PHYSICAL REVIEW D VOLUME 11, NUMBER 5 1 MARCH 1975

Resonant partial-wave amplitudes in $\pi N \rightarrow \pi\pi N$ according to the naive quark-pair-creation model

A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal

Laboratoire de Physique Théorique et Hautes Energies, Université de Paris Sud, Bâtiment 211, 91405 Orsay, France*

(Received 23 April 1974)

We make an extensive comparison of the naive quark-pair-creation model of strong-interaction vertices for $\pi N \rightarrow N^* \rightarrow \Delta\pi$, ρN, $\sigma N \rightarrow \pi\pi N$ with recent experimental results. The model describes any vertex in terms of a quark-pair creation in the ${}^3P_0$ state and a rearrangement leading to final states. The hadron states are given by SU(6) harmonic-oscillator wave functions. We show that the model correctly describes, without any additional assumption, the centrifugal barrier effect and also the "anti-SU(6)$_W$" coupling signs, features that are present in most reactions (with the important exception of $FP_{15} \rightarrow \Delta\pi$ for the second feature) and are included phenomenologically in the works of Rosner *et al.* Both facts are related to the presence, in the limit of elementary meson emission, of recoil terms depending on internal quark momenta. Contrary to other works, the model predicts the relative coupling signs for all the two-body baryonic decays leading to $\pi\pi N$. Concerning $\Delta\pi$ and ρN channels, among 18 predictions, 15 agree with experiment, comparing our results to the last report of the Particle Data Group, and three disagree. One of these three is the very serious failure $FP_{15} \rightarrow \Delta\pi$, which is among the best determined experimental coupling signs. We compare our predictions with parallel works. Unlike the model of Feynman, Kislinger, and Ravndal, in the limit of elementary ρ emission, our model presents a recoil term in the spin part of the interaction, $\vec{\sigma}(i) \cdot [(\vec{k}_\rho - \vec{k}_i) \times \vec{\epsilon}_\rho]$. Thus, for $FP_{35} \rightarrow N\rho_3$, our model predicts an "anti-SU(6)$_W$" sign, in agreement with experiment and contrary to the prediction of Moorhouse and Parsons. For the controversial σ production, the model is contradicted by experiment; we show, however, that the model could be compatible with experiment if the $I = 0$, $J^P = 0^+$ dipion observed is not really a resonance.

I. INTRODUCTION

Important progress has been made recently in the knowledge of hadronic vertices. Strong meson and baryon decays have been submitted to extensive experimental studies. A number of experimental results concerning $\pi N \rightarrow \pi\pi N$ have recently been published[1] [we will refer to them as experiment I]; not only the coupling strength but also the coupling signs of vertices $N^*\pi\Delta$, $N^*\rho N$, and $N^*\sigma N$ are given. Such signs are very discriminative tests of any model trying to describe hadronic vertices.

Experiment has ruled out the former SU(6)$_W$ vertex symmetry for two main reasons: (i) When the same decay channel implies two different partial waves, experiment shows in general a damping of the highest wave, presumably due to a centrifugal barrier effect; (ii) the relative sign of the two partial waves seems in most cases to be the opposite of the one predicted by SU(6)$_W$. This fact has been called an "anti-SU(6)$_W$" relative sign.[2]

Stimulated mainly by Rosner, a great deal of theoretical work has been made in the frame of the SU(6)$\otimes$O(3) classification of hadrons to overcome the failure of SU(6)$_W$. Various models have been proposed. Petersen and Rosner,[2] Faiman and Plane,[3] and Faiman and Rosner[2] have developed the "l-broken" SU(6)$_W$ model of hadronic vertices. Moorhouse and Parsons[4] have used the elementary-meson-emission quark model of Feynman, Kislinger, and Ravndal (FKR).[5] The "naive" quark-pair creation model (QPCM) of Micu[6] has been developed by us[7] with the help of SU(6) harmonic-oscillator wave functions and an explicit ${}^3P_0$ quark-pair creation operator. It has the advantage of making definite predictions for all hadron vertices. It includes automatically the centrifugal barrier effect. An algebraic approach based on PCAC (partially conserved axial-vector current) and the Melosh transformation from constituent to current quarks[8] has been used by Gilman, Kugler, and Meshkov.[9]

All these models have some common features related to the SU(6)$\otimes$O(3) classification, namely, the recoupling coefficients of unitary spin, quark spin, and quark orbital angular momenta. But they differ in the detailed dynamical description. These finer features of various models appear clearly in the predictions for different partial-wave decay amplitudes.

Faiman and Rosner[2] have confronted their model with the recent experimental analysis (I) of $\pi N \rightarrow \pi\Delta$. The results concerning $\pi N \rightarrow \pi\Delta, \rho N$ have been studied by Moorhouse and Parsons.[4] Our model was confronted to partial data on $\pi N \rightarrow \pi\Delta \rightarrow \pi\pi N$.[10] The Melosh approach has been compared to experiment I on $\pi N \rightarrow \pi\Delta$.[9]

Experiment I seems to contradict not only specific predictions of models, but also some common predictions of the whole class of models based on the $SU(6)\otimes O(3)$ classification. At the Aix-en-Provence Conference of 1973, Butterworth[11] reported a new solution fitting the experimental data and which agrees much better with the common features of all classes of models. We will refer to this solution as II. Recently, some new results have been published by the Particle Data Group,[12] to which we will refer as R.

In the present paper we intend to make a systematic comparison between these last experimental results and QPCM predictions concerning coupling signs and coupling strengths. The model predicts the relative signs for all $\pi N \to \pi\pi N$ channels, contrary to the above-mentioned models, which cannot predict, for instance, a relative sign of $\pi N \to \pi\Delta$ and $\pi N \to \rho N$. We also make a study of $N^* \to \sigma N$ which, as far as we know, has not yet been done.

In Sec. II we recall the basis of the model and we write the general form of the matrix elements. In Sec. III we derive the centrifugal barrier and the anti-$SU(6)_W$ relative signs. In Sec. IV we formulate selection rules for the couplings $N^*\sigma N$. In Sec. V we compare our predictions to experiment. Section VI is devoted to a comparison of our model with parallel works.

II. GENERAL FORM OF THE MATRIX ELEMENTS

The principles of the model have been described in detail elsewhere.[7] An intuitive picture underlying it is given by the diagram of Fig. 1. The spectator quarks are supposed not to change their SU(3) quantum numbers, nor their momentum and spin. The "created" pair $q_4\bar{q}_5$ must be therefore in a $^3P_0$ $(C=+1)$, SU(3) singlet state of null momentum $\vec{k}_4+\vec{k}_5=0$.

Then, defining the $\hat{R}$ operator

$$\langle B, M|\hat{T}|A\rangle = 2\pi\delta(E_A - E_B - E_M)\langle B, M|\hat{R}|A\rangle, \quad (2.1)$$

we write

$$\hat{R} = \sum_{i,j} \int d\vec{k}_q\, d\vec{k}_{\bar{q}} \left\{ \gamma\,\delta^{(3)}(\vec{k}_q + \vec{k}_{\bar{q}}) \sum_m C_{11}(0,0;m,-m)\mathcal{Y}_1^m(\vec{k}_q - \vec{k}_{\bar{q}})(\chi_1^{-m}\phi_0)_{ij} \right\} a_i^\dagger(\vec{k}_q) b_j^\dagger(\vec{k}_{\bar{q}}) , \quad (2.2)$$

where i,j are SU(6) indices, $a^\dagger$ and $b^\dagger$ are creation operators of quarks and antiquarks, ϕ_0 is for an SU(3) singlet, χ_1^{-m} is a triplet state of spin, $\mathcal{Y}_1^m$ reflects the $L=1$ angular momentum of the pair, and γ is a dimensionless constant. We take the matrix elements of $\hat{R}$ between the SU(6) harmonic-oscillator quark wave functions of hadrons A and B, M.

Now, let us get the general form of the matrix element for $A \to B+M$, where A and B are baryons and M is a meson. We shall use the notations J_X, L_X, S_X, I_X $(X=A, B, M, P)$ for the spin, internal quark orbital momentum, total quark spin, and isospin of the hadron (A, B, M) or of the pair (P), with the coupling $J_X = L_X + S_X$. The partial decay amplitudes $M(l, s)$ for the decay $A \to B+M$ are defined with the following conventions for adding angular momenta and isospins

$$\begin{aligned} &\vec{J}_B + \vec{J}_M = \vec{s} ,\\ &\vec{l} + \vec{s} = \vec{J}_A , \\ &\vec{I}_B + \vec{I}_M = \vec{I} = \vec{I}_A . \end{aligned} \quad (2.3)$$

l is the orbital angular momentum between M and B. Then, these amplitudes are expressed in terms of spatial, spin, and isospin reduced matrix elements by

$$\begin{aligned} M(l,s) = \mathcal{J}^{(I_A;I_B I_M)} \sum_{L, L_f, S} &\mathcal{L}_{L L_f; l}^{(L_A;L_B L_M)} \mathcal{S}_S^{(S_A;S_B S_M)} (-1)^{L_A+S+J_A+S_P} [\tfrac{1}{3}(2L+1)(2S+1)]^{1/2} \begin{Bmatrix} L_A & S_A & J_A \\ S & L & L_P \end{Bmatrix} \\ &\times (-1)^{S+J_A-l-L_f} [(2J_B+1)(2J_M+1)(2L_f+1)(2L+1)(2S+1)(2s+1)]^{1/2} \\ &\times \begin{Bmatrix} L_f & l & L \\ J_A & S & s \end{Bmatrix} \begin{Bmatrix} L_B & S_B & J_B \\ L_M & S_M & J_M \\ L_f & S & s \end{Bmatrix} . \end{aligned} \quad (2.4)$$

The reduced spatial matrix elements $\mathcal{L}_{L L_f; l}^{(L_A; L_B L_M)}$ are defined from the spatial integral

$$\begin{aligned} I_{M_A m_p; M_B m_M}^{(L_A; L_B L_M)}(\vec{k}_M) = 3\gamma \int d\vec{k}_1 \cdots d\vec{k}_5\, &\psi_B^{M_B *}(\vec{k}_1, \vec{k}_2, \vec{k}_4)\delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_4 + \vec{k}_M) \\ &\times \psi_M^{m_M *}(\vec{k}_3, \vec{k}_5)\delta(\vec{k}_3 + \vec{k}_5 - \vec{k}_M)\mathcal{Y}_1^m(\vec{k}_4 - \vec{k}_5)\psi_A^{M_A}(\vec{k}_1, \vec{k}_2, \vec{k}_3)\delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \end{aligned} \quad (2.5)$$

by

$$I^{(L_A;\,L_B L_M)}_{M_A m_p;\,M_B m_m}(\vec{k}_M)=\sum_{L L_f l}\mathfrak{L}^{(L_A,\,L_B L_M)}_{L,\,L_f;\,l}(|\vec{k}_M|)\sum_{M M_f m}C_{L_A L_P}(LM;M_A m_P)C_{L_B L_M}(L_f M_f;M_B m_M)C_{l L_f}(LM;mM_f)Y_l^m(\hat{k}_M)\tag{2.6}$$

corresponding to the couplings

$$\vec{L}_A+\vec{L}_P=\vec{l},\quad \vec{L}_B+\vec{L}_M=\vec{L}_f,\quad \vec{l}+\vec{L}_f=\vec{L}\,,\tag{2.7}$$

and the spin and isospin reduced matrix elements $\mathcal{S}_S^{(S_A;\,S_B S_M)}$ and $\mathcal{G}^{(I_A;\,I_B I_M)}$ are defined by

$$\langle\chi_B^{\mu_B}(124)\chi_M^{\mu_M}(35)|\chi_A^{\mu_A}(123)\chi_P^{\mu_P}(45)\rangle=\sum_{S,\mu}\mathcal{S}_S^{(S_A;\,S_B S_M)}C_{S_A s_P}(S\mu;\mu_A\mu_P)C_{S_B S_M}(S\mu;\mu_B\mu_M)\tag{2.8}$$

and

$$\langle\phi_B^{i_B}(124)\phi_M^{i_M}(35)|\phi_A^{i_A}(123)\phi_0(45)\rangle=\mathcal{G}^{(I_A;\,I_B I_M)}\,C_{I_B I_M}(I_A i_A;\,i_B i_M)\,,\tag{2.9}$$

where χ and ϕ are spin and isospin wave functions. This corresponds to the couplings

$$\begin{aligned}&\vec{S}_A+\vec{S}_P=\vec{S}\,,\\&\vec{S}_B+\vec{S}_M=\vec{S}\\&\vec{I}_A+\vec{I}_P=\vec{I}=\vec{I}_A,\qquad(\vec{I}_P\equiv 0)\,,\\&\vec{I}_B+\vec{I}_M=\vec{I}=\vec{I}_A\,.\end{aligned}\tag{2.10}$$

Denoting by S_{12} the spin of the diquark (q_1q_2) (which must be the same in A and B) we can write an expression for the spin matrix elements

$$\mathcal{S}_S^{(S_A;\,S_B S_M)}=[(2S_B+1)(2S_M+1)(2S_A+1)(2S_p+1)]^{1/2}\times\begin{Bmatrix}S_{12}&\frac{1}{2}&S_B\\ \frac{1}{2}&\frac{1}{2}&S_M\\ S_A&S_P&S\end{Bmatrix}\,,\tag{2.11}$$

and for the isospin, taking into account $\vec{I}_P=0$, we get analogously

$$\mathcal{G}^{(I_A;\,I_B I_M)}=(-1)^{I_{12}+I_M+I_A+1/2}\times[\tfrac{1}{2}(2I_M+1)(2I_B+1)]^{1/2}\times\begin{Bmatrix}I_{12}&I_B&\frac{1}{2}\\ I_M&\frac{1}{2}&I_A\end{Bmatrix}\tag{2.12}$$

In formula (2.4), we have omitted for simplicity an additional summation due to the fact that the hadron wave functions are not of the simple form $\phi(\chi_S\psi_L)_J$ but are linear combinations of such products.

Let us now consider the case $L_B=L_M=0$, i.e., the baryon B is N or Δ, the meson M is π or ρ. Formula (2.4) reduces to

$$M(l,s)=\mathcal{G}^{(I_A;\,I_B I_M)}\mathfrak{L}^{(L_A;0,0)}_{l,0;\,l}\mathcal{S}_s^{(S_A;\,S_B S_M)}\times(-1)^{L_A+J_A+s+1}[\tfrac{1}{3}(2l+1)(2s+1)]^{1/2}\times\begin{Bmatrix}L_A&S_A&J_A\\ s&l&1\end{Bmatrix}\,,\tag{2.13}$$

and formula (2.6) reduces to

$$I^{(L_A;0,0)}_{M_A m_P;00}(\vec{k}_M)=\sum_l\mathfrak{L}^{(L_A;00)}_{l0;\,l}C_{L_A L_P}(l0;M_A m_P)\times Y_l^0(\hat{k}_M)\,,\tag{2.14}$$

where we take $\hat{k}_M$ as the axis of quantization.

III. SOME SPECIFIC PREDICTIONS OF THE MODEL: CENTRIFUGAL BARRIER AND ANTI-SU(6)$_W$ SIGNS

For a resonance A of a supermultiplet of quark orbital angular momentum L_A, decaying into $\underline{35}(L_M=0)+\underline{56}(L_B=0)$, we have, in general, two partial waves allowed, $l=L_A\pm 1$. The corresponding spatial matrix elements are given by [from (2.14) and the Appendix]

$$\mathfrak{L}^{(L_A;00)}_{L_A+1,0;\,L_A+1}=\frac{(+K)}{(2L_A+3)^{1/2}}(-xy)\vec{k}_M^{\,2}\times C_{L_A L_P}(L_A+1,0;00)\,,\tag{3.1}$$

$$\mathfrak{L}^{(L_A;00)}_{L_A-1,0;\,L_A-1}=\frac{(+K)}{(2L_A-1)^{1/2}}\left(\frac{2L_A+1}{\rho^2}-xy\vec{k}_M^{\,2}\right)\times C_{L_A L_P}(L_A-1,0;00)\,,\tag{3.2}$$

where

$$\begin{aligned}K&=3\gamma N_A N_B N_M 2^{L_A+1}\exp\left[-\vec{k}_M^{\,2}\,\frac{R_B^{\,2}(12R_A^{\,2}+5R_M^{\,2})}{24(3R_B^{\,2}+R_M^{\,2})}\right]\times\frac{[3(2L+1)]^{1/2}}{4\pi}\left(\frac{2\pi}{\rho^2}\right)^{3/2}y^{L_A-1}|\vec{k}_M|^{L_A-1}\,,\\ \rho^2&=3R_B^{\,2}+R_M^{\,2},\quad x=\frac{4R_B^{\,2}+R_M^{\,2}}{2\rho^2}\,,\\ y&=\frac{2R_B^{\,2}+R_M^{\,2}}{2\rho^2}\,.\end{aligned}\tag{3.3}$$

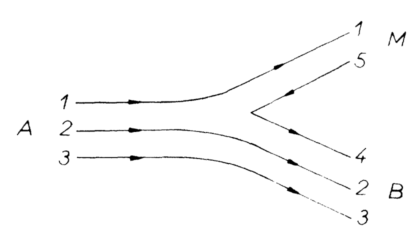

FIG. 1. Intuitive representation of the quark-pair creation model.

R_B and R_M are baryon and meson wave-function radii, and N_A, N_B, N_M are normalization constants.

Let us define

$$\mathcal{L}^+ = \mathcal{L}^{(L_A;00)}_{L_A+1,0;L_A+1}(2L_A+3)^{1/2} \times [C_{L_A L_p}(L_A+1, 0; 00)]^{-1} , \quad (3.4)$$

$$\mathcal{L}^- = \mathcal{L}^{(L_A;00)}_{L_A-1,0;L_A-1}(2L_A-1)^{1/2} \times [C_{L_A L_p}(L_A-1, 0; 00)]^{-1} . \quad (3.5)$$

Then, we obtain easily

$$\frac{\mathcal{L}^+}{\mathcal{L}^-} = \frac{-\vec{k}_M^{\,2}}{(2L_A+1)/xy\rho^2 - \vec{k}_M^{\,2}} . \quad (3.6)$$

The vertex symmetry $SU(6)_W$ predicts $\mathcal{L}^+/\mathcal{L}^- = +1$. This can be easily understood: $SU(6)_W$ is obtained when we neglect the quark movement in the directions transversal to the momentum $\vec{k}_M$. This is the case when the radii of the wave functions go to infinity (that is, by the uncertainty relation, no internal momenta for the quarks); then the first term in the denominator of (3.6) vanishes.

Now, we see that the model predicts the relative sign as anti-$SU(6)_W$ (that is, $\mathcal{L}^+/\mathcal{L}^-$ negative) if

$$\vec{k}_M^{\,2} < \frac{2L_A+1}{xy\rho^2} .$$

In fact, if we take realistic radii, that is $R_B^2 = 6$ GeV^{-2}, $R_M^2 = 8$ GeV^{-2},[13] then,

$$\frac{1}{xy\rho^2} = 0.163 \text{ GeV}^{-2}$$

and it happens that *in all reactions studied in experiment I we are in the case of anti-$SU(6)_W$ relative signs*. Let us emphasize that we predict this effect for $\pi\Delta$ as well as for ρN decays.

We also deduce easily from (3.6) and the values of the radii we have adopted *the centrifugal barrier effect*, since, when k_M^2 is small,

$$\frac{\mathcal{L}^+}{\mathcal{L}^-} \sim (rk_M)^2, \quad \text{with } r^2 = \frac{xy}{2L_A+1}\rho^2 . \quad (3.7)$$

Relations (3.1) and (3.2) imply also a cancellation of the lowest partial wave when

$$\vec{k}_M^{\,2} = \frac{2L_A+1}{xy}\rho^{-2} .$$

There is no clear experimental evidence for such an effect. One might be tempted to explain the experimental suppression of the wave FP_{35} (the notation is the standard one for partial wave; see for instance Ref. 12) by this effect, but one must then suppose R_B^2 and R_M^2 to be about twice as big as the adopted values.

As we have pointed out in Ref. 7, these phenomena which contradict $SU(6)_W$ can be phenomenologically described in terms of the presence of recoil terms (dependent on the internal quark momenta $\vec{k}_i$) in effective interactions corresponding to elementary meson emission. An example of these is the recoil term of Mitra and Ross in pion emission,[14] introduced through Galilean invariance,

$$\left[\vec{\sigma}(i)\cdot\left(\vec{k}_\pi - \frac{\omega_\pi}{m_q}\vec{k}_i\right)\right][\vec{\tau}(i)\cdot\vec{\pi}] . \quad (3.8)$$

We have shown in Ref. 7 that our model, in the limit of elementary meson emission, leads indeed to effective interactions of the forms for π emission,

$$[\vec{\sigma}(i)\cdot(\vec{k}_\pi - \vec{k}_i)][\vec{\tau}(i)\cdot\vec{\pi}] , \quad (3.9a)$$

and for ρ emission,

$$\{\tfrac{1}{2}(\vec{k}_\rho - \vec{k}_i)\cdot\vec{\epsilon}_\rho + \tfrac{1}{2}i\vec{\sigma}(i)\cdot[(\vec{k}_\rho - \vec{k}_i)\times\vec{\epsilon}_\rho)\}[\vec{\tau}(i)\cdot\vec{\pi}] . \quad (3.9b)$$

These limits of our model will be useful for the comparison with models of elementary meson emission, as we shall see in Sec. VI.

IV. SELECTION RULE FOR σ EMISSION

Let us consider the process $A \to B + \sigma$, B being N or Δ. Let l be the orbital angular momentum between B and σ, and J_A, P_A, and L_A the spin, parity, and relative quark angular momentum of the hadron A. l is determined by J_A and P_A according to the relations.

$$|J_A - J_B| \le l \le J_A + J_B, \quad (-1)^l = P_A . \quad (4.1)$$

On the other hand, as we shall show below, the QPCM, on the basis of $P_A = (-1)^{L_A}$, implies the rule

$$l = L_A . \quad (4.2)$$

Therefore, according as relations (4.1) and (4.2) are or are not in agreement, the decay $A \to B + \sigma$ will be allowed or forbidden.

If we look at the experimental results I, we see that in fact all observed $N\sigma$ channels are in agreement with (4.2) if we take the usually admitted values of L_A. For instance, the P_{11} resonances at 1470 and 1780 MeV ($L_A = 0$) decay in the PS_{11} wave ($l = 0$). On the other hand, the DF_{15} appears to be nonresonant, in agreement with the usually admitted $L_A = 1$ for the D_{15}. However, this absence could also be explained by a centrifugal barrier effect. Moreover, the DF_{15} suppression is the only one to be predicted for the well-established resonances. Therefore, experimental information about the $\Delta\sigma$ channel would be very interesting to test this selection rule. Besides, the success of this selection rule cannot be considered as very significant because, as we shall see in the following section, the quantitative predictions of the QPCM for σ emission are not in agreement with experiment. On the experimental side, the situation is controversial; the σ produced is essentially a dipion in the s state whose resonant part is not really isolated.

In spite of all these uncertainties, we will nevertheless derive the selection rule (4.2) since it has a theoretical interest in our model and may be useful for forthcoming experiments. Instead of deriving it from the machinery of the preceding section, we will give a more direct proof which shows how it depends on the angular structure of the model.

For the structure of the σ in the quark model, we take a $^3P_0$ state of a quark and an antiquark with null isospin.

Let us consider $\vec{S}_X, \vec{L}_X, \vec{J}_X$ ($X=A, B, M, P$), and $\vec{l}$ as operators acting in the space E of wave functions of four quarks and one antiquark (1, 2, 3, 4, 5). For example, $\vec{l}$ is the relative orbital angular momentum between (124) and (35). Within this space, the transition matrix element can be expressed as the scalar product of the initial state A plus the quark pair $^3P_0$ created by $\hat{T}$ and the final state $|B, M\rangle$. Let P_i be the projector on the subspace P_iE satisfying

$$\vec{J}_P = 0 \, , \qquad \vec{L}_A{}^2 = L_A(L_A+1) \, , \qquad \vec{l}_{AP} = 0 \tag{4.3}$$

($\vec{l}_{AP}$ is the orbital angular momentum between A and P). The system $|A, P\rangle$, i.e., the initial hadron A plus the $^3P_0\, q\bar{q}$ pair, belongs to P_iE. Similarly, let P_f be the projector on the subspace

$$\vec{J}_M = 0 \, , \qquad \vec{l}^{\,2} = l(l+1) \, , \qquad \vec{L}_B = 0 \, , \tag{4.4}$$

in which a typical state is the final state $|B, M\rangle$.

The transition will then be forbidden if $P_f P_i = 0$. Then, the selection rule will be proved if we have $P_f P_i = 0$ for $|l - L_A| \geq 2$ (the possible values of l being *a priori* $L_A, L_A \pm 2, \ldots$).

From the definitions of the projectors P_i and P_f, we have the relations

$$\vec{J}_P P_i = 0, \quad \vec{l}_{AP} P_i = 0 \, , \qquad P_f \vec{J}_M = 0, \quad P_f \vec{L}_B = 0 \, , \tag{4.5}$$

and from the identity

$$\vec{L}_P + \vec{L}_A + \vec{l}_{AP} \equiv \vec{L}_M + \vec{L}_B + \vec{l}$$

we get, using relations (4.5),

$$P_f(\vec{L}_A + \vec{S}_M)P_i = P_f(\vec{l} + \vec{S}_P)P_i \ .$$

According to

$$\vec{S}_M = \vec{s}_3 + \vec{s}_5, \quad \vec{S}_P = \vec{s}_4 + \vec{s}_5$$

($\vec{s}_i$ is the spin of a quark or an antiquark), we have finally

$$P_f(\vec{L}_A + \vec{s}_3)P_i = P_f(\vec{l} + \vec{s}_4)P_i \ . \tag{4.6}$$

$\vec{L}_A + \vec{s}_3 = \vec{l} + \vec{s}_4$ implies $|l - L_A| \leq 1$. So one could conclude directly that $P_f P_i = 0$ when $|l - L_A| \geq 2$ if P_i and P_f were commuting operators. However, this is not the case and the conclusion is thus not so straightforward. We have still to notice that $\vec{L}_A + \vec{s}_3$ commutes with P_i and $\vec{l} + \vec{s}_4$ with P_f. Then, from (4.6) we get[15]

$$P_f(\vec{L}_A + \vec{s}_3)^2 P_i = P_f(\vec{l} + \vec{s}_4)^2 P_i \ . \tag{4.7}$$

Decomposing P_iE and P_fE into eigenstates of $(\vec{L}_A + \vec{s}_3)^2$ and $(\vec{l} + \vec{s}_4)^2$, relation (4.7) implies that two such eigenstates are orthogonal unless they correspond to the same eigenvalue. But, under the condition $|L_A - l| \geq 2$ there can be no equal eigenvalues of $(\vec{L}_A + \vec{s}_3)^2$ and $(\vec{l} + \vec{s}_4)^2$.

Let us now quote another selection rule useful for the study of the transitions $P_{11} \to \sigma N$: a $\underline{70}$, $L_A = 0$ resonance cannot decay into σN (nor $\sigma\Delta$). We briefly indicate how it can be proved: With $\vec{L}_A = 0$ and $\vec{l} = 0$, relation (4.6) gives

$$P_f \vec{s}_3 P_i = P_f \vec{s}_4 P_i \ .$$

Making further P_i and P_f projections on states $\vec{I}_P = 0$ and $\vec{I}_M = 0$, we have also

$$P_f \vec{I}_3 P_i = P_f \vec{I}_4 P_i$$

From these two relations, we can deduce[15] the equality of the SU(6) generators and then the equality of the Casimir operators of A and B (taken between P_i and P_f), so that A and B must belong to the same representation of SU(6).

V. COMPARISON WITH EXPERIMENT

The quark-pair creation model predicts unambiguously the relative signs of the $\Delta\pi$, $N\rho$, and $N\sigma$ channels in $\pi N \to \pi\pi N$. Indeed, all the couplings depend on the pair creation constant γ which remains the only free parameter. The relative signs do not depend on the signs of the hadronic wave functions either, since all the wave functions appear twice (the wave function and its complex conjugate) in the calculation, except the pion one which appears three times in all the reactions. However, since the σ particle brings in some difficulties, we start discussing the results for $\Delta\pi$ and $N\rho$ channels, and make a separate discussion of σ production.

A. Coupling signs for $\Delta\pi$ and $N\rho$ channels

Let us consider the 13 coupling signs unambiguously determined by experiment II and corresponding to particles whose quark-model classification is well known (see Fig. 2). Then, 12 signs are in agreement with experiment and one is in contradiction with it. However, this disagreement is a serious one since it concerns the $FP_{15} \to \Delta\pi$, which is one of the most solid amplitude signs of the experimental analysis.

In (R) four signs appear which were not reported in II and which correspond to unambiguously classified resonances or to a classification-independent QPCM prediction ($SD_{11} \to \Delta\pi$). Of these, three agree with QPCM predictions ($FF_{15} \to \Delta\pi$, $DD_{33} \to \Delta\pi$, $SD_{11} \to \Delta\pi$) and one disagrees ($DS_{33} \to N\rho_3$). Now let us discuss in some detail the dubious cases.

$FF_{37} \to \Delta\pi$, $N\rho_3$. The Argand diagrams plotted in experiment I show the resonant point almost on the same horizontal line as the center of the Argand circle. So, it is difficult to say whether the coupling sign is plus or minus. These signs are usually tabulated as plus for $\Delta\pi$ and minus for $N\rho_3$ because of a large repulsive background, although the two Argand diagrams are quite similar. The QPCM predicts plus in both cases.

$PP_{33} \to \Delta\pi$. The resonance $P_{33}(1680)$ can be considered as an $L_q = 2$ excitation or as a radial excitation with $L_q = 0$. The experimental signs agree with $L_q = 0$.

$SS_{11}(1535)$ *and* $SS_{11}(1700) \to N\rho_1$. Both experimental coupling signs [the first one given in (R)] agree with a total quark spin $\frac{3}{2}$ for $SS_{11}(1535)$ and $\frac{1}{2}$ for $SS_{11}(1700)$. This implies a mixing angle near 90° with the usual notations. Faiman and Hendry,[16] fitting strong decays in an elementary pion emission model, find two solutions, $\theta = 35°$ and $\theta = 90°$. Copley, Karl, and Obryk,[17] studying pion photoproduction, found 70°, and Knies, Moorhouse, and Oberlack,[18] recently studying resonant photoproduction in the FKR model, concluded that there is a strong mixing. Our recent unpublished study of $S_{11}(1535)$ electroproduction comes to the same conclusion.

$DD_{13}(1520)$ *and* $DD_{13}(1700) \to \Delta\pi$; $DS_{13}(1520)$ *and* $DS_{13}(1700) \to N\rho_3$. These four coupling signs agree with experiment if one takes $S_q = \frac{1}{2}$ for $D_{13}(1520)$ and $S_q = \frac{3}{2}$ for $D_{13}(1700)$, that is to say, a small mixing angle. Faiman and Hendry[16] found two solutions: 35° and 127°, which are bigger than our solution. But Copley, Karl, and Obryk[17] as well as Knies, Moorhouse, and Oberlack[18] conclude, as we do, that the mixing angle is small. Our study of $D_{13}(1520)$ electroproduction comes also to the same conclusion.

B. Partial widths for $N\rho$ and $\Delta\pi$ decays

We start from the formula (nonrelativistic phase space)

$$\Gamma(l, s) = 2\pi \frac{E_B E_M k}{M_A} |M(l, s)|^2 , \qquad (5.1)$$

where $M(l, s)$ is the matrix element for a partial wave decay, introduced in Sec. II. Factorizing the quark-pair creation constant γ, $M(l, s) = \gamma\sqrt{3} A(l, s)$, where $A(l, s)$ is completely deter-

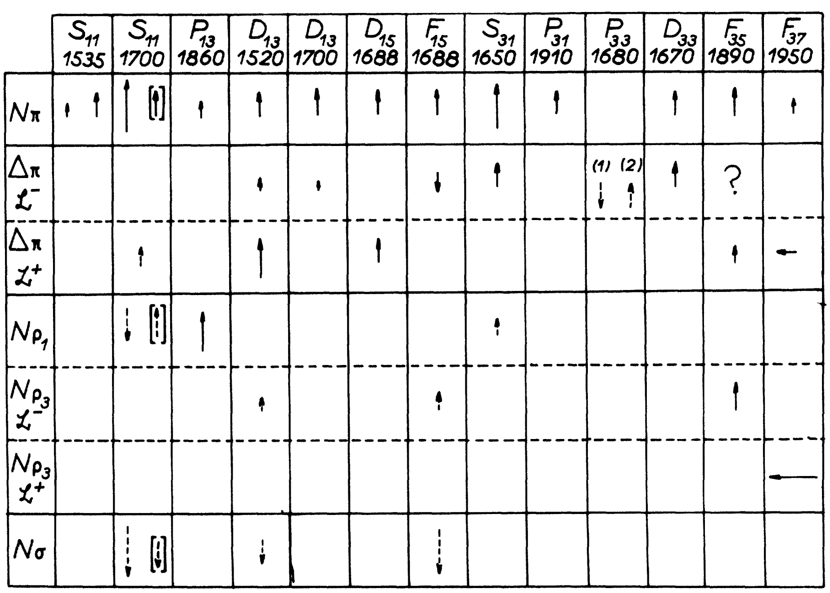

FIG. 2. Summary of comparison of the QPCM with experiment for various production processes. The arrows point upwards when predicted and experimental coupling signs agree, and downwards when they disagree. The length of the arrow is proportional to the ratio r. Dashed arrows correspond to productions below threshold (r infinite) or to unknown partial widths, the sign convention being the same as for finite r. For S_{11}, the bracketed arrows correspond to a mixing angle of 90°. For the dubious case (F_{37}), the arrows point horizontally.

mined by the model, we compute for all observed partial decay widths the ratio

$$\gamma' = \left(\frac{3\Gamma}{8\pi^2(E_B E_M k/M_A)|A|^2}\right)^{1/2} . \tag{5.2}$$

This ratio gives an estimate of $\gamma(3/4\pi)^{1/2}$. Note that there is no theoretical value of γ. We simply hope that γ' will not vary too much from one decay to another. Let us first consider $N\pi$ and $\Delta\pi$ channels.

Let us take the median of the γ' distribution, that is, $\gamma'_m = 1.45$, and define a "theoretical" partial width Γ_{th} from the formula (5.1), where we take γ'_m for γ. We then plot (Fig. 3) $r = \Gamma_{\text{exp}}/\Gamma_{\text{th}}$ for all partial widths in experiment II. Most values of r are inside the interval [0.38, 1.8]. Note that we calculated the "theoretical" width using the mixing angles and classifications determined in Sec. V A. Four waves fall outside the above-noted interval: These are $DS_{13}(1520) \to \Delta\pi$, $DS_{13}(1700) \to \Delta\pi$, and $DD_{13}(1520) \to \Delta\pi$, which contradict the centrifugal barrier effect, and $S_{31}(1650) \to N\pi$, whose ratio r is very large.[19]

We did not plot in Fig. 3 the new results in (R) because experimental uncertainties seem large in these cases. One can see in Table I that among seven such new widths, five fall inside the same interval and two outside: $SS_{11}(1535)$ and $SS_{11}(1700) \to \Delta\pi$.

If we extend the calculation to the partial widths of hadrons with $L_q = 0$, that is, to $\Delta \to N\pi$ and to $\rho \to \pi\pi$ [and SU(3)-related decays], we find that r falls between 1 and 1.5.

Concerning $N^* \to N\rho$, it happens that most of the observed reactions are below the ρ-production on threshold or just at threshold. Therefore, the predicted partial width would vanish if the ρ had zero width because of the kinematical factor k, and the ratio r would be infinite. Note that this problem has nothing to do with the specific features of the QPCM. Assuming that the integral of the phase space over the tail of the Breit-Wigner distribution is approximately constant, we compute $r' = rk^{1/2}$, which should be roughly constant. The five observed reactions give r' between 0.28 and 0.9. Once more, the DS_{13} partial width is smaller than expected. $FP_{15} \to N\rho_3$ is observed although the predicted partial width has a k^3 factor at threshold.

When ρ production happens above threshold (in three cases), the ratio r lies between 1.5 and 5, relatively larger than in $N\pi$ and $\Delta\pi$ channels. So, in all cases of ρ production, the experimental partial widths are larger than the predictions, but this may be due to phase-space effects when one goes beyond the narrow-width approximation.

Another positive test for the partial-decay width predictions of the model is the following: The amplitudes are observed when their computed values are large (see Table II). Defining

$$a = |A| \left(8\pi^2 \frac{kE_B E_M}{M_A}\right)^{1/2} , \tag{5.3}$$

we see that almost all observed amplitudes have $a > 0.04$. The only exceptions are the above-mentioned ρ-production reactions below threshold. All amplitudes with $a > 0.17$ are observed, with the exception $FP_{35} \to \Delta\pi$.

C. σ production

If we assume that the σ resonance is really produced and dominant in the reactions labeled as σ-resonance production in experiments I and II, the QPCM is in big trouble in this respect. Three decays are observed, all at threshold. The experimental coupling signs are opposite to the predictions of the model. The wave $PD_{13} \to N\sigma$ is not observed, contrary to $FD_{15} \to N\sigma$, although the model predicts the latter to be much smaller than the former, because there is a cancellation effect for $FD_{15} \to N\sigma$.

In fact, as has been noted by experimentalists in I, it is not at all sure that a σ resonance is observed, but there may be rather a nonresonant dipion with the σ quantum numbers. We make the following hypothesis: We assume that the decays occur through an intermediate baryonic resonance (not included in the phenomenological analysis I or II)

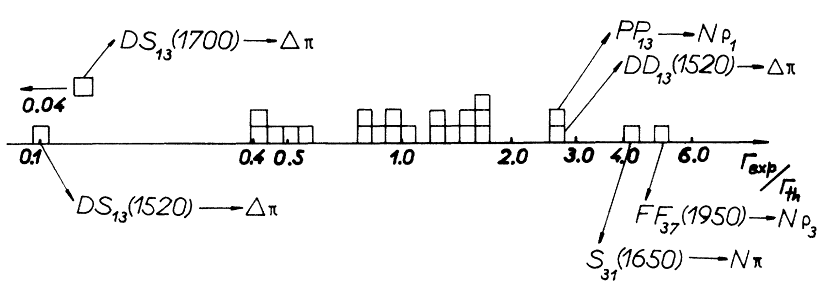

FIG. 3. We plot the ratio r on a logarithmic scale. Each square represents one reaction whose r is defined.

TABLE I. This table gives for each reaction the ratio r defined in Sec. V. If the sign preceding r is +, this means that experimental and theoretical coupling signs agree; − means that they do not agree. $a(N^* \to PD, N\pi)$ is defined from the amplitude by $a(N^* \to PD, N\pi) = Ak^{1/2}(E_B E_M 8\pi^2/MA)^{1/2}$. A is the amplitude $N^* \to DP$ with the total coupling sign at resonance. It includes the kinematical factors. The "additional coefficients" are defined in the Appendix. They are necessary to take into account the conventions, and the second-generation decay coupling sign. Experimental partial widths have been taken from Table III of Cashmore (Ref. 1), and signs have been taken from experiment II. Partial widths in parentheses have been taken elsewhere, often from the pole residue estimation (Table IV of first paper of Ref. 1). Signs in parentheses with subscripts R refer to new results in (R).

Reaction	S_q	$M(l,s)/\sqrt{3}$	Additional coefficients	$A(N^* \to PD)$ or $A(N^* \to N\pi)$	$a(N^* \to PD)$ or $a(N^* \to N\pi)$	Experimental sign $\times \Gamma^{1/2}$	r
$S_{11}(1535) \to N\pi$	$\frac{1}{2}$	$+0.13-0.26k^2$		+0.07	+0.24	+0.19	+0.8
	$\frac{3}{2}$	$+0.063-0.13k^2$		+0.035	+0.12		+1.6
$SD_{11} \to \Delta\pi$	$\frac{1}{2}$	$-0.37k^2$	+	−0.016	+0.030	$(-)_R$	$(+)_R$
$SS_{11} \to N\rho_1$	$\frac{1}{2}$	$-0.036+0.075k^2$	−	+0.036	+0.0	(0.10)	$(0.46/k^{1/2})$
	$\frac{3}{2}$	$+0.036-0.075k^2$	−	−0.036		(0.10)	$(+)_R$
$SD_{11} \to N\rho_3$	$\frac{1}{2}$	$-0.053k^2$	−	+0.0	+0.0		
$SP_{11} \to N\sigma$	$\frac{1}{2}$	$-0.085k$	−	+0.0	+0.0	$(-)_R(0.18)$	$(0.35/k^{3/2})(-)_R$
$S_{11}(1700) \to N\pi$	$\frac{3}{2}$	$+0.063-0.13k^2$		+0.02	+0.08	+0.27	+3.4
	$\frac{1}{2}$	$+0.13-0.26k^2$		+0.02	+0.16		+1.7
$SD_{11} \to \Delta\pi$	$\frac{3}{2}$	$-0.18k^2$	+	−0.03	−0.10		+
	$\frac{1}{2}$	$-0.37k^2$	+	−0.06	−0.20	−	+
$SS_{11} \to N\rho_1$	$\frac{3}{2}$	$+0.036-0.075k^2$	−	−0.036	−0.0	+0.10	$-0.48/k^{1/2}$
	$\frac{1}{2}$	$-0.036+0.075k^2$		+0.036	+0.0		$+0.48/k^{1/2}$
$SD_{11} \to N\rho_3$	$\frac{3}{2}$	$+0.053k^2$	−	−0.0	−0.0		
$SP_{11} \to N\sigma$	$\frac{3}{2}$	$-0.042k$	−	+0.0	+0.0	−0.34	$-1.4/k^{3/2}$
		$-0.085k$		+0.0	+0.0	−0.34	$-0.7/k^{3/2}$
$P_{13}(1860) \to N\pi$	$\frac{1}{2}$	$+0.019k-0.23k^3$		+0.055	+0.27	+0.25	+0.93
$PP_{13} \to \Delta\pi$	$\frac{1}{2}$	$+0.069k-0.085k^3$	+	+0.024	+0.10		
$PF_{13} \to \Delta\pi$	$\frac{1}{2}$	$+0.25k^3$	+	+0.037	+0.15		
$PP_{13} \to N\rho_1$	$\frac{1}{2}$	$-0.16k+0.19k^3$	+	−0.050	−0.18	−0.44	+2.4
$PP_{13} \to N\rho_3$	$\frac{1}{2}$	$-0.079k+0.098k^3$	+	−0.024	−0.08		
$PF_{13} \to N\rho_3$	$\frac{1}{2}$	$-0.29k^3$	+	−0.015	−0.06		
$PD_{13} \to N\sigma$	$\frac{1}{2}$	$-0.074k^2+0.065k^4$	+	−0.010	−0.04		
$D_{13}(1520) \to N\pi$	$\frac{1}{2}$	$-0.26k^2$		+0.055	+0.19	+0.26	+1.3
$DS_{13} \to \Delta\pi$		$-0.13+0.26k^2$	+	+0.11	+0.23	+0.12	+0.52
$DD_{13} \to \Delta\pi$		$-0.026k^2$	+	+0.017	+0.04	+0.095	+2.4
$DD_{13} \to N\rho_1$		$+0.075k^2$	−	+0.0	+0.0		
$DS_{13} \to N\rho_3$		$-0.18+0.37k^2$	−	−0.18	−0.0	−0.14	$+0.13/k^{1/2}$
$DD_{13} \to N\rho_3$		$-0.37k^2$	−	−0.0	−0.0		
$DP_{13} \to N\sigma$		$+0.042k$	−	+0.0		(0.055)	$(0.22/k^{3/2})$
$D_{13}(1700) \to N\pi$	$\frac{3}{2}$	$+0.041$		+0.0145	+0.060	+0.115	+1.9
$DS_{13} \to \Delta\pi$		$+0.20-0.40k^2$	+	+0.135	+0.42	+0.13	+0.31
$DD_{13} \to \Delta\pi$		$-0.33k^2$	+	−0.053	−0.16	$(-)_R(0.055)$	$(+)_R(0.34)$
$DD_{13} \to N\rho_1$		$+0.024k^2$	−	−0.0	−0.0		
$DS_{13} \to N\rho_3$		$-0.058+0.12k^2$	−	+0.058	+0.0	$(+)_R(0.55)$	$(+)_R(0.17/k^{1/2})$
$DD_{13} \to N\rho_3$		$-0.095k^2$	−	+0.0			
$DP_{13} \to N\sigma$		$-0.067k$	−	+0.0	+0.0	−0.32	$-0.8/k^{3/2}$
$D_{15}(1670) \to N\pi$	$\frac{3}{2}$	$+0.10k^2$		+0.03	+0.13	+0.24	+1.8
$DD_{15} \to \Delta\pi$		$-0.37k^2$	+	−0.048	−0.15	−0.27	+1.8
$DD_{15} \to N\rho_1$		$+0.058k^2$	−	−0.0	−0.0		
$DD_{15} \to N\rho_3$		$+0.11k^2$	−	−0.0	−0.0		
$F_{15}(1688) \to N\pi$	$\frac{1}{2}$	$-0.24k^3$		+0.045	+0.19	+0.28	+1.85
$FP_{15} \to \Delta\pi$		$+0.17k+0.21k^3$	+	−0.052	−0.16	+0.145	−0.9
$FF_{15} \to \Delta\pi$		$+0.17k^3$	+	−0.009	−0.035	$(-)_R$	$(+)_R$
$FF_{15} \to N\rho_1$		$+0.19k^3$	+	−0.0	−0.0		
$FP_{15} \to N\rho_3$		$-0.19k+0.24k^3$	+	+0.0	+0.0	+0.13	$+0.11/k^{3/2}$
$FF_{15} \to N\rho_3$		$-0.20k^3$	+	+0.0	+0.0		
$FD_{15} \to N\sigma$		$+0.0023k^2$	+	−0.0	−0.0	+0.12	$-\infty$

TABLE I. *(Continued)*

Reaction	S_q	$M(l,s)/\sqrt{3}$	Additional coefficients	$A(N^*\to PD)$ or $A(N^*\to N\pi)$	$a(N^*\to PD)$ or $a(N^*\to N\pi)$	Experimental sign $\times\Gamma^{1/2}$	r
$S_{31}(1650)\to N\pi$	$\frac{1}{2}$	$+0.045-0.91k^2$		+0.018	+0.075	+0.22	+2.9
$SD_{31}\to\Delta\pi$		$+0.41k^2$	−	−0.053	−0.16	−0.23	+1.4
$SS_{31}\to N\rho_1$		$-0.13+0.26k^2$	−	+0.13	+0.0	+0.22	$+0.28/k^{1/2}$
$SD_{31}\to N\rho_3$		$+0.15k^2$	+	+0.0	+0.0		
$P_{31}(1910)\to N\pi$	$\frac{3}{2}$	$+0.15k-0.19k^3$		+0.035	+0.18	+0.27	+1.5
$PP_{31}\to\Delta\pi$		$+0.086k-0.11k^3$	−	−0.030	−0.12		
$PP_{31}\to N\rho_1$		$+0.089k-0.11k^3$	+	+0.020	+0.07		
$PP_{31}\to N\rho_3$		$-0.063k+0.077k^3$	+	−0.020	−0.07		
$P_{33}(1680)\to N\pi$	$\frac{3}{2}$	$+0.11k-0.13k^3$		+0.038	+0.16		
	$L_q=2$						
	$L_q=0$	$+0.27k$		+0.14	+0.60		
$PP_{33}\to\Delta\pi$	$L_q=2$	$+0.15k-0.19k^3$	−	−0.045	−0.13	+	−
	$L_q=0$	$-0.49k$		+0.20	+0.58		+
$PF_{33}\to\Delta\pi$	$L_q=2$	$-0.14k^3$	−	+0.0065	+0.02		
$PP_{33}\to N\rho_1$	$L_q=2$	$+0.062k-0.077k^3$	+	+0.0	+0.0		
$PF_{33}\to N\rho_3$		$+0.10k^3$	+	+0.0	+0.0		
$D_{33}(1670)\to N\pi$	$\frac{1}{2}$	$-0.091k^2$		+0.028	+0.12	+0.17	+1.4
$DS_{33}\to\Delta\pi$		$+0.14-0.29k^2$	−	+0.105	+0.31	+0.415	+1.3
$DD_{33}\to\Delta\pi$		$+0.29k^2$	−	+0.035	+0.105	$(+)_R$	$(+)_R$
$DD_{33}\to N\rho_1$		$+0.26k^2$	−	+0.0	+0.0		
$DS_{33}\to N\rho_3$		$+0.05-0.11k^2$	−	+0.05	+0.0	$(-)_R(0.23)$	$(0.85/k^{1/2})(-)_R$
$F_{35}(1890)\to N\pi$	$\frac{3}{2}$	$+0.07k^3$		+0.025	+0.12	+0.20	+1.65
$FP_{35}\to\Delta\pi$		$+0.18k-0.22k^3$	−	−0.063	−0.24		0
$FF_{35}\to\Delta\pi$		$-0.20k^3$	−	+0.031	+0.12	+0.12	+1
$FF_{35}\to N\rho_1$		$+0.04k^3$	+	+0.0028	+0.009		
$FP_{35}\to N\rho_3$		$-0.13k+0.16k^3$	+	−0.053	−0.21	−0.37	+1.75
$FF_{35}\to N\rho_3$		$+0.15k^3$	+	+0.010	+0.04		
$F_{37}(1950)\to N\pi$	$\frac{3}{2}$	$+0.15k^3$		+0.061	+0.31	+0.29	+0.92
$FF_{37}\to\Delta\pi$		$-0.21k^3$	−	+0.041	+0.18	(+ ?)0.19	(+?)1.05
$FF_{37}\to N\rho_1$		$+0.09k^3$	+	+0.009	+0.036		
$FF_{37}\to N\rho_3$		$+0.15k^3$	+	+0.015	+0.06	(− ?)0.19	(− ?)3.2

$$N^*\to N^{*\prime}\pi\to N\pi\pi$$

rather than through a σ. We calculate the amplitude for such a reaction to give two pions with the σ quantum numbers. As $N^{*\prime}$ we take $P_{11}(1470)$, $S_{11}(1535)$, and $D_{13}(1520)$ [we assume $S_q=\frac{3}{2}$ for $S_{11}(1535)$]. We then easily see that these processes must theoretically dominate direct σ production, not only because of kinematical factors (see Table III), but essentially because the predicted amplitudes are bigger.

Nevertheless, the F_{15} amplitude still remains small as compared to the observed widths because of centrifugal barrier effects (other mechanisms, such as virtual nucleon exchange could explain such a partial width with a good coupling sign). As far as signs are concerned, for D_{13} and F_{15} we get signs in agreement with experiment, but for $S_{11}(1700)$ we still obtain a bad sign.

For P_{13}, the predicted sign is plus, and nothing explains why it is not observed. Note, however,

TABLE II. Number of observed and unobserved amplitudes as a function of a, where $a=a(N^*\to N\pi$ or $DP)$ defined in Table I.

	$a<0.04$	$0.04<a<0.1$	$0.1<a<0.17$	$0.17<a$	Total
Observed amplitudes	4 below ρ-production threshold	3	11	11	29
Unobserved amplitudes	27	6	4	1	38
Total	31	9	15	12	67

TABLE III. Production of ND, where D is a dipion with $I=0$, $S=0$, and comparison of amplitudes through σ-resonance production and through baryonic resonance. Recoupling coefficients are due to the fact that we take only $I=0$, $S=0$ dipions in all reactions. Additional coefficients are due to conventions and to the sign of the N^* production amplitude. A and a are defined in formulas (5.1) and (5.3).

Reaction	$M(N^* \to N'\pi, \sigma\pi)/\sqrt{3}$	$\epsilon(N' \to N\pi)$ $\epsilon(\sigma \to \pi\pi)$	Recoupling coefficients	Additional coefficients	$A(N^* \to N\pi\pi)$	Dominant sign	Experimental sign
$S_{11}(1700) \to S_{11}(1520) + \pi$	$+0.19k - 0.06k^3$	+	$-1/\sqrt{3}$	+	−		
$S_{11}(1700) \to D_{13}(1520) + \pi$	$+0.25k - 0.23k^3$	−	$-1/\sqrt{3}$	+	+		
$S_{11}(1700) \to P_{11}(1470) + \pi$	$+0.13 - 0.26k^2$	−	$-1/\sqrt{3}$	+	+	+	−
$S_{11}(1700) \to N\sigma$	$-0.085k$	−	$+1$	+	+		
$F_{15}(1688) \to S_{11}(1520) + \pi$	$+0.23k^2 + 0.046k^4$	+	$-1/\sqrt{3}$	+	+		
$\to D_{13}(1520) + \pi$	$D\ +0.05k^4$	−	$1/\sqrt{15}$	+	−		
	$G\ -0.12k^4$		$\sqrt{2}/3$		+	0+	+
$\to P_{11}(1520) + \pi$	$-0.23k^3$	−	$\sqrt{5}/3\sqrt{3}$	+	+		
$\to N\sigma$	$+0.0023k^2$	−	$+1$	+	−		
$D_{13}(1700) \to S_{11}(1520) + \pi$	$+0.13k + 0.01k^3$	+	$-1/\sqrt{3}$	+	−		
$\to D_{13}(1520) + \pi$	$+0.13k + 0.01k^3$	−	$1/\sqrt{30}$	+	−		
$\to P_{11}(1470) + \pi$	$+0.04k^2$	−	$1/\sqrt{6}$	+	−	−	−
$\to N\sigma$	$-0.06k$	−	$+1$	+	+		
$P_{13}(1860) \to S_{11}(1520) + \pi$	$-0.004k^2 + 0.016k^4$	+	$-1/\sqrt{3}$	−	−		
$\to D_{13}(1520) + \pi$	$S\ -0.07k^2 + 0.09k^4$	−	$-1/\sqrt{3}$	−	+		
	$D\ +0.07k^2 - 0.09k^4$	−			−	+	
$\to P_{11}(1520) + \pi$	$+0.19k - 0.23k^3$	−	$-\sqrt{5}/18$	−	+		
$\to N\sigma$	$-0.0 - 0.074k^2$	−	$+1$	−	−		

that the problem is general for all P_{13} decay channels, since the partial decays P_{13} are unobserved although the calculated decay widths are relatively large (third column of Table II).

We can tentatively conclude that the observed reactions do not occur through a σ resonance, but rather through a nonresonant dipion system with the same quantum numbers of the σ. However, the production mechanism of this dipion state is still an open problem.

VI. COMPARISON WITH OTHER WORKS

All the models which try to discuss systematically the strong-interaction vertices start from the $SU(6)\otimes O(3)$ classification of hadrons. But the description of the decay process itself is quite different in each model. First, there are models like the QPCM or "l-broken" $SU(6)_W$ of Rosner *et al.*[2,3] which treat directly the strong-interaction process, considering all the hadrons on the same footing. Second, there is the current-algebra approach of Melosh[8,9] which takes the path of replacing one meson by a current according to PCAC, for instance. The covariant quark model of Feynman, Kislinger, and Ravndal[5,4] lies somewhere between the two approaches. On the other hand, the Melosh approach and l-broken $SU(6)_W$ take a purely algebraic point of view, while the QPCM and the FKR model consider the internal motion of quarks inside the hadrons in a realistic way.

The different models will not be different at the level of the SU(6) vector coupling coefficients, but will differ in their predictions of the ratio between the independent partial waves, $\mathcal{L}^+/\mathcal{L}^-$, which consequently must be carefully discussed.[20]

A. The "l-broken" $SU(6)_W$ model of Faiman and Rosner

Faiman and Rosner have studied the reaction $\pi N\to N^*\to\pi\Delta$ in an algebraic approach, namely, the l-broken $SU(6)_W$ model.[2,3]

For strict $SU(6)_W$, the pion emission operator is such that $L_z=0$ between the initial and final baryons ($0z$ is the direction of the pion momentum). $SU(6)_W$ then implies $\mathcal{L}^+/\mathcal{L}^-=+1$ [see formula (3.6)]. Now, l-broken $SU(6)_W$ admits both $L_z=0$ and $L_z=\pm1$ transitions with an arbitrary amount of each [*a priori* different for each SU(6) supermultiplet], making the ratio $\mathcal{L}^+/\mathcal{L}^-$ arbitrary. The coupling sign will be the same as in the QPCM if $\mathcal{L}^+/\mathcal{L}^-$ is chosen to be negative, that is, anti-$SU(6)_W$ relative signs. If $\mathcal{L}^+/\mathcal{L}^-$ is chosen to be positive, the coupling signs will be different from those predicted by the QPCM.

The model of Faiman and Rosner was in contradiction with the former results of I for wave D_{13}. If one reverses the coupling signs below 1600 MeV, as was done in II, then the Faiman and Rosner conclusion is that one has anti-$SU(6)_W$ signs for the ($\underline{70}$, $L=1^-$) supermultiplet, since DS_{13}, SD_{31}, and DS_{33} agree with this sign. On the other hand, they choose $SU(6)_W$ signs for the supermultiplet ($\underline{56}$, $L=2^+$) since FP_{15} agrees with the $SU(6)_W$-like solution.[21] Such a conclusion is not contradictory with the general algebraic frame of the l-broken $SU(6)_W$ symmetry, although, in the quark model, such different behaviors for ($\underline{70}$, $L=1^-$) and ($\underline{56}$, $L=2^+$) seems puzzling. As we have seen in Sec. III, we get the same phenomenon of anti-$SU(6)_W$ signs for the resonances belonging to ($\underline{70}$, $L=1^-$) as well as for those belonging to ($\underline{56}$, $L=2^+$). The FP_{15} coupling sign, if it is confirmed, will be a serious drawback for the QPCM or it will imply reconsidering the classification of the F_{15} wave.

Recently[20] Faiman analyzed the results of $N\rho$ production and concluded that experimental signs agree with $SU(6)_W$. This seems violently contradictory with our conclusion since we found only two signs in contradiction with anti-$SU(6)_W$ (see Table IV). In order to show why we come to such different conclusions we have reported in Table V the respective predictions of the QPCM, $SU(6)_W$, the experimental results (R), and the experimental results reported by Faiman, who uses particular criteria in interpreting the signs of the amplitudes. Notice that the convention used in Faiman's paper (baryon first) and the one we use here are different. The signs to change conventions can be found in (R), page 118.

Faiman, using, contrary to us, the l-broken model, does not predict the relative signs of amplitudes $N\pi\to N^*\to\Delta\pi\to\pi\pi N$ to $N\pi\to N^*\to\rho N\to\pi\pi N$. So he can multiply all signs of the $N\rho$ channel by (-1) without changing the signs of $\Delta\pi$. In the QPCM this is impossible without getting wrong all $\Delta\pi$ signs. In fact, Faiman's predictions have such a (-1) coefficient multiplying all $N\rho$ channel as compared to our predictions. Now going from $SU(6)_W$ to anti-$SU(6)_W$ one multiplies by (-1) all waves whose initial and final orbital numbers are not the same ($FP, DS, PF, SD, \ldots$) and we do not change those whose orbital number are the same ($SS, PP, DD, FF, \ldots$). Then combining this sign with the over-all sign we mentioned above, we conclude that our predictions will differ from Faiman's in this second case ($SS, PP, \ldots$).

If we exclude the PP_{11} waves which we do not consider in our analysis [as compared to (R), one sign is compatible with Faiman's prediction and one with the QPCM's], there are five waves of the second case: both $SS_{11}\to N\rho_1$, $SS_{31}\to N\rho_1$, $PP_{13}\to N\rho_1$, and $FF_{37}\to N\rho_3$. SS_{11} predictions depend on the mixing angle. Experiment agrees with Faiman's prediction for an angle of 0° and with the QPCM's prediction for 90° (see Sec. V A). SS_{31} disagrees with Faiman's prediction as noticed in Ref. 20. PP_{13} disagrees also with Faiman's prediction if one takes the sign reported in (R), but it agrees if one uses Faiman's criteria for interpreting the experiment. FF_{37} disagrees with the QPCM. $DS_{33} \rightarrow N\rho_3$, as reported in (R) disagrees with both models, but following Faiman's criteria it agrees with both.

As a conclusion one can say that, as far as only the $N\rho$ channel is concerned, Faiman's predictions disagree with results of (R) in three cases, and the results of the QPCM disagree in two cases. So we can say that the $N\rho$ channel alone is not conclusive. We would like to stress once more here that one advantage of the QPCM is that it has no freedom to change an over-all sign with respect to the $\Delta\pi$ channel.

B. Algebraic approach based on the Melosh transformation

In this model, used by Gilman, Kugler, and Meshkov[9] to study the $N^* \rightarrow \Delta\pi$ couplings, pion emission is related via PCAC to the axial-charge Q_5^α matrix elements. The algebraic behavior of Q_5^α under the SU(6) group of the hadron classification is then abstracted from the quark model. More precisely, if the hadron states are described by the definite SU(3)⊗SU(3) representation of the quark-model classification, the axial charge is the sum of two terms, one behaving like $[(8,1)-(1,8)]$ and one like $[(3,\bar{3})-(\bar{3},3)]$. In terms of the quark model, the operator $[(8,1)-(1,8)]$ corresponds to $\Delta L_z = \Delta S_z = 0$, and $[(3,\bar{3})-(\bar{3},3)]$ to $\Delta L_z = \Delta S_z = \pm 1$. Strict $SU(6)_W$ is equivalent to the contribution of the term $[(8,1)-(1,8)]$ alone. As shown in Ref. 22 by Hey, Rosner, and Weyers, this model is indeed equivalent to the l-broken symmetry scheme, as long as we deal with pionic reactions. As in l-broken $SU(6)_W$, there are two arbitrary parameters for each irreducible representation of SU(6).

C. Calculations in the quark model of Feynman, Kislinger, and Ravndal

This model[5] has been used by Moorhouse and Parsons[4] to study the vertices $N^*\Delta\pi$ and $N^*N\rho$. In this model, the emission of a pion (or a ρ) is treated through PCAC (or vector-meson dominance) by the coupling of an axial-vector current (or a vector current) to the quarks. The mesons are then treated as elementary quanta. FKR have shown that, for pion emission, their model is equivalent to the interaction of Mitra and Ross[14]

$$\left[\vec{\sigma}(i)\cdot\left(\vec{k}_\pi - \frac{E_\pi}{m_q}\vec{k}_i\right)\right][\vec{\tau}(i)\cdot\vec{\pi}] \;. \tag{6.1}$$

Now, as we recalled in Sec. III, we have shown in a preceding paper[7] that, in the limit of elementary pion emission, the QPCM is also similar to the Mitra and Ross interaction. In this limit we have obtained indeed the coupling

$$[\vec{\sigma}(i)\cdot(\vec{k}_\pi - \vec{k}_i)][\vec{\tau}(i)\cdot\vec{\pi}] \tag{6.2}$$

(where $\vec{k}_i$ is the initial momentum of the emitting quark). $SU(6)_W$ is equivalent to the coupling without recoil term,

$$[\vec{\sigma}(i)\cdot\vec{k}_\pi][\vec{\tau}(i)\cdot\vec{\pi}] \tag{6.3}$$

So, we understand why Moorhouse and Parsons find, for $N^* \rightarrow \Delta\pi$ decays, anti-$SU(6)_W$ signs as in our model.

Now, what happens for ρ emission? Moorhouse and Parsons use a coupling which is equivalent to the nonrelativistic one for the electromagnetic interaction

$$-(\vec{k}_i - \tfrac{1}{2}\vec{k}_\rho)\cdot\vec{\epsilon}_\rho + i\,\tfrac{1}{2}\vec{\sigma}(i)\cdot(\vec{k}_\rho \times \vec{\epsilon}_\rho) \tag{6.4}$$

to be compared with the elementary ρ-emission limit of our model[7]

$$\tfrac{1}{2}(\vec{k}_\rho - \vec{k}_i)\cdot\vec{\epsilon}_\rho + i\,\tfrac{1}{2}\vec{\sigma}(i)\cdot[(\vec{k}_\rho - \vec{k}_i)\times\vec{\epsilon}_\rho] \;. \tag{6.5}$$

Both models disagree with $SU(6)_W$, which would correspond to[23]

$$\vec{k}_\rho\cdot\vec{\epsilon}_\rho + i\vec{\sigma}(i)(\vec{k}_\rho\times\vec{\epsilon}_\rho) \;. \tag{6.6}$$

However, they are not similar in the case of ρ emission because *the spin term* is different in our model: It contains a recoil term absent in the FKR model or $SU(6)_W$. Then both models will lead to different predictions in transitions $N^* \rightarrow N\rho$, where the total quark spin for the N^* is $S_q = \frac{3}{2}$ since then only the spin term of the interaction contributes.

Moorhouse and Parsons calculate the following resonance decays to $N\rho$: $DS_{13}(1520) \rightarrow N\rho_3$, $FP_{15} \rightarrow N\rho_3$, $FP_{35} \rightarrow N\rho_3$, $PP_{13} \rightarrow N\rho_1$, $FF_{37} \rightarrow N\rho_3$. In the case of $FP_{35} \rightarrow N\rho_3$, since the quark spin is $\frac{3}{2}$ in the initial state, only the spin term contributes.

TABLE IV. Comparison of predictions of QPCM and those of Moorhouse and Parsons with experiment II for ρ production.

Reaction	Moorhouse and Parsons	QPCM	Experiment II
$DS_{13}(1520) \rightarrow N\rho_3$	−	−	−
$FP_{15}(1688) \rightarrow N\rho_3$	+	+	+
$FP_{35}(1890) \rightarrow N\rho_3$	+	−	−
$PP_{13}(1860) \rightarrow N\rho_1$	−	−	−
$FF_{37}(1950) \rightarrow N\rho_3$	+	+	−?

Indeed, in these cases, Moorhouse and Parsons get SU(6)$_W$-like signs. The predicted sign is different [anti-SU(6)$_W$] in our model only for FP_{35}, since for FF_{37} the sign is fixed by spin and isospin vector coupling coefficients. For the other three resonances, both convective and spin terms come in. The convective term of interaction (6.4) (dependent on k_i) is bigger in magnitude than the spin term. Then, for these transitions one can expect anti-SU(6)$_W$ signs in the FKR model as well as in the QPCM, although the emission interaction is different. The predictions of Moorhouse and Parsons agree indeed with our model for these resonances.

In conclusion, we think that our model compares favorably with the model used by Moorhouse and Parsons since it solves the problem with the FP_{35} wave and leaves unexplained only the FF_{37} sign, which seems dubious on the experimental side.

VII. CONCLUSION

The naive quark-pair creation model describes completely the structure of baryon two-body decays with the help of a very simple hypothesis, and introducing only one over-all arbitrary factor: the pair creation constant γ.

If we take the signs reported in (R) concerning the channels $N\rho$ and $\Delta\pi$, and compare them to the predictions of the QPCM which do not depend on questionable resonance classifications or mixing angles, we find that 15 signs agree and three disagree ($FP_{15}\rightarrow\Delta\pi$, $DS_{33}\rightarrow N\rho_3$, and $FF_{37}\rightarrow N\rho_3$). The trouble is that one of these three is $FP_{15}\rightarrow\Delta\pi$, which is usually considered as one of the most firmly determined of the experimental analysis, and it is the only sign which discriminates between SU(6)$_W$ and anti-SU(6)$_W$ for $L_q=2$ baryons decaying into $\Delta\pi$. Concerning the $N\rho$ channel, if one considers this channel by itself one cannot discriminate now between SU(6)$_W$ and anti-SU(6)$_W$ signs. Our model predicts the relative coupling signs between $\Delta\pi$ and $N\rho$ channels, and predicts anti-SU(6)$_W$ for $N\rho$, too. This prediction disagrees with 2 experimental signs in (R) (these two do not seem to be very firmly determined experimentally) and agrees with four classification-independent signs and three more signs with proper mixing angles. Concerning σ production, the QPCM predictions are systematically in contradiction with experimental results. However, the interpretation of experiment as σ production is still very dubious.

TABLE V. Comparison of QPCM and Faiman's predictions with experimental results (R) and with experiment as reported by Faiman. We have multiplied all results in (R) by (−1) in order to make the sign convention agree with that of Table I. All signs in Faiman's paper are in the "baryon-first convention". The signs to be multiplied in order to change to the conventions we use here can be read in (R), p. 118.

Reaction	SU(6)$_W$	QPCM	Experiment reported in (R) multiplied by (−1)	Experiment reported by Faiman
$P_{11}(1470)\rightarrow N\rho_1$	+	−	−	+ ?
$P_{11}(1700)\rightarrow N\rho_1$	+	−	+	+ ?
$S_{11}(1520)\rightarrow N\rho_1$				
$S_q=\frac{1}{2}$	−	+	−	−
$S_q=\frac{3}{2}$	+	−		
$SS_{11}(1700)\rightarrow N\rho_1$				
$S_q=\frac{1}{2}$	+	−	+ (−)	+ ?
$S_q=\frac{3}{2}$	−	+		
$DS_{13}(1520)=N\rho_3$				
$S_q=\frac{1}{2}$	−	−	−	−
$DS_{13}(1700)\rightarrow N\rho_3$				
$S_q=\frac{3}{2}$	+	+	+ ?	+
$SS_{31}(1650)\rightarrow N\rho_3$	−	+	+	+ ?
$FP_{15}(1690)\rightarrow N\rho_3$	+	+	+	+
$DS_{33}(1670)\rightarrow N\rho_3$	+	+	−	+
$PP_{13}(1860)\rightarrow N\rho_1$	+	−	−	+ ?
$FP_{35}(1890)\rightarrow N\rho_3$	−	−	−	−
$FF_{37}(1950)\rightarrow N\rho_3$	−	+	−	−

Alternative interpretations of the data compatible with the QPCM exist except for one coupling.

Compared to the other models which have actually tried to explain the baryon decays, the model has the great advantage of giving a much more complete description of hadron states and of the decay process. It is able to include the successful aspects of various other approaches, particularly the phenomenological properties emphasized by Rosner *et al.*: coplanar $SU(3) \otimes SU(3)$ symmetry[24] and anti-$SU(6)_W$ signs.[2] At the same time, it goes beyond any of these approaches, since it fixes in the right way the quantities that are left arbitrary by the algebraic approaches (thus the centrifugal barrier is predicted quantitatively). Moreover, it treats in a unified manner the various possible meson emissions which are treated separately in the approaches based on currents. Thus, the general relative phase between the $\Delta\pi$, $N\rho$, and $N\sigma$ decays is predicted.

In fact, no freedom is left. For instance, choosing $SU(6)_W$ or anti-$SU(6)_W$ signs according to the SU(6) multiplets is not allowed. Anti-$SU(6)_W$ signs are implied for all the known cases. These more stringent predictions of the model are in general confirmed by experiment for the most reliable decays $N^* \to \Delta\pi$ and $N\rho$ and not for the $N\sigma$ decays which are dubious. In one case, the model is in open conflict with the definite predictions of another model (Moorhouse and Parsons, for $FP_{35} \to N\rho_3$) and there the data favor the QPCM.

The model, in its present state, is too simple mainly because it does not treat relativistically the rapid motions of quarks and hadrons and because it lacks crossing symmetry. It must be considered as a first step in a complete treatment of the decay process. This first step consists in building up one relativistic phenomenon: quark-pair creation in hadrons, which seems of most immediate consequence for the decay process. Other steps should not destroy the good features obtained in the first step. We have given in other papers[25] some other elements of a relativistic treatment of hadron phenomena, concerning the spin and the spatial wave functions of hadrons. These aspects certainly have to be included in these further steps.

Our method, if not our results, is rather different from another approach to a relativistic quark model of hadron decays, using Bethe-Salpeter amplitudes and the Mandelstam formalism. This approach has been developed mainly by Kitazoe *et al.*,[26] Kaiba *et al.*,[27] and Böhm, Joos, and Kramers.[28] One advantage of this approach is that covariance and also crossing symmetry are automatically introduced with the field theory. The general description of the process (triangle graphs) has something in common with the QPCM. Moreover, under its most recent version,[29] the structure of the matrix element and some quantitative results, contrary to older versions, are rather similar to the QPCM for $A_1 \to \rho\pi$ and $B \to \omega\pi$, although there remain definite divergences.

However, the formalism and the calculations are very difficult and there is a rather wide range of possible types of potentials and couplings. So the comparison with experiment and the test of the various dynamical hypothesis is by no means direct and easy (baryon decay calculations would be very lengthy and have not yet been done).

We think it better at present to develop the relativistic quark model with a much lighter and simpler formalism such as the QPCM for describing the pair creation.

APPENDIX: CONVENTIONS

In order to compare all $\pi N \to \pi\pi N$ channels and to compute the second-generation decay amplitudes $\Delta \to N\pi$, $\rho \to \pi\pi$, and $\sigma \to \pi\pi$, we have to make precise conventions. We will choose our conventions so as to make the comparison with experiment I easy. Let us define $A_l(\pi N \to N^*)$ by

$$\langle N^*(J_z, I_z) | N(\mu_N, I_z^N); \pi(I_z^\pi) \rangle = \sum_{l,m} C_{l,1/2}(JJ_z; m, -\mu_N) C_{1,1/2}(II_z; I_z^\pi I_z^N) \times A_l(\pi N \to N^*) Y_l^m(\hat{k}_\pi) .$$

Now define $A_{l,S}(N^* \to PD)$, where P is a stable particle (N or π) and D a resonance (Δ, ρ, σ):

$$\langle P(S_z^P, I_z^P); D(S_z^D, I_z^D) | N^*(J_z I_z) \rangle = \sum_{l,S,m} C_{lS}(JJ_z; m, S_z^P + S_z^D) C_{S^P S^D}(S, S_z^P + S_z^D; S_z^P, S_z^D) \times C_{I^D I^P}(II_z; I_z^D I_z^P) A_{l,S}(N^* \to PD) Y_l^m(\hat{k}_P) .$$

Let us define the second-generation decay amplitude $A_{l',S_{ab}}(D \to P_a P_b)$, where P_a is a nucleon N when D is a Δ ($l'=1$ for ρ or Δ, $l'=0$ for σ),

$$\langle P^a(S_z^a I_z^a), P^b(S_z^b I_z^b) | D(S_z^D I_z^D) \rangle = \sum_{l', m', S^{ab}} C_{S^a S^b}(S^{ab} S_z^{ab}; S_z^a S_z^b) C_{l' S^{ab}}(S^D S_z^D; m' S_z^{ab}) \times C_{I^a I^b}(I^D I_z^D; I_z^a I_z^b) A_{l', S^{ab}}(D \to P^a P^b) \times Y_{l'}^{m'}(\hat{k}_a) .$$

In the preceding formulas, angular momenta are quantized along $\vec{k}$, $\vec{k}_p$, and $\vec{k}_a$, respectively. Using the formulas of Sec. I, we find

$$M(\Delta \to N\pi) = \frac{-1}{3\sqrt{6}} \mathcal{L}_{10;1}^{(0;00)} .$$

Further on we shall denote by $\epsilon(x)$ the sign of any real number x. From (3.1), we see that

$$\epsilon(M(\Delta\to N\pi))=\epsilon(\gamma)\ ,$$

where γ is the quark-pair creation constant. Furthermore, comparing formulas of Sec. II and the convention we fixed above, we easily see that we have to permute I_π and I_N in the isospin Clebsch-Gordan coefficients in $\pi N\to N^*$. We have also to replace $\vec{k}_\pi$ with $\vec{k}_N$ in the $\mathcal{Y}_1^m(\vec{k}_\pi)$ of $\Delta\to N\pi$. Then we have

$$\epsilon[A_l(\pi N\to N^*)A_{l',3/2}(N^*\to\Delta\pi)A_{1,1/2}(\Delta\to N\pi)]$$
$$=(-1)^{J_{N^*}-3/2}[-\epsilon(\gamma)]$$
$$\times\epsilon(M(l,\tfrac{1}{2})_{N^*\to N\pi})\epsilon(M(l',\tfrac{3}{2})_{N^*\to\Delta\pi})\ .$$

With formulas similar to those of Sec. II, we get

$$\epsilon[A_{1,0}(\rho\to\pi\pi)]=-\epsilon(\gamma)\ ,$$
$$\epsilon[A_{0,0}(\sigma\to\pi\pi)]=-\epsilon(\gamma)\ .$$

Comparing Sec. II with the above-mentioned conventions we see that we have to permute I_π and I_N, I_M and I_N, and to replace $\mathcal{Y}_{l'}^{m'}(\vec{k}_M)$ with $\mathcal{Y}_{l'}^{m'}(\vec{k}_N)$. Then

$$\epsilon[A_l(\pi N\to N^*)A_{l'}(N^*\to\rho N)A_{1,0}(\rho\to\pi\pi)]$$
$$=[-\epsilon(\gamma)]\epsilon[M(l,\tfrac{1}{2})_{N^*\to N\pi}]\epsilon[M(l',s)_{N^*\to N\rho}](-1)^{l'}\ ,$$

$$\epsilon[A_l(\pi N\to N^*)A_{l'}(N^*\to\sigma N)A_{00}(\sigma\to\pi\pi)]$$
$$=(-1)[-\epsilon(\gamma)]\epsilon[M(l,\tfrac{1}{2})_{N^*\to N\pi}]$$
$$\times\epsilon[M(l',\tfrac{1}{2})_{N^*\to N\sigma}](-1)^{l'}\ .$$

---

*Laboratoire associé au Centre National de la Recherche Scientifique. Postal address: Université de Paris Sud, Bâtiment 211, 91405 Orsay, France.

[1] D. J. Herndon *et al.*, LBL Report No. LBL-1065 (unpublished); SLAC Report No. SLAC-PUB-1108, 1972 (unpublished); R. J. Cashmore, in *Baryon Resonances—73*, proceedings of the Purdue Conference on Baryon Resonances, edited by E. C. Fowler (Purdue Univ. Press, Lafayette, Indiana, 1973), p. 53.

[2] D. Faiman and J. Rosner, Phys. Lett. 45B, 357 (1973); W. Petersen and J. Rosner, Phys. Rev. D 6, 820 (1972); 7, 747 (1973); E. Colglazier and J. Rosner, Nucl. Phys. B27, 349 (1971).

[3] D. Faiman and D. Plane, Phys. Lett. 39B, 358 (1972); Nucl. Phys. B50, 379 (1972).

[4] R. G. Moorhouse and N. Parsons, Phys. Lett. 47B, 24 (1973).

[5] R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D 3, 2706 (1971).

[6] L. Micu, Nucl. Phys. B10, 521 (1969).

[7] A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal, Phys. Rev. D 8, 2223 (1973); 9, 1415 (1974).

[8] H. J. Melosh IV, Caltech thesis, 1973 (unpublished).

[9] F. J. Gilman, M. Kugler, and S. Meshkov, Phys. Lett. 45B, 481 (1973).

[10] A. D. Brody *et al.*, Phys. Lett. 34B, 665 (1971); Phys. Rev. D 4, 2693 (1971).

[11] I. Butterworth, in proceedings of the Second International Conference on Elementary Particles, Aix-en-Provence, 1973 [J. Phys. (Paris) Suppl. 34, C1-173 (1973)].

[12] Particle Data Group, Phys. Lett. 50B, 120 (1974).

[13] A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal, Orsay Report No. LPTHE 72/6, 1972 (unpublished). Note that the "anti-SU(6)$_W$" signs are, in this model, a consequence of the physical value of the radii which has been determined from a lot of different phenomena, all in rough agreement. If the radii were, say, twice bigger, than a lot of signs would be reversed.

[14] A. Mitra and M. Ross, Phys. Rev. 158, 1630 (1967).

[15] Let P_i and P_f be projectors; let A and A' (B and B') be operators commuting with P_i (P_f); then

$$\left.\begin{array}{l}P_fAP_i=P_fBP_i\\ P_fA'P_i=P_fB'P_i\end{array}\right\}\Rightarrow P_fAA'P_i=P_fBB'P_i\,.$$

[16] D. Faiman and A. W. Hendry, Phys. Rev. 180, 1572 (1969).

[17] L. A. Copley, G. Karl, and E. Obryk, Phys. Lett. 29B, 117 (1969); Nucl. Phys. B31, 303 (1969).

[18] G. Knies, R. G. Moorhouse, and H. Oberlack, Phys. Rev. D 9, 2680 (1974).

[19] See Cashmore (Ref. 1).

[20] H. J. Lipkin, Phys. Rev. D 9, 1579 (1974).

[21] D. Faiman, Nucl. Phys. B77, 443 (1974); Phys. Lett. 49B, 365 (1974); in proceedings of the Second International Conference on Elementary Particles, Aix-en-Provence, 1973 [J. Phys. (Paris) Suppl. 34, C1-169 (1973)].

[22] A. J. G. Hey, J. L. Rosner, and J. Weyers, Nucl. Phys. B61, 205 (1973).

[23] J. Uretsky, quoted in R. H. Dalitz, in *Proceedings of the Hawaii Topical Conferencein Particle Physics*, edited by S. Pakvasa and S. F. Tuan (Hawaii Univ. Press, Honolulu, Hawaii, 1968), p. 388.

[24] J. L. Rosner, Phys. Rev. D 6, 1781 (1972).

[25] A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal, Nucl. Phys. B37, 552 (1972); Phys. Rev. D 9, 2636 (1974); Ann. Phys. (N. Y.) 88, 242 (1974).

[26] T. Kitazoe and T. Teshima, Nuovo Cimento 57A, 497 (1968).

[27] M. Kaiba, K. Fukushima, and S. Hori, Prog. Theor. Phys. 41, 182 (1969).

[28] M. Böhm and T. Gudehus, Nuovo Cimento 57A, 578 (1968).

[29] M. Böhm, H. Joos, and M. Krammer, Nucl. Phys. B69, 349 (1974).