Parity violation in three-triplet gauge models

A. Andraši, J. Missimer*, and D. Tadić
Institute "Rudjer Bošković" and University of Zagreb, Zagreb, Croatia, Yugoslavia
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Enhancement of the parity-violating isospin-changing one-pion-exchange potential for three-triplet gauge models has been found with respect to the standard weak-Hamiltonian model. In the case of the Bég-Zee model, which allows for the neutral hadron-current coupling to neutrinos, there is an enhancement by at least a factor of 10. The Lee-Prentki-Zumino and the Georgi-Glashow models are also discussed. The role of Fierz transformations has been investigated in detail. It has been suggested that the detection of an angular γ asymmetry comparable with circular γ polarization in thermal-neutron capture, as well as larger resonance than expected in the decay process of lithium, can be meaningful substantiation of unified theories of weak and electromagnetic interactions.

A recent analysis of three-triplet gauge models of weak and electromagnetic interactions[1] has indicated a large enhancement of parity violation in nuclear processes relative to the estimate following from the Cabibbo theory. While remarking the importance of the color-octet parts of currents, the analysis employed a model which was designed to exclude the neutral hadron-current coupling to neutrinos, the existence of which has been indicated in recent experiments.[2] Moreover, it suggests that the symmetry of a model with respect to the interchange of vector and axial-vector currents ($V \leftrightarrow A$) and its susceptibility to Fierz transformations are essential in determining the SU(3) structure.

The present paper shows that parity-violating effects are also enhanced in a model proposed by Bég and Zee,[3] which does exhibit a neutral hadron-current coupling to neutrons, and estimates the magnitude of the enhancement. The paper also examines the role of Fierz transformations (FT) and the $V \leftrightarrow A$ symmetry of a model in the analysis of its SU(3) transformation properties. In order to illustrate the general features of three-triplet models, we present a summary of the previously analyzed Lee-Prentki-Zumino (LPZ)[4] model and a complete analysis of the Georgi-Glashow (GG)[5] model.

The gauge group of the model proposed by Bég and Zee is $SU(2)\otimes U(1)$. The quarks are assigned to four doublets

$$\begin{pmatrix} p_{1L} \\ n_{1L}(\theta_C) \end{pmatrix}, \begin{pmatrix} p_{2L} \\ \lambda_{1L}(\theta_C) \end{pmatrix}, \begin{pmatrix} p_{3R} \\ n_{2R} \end{pmatrix}, \begin{pmatrix} p_{2R} \\ n_{3R} \end{pmatrix}, \tag{1}$$

and the complementary ten singlets. The symbols (p, n, λ) denote the transformation properties of the quarks under the conventional group SU(3), while the indices (1, 2, 3) characterize their transformation properties under the color group SU(3)′. The subscripts L and R indicate left- and right-handed projections of the quarks, and as is now conventional

$$\begin{aligned} n_{1L}(\theta_C) &= n_{1L}\cos\theta_C + \lambda_{1L}\sin\theta_C , \\ \lambda_{1L}(\theta_C) &= -n_{1L}\sin\theta_C + \lambda_{1L}\cos\theta_C . \end{aligned} \tag{2}$$

In order to exhibit the SU(3) structure of the hadron currents of the model, we use meson symbols to label charged currents and capitalized SU(3)-quark symbols to denote neutral currents, i.e.,

$$\begin{aligned} \Pi^{+i}_{Lj} &= \bar{n}_{jL}\gamma_\mu p_{iL} \\ &= \bar{n}_j\gamma_\mu \tfrac{1}{2}(1+\gamma_5)p_i , \end{aligned} \tag{3}$$

$$P^{\,i}_{Lj} = \bar{p}_{jL}\gamma_\mu p_{iL} , \tag{4}$$

where P^i_{Lj} transforms as $(1/\sqrt{6})\eta^0 + (1/\sqrt{2})\pi^0 + \frac{1}{3}\sigma^0$, σ^0 being an SU(3) scalar. In order to display the color structure of the currents, we decompose each current into color-octet (script letter) and color-singlet (capitalized without color indices) parts, i.e.,

$$P^i_{Lj} = \mathcal{P}^{\,i}_{Lj} + \tfrac{1}{3}P_L , \tag{5}$$

where $P_L = \sum_i \mathcal{P}^{\,i}_{Li}$ is the color-singlet piece. Lorentz indices will be suppressed.

In this notation, the charged current of the model is

$$\begin{aligned} J^+_\mu = \Pi^{-1}_{L1}\cos\theta_C + K^{-1}_{L1}\sin\theta_C + \Pi^{-2}_{R3} - \Pi^{-1}_{L2}\sin\theta_C \\ + K^{-1}_{L2}\cos\theta_C + K^{-2}_{R3} . \end{aligned} \tag{6}$$

The color-singlet part of the charged current

$$|J^+_\mu|_{CS} = \tfrac{1}{3}|\Pi^-_L\cos\theta_C + K^-_L\sin\theta_C| \tag{7}$$

clearly exhibits the current structure of the Cabibbo model. The neutral hadronic current is

$$\begin{aligned} J^Z_\mu &= J^3_\mu - \sin^2\theta_W J^\gamma_\mu \\ &= \tfrac{1}{2}(P^1_{L1} + P^2_2 + P^3_{R3} - N^1_{L1} - N^2_{R2} - \Lambda^1_{L1} - \Lambda^3_{R3}) \\ &\quad - \sin^2\theta_W\, \tfrac{1}{3}(2P - N - \Lambda) , \end{aligned} \tag{8}$$

whose color-singlet part is

$$|J_\mu^Z|_{CS}=\tfrac{1}{2}\cos 2\theta_W\, J_\mu^\gamma\ , \tag{9}$$

a vector current which cannot contribute to parity-violating processes. Hence, the enhancement of such processes, which is a consequence of the neutral current, arises solely from the color-octet piece.

In terms of charged and neutral currents, the hadronic component of the Lagrangian is

$$\mathcal{L}_n=-eJ_\mu^\gamma A_\mu+\frac{1}{\sqrt{2}}\frac{e}{\sin\theta_W}(J_\mu^+W_\mu^-+J_\mu^-W_\mu^+)+\frac{2e}{\sin 2\theta_W}J_\mu^Z Z_\mu\ . \tag{10}$$

Despite the apparently formidable mixture of left- and right-handed currents in both charged and neutral currents, evaluation of the products of currents appearing in the effective Hamiltonian yields a simple result. The parity-violating (pv) pieces of the Hamiltonian are comprised in

$$H_W^{\rm pv}=-\frac{1}{48}\frac{1}{M_W^2}\frac{e^2}{\sin^2\theta_W}\{\cos^2\theta_C|\pi_{Lj}^{-i}\pi_{Li}^{+j}-\tfrac{8}{3}\Pi_L^-\Pi_L^+|+\sin^2\theta_C|\mathcal{K}_{Lj}^{-i}\mathcal{K}_{Li}^{+j}-\tfrac{8}{3}K_L^-K_L^+|$$
$$+\sin\theta_C\cos\theta_C|\pi_{Lj}^{-i}\mathcal{K}_{Li}^{+j}+\mathcal{K}_{Lj}^{-i}\pi_{Li}^{+j}-\tfrac{8}{3}|\Pi_L^-K_L^++K_L^-\Pi_L^+)|+(+\leftrightarrow-)\} \tag{11}$$

and

$$H_Z^{\rm pv}=\frac{1}{8}\frac{1}{M_Z^2}\frac{e^2}{\sin^2 2\theta_W}|(\mathfrak{N}_{Lj}^i\lambda_{Li}^j+\lambda_{Lj}^i\mathfrak{N}_{Li}^j)+(\mathcal{P}_{Rj}^i\mathfrak{N}_{Li}^j+\mathfrak{N}_{Lj}^i\mathcal{P}_{Ri}^j)|\ . \tag{12}$$

The indices i and j are summed over, indicating the color-singlet character of the products of color octets. Such summations always appear upon extracting the color singlet from a product of color octets $\mathfrak{M}_\beta^\alpha$ and $\mathfrak{N}_\epsilon^\gamma$ because

$$|\mathfrak{M}_\beta^\alpha\mathfrak{N}_\epsilon^\gamma|_{CS}=\tfrac{1}{24}(3\delta_{\alpha\epsilon}\delta_{\beta\gamma}-\delta_{\alpha\beta}\delta_{\gamma\epsilon})\mathfrak{M}_j^i\mathfrak{N}_i^j\ . \tag{13}$$

When we apply this relation to the product of currents appearing in the effective Hamiltonian and use the identity

$$|\mathfrak{M}_L\mathfrak{N}_L+\mathfrak{M}_L\mathfrak{N}_R+\mathfrak{M}_R\mathfrak{N}_L+\mathfrak{M}_R\mathfrak{N}_R|_{\rm pv}=|(\mathfrak{M}_L+\mathfrak{M}_R)(\mathfrak{N}_L+\mathfrak{N}_R)|_{\rm pv}=|\mathfrak{M}\mathfrak{N}|_{\rm pv}=0 \tag{14}$$

repeatedly, Eqs. (11) and (12) result. The subscript pv on the identity denotes parity-violating terms.

The effective Hamiltonians of the Bég-Zee model display the general features of the SU(3) structure of any three-triplet model. In terms possessing $V\leftrightarrow A$ symmetry, both products of color-octet and of color-singlet tensors produce color-singlet tensors of two types: those resulting from the product decomposition of the SU(3) octet with itself and those from the decomposition of the octet with a singlet. Color-octet terms, such as $\mathcal{P}_{Rj}^i\mathfrak{N}_{Li}^j+\mathfrak{N}_{Lj}^i\mathcal{P}_{Ri}^j$, which are antisymmetric under $V\leftrightarrow A$, produce the two types of tensors as well. However, the same linear combination of tensors always appears, so effectively there is only one new tensor. Color singlets producing terms antisymmetric under $V\leftrightarrow A$ cannot appear, since $V+A$ currents in semileptonic decays would result. Since terms appearing in the Bég-Zee model which are antisymmetric under $V\leftrightarrow A$ are not relevant to the following discussion of parity violation, we defer discussing them until we consider the Georgi-Glashow model, where they are relevant.

We will denote the tensors symmetric under $V\leftrightarrow A$ according to Table I. Some additional details can be found in the Appendix. Examination of the effective Hamiltonian of the Bég-Zee model reveals that these tensors are present only in two combinations. The charged current exhibits only the combination $U^d=O^d-\tfrac{8}{3}T^d$, while the neutral current is characterized by the tensor V^d. For the octet representation $V^8=O^8-(\tfrac{5}{6})^{1/2}Q^8$. The tensor decomposition and the assumption of octet dominance then imply the following form of the parity-violating weak Hamiltonian:

$$H^{\rm pv}=-\frac{1}{24}\left(\frac{3}{10}\right)^{1/2}\frac{e^2}{\sin^2\theta_W}\frac{1}{M_W^2}\sin\theta_C\cos\theta_C\left[U^8_{1\,1/2\,-1/2}+(\tfrac{1}{2})^{1/2}\tan\theta_C\,U^8_{010}-\left(\frac{1}{2}\right)^{1/2}\frac{M_W^2}{M_Z^2}\frac{1}{\cos^2\theta_W}\frac{1}{\sin\theta_C\cos\theta_C}V^8_{010}\right]\ . \tag{15}$$

TABLE I. The symbols C and R label color and regular SU(3) representations, respectively. d is the dimension of the representation.

C \ R	8×8	8×1
$[8\times 8]_{CS}$	O^d	Q^d
1×1	T^d	S^d

We have neglected any contributions that Higgs scalars might make in the model. We have also discarded terms with tensors U^8_{000}, V^8_{000}, and $U^8_{-1\,1/2\,1/2}$, since they are not required in the following discussion.

The implications for parity-violating nuclear processes of the SU(3) structure of a Hamiltonian are expressed in the sum rule

$$(\tfrac{3}{2})^{1/2}A(n^0_-) - F[2A(\Lambda^0_-)+A(\Xi^-_-)] = 0\ , \qquad (16)$$

relating $\Delta S=1$ to $\Delta S=0$ weak processes.[6] The Cabibbo model predicts $F_C = \tan\theta_C$. On the other hand, the Bég-Zee model implies a sum rule in terms of two reduced matrix elements, $u=\langle\|U^8\|\rangle$ and $v=\langle\|V^8\|\rangle$, with

$$F_{BZ} = \tan\theta_C - \frac{v}{u}\frac{M_W^2}{M_Z^2}\frac{1}{\cos^2\theta_W}\frac{1}{\sin\theta_C\cos\theta_C}\ . \qquad (17)$$

In the Bég-Zee model, $M_W^2 > M_Z^2\cos^2\theta_W$. Thus, for $\sin\theta_C \approx 0.22$

$$|F_{BZ}/F_C| > |1-20v/u|\ . \qquad (18)$$

The computation of F_{BZ} has not employed Fierz transformations of the color-octet contributions to the Hamiltonian, a procedure which analyses of SU(3) in three-triplet models commonly assume necessary. Only the determination of the color-singlet part of the product of two color octets is necessary in a discussion of the SU(3) content of a product of currents, since the SU(3) properties of those currents are independent of its SU(3)′ properties. Hence, the result (17) does not depend on the assumption that one deals effectively with the local current-current structure. The employment of FT is vitally dependent on this assumption. However, Fierz transformations do determine the relationship between the tensors O^d and Q^d arising from color octets and the tensors T^d and S^d arising from color singlets. Relegating the discussion of FT to the Appendix, we now exhibit the relevant deductions. For the octet representations of regular SU(3), the relation is

$$O^8_\alpha = -T^8_\alpha + \tfrac{1}{3}(\tfrac{10}{3})^{1/2}S^8_\alpha\ ,$$
$$Q^8_\alpha = (\tfrac{5}{6})^{1/2}T^8_\alpha + \tfrac{1}{3}S^8_\alpha\ ,$$
$$\alpha = (YII_3)\ . \qquad (19)$$

Applying the equalities to the tensors U and V of Eq. (15) yields

$$U^8_\alpha = 2V^8_\alpha = -\tfrac{11}{3}(\tfrac{1}{5})^{1/2}T^8_\alpha + (\tfrac{2}{3})^{1/2}S^8_\alpha\ . \qquad (20)$$

Hence, Fierz transforming determines the ratio $V/U=\frac{1}{2}$ of Eq. (18) and indicates that the Bég-Zee model produces an enhancement relative to the Cabibbo model of at least a factor of 10. The enhancement is comparable with that predicted by the d'Espagnat model,[7] but is five or six times smaller than that reported by Bailin *et al.*[1] in their analysis of the Lee-Prentki-Zumino model.

The analysis of the LPZ model can be summarized as follows. The charged and neutral currents of the model are

$$|J^+|_{LPZ} = \cos\alpha_n\, N^1_{L3} + \sin\alpha_n\, N^3_{L2} - \cos\alpha_n\, \Pi^{-1}_{L1} - \sin\alpha_n\, \Pi^{-2}_{L1} + \cos\alpha_\lambda\, \Lambda^3_{L1} + \sin\alpha_\lambda\, \Lambda^3_{L2} - \cos\alpha_\lambda\, K^{-1}_{L2} - \sin\alpha_\lambda\, K^{-2}_{L2} \qquad (21a)$$

and

$$J^Z = 2(P^1_{L1}+P^2_{L2}-N^3_{L3}-\Lambda^3_{L3}) - 2\sin^2\theta_W(P^1_1+P^2_2-N^3_3-\Lambda^3_3)\ , \qquad (21b)$$

respectively. Since the quarks are integrally charged in this model, even currents diagonal in SU(3)-quark labels can carry charge, hence, the appearance of N and Λ in the charged current. Application of Eq. (13) to the products of charged and neutral currents yields effective interactions comparable with those found by Bailin *et al.*, but the SU(3) decomposition produces a slightly different ratio

$$F_{LPZ} = \frac{|-[13-11\sin^2\alpha_\lambda + 22\cos(\alpha_n-\alpha_\lambda) - 24\cos2\theta_W]t + [1+\sin^2\alpha_\lambda - 2\cos(\alpha_n-\alpha_\lambda)]s|}{\cos\alpha_n\sin\alpha_\lambda(11t+s)}\ . \qquad (22)$$

Here s and t refer to the reduced matrix elements of tensors S^8 and T^8 defined in Table I and in the Appendix. The corrections imply quantitatively, but not qualitatively, different enhancements relative to the Cabibbo model. The enhancement is estimated by assuming $\alpha_n = \alpha_\lambda = \theta_C$ and $\sin\theta_C = 0.22$, for which

$$\frac{F_{LPZ}}{F_C} \simeq -\frac{1}{\sin^2\theta_C}\frac{35-24\cos2\theta_W+\sigma}{11+\sigma}\ , \qquad (23)$$

where $\sigma = st^{-1}$. For $\theta_W=\frac{1}{4}\pi$, the dominance of the octet tensor from the decomposition of 8×8

($s=0$) effects an enhancement by a factor of 70 and the dominance of the octet tensor of the 8×1 decomposition ($t=0$) produces an enhancement by a factor of 20. Bailin *et al.* found factors of 50 and 60, respectively, with the same values of the Cabibbo and the Weinberg angles. Reasoning based on the separability of the SU(3) and SU(3)′ representations, as outlined in the Appendix, fixes $\sigma = 15/\sqrt{30}$; thus, for $\theta_W = \frac{1}{4}\pi$ it effects an enhancement of a factor of 55. Equation (18) indicates an enhancement by at least a factor of 10 for a wide range of values of σ.

Comparison of the sum rule determined by the LPZ model with that of the Bég-Zee model reveals two interesting differences. Whereas the neutral current does not contribute at all to the enhancement of the LPZ model, the neutral current of the Bég-Zee model produces the enhancement. Moreover, the tensors of the effective LPZ Hamiltonian do not combine in as fortuitous a manner as they do in the Bég-Zee Hamiltonian. The relative importance of octet tensors from 8×8 and 8×1 is totally irrelevant in the Bég-Zee model, but their relative importance in the LPZ model means a difference in enhancement of orders of magnitude, F_{LPZ}/F_C.

The LPZ model is particularly simple to analyze because it employs currents of only one chirality. Hence, the products of currents exhibit a $V \leftrightarrow A$ symmetry which implies at most two distinct tensors in the interactions. A model which employs currents of both chirality and exhibits an additional $V \leftrightarrow A$ antisymmetric tensor in the sum rule is the Georgi-Glashow model. The Georgi-Glashow model employs no neutral current at all; its charged current is

$$J_\mu^+ = |J_\mu^+|_{\text{LPZ}} + N_{R2}^3 - \Pi_{R1}^{-2} + \Lambda_{R1}^3 - K_{R2}^{-1} \ . \tag{24}$$

As the reference to $|J^+|_{\text{LPZ}}$ indicates, only the last four terms distinguish it from the LPZ charged current. The effective nonleptonic Hamiltonian of the model is simply

$$H_{\text{eff}}^W = (e^2/M_W^2)|J_\mu^+ J_\mu^- + J_\mu^- J_\mu^+| \ . \tag{25}$$

After the singlet contribution to the product of currents has been extracted, the parity-violating part of the Hamiltonian derives from the terms

$$\begin{aligned} H_W^{\text{pv}} = \frac{1}{24}\frac{e^2}{M_W^2}\{ &(\sin^2\alpha_n - 1)N_L P_L + 3(1-\sin^2\alpha_\lambda)\Pi_L^- \Pi_L^+ + 3\sin^2\alpha_\lambda K_L^- K_L^+ \\ &- 3\sin^2\alpha_\lambda \Lambda_L P_L + 2\cos(\alpha_n - \alpha_\lambda)|3K_L^0 \bar{K}_L^0 - N_L\Lambda_L| \\ &- \cos\alpha_n \sin\alpha_\lambda |P_L \bar{K}_L^0 + P_L K_L^0 - 3(\Pi_L^- K_L^+ + K_L^- \Pi_L^+)| + 6(\cos\alpha_n - \sin\alpha_\lambda)\mathfrak{N}_{Lj}^i \lambda_{Ri}^j + (\text{i.c.})\} \ , \end{aligned} \tag{26}$$

where (i.c.) indicates additional terms obtained by interchanging the order of the currents in the equation. All terms except the last have been Fierz-transformed to facilitate the decomposition of the Hamiltonian into the T and S tensors introduced in Table I. The last term

$$6(\cos\alpha_n - \sin\alpha_\lambda)(\mathfrak{N}_{Lj}^i \lambda_{Ri}^j + \lambda_{Rj}^i \mathfrak{N}_{Li}^j)$$

is antisymmetric under $V \leftrightarrow A$, and Fierz transformations do not relate it to S or T tensors. The term, therefore, necessitates the introduction of another tensor. We denote its reduced matrix by a.

With this additional reduced matrix element, the sum rule predicted by the Georgi-Glashow-model Hamiltonian of Eq. (26) is

$$F_{\text{GG}} = \frac{\{[15\sin^2\alpha_\lambda - 22\cos(\alpha_n - \alpha_\lambda)]t + [3\sin^2\alpha_\lambda - 2\cos(\alpha_n - \alpha_\lambda)]s + 12(\cos\alpha_n - \sin\alpha_\lambda)a\}}{\cos\alpha_n \sin\alpha_\lambda(11t+s)} \ . \tag{27}$$

With the values of α_n and α_λ employed earlier, the enhancement is

$$\frac{F_{\text{GG}}}{F_C} = -\frac{1}{\sin^2\theta_C}\left(2 - \frac{1+9\alpha}{11+\sigma}\right) , \tag{28}$$

where α and σ are ratios, $\alpha = at^{-1}$ and $\sigma = st^{-1}$, respectively. If the $V \leftrightarrow A$ antisymmetric character of a tensor indicates that its reduced matrix elements are negligible compared with the corresponding symmetric tensors, then the Georgi-Glashow model predicts an enhancement by a factor of 20 for almost any value of σ. If antisymmetric tensors produce non-negligible effects, any enhancement might result. Regardless of the magnitude, an enhancement is, as conjectured by Bailin *et al.*, as likely a consequence of the Georgi-Glashow model as of the LPZ model.

As the sum rules of Eqs. (22) and (27) show, the enhancements occurring in the LPZ and Georgi-Glashow models are consequences of the charged currents. On the other hand, the charged currents of the Bég-Zee model imply a Hamiltonian whose structure resembles that of the Cabibbo model. Hence, the charged currents produce none of the enhancement. The neutral current, exhibiting the coupling of hadrons to neutrinos which the other models were designed to exclude, is solely responsible for the enhancement, and its magnitude is unaffected by the relative strength of 8×1 or 8×8 octet tensors.

The Bég-Zee model, as well as the other two three-triplet models, effects an enhancement relative to the Cabibbo model which is comparable with the $(\sin^2\theta)^{-1}$ enhancement predicted by the d'Espagnat model and has similarly important experimental consequences.

As originally remarked by Danilov[8] and substantiated under numerous wider assumptions by others,[9] the circular polarization of photons emitted in thermal-neutron capture is predominantly a consequence of the isoscalar part of the parity-violating weak interaction, whereas the angular asymmetry results from the isovector part. Tadić[10] has approximately estimated that an enhancement of the isovector part by a factor of 10 implies an asymmetry comparable with the circular polarization. Hence, both observables, instead of only the already measured[11] circular polarization, would be measurable with present techniques.

An enhancement of the isovector component would also increase the observability of the $\Delta I = 1$ parity-violating decay of the 3.562-MeV excited state of lithium in the process

$$\alpha + d \rightarrow {}^6\text{Li}^* \rightarrow {}^6\text{Li} + \gamma \ .$$

This process was originally studied by Wilkinson[12] as a candidate for detecting parity violation in nuclear processes, but was undetectable at that time. Recently, Fiorini *et al.*[13] have undertaken a precise determination of the lifetime with modern techniques.

The enhancement predicted by any of the three-triplet models is insufficient to resolve the discrepancy of two orders of magnitude between theoretical predictions and experimental measurements in heavy nuclei.[14] However, the detection of an angular asymmetry comparable with the circular polarization in thermal-neutron capture, as well as larger resonance than expected in the decay process of lithium, would be meaningful substantiation of unified theories of weak and electromagnetic interactions.

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APPENDIX

Nine quarks of three-triplet gauge models transforming as a direct $SU(3)\times SU(3)'$ product can be understood to belong to the product representation; for example,

$$p_1 = \chi_1 \psi_p \ , \tag{A1}$$

where

$$\chi = \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix} \tag{A2}$$

and

$$\psi = \begin{pmatrix} \psi_p \\ \psi_n \\ \psi_\lambda \end{pmatrix} \tag{A3}$$

are the triplet representation of $SU(3)'$ and $SU(3)$, respectively.

The separability assumption (A1), which seems to be acceptable as long as all physical interactions are color scalars, is not needed for the derivation of the relation (19), for which invariance under Fierz transformations alone is sufficient.

In general, a current density B^i_j transforms as a color octet M^i_j and as a color singlet B^k_k

$$B^i_j = M^i_j + \tfrac{1}{3}\delta_{ij} B^k_k \ . \tag{A4}$$

It means that a term of the type $B^1_1 C^1_1$, for example, contains two color singlets

$$B^1_1 C^1_1 = -\frac{\sqrt{2}}{6} T^S + \tfrac{1}{9}\hat{T}^S + (\text{nonsinglets}) \ ,$$

$$T^S = -\frac{1}{2\sqrt{2}} M^i_j N^j_i \ , \tag{A5a}$$

$$\hat{T}^S = B^k_k C^l_l \ . \tag{A5b}$$

Terms such as $N^i_j N^j_i$ will further contain two real SU(3) octets, obtained by the reductions

$$8\times 8 \rightarrow 8 \oplus \cdots \Rightarrow T^8 \ , \tag{A6a}$$

and

$$8\times 1 \rightarrow 8 \oplus \cdots \Rightarrow \hat{T}^8 \ , \tag{A6b}$$

respectively.

If the separable representation (A1) is assumed, the correspondence with tensors given in Table I is

$$\begin{array}{ll} O^8 & -2\sqrt{2}\, T^S T^8 \ , \\ Q_8 & -2\sqrt{2}\, T^S \hat{T}^8 \ , \\ T^8 & \hat{T}^S T^8 \ , \\ S^8 & \hat{T}^S \hat{T}^8 \ . \end{array} \tag{A7}$$

In Ref. 1 the FT was used to express everything in the color scalar T^S [(A6b)] by transforming

$$(\bar{\psi}_a^i \psi_j^b)(\psi_C^j \psi_i^d) \overset{\mathrm{FT}}{\to} (\bar{\psi}_a^i \psi_i^d)(\bar{\psi}_C^j \psi_j^b) .$$

Here i, j and a, b, c, d are SU(3)′ and SU(3) indices, respectively, while the time-space structure has to be understood.

Invariance under Fierz transformations obviously means a property of the theory beyond the mere SU(3) or SU(3)′ transformation properties, so it must result in relations among invariant tensors. They are simply found by equating the reduced expression before and after FT.

If, in addition, the separability assumption (A1) is used, all four tensors from (A8) can be connected. For example,

$$N_j^i N_i^j \to (-2\sqrt{2}\, T^S + \tfrac{1}{3}\hat{T}^S)(\bar{\psi}_n \psi_n)$$

without FT and

$$N_j^i N_i^j \to \hat{T}^S(\bar{\psi}_n \psi_n)$$

after FT, thus implying

$$\hat{T}^S = -3\sqrt{2}\, T^S . \tag{A8}$$

Similarly, if one studies the expression

$$(\bar{p}^1 n_1)(\bar{n}^1 p_1) \to (B_1^1 B_1^1)(\bar{\psi}_p \psi_n)(\bar{\psi}_n \psi_p) \overset{\mathrm{FT}}{\to} (B_1^1 B_1^1)(\bar{\psi}_p \psi_p)(\bar{\psi}_n \psi_n) ,$$

in which the FT leaves the color part invariant, one can, after the real SU(3) reduction, establish the relation

$$\hat{T}^8 = \frac{15}{\sqrt{30}} T^8 . \tag{A9}$$

Thus all tensors in (A7) are connectible, as was used in discussing formula (23).

It is possible to show that relations of the type (A9) hold for SU(3) tensors of other dimensions, i.e., SU(3) singlets and 27-plets.

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*Present address: Carnegie-Mellon University, Pittsburgh, Pennsylvania, 15213.

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