag{B21}$$

$$\gamma_{+-}=\gamma_{-+}=\frac{g_{12}}{\Delta\alpha}(\beta_1-\beta_2)\ , \tag{B22}$$

and

$$\gamma_{--}=\frac{1}{\Delta\alpha}[g_{22}(\alpha_+-\bar{\beta}_2)-2(\alpha_+-\bar{\beta}_2)(\alpha_+-\bar{\beta}_1)+g_{11}(\alpha_+-\bar{\beta}_1)]\ . \tag{B23}$$

Notice that using these we may check the equations in the text for the multiplicity moments since in the Mueller-Regge formalism we have

$$\frac{f_1^{\text{ch}}}{Y}\sim\gamma_{++} \tag{B24}$$

and

$$\frac{f_2^{\text{ch}}}{Y}\sim\frac{2\gamma_{+-}{}^2}{\Delta\alpha}\ . \tag{B25}$$

Using the above form of S we find that

$$[S^T G\hat{F}(r)GS]_{++}=\frac{1}{\Delta\alpha}\,[(\alpha_+-\bar{\beta}_2)(\alpha_+-\beta_1)^2e^{(\beta_1-1)r}+(\alpha_+-\bar{\beta}_1)(\alpha_+-\beta_2)^2e^{(\beta_2-1)r}]\ . \tag{B26}$$

This allows us to write

$$\frac{dn}{dr}(r)=A_1e^{-(\alpha_+-\beta_1)r}+A_2e^{-(\alpha_+-\beta_2)r}\ , \tag{B27}$$

with

$$A_1=\frac{(\alpha_+-\bar{\beta}_2)(\alpha_+-\beta_1)^2}{(\alpha_+-\alpha_-)\gamma_{++}} \tag{B28}$$

and

$$A_2=\frac{(\alpha_+-\bar{\beta}_1)(\alpha_+-\beta_2)^2}{(\alpha_+-\alpha_-)\gamma_{++}}\ , \tag{B29}$$

with the denominator $\gamma_{++}\Delta\alpha$ determined from Eq. (B21).

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[1]Perhaps the most ambitious attempt is by S. S. Pinsky and G. H. Thomas, Phys. Rev. D 9, 1350 (1974).

[2]S. S. Pinsky, D. R. Snider, and G. H. Thomas, Phys Lett. 47B, 505 (1973).

[3]G. F. Chew and A. Pignotti, Phys. Rev. 176, 2112 (1968).

[4]H. D. I. Abarbanel, G. F. Chew, M. L. Goldberger, and L. M. Saunders, Phys. Rev. Lett. 26, 973 (1971); Ann. Phys. (N.Y.) 73, 156 (1972).

[5]W. R. Frazer, D. R. Snider, and C.-I Tan, Phys. Rev. D 8, 3180 (1973); and references therein.

[6]P. W. Coulter and D. R. Snider, Phys. Rev. D 8, 4055 (1973).

[7]H. W. Wyld, Nuovo Cimento 7A, 632 (1972).

[8]Chung-I Tan has studied some other aspects of this model. See Phys. Rev. D 8, 935 (1973).

[9]This preliminary distribution is from a sample of the 205-GeV/c pp bubble chamber data described in Ref. 12.

[10]G. F. Chew and D. R. Snider, Phys. Lett. 31B, 73 (1970).

[11]See, for example, the figures in J. Whitmore, Phys. Rep. 10C, 274 (1974), on page 298.

[12]R. Singer *et al.*, Phys. Lett. B49, 481 (1974).

[13]S. R. Amendolia *et al.*, Phys. Lett. B48, 359 (1974).

[14]W. Frazer, R. Peccei, S. Pinsky, and C.-I Tan, Phys. Rev. D 7, 2647 (1973).

[15]Because the end particles are not included P_m is the transform of $\langle(n-2)(n-3)\ldots(n-m-1)\rangle$.