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Quark-parton models and the neutron charge radius

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We point out that in parton models of nucleons, including standard constituent-quark models, the nonvanishing of the neutron charge radius implies an essential mixing of the space part of the nucleon wave function with those parts involving isospin and other discrete quantum numbers. We state these conditions precisely, and give a new value for this charge radius, derived from precise electron-deuteron scattering data.

Composite models of the hadrons have achieved considerable popularity, primarily because the most elementary, nonrelativistic quark pictures account remarkably well for most of the available data on hadronic physics.[1] However, recent electron-positron annihilation data[2] may indicate a breakdown of the quark-parton model's predictions of large-momentum transfer electromagnetic properties of hadrons. In this note we point out that if simple constituent-quark models are to account for even low-momentum-transfer data on the electromagnetic structure of the neutron, they will have to be restricted in nonobvious ways. [In this note, the term "quark model" will be used to refer to any constituent model of the nucleon in which the spin-isospin wave function is a representation of a symmetry such as SU(3) or SU(6). In addition, we want to emphasize that we are studying conventional low-energy constituent-quark models rather than direct manifestations of infinite-momentum-frame parton models. These sets of models may or may not be the same.] These data imply that such models must incorporate either the admixture of quark-antiquark pairs, or the relaxation of spatial symmetry of the wave functions, or both. In other words, the space and discrete quantum number parts of the wave function must be mixed. Such changes will at least marginally affect other predictions of the model (such as magnetic-moment ratios), as well as its esthetic appeal.

Several years ago it was shown[3] that under SU(6) the ratio of neutron and proton form factors of a given type (electric or magnetic) is independent of momentum transfer, and therefore can be evaluated at any convenient value of q^2, say $q^2=-\infty$ or $q^2=0$. This result implies that the neutron's electric form factor, $G_E^n(q^2)$, vanishes identically.

It has been known for some time that scattering of thermal neutrons from atomic electrons predicts finite charge radius $\langle r_n^2\rangle$ for the neutron. The nonzero value of the radius, in turn, implies a nonvanishing electric form factor for the neutron at $q^2<0$ (we take $q^2=t$) since

$$\langle r_n^2\rangle = -6\frac{d}{dq^2}G_E^n(q^2)\bigg|_{q^2=0}\,.$$

The most recent[4] determination of thermal neutron-electron interaction predicts a value of

$$\frac{dG_E^n}{dq^2}\bigg|_{q^2=0} = 0.0189\pm0.0004\ \mathrm{fm}^2\,.$$

Elastic electron-deuteron scattering may also be used to measure the value of G_E^n, including its sign, at larger values of q^2. Apparent discrepancies between low-q^2 electron scattering and thermal-neutron data have now been shown to arise from inadequate (deuteron) wave functions used in the analysis of electron-deuteron scattering.[5] Using wave functions derived from any realistic nucleon-nucleon potential, and with the proper binding energy, the value derived from low-q^2 e-d scattering is now known to be consistent with those from thermal neutrons.

One of us (J.S.M.) has recently analyzed very precise electron-deuteron and electron-proton scattering extending in q^2 from 0.5 to 10 fm^{-2}.[6] The low-q^2 part of the data has been used to extract the slope of G_E^n at $q^2=0$, the value obtained being

$$\frac{dG_E^n}{dq^2}\bigg|_{q=0} = 0.024\pm0.003\ \mathrm{fm}^2\,,$$

which is in essential agreement both with other very-low-q^2 scattering experiments[7] and with thermal-neutron data. The measured value of G_E^n is nonzero in this entire range, and the high-q^2 measurements are consistent with earlier experiments.[8]

The neutron charge radius is far from small on the scale of the proton, with limits $0.11 \leq |r_n{}^2| \leq 0.16$. The new data from e-d scattering imply that

$$\frac{\langle r_n{}^2\rangle}{\langle r_p{}^2\rangle} = -0.225 \pm 0.035\,,$$

where $\langle r_p{}^2\rangle$ is the corresponding charge radius of the proton.

If we suppose that the nucleon is composed of several nonrelativistic pointlike particles, each with a charge and intrinsic magnetic moment, the charge form factor may be written

$$G_E(q^2) = \left\langle \Psi(1,\ldots,N) \right| \sum_{i=1}^{N} Q_n e^{-i\vec{q}\cdot\vec{r}_i} \left| \Psi(1,\ldots,N) \right\rangle, \tag{1}$$

where $\Psi(1,\ldots,N)$ is the wave function of the constituents, and Q_i is the charge of the ith constituent. For conceptual simplicity we have expressed the charge form factor in the rest frame of the nucleon. Since[7] G_E is the Fourier transform of the zeroth component of the current density in the Breit frame, and since the Breit frame reduces to the rest frame when q vanishes, Eq. (1) is to be understood in the sense of a Taylor series for G_E about $q^2 = 0$. We now note that if the wave function of the neutron can be written as a sum of products,

$$\Psi(1,\ldots,N) = \sum_a \phi_a(1,\ldots,N)\chi_a(1,\ldots,N)\,,$$

where the ϕ_a's are space wave functions and the χ_a's incorporate all other quantum numbers such as spin, isospin, and color, then under the following two circumstances the form factor will be proportional to the total charge (and hence, in the case of the neutron, zero): (a) Each χ_a has a definite symmetry (either odd or even) under interchange of a pair of quarks, but has (the same) definite total quantum numbers as Ψ; or (b) the total wave function is either even or odd under interchange of the space coordinates of any pair. These two cases include but are not limited to all standard quark models. For example, the usual nonrelativistic quark model puts all three quarks in the nucleon in identical $l = 0$ harmonic-oscillator states.

The vanishing of the neutron charge radius (and indeed, of all moments of its charge distribution) in standard quark-type models[1] is of course an artifact of these models and is in no sense fundamental. For example, there is no reason, other than economy or esthetics, to assume that the quarks occupy identical orbitals with $l = 0$. As shown in Table I, we have constructed, *ad hoc,* several models which yield reasonable values of the ratio of neutron and proton charge radii, without doing excessive violence to the predicted ratios of their magentic moments. These models include nonsymmetric s^2s' and sp^2 configurations (positive-parity quarks) as well as s^2p (negative-parity quarks). (The s^2p model listed in Table I also predicts $\mu_p/\mu_n = -1.64$ when $g_{\text{quark}} = 1$ and -1.52 when $g_{\text{quark}} = 2$. These are hardly drastic modifications of the usual quark-model ratio of -1.5.)

We must point out, however, that changing the nature of the ground state from the expected lowest-energy state of the harmonic oscillator diminishes our ability to predict, in any natural way, the spectrum of the excited states. Such prediction has been regarded as one of the signal successes of the quark model.

Another way out is provided by the admixture of quark-antiquark pairs. For example, to the usual s^3 ground state, we add (in the same harmonic oscillator) a $q\bar{q}$ pair which is in a relative s-wave state and in a collective $1p$ state with respect to q's in the qqq group. Since we are treating the problem nonrelativistically, we neglect the small components of the quark spinors and thereby any cross terms between the qqq state and the $qqqq\bar{q}$ state. Hence the entire neutron charge radius results from the diagonal terms in the $qqqq\bar{q}$ state in Eq. (1). The calculation, which is straightforward, leads to the interesting bound $|\langle r_n{}^2\rangle/\langle r_p{}^2\rangle| \leq \frac{1}{9}$. This bound is saturated only when the ratio of qqq state to $qqqq\bar{q}$ state in the physical nucleon vanishes *and* the average charge of the $\bar{q}$ is $-\frac{2}{3}$.

While the addition of $q\bar{q}$ pairs thus provides a way out in principle, the above result suggests that either a single $q\bar{q}$ pair must be in a state with high principal quantum number or that many $q\bar{q}$ pairs are required. This in turn leads to the alternative of returning to simple meson-nucleon composite models, which also have no difficulty accounting for the observed ratios. The results of some of these calculations are also presented in Table I, not because we take them very seriously, but rather to illustrate that it is not hard to fit the data in several reasonable, albeit rather model-dependent, ways. Of course meson-nucleon models do not immediately yield the other good predictions of the quark model.

A relativistic model must be subject to the same theorem we have stated above. Such models will in fact normally mix space and spin parts of the wave function; but one would have to test any particular relativistic model against the details of the theorem.

Our conclusion that low-energy data require some mixing of the space and discrete quantum

TABLE I. A list of models and the results they give for the neutron charge radius. In the quark models the harmonic-oscillator potential is the same for all particles.

Description of model	Neutron wave function	Proton wave function	$\langle r_n^2\rangle / \langle r_p^2\rangle$
(1) Symmetric quarks in harmonic oscillator.	3 quarks in s state	same	0
(2) Asymmetric quark model, equal-mass quarks of odd parity.	2 $\mathcal{N}$ quarks in relative s state, $\mathcal{P}$ in P state relative to 2 $\mathcal{N}$ c.m.	same, but with $\mathcal{P}$ and $\mathcal{N}$ interchanged	−0.27
(3) 100% $q\bar{q}$ admixture, with average $\bar{q}$ charge of Q, even-parity quarks of equal mass.	3 quarks in s states, $q\bar{q}$ in relative s state, but with c.m. in a p state relative to the other three quarks.	same, but with $\mathcal{P}$ and $\mathcal{N}$ interchanged	$Q/(\frac{20}{3}+Q)$
(4) Pion-nucleon composite, with all internal particles pointlike, and with masses given their experimental values.	$(\frac{1}{3})^{1/2}(\pi^0\times n)$ $-(\frac{2}{3})^{1/2}(\pi^-\times p)$	$-(\frac{1}{3})^{1/2}(\pi^0\times p)$ $+(\frac{2}{3})^{1/2}(\pi^+\times p)$	−0.98
(5) Same as above, but with π replaced by ρ. All masses given their experimental values.	same as (4) with $\pi\to\rho$	same as (4) with $\pi\to p$	−0.25
(6) Pion-nucleon bootstrap, all particles taken to be physical and composite, with all internal particles given their physical charge radii and masses.	same as (4)	same as (4)	$\langle r_p^2\rangle \geq \langle r_n^2\rangle$

number parts of the wave function constitutes one piece of evidence in the quark-model jigsaw puzzle.[10] The only other such piece comes from deep-inelastic scattering from nucleons, where the fact that $\nu W_2^{\text{neutron}}/\nu W_2^{\text{proton}} \sim 0.3$ near the Bjorken variable $x=1$ implies that the $\mathcal{N}$-quark distribution falls more quickly than the $\mathcal{P}$-quark distribution within the proton near $x=1$.[11]

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[11]We are indebted to Professor R. P. Feynman for this point.