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Constraints on disfavored fragmentation

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Further constraints are obtained for the behavior of the inclusive spectrum near the phase-space boundary.

I. INTRODUCTION

Several years ago Chou and Yang[1] proposed to separate the type of fragments in hadron collisions into two kinds: favored fragmentation and disfavored fragmentation. Let the inclusive spectrum be denoted by

$$\rho_n(p_1,p_2\cdots p_n)\prod_i d^3p_i = \lim_{E\to\infty} (\text{partial cross section that } n \text{ particles with momenta } p_i,\ldots,p_n \text{ are emitted}) . \quad (1)$$

Then ρ_n can be separated into two parts,

$$\rho_n = \sigma_n + \tau_n , \quad (2)$$

where σ_n contains a δ function at the phase-space boundary and is contributed from n-particle exclusive final states. The τ_n is contributed from more-than-n-particle final states. The favored fragmentation consists of those fragments like $p\to p$, $\pi^\pm\to\pi^\pm$, $p\to p\pi^+\pi^-$, where the final particles together have the same quantum number (Q, B, G, I) as the initial projectile. Then σ_n does not vanish for the favored fragmentation. For the disfavored fragmentation like $\pi^+\to\pi^-$, $p\to\pi^-$, σ_n vanishes and τ_n approaches zero near the phase-space boundary. Experimental evidence[2] has so far strongly supported this classification and also possibly the even finer classification from the quark-model hypothesis among the disfavored fragmentations.[3]

In this paper the author wishes to go one step further and investigate the constraints placed on the rate that τ_n can approach zero near the phase boundary from kinetic considerations and some general dynamic considerations.

II. PHASE-SPACE MODEL

To introduce the notation let us briefly reivew a model of fragmentation based on phase space[4,5] alone. To be specific let us consider the case that the proton only fragments into proton and π^0:

$$\begin{aligned} &p\to p ,\\ &p\to p\pi^0 ,\\ &p\to p+n(\pi^0) . \end{aligned} \quad (3)$$

Then the single-particle inclusive ρ_1 of the proton consists of

$$\rho_1 = \sigma_1 + \tau_1 . \quad (4)$$

Then σ_1 comes from the fragment $p\to p$ and is

$$\sigma_1 = \text{const}\times\delta(1-x_1) ,$$

where $x_1 = p_{1\parallel}/p_0^*$ and $p_{1\parallel}, p_0^*$ are the longitudinal momenta of the final and initial protons in the c.m. system. The τ_1 comes from a summation of fragmentations $p+(l-1)\pi^0$:

$$\tau_1(x_1) = \sum_{l=2}^{n} \rho_{1l}(x_1) , \quad (5)$$

where ρ_{1l} is the contribution to the inclusive spectrum by exactly l final particles. From pure phase-space factors one gets

$$\rho_{1l}(x_1) = a_l \int \prod_{i=2}^{l} dx_i\, \delta\Big(1-\sum_{j=1}^{l} x_j\Big) \quad (6)$$

which has the normalization

$$\int \rho_{1l}(x_1)dx_1 = l\,\sigma(l) , \quad (7)$$

where $\sigma(l)$ is the cross section for the proton to fragment into exactly l particles. It is easy to see that the inclusive spectrum for $p\to\pi^0$ is

$$\tau_1(x_1) = \sum_{l=2}^{\infty} \sigma(l)l(l-1)(1-x_1)^{l-2} . \quad (8)$$

As $x_1\to 1$, the contribution comes from $\sigma(2)$ only,

$$\tau_1(x_1)\to 2\sigma(2) , \quad (9)$$

and all contributions from $l>2$ vanish because of the high power of $(1-x_1)$. In general,

$$\tau_n = \sum_{l=l_{\min}}^{l_{\max}} \frac{\sigma(l)l!}{(l-n-1)!}\left(1-\sum_{i=1}^{n} x_i\right)^{l-n-1} . \quad (10)$$

Near the phase-space boundary, $\sum x_i = 1-\epsilon$, each term of τ_n approaches zero like ϵ^{l-n-1}, and

$$\tau_n \sim \epsilon^{l_{\min}-n-1} . \tag{11}$$

There are two points that are worth pointing out:

(a) These results depend crucially on the existence of the $\delta(1-\sum x_i)$, which is given by the minimal rule[5]: In fragmentation processes the two colliding hadrons exchange as little longitudinal momenta as possible. There is already direct experimental evidence[6,7] for the minimal rule.

(b) The result does not depend on the distribution of $\sigma(l)$, whether it goes like $1/l^2$ or like a Poisson distribution, or like a combination of both. It also does not depend on the energy variation of $\sigma(l)$.

III. REALISTIC MODEL

It is obvious that a realistic model must be of the form

$$\rho_{1l}(x_1) = a_l \int \prod_{i=2}^{l} dx_i |M_{fi}|^2 \delta\left(1 - \sum_{j=1}^{l} x_j\right), \tag{12}$$

where M_{fi} is the matrix element for the proton to fragment into l particles.

To investigate the general structure of M_{fi}, let us examine the single inclusive spectrum ρ_1. All existing theoretical models and experimental data agree that, near $x_1 \sim 1$, $\tau_1 \propto (1-x_1)^{\alpha_1}$, with $\alpha_1 > 0$ for disfavored fragmentation. Hence, M_{fi} cannot be a function of highly singular nature or even an exponential function of x. Otherwise, upon integration it will not give a $(1-x_1)^{\alpha_1}$ behavior. Furthermore, phenomenologically a model[6] with $|M_{fi}|^2 \sim x_{\text{nucl}}^{\ 3}$ is sufficient to fit all available data for $pp \to \pi^\pm$ + anything at all energies.

Hence, let us *assume* that the most general form of the matrix element is a polynomial in x:

$$|M_{fi}|^2 = \sum_{\{\beta\}} C_{\alpha_1 \cdots \alpha_r} x_1^{\beta_1} \cdots x_n^{\beta_n} , \tag{13}$$

where $\{\beta\}$ is the set of all positive integers, and C are some constants. Then it is easy to prove the following theorem (see Appendix).

IV. THEOREM

If τ_n is the inclusive spectrum of n-particle final states, and the matrix element $|M_{fi}|^2$ is a polynomial in x, then

$$\text{(a):}\quad \tau_n \sim \epsilon^{\alpha_n} \text{ as } \epsilon \to 0, \text{ where } \epsilon = 1 - \sum_{i=1}^{n} x_i , \tag{14}$$

and

(b): α_n is some number, which has a lower bound:

$$\alpha_n \geq l_{\min} - n - 1 , \tag{15}$$

where $l_{\min}$ is the minimum number of final particles to produce a nonvanishing τ_n spectrum.

The results (14) and (15) are the same as those in Eq. (11) which are obtained from pure phase-space considerations alone. They have the practical consequences that for $p \to K^-$, $\alpha_1 > 1$, and $p \to K^-K^-$, $\alpha_2 > 2$. Some more examples are listed in Table I.

Let us consider the following inclusive spectra:

$$\begin{aligned} p &\to \pi^+ + \text{anything} \\ &\to \pi^+\pi^+ + \text{anything} \\ &\to n(\pi^+) + \text{anything} . \end{aligned} \tag{16}$$

Near the phase-space boundary, i.e., $\sum x_i \sim 1$, these inclusive spectra are contributed only from the following exclusive processes:

TABLE I. Lower bound for the rate of τ_n approaching the phase-space boundary from phase space.

n No. of final particles in the inclusive spectra	Examples	$l_{\min}$ Min. No. of final particles for the exclusive processes	Examples	Lower bound for α_n
1	$p \to \pi^+$	2	$p \to \pi^+ n$	0
	$p \to K^-$	3	$p \to K^-K^+p$	1
2	$p \to \pi^+\pi^-$	3	$p \to \pi^+\pi^- p$	0
	$p \to K^-K^-$	5	$p \to K^-K^+K^-K^+p$	2
3	$p \to \pi^+\pi^+\pi^-$	4	$p \to \pi^+\pi^+ p \pi^-$	0
	$p \to \pi^+\pi^+\pi^+$	6	$p \to \pi^+\pi^+\pi^+\pi^-\pi^- n$	2
	$p \to K^-K^-K^-$	7	$p \to 3(K^+K^-)p$	3

$$p \to \text{neutron} + \pi^+$$
$$\to \text{neutron} + \pi^+ + (\pi^+\pi^-)$$
$$\vdots$$
$$\to \text{neutron} + \pi^+ + (n-1)(\pi^+\pi^-) \ . \qquad (17)$$

The inclusive spectra behave like

$$\tau_n \sim \epsilon^{\alpha_n} ,$$

with

$$\alpha_n = \beta_{n+1} + \beta_{n+2} + \cdots \beta_{2n} + (n-1) \ . \qquad (18)$$

Then if one assumes that the mechanism for creating each additional pion pair is similar, then one has

$$\beta_{2n-1} = \beta_{2n-3} = \beta_{2n-5} = \cdots ,$$
$$\beta_{2n-2} = \beta_{2n-4} = \beta_{2n-6} = \cdots \ . \qquad (19)$$

Hence, one has the following equality among the α_n:

$$(\alpha_2 - \alpha_1) = (\alpha_3 - \alpha_2) = (\alpha_4 - \alpha_3) = \cdots = (\alpha_{n-1} - \alpha_n) = \cdots, \qquad (20)$$

which can be checked by experimental measurements in the future. It is easy to see also that if pions are created as triplets $(\pi^+\pi^-\pi^0)$ instead of pairs at any one time, the above equation (20) still holds. The equation can also be generalized to hold for the inclusive spectra

$$p \to n(K^+) + \text{anything}$$
$$\to n(\bar{p}) + \text{anything}$$
$$\to n(\Lambda) + \text{anything, etc.} \qquad (21)$$

V. DISCUSSION

Our discussions above come from the consideration of phase space alone and do not involve any specific behavior of the dynamics. In Ref. 1 Chou and Yang propose a selection rule on the behavior of τ_n due to dynamics. They argue that for disfavored fragments $a \to c$, if one sits in the rest frame of a, and if c does not have the same quantum number as a, the acceleration of c would be difficult; hence, the matrix element should vanish at the phase-space boundary. If the matrix element goes like

$$|M|^2 \sim x_1^{\beta_1} \cdots x_n^{\beta_n} ,$$

then

$$\tau_1 \propto \epsilon^{\beta_2 + \beta_3 + \cdots \beta_n + (n-2)} \ . \qquad (22)$$

The Chou-Yang selection rule, in our notation, simply implies

$$\beta_2 + \beta_3 + \cdots \beta_n > 0 \qquad (23)$$

for disfavored fragmentation. Hence, their results are quite different from the result obtained from above.

In short our results (15) are very general and should be true for any "reasonable" dynamical model.

APPENDIX

Let the inclusive spectrum τ_n of n particles be a summation of the exclusive processes: From a number l of final particles, where l runs from $l_{\min}$ to $l_{\max}$,

$$\tau_n(x_1, x_2 \cdots x_n) = \sum_{l=l_{\min}}^{l_{\max}} \rho_{nl} , \qquad (A1)$$

where

$$\rho_{nl} = a_l \int \prod_{i=n+1}^{l} dx_i |M|^2 \delta\left(1 - \sum_{i=1}^{l} x_i\right) \ . \qquad (A2)$$

The matrix element squared is assumed to consist of only polynomials in x_i. Hence, the most general form is

$$|M|^2 = \sum_{\{\beta\}} C_{\beta_1 \cdots \beta_n} x_1^{\beta_1} x_2^{\beta_2} \cdots x_n^{\beta_n} \ . \qquad (A3)$$

One can then substitute Eq. (A3) into Eq. (A2) and perform the integration. Let us first note that the following integral can be performed exactly:

$$I_n(x_1, \ldots, x_n) = \int dx_{n+1} dx_{n+2} \cdots dx_l \, x_1^{\beta_1} x_2^{\beta_2} \cdots x_l^{\beta_l} \, \delta\left(1 - \sum_{i=1}^{l} x_i\right)$$
$$= d_{l-1} d_{l-2} \cdots d_{n+1} x_1^{\beta_1} x_2^{\beta_2} \cdots x_n^{\beta_n} \left(1 - \sum_{i=1}^{n} x_i\right)^{\beta_{n+1} + \beta_{n+2} + \cdots + \beta_2 + (l-n-1)} , \qquad (A4)$$

where d are some constants given by

$$d_{n-k} = d(\beta_{l-k}, \beta_l + \beta_{l-1} + \cdots + \beta_{l-k+1} + k - 1) \qquad (A5)$$

and

$$d(a, b) = \sum_{k=0}^{b} \frac{(-1)^k b!}{(b-k)!\,k!\,(a+k+1)} \ . \qquad (A6)$$

Secondly, each exclusive process must normalize to the partial cross section $\sigma(l)$:

$$\int_{-1}^{1} \rho_{1l}(x_1)dx_1 = l\sigma(l) \ . \tag{A7}$$

Then the constant a_l has the following value:

$$a_l = \frac{\sigma(l)l}{\sum C_{\beta_1 \cdots \beta_n} d_{n-1} d_{n-2} \cdots d_1} . \tag{A8}$$

Finally, the behavior of the inclusive spectrum near the phase-space boundary can then be easily seen to be

$$\rho_{nl} \sim \epsilon^{\beta_{n+1}+\beta_{n+2}+\cdots \beta_l + (l-n-1)} \tag{A9}$$

and

$$\tau_n \sim \epsilon^{\beta_{n+1}+\beta_{n+2}+\cdots \beta_{l\min}+(l_{\min}-j-1)} \tag{A10}$$

when

$$\epsilon = 1 - \sum_{i=1}^{n} x_i \to 0 \ .$$

In particular, for the single-particle inclusive spectrum one has

$$\tau_1 \sim \epsilon^{\beta_2+\beta_3+\cdots \beta_{l\min}+(l_{\min}-n-1)} \ . \tag{A11}$$

It is also clear from the above derivation that the β_i do not have to be integers. They can be any positive number.

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