Counting hadron states

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The question of how to count hadron states is explored. It is suggested that there may be states which should be ignored in classification schemes such as SU(3) or the quark model.

The purpose of this note is to point out that there may be hadron states which ought to be ignored in classification schemes such as SU(3) or the quark model. In a well-defined sense, such states do not exist if we look at an average coarse-grained mass spectrum. The deuteron will be shown to be a noncontroversial example of such a state. Whether or not more interesting examples exist among the many meson and baryon resonances is not yet known.

The basic idea, which is implicit in some recent work in statistical mechanics,[1] is best illustrated by an example.

The deuteron is supposed to belong to a $\overline{\overline{10}}$ representation of SU(3).[2] This would imply low-energy bound states or resonances in hyperon-nucleon and hyperon-hyperon channels. It may be, however, that these presumed partners of the deuteron do not exist. What would this mean?

To answer this question let us begin with a precise definition of SU(3) for the hadron spectrum. Let $\rho_Q(M)$ be the density per unit mass of hadron states with quantum numbers Q. A stable state with mass M_0 clearly contributes a δ function $\delta(M-M_0)$ to ρ. We will see later how to compute ρ for continuum states. Here we need only mention that in the continuum ρ is to be the difference in density of states for interacting as opposed to free particles and need not be positive. Now define $\bar{\rho}_Q(M)$ as the average of ρ_Q over a mass interval ΔM which is typical of SU(3) breaking; e.g., one could set

$$\bar{\rho}_Q(M)=\frac{1}{\Delta M}\int_{-\Delta M/2}^{\Delta M/2}\rho_Q(M+M')dM' \ . \tag{1}$$

From the baryon mass differences we expect that $\Delta M\sim 200$ MeV would be an appropriate averaging scale for SU(3) considerations.

If states with quantum numbers Q belong to an SU(3) multiplet containing states with quantum numbers Q', Q'', etc., then SU(3) requires that

$$\bar{\rho}_Q(M)\approx\bar{\rho}_{Q'}(M)\approx\bar{\rho}_{Q''}(M)\cdots \ . \tag{2}$$

Note that averaging has gotten rid of mass differences so that our criterion is just that there be the correct number of states in various channels.

With this definition of SU(3) the absence of partners for the deuteron will be bad news unless the average density $\bar{\rho}_D$ in the deuteron channel somehow succeeds in vanishing. Actually this is just what happens. The δ function from the deuteron is canceled in the average by a hole in the n-p continuum states. To see how this goes consider the p-n system confined to a large spherical box of radius R in the center-of-mass system. If we concentrate on that partial wave which contains the deuteron, then on the edge of the box the wave function is approximately $\psi\approx\sin[kR+\delta(\epsilon)]$ where $\delta(\epsilon)$ is the phase shift as a function of energy ϵ and we have assumed that R is much larger than the range of the interaction. Making the wave function vanish at R yields

$$kR+\delta(\epsilon)=N\pi \ , \tag{3}$$

where N is an integer and

$$\begin{aligned}\frac{dN}{d\epsilon}&=\frac{1}{\pi}\left[R\left(\frac{dk}{d\epsilon}\right)+\frac{d\delta}{d\epsilon}\right]\\&=\frac{dN}{d\epsilon}\bigg|_0+\frac{1}{\pi}\frac{d}{d\epsilon}\delta(\epsilon) \ ,\end{aligned} \tag{4}$$

where $dN/d\epsilon\,|_0$ is the density of states for noninteracting particles. An approximate definition of ρ is evidently[3,4]

$$\rho=\frac{dN}{d\epsilon}-\frac{dN}{d\epsilon}\bigg|_0=\frac{1}{\pi}\frac{d}{d\epsilon}\delta \ , \tag{5}$$

which is independent of the radius of the box R as it should be. For the deuteron $\bar{\rho}_D$ is then easily seen to be

$$\bar{\rho}_D=\frac{1}{\Delta\epsilon}\left\{1+\frac{1}{\pi}\left[\delta(\Delta\epsilon)-\delta(0)\right]\right\} \ , \tag{6}$$

where the one comes from the deuteron and the difference in phase shifts is the result of integrating the expression in Eq. (5) over an averaging energy $\Delta\epsilon \sim \Delta M \sim 200$ MeV. The phase shift in the deuteron channel is π at threshold ($\epsilon = 0$) and falls rapidly, being nearly zero at an energy $\Delta\epsilon \sim 200$ MeV above threshold. Thus $\bar{\rho}_D$ nearly vanishes, which can be understood as the statement that in the region from the deuteron up to about 200 MeV, the net number of states is the same as for two free nucleons.

Evidently, there is a well-defined sense in which the deuteron does not exist if we look over a mass interval typical of SU(3) breaking. The possible lack of strange partners for the deuteron is not a serious matter.

It is no accident that $\bar{\rho}$ vanishes for the deuteron. The deuteron and low-energy p-n scattering are adequately described by nucleons interacting through a static potential. In potential theory one can derive Levinson's theorem which says that[5]

$$0 = N_B + \frac{1}{\pi}\int_0^\infty \left[\frac{d}{d\epsilon}\,\delta(\epsilon)\right] d\epsilon$$

$$= N_B + \frac{1}{\pi}\,[\delta(\infty) - \delta(0)] \ , \qquad (7)$$

where N_B is the number of bound states in the partial wave under consideration. On a nuclear physics level, an energy of order 200 MeV is essentially infinite and we see that $\bar{\rho}_0$ vanishes as a consequence of Levinson's theorem.

Levinson's theorem can be interpreted as the statement that the interaction does not change the net number of states in any channel. Thus the nuclear force can bind the deuteron only at the expense of a compensating depletion in continuum states. The idea which we want to abstract from this example is that certain interactions like a nucleon-nucleon potential do not make new states but just shuffle around the states which are already there. All this reshuffling takes place on an energy scale which is small on a hadronic scale of, say, one GeV.

The general formula for $\rho_Q(M)$ can be shown to be[4,6]

$$\rho_Q(M) = \sum_i \delta(M - M_i) + \frac{1}{2\pi i}\frac{d}{dM}\,\mathrm{tr}_Q[\ln S(M)]_c \ , \qquad (8)$$

where the sum runs over stable particles with quantum numbers Q, $[\ln S(M)]_c$ is the corrected part of the logarithm of the S matrix at center-of-mass energy M, and tr_Q is a suitably defined trace[7] over all channels with quantum numbers Q. For a single partial wave S is $e^{2i\delta}$ so that our definition of ρ agrees with Eq. (5) in this special case. In general, ρ can be thought of as the (infinite) sum over derivatives of eigen-phase shifts.[6]

To see that Eq. (8) is a reasonable definition let us consider a resonance. At a sharp resonance some eigen-phase shift jumps rapidly by 180° so that

$$\delta_r(M) \approx \text{const} + \pi\theta(M - M_r) \qquad (9a)$$

and

$$\frac{1}{\pi}\frac{d}{dM}\,\delta_r(M) \approx \delta(M - M_r) \ . \qquad (9b)$$

Therefore, a sharp resonance acts just like a stable particle. A wide resonance puts a broad bump of unit area in ρ.

The multiparticle generalization of Levinson's theorem would read

$$\int_0^\infty \rho_Q dM = 0 \quad \text{(Levinson's theorem)} \ . \qquad (10)$$

This is an established result for multiparticle potential scattering.[6] However, hadronic reactions do not appear to satisfy Levinson's theorem, at least for any reasonable cutoff on the integral. The dual model violates Levinson's theorem for any cutoff and it is probably not satisfied in field theory. Thus, whatever the fundamental hadron interaction is, it is unlikely to satisfy Eq. (10) at any reasonable energy. Nevertheless the notion behind Levinson's theorem that some interactions just move states around as opposed to making new states could, as we will see below, be a useful concept in hadron spectroscopy.

Let us take a coarse-grained view of the hadron spectrum by looking at $\bar{\rho}$ with an averaging interval ΔM of a few hundred MeV.

We already know that the deuteron appears in the local density ρ but not in the average $\bar{\rho}$. The 3-3 resonance Δ will, however, appear as a bump in both ρ and $\bar{\rho}$.[8] *Thus we see that there can be two kinds of hadrons*. The first kind which we call *true hadrons* remain as bumps in $\bar{\rho}$ as well as ρ. The second type which we will call *accidental states* disappear when we average over a mass interval which is small on a hadronic scale. The name accidental is chosen to suggest that such states might be produced accidentally by relatively weak long-range effects such as π exchange in the case of a deuteron.

What we have in mind here is something like the following. Simple N/D calculations based on a few channels and keeping only light-particle exchanges can often produce resonances. The appearance of such a resonance is often almost a kinematic effect due to, say, the opening of a new inelastic channel. Any kind of dynamics like this which is localized in energy ought to satisfy Levinson's theorem locally.[9] This sort of dynamics should not lead to a net change in number of states in any given energy range, but will just reshuffle those states which were already there. We know nowadays that whatever the fundamental hadron dynamics is, it is not of this simple type. However, it is quite possible that such effects do exist on top of whatever one's theory of fundamental hadron physics is.[10,11]

Do accidental states other than the deuteron really exist? At present one simply does not know. It is easy to convince oneself that the major well-studied states like ρ, Δ, etc., are *not* accidental.

Where might we look for accidental states?[11] Probably the meson channels are more likely because they are eigenstates of more quantum numbers so inelastic channels (where a fluctuation in number might drive an accidental resonance in some channel) frequently decouple.

Perhaps a likely case is the B meson multiplet. The evidence for the B and its properties is not controversial. It is coupled to $\pi\omega$, $\eta\rho$, πA_2, plus higher-mass channels and strange-particle channels. The isoscalar partner of the B is coupled to $\pi\rho$, $\eta\omega$ plus higher-mass and strange-particle channels, missing the equivalent of πA_2 (which becomes a strange-particle channel 500 MeV higher in mass). If the B is strongly coupled to πA_2, as is likely from data on B production by isovector exchange, then perhaps the B is an accidental particle which has no isoscalar SU(3) partner.[12,13] These ideas can be tested by studying the $\pi\omega$ phase shift in the 1^+ channel, looking for Levinson's theorem behavior, and by studying the 1^+, isoscalar, odd-G channel.[11]

In baryon channels one might look at the Roper resonance $N^*(1450)$ as a possible accidental particle, driven by the strong nearby $\pi\Delta$ and ϵN thresholds.[10] One could study the p_{11} phase shift, perhaps best in the $\pi\Delta$ channel since the inelasticity is large, looking for Levinson's theorem behavior. Also, if the coupling of $N^*(1450)$ to strange-particle channels such as KY^* is important (as is often the case for the higher mass one when two resonances N, N^* have the same quantum numbers) it might happen that the Λ^* SU(3) partner of $N^*(1450)$ would not exist since it is missing the $\bar{K}\Delta$ $(I=1)$ channel.

It is reasonable to assume that SU(3), the quark model, or any other classification scheme should apply to the strong short-range interactions[10] which produce the true hadrons in $\bar{\rho}$. If accidental states exist they could perfectly well not fit into one's classification scheme. This is no real loss since if we average over a reasonable mass interval the peripheral states disappear. In fact, the existence of accidental states is likely to be extremely sensitive to symmetry-breaking effects. For example, if the π mass were as large as the K mass the deuteron would probably not exist. Also, the kind of N/D dynamics discussed above is extremely sensitive to mass differences at thresholds and in exchange forces.

We close with the obvious remarks that the existence of an incomplete SU(3) multiplet of accidental mesons would be a perfectly acceptable circumstance and that the quark model need not fear the establishment of a Z^* if such a state turns out to be accidental.

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[1]R. Dashen and R. Rajaraman, Phys. Rev. D 10, 694 (1974); 10, 708 (1974).

[2]R. Oakes, Phys. Rev. 131, 2239 (1963).

[3]This is a standard result which can be found in many textbooks.

[4]R. Dashen, S. Ma, and H. Bernstein, Phys. Rev. 187, 345 (1969).

[5]This assumes that there are no elementary particles in the channel as is the case in normal potential theories.

[6]J. Wright, Phys. Rev. 139, B137 (1965).

[7]The δ functions of energy and momentum conservation are assumed to have been factored out of S. Defining what is meant by the trace still takes some care in the multibody case; see R. Dashen and S. Ma, J. Math. Phys. 12, 689 (1971). In the present context we are always interested in the density of states with a given angular momentum J, in which case the trace in Eq. (8) is always well defined.

[8]This is easily seen by looking at the experimental (3, 3) πN phase shift.

[9]We are explicitly not talking about an S-matrix dynamics where the continuous openings of more and more new channels lead to infinitely rising Regge trajectories. This sort of thing is not local in energy.

[10]In addition to quarks, etc., we would place the kind of

dynamics mentioned in the previous footnote in this category.

[11]In practice the use of Eq. (8) will presumably be restricted to cases where the scattering process can be approximated by a coupled set of quasi-two-body reactions. Equation (8) is then a straightforward generalization of Eq. (5). As a matter of consistency, it may be shown that S-matrix elements involving an accidental particle in either the initial or final states will cancel out in the averaged $\bar{\rho}$. (See Ref. 1.) Thus $\bar{\rho}$ is determined by the scattering of "true hadrons" as it should be. S-matrix elements containing accidental particles do contribute to ρ, however. Thus accidental particles can make further accidental particles but not true hadrons.

[12]Another possibility is that the unusual structure of the inelastic channels pushes the partner of the B up to a mass which is considerably higher than one would normally expect.

In general, not only the existence of accidental states but also the properties of all particle states will be influenced by fluctuations in inelastic channel density. Masses and widths will be shifted from values expected on the basis of symmetry predictions, such as SU(3) couplings and mass formulas. Without a quantitative multichannel theory it is hard to estimate general results, but in particular cases of interest it should be possible to analyze situations in a useful way. See the following reference.

[13]For example, the analysis of F. S. Henyey and G. L. Kane, Phys. Rev. D 9, 302 (1974), shows how one can study the effect of inelastic channels on the width of the Δ.