Chiral symmetry and the quark model*

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The construction of chiral-symmetric quark models is investigated. We show that the Nambu-Goldstone realization of chiral symmetry leads naturally to representation mixing of quark states and of their composites. Constructing SU(6) wave functions for hadron states, we obtain a mixing scheme algebraically equivalent to the Melosh transformation. The resulting phenomenology for pionic decays and for weak and electromagnetic vertices is known to be very successful. The present work provides a theoretical basis for these results and opens new avenues for the study of SU(6) symmetry and the investigation of deep-inelastic scattering processes.

I. INTRODUCTION

The weak and electromagnetic interactions of hadrons measure matrix elements of the vector and axial-vector currents. According to the current-algebra hypothesis of Gell-Mann[1] the charges $Q_a(t)$ and $Q_a^5(t)$ associated with these currents generate at fixed time the transformation group SU(3)×SU(3). The vector currents are approximately conserved, and the associated charges generate the familiar symmetry group SU(3). All known hadronic states fall nicely into approximately mass-degenerate multiplets of this group. The predictions of SU(3) for vertices of the strong, electromagnetic, and weak interactions are also well verified.

As for the axial-vector currents, it is only natural to ask if they, too, are conserved, and if the strong interactions therefore enjoy an approximate invariance under the full chiral group SU(3) ×SU(3). The evidence for mass-degenerate chiral multiplets is negative, since such multiplets could be formed only from unobserved parity doublets. An alternative is provided by the hypothesis of partially conserved axial-vector currents (PCAC),[2,3] whereby approximate conservation of the axial-vector currents is related to the small masses of the pseudoscalar mesons. Chiral symmetry is then said to be realized in the Nambu-Goldstone[3,4] mode. The symmetry of the vacuum is now spontaneously broken, and the physical states no longer fall into simple degenerate multiplets.[5]

In this picture, decays involving the pseudoscalar mesons are governed by matrix elements of the axial-vector currents. Consequently, systematic studies of such decays can serve to map out the structure of SU(3)×SU(3) multiplets. Phenomenological analyses[6] of this type all seem to indicate that physical hadrons transform as complex mixtures of chiral representations. Indeed, each irreducible chiral representation seems to involve a mixture of infinitely many physical states.[7]

We can contrast this complex picture with the simple quark-model description of the hadron spectrum. All known states fall nicely into multiplets of the group SU(6).[8] Of course the extension of SU(6) to the description of interaction vertices does not seem very straightforward, and the dynamical basis of the quark model remains unclear.[9] Recent progress has been made, however, following a proposal of Melosh[10] to describe the SU(6) transformation properties of the vector and axial-vector currents. This permits the calculation of weak and electromagnetic vertices and—with the assumption of PCAC—of strong pionic vertices as well. The systematic application[11] of this idea has met with considerable phenomenological success and has explained many facets of the chiral multiplet structure cited above. Unfortunately, Melosh's results were derived for the free-quark model where there is no SU(3) ×SU(3) symmetry, no PCAC relation, and no reason for the existence of chiral multiplets at all. The dynamical origin of the assumed transformation (or mixing) hence remains obscure.

In this paper[12] we will study quark models in which chiral symmetry is exact. Assuming the symmetry to be realized in the Nambu-Goldstone mode, we demonstrate that representation mixing of the quark fields is a natural consequence. Constructing composite states of quarks, we show how the Melosh transformation[10] arises and indicate the limits of its applicability. The resulting picture of hadrons is a rich one, which provides a framework suitable for further studies of SU(6) and of deep-inelastic and other high-energy scattering processes.

In Sec. II we review how the PCAC relation arises from the Nambu-Goldstone realization of chiral symmetry. This relation will underscore the utility of chiral charges defined by suitable integration over the null plane. These charges can be constructed explicitly in the σ model, as we demonstrate in Sec. III. As shown there, representation mixing of the canonical fermion fields in the model follows naturally from the Nambu-Goldstone realization of chiral symmetry. Implications of this result are discussed in Sec. IV in the general context of chiral-symmetric quark models.

In Sec. V we introduce hadronic wave functions which incorporate the empirical features of an SU(6) classification. This leads to a chiral mixing scheme equivalent to that first proposed by Melosh. The successful applications of this scheme are briefly reviewed in Sec. VI. In Sec. VII our interpretation of the quark model is described at greater length, and analogies with systems of solid-state physics are briefly developed. The relevance of these ideas to deep-inelastic scattering is discussed in Sec. VIII.

II. CHIRAL SYMMETRY AND NULL-PLANE CHARGES

Throughout this paper we consider theories with *exact* chiral symmetry in the Nambu-Goldstone mode.[5] Both the vector currents $V_a^\mu(x)$ and axial-vector currents $A_a^\mu(x)$ are divergenceless, and the corresponding charges,

$$Q_a(x^0) = \int d^4y\, \delta(x^0 - y^0)\, V_a^0(y)$$

and (2.1)

$$Q_a^5(x^0) = \int d^4y\, \delta(x^0 - y^0) A_a^0(y)\ ,$$

are therefore conserved:

$$\frac{d}{dx^0}\, Q_a(x^0) = \frac{d}{dx^0}\, Q_a^5(x^0) = 0\ . \qquad (2.2)$$

These charges generate the symmetry group SU(3)×SU(3), and the canonical fields of the theory typically transform as irreducible representations of this group. If, for example, there exists a fermion coupled to the axial-vector current, its bare mass will vanish and its right- and left-handed components will transform as the representations (3,1) and (1,3), respectively. When the symmetry associated with the axial charges is realized by the Nambu-Goldstone mechanism,[5] the following state of affairs is known to result:

(i) The ground state is degenerate, so that

$$Q_a^5\,|0\rangle \neq 0\ ; \qquad (2.3)$$

(ii) Q_a^5 excites *massless* pseudoscalar mesons ("pions");

(iii) the physical (quasiparticle) fermion states acquire a mass m.

Because Q_a^5 excites a number of pions when operating on any given state, the vacuum state and all physical states built on it display complicated transformation properties under this operator. Fortunately, as we will now proceed to demonstrate, the operators relevant for studying the chiral structure of hadrons are not these static charges, but some related operators defined by null-plane integrals of the axial-vector currents.

Let us examine an arbitrary matrix element of the axial-vector current,[13]

$$\langle\beta\,|A_a^\mu(0)|\,\alpha\rangle = if_\pi\langle\beta,\pi_a\,|\,\alpha\rangle\, q^\mu/q^2 + \langle\,\beta\,|A_a^\mu|\,\alpha\rangle_N\ , \qquad (2.4)$$

where $q = p_\alpha - p_\beta$. Figure 1 illustrates this explicit separation of the pion-pole term from the nonpole term $\langle\beta\,|A_a^\mu|\,\alpha\rangle_N$. The conservation condition $\partial_\mu A_a^\mu(x) = 0$ relates these terms by the PCAC constraint,

$$\langle\beta,\pi_a\,|\,\alpha\rangle = -(if_\pi)^{-1} q_\mu\langle\beta\,|A_a^\mu|\,\alpha\rangle_N\ . \qquad (2.5)$$

This expression can be conveniently rewritten in terms of the full axial-vector current if one carefully selects Fourier components to which the pion pole does not contribute. Introducing the null-plane components $q^\pm = (q^0 \pm q^3)/\sqrt{2}$, etc. (see Appendix B), one easily verifies from Eq. (2.4) that

$$\langle\beta\,|A_a^+(0)|\,\alpha\rangle_N = \lim_{q^\perp\to 0}\ \lim_{q^+\to 0}\ \langle\beta\,|A_a^+(0)|\,\alpha\rangle\ . \qquad (2.6)$$

Note that the order of limits here is important, since we want the pole term $q^+/(q_\perp{}^2 - 2q^+q^-)$ not to contribute. We thus arrive at the important relation

$$\langle\beta,\pi_a\,|\,\alpha\rangle = \frac{m_\alpha{}^2 - m_\beta{}^2}{if_\pi\langle p_\beta\,\|\,p_\alpha\rangle}\,\langle\beta\,|\,\hat{Q}_a^5(0)\,|\,\alpha\rangle\ , \qquad (2.7)$$

where

$$\langle p_\beta\,\|\,p_\alpha\rangle = (2\pi)^3\, 2p_\alpha^+\,\delta(p_\alpha^+ - p_\beta^+)\,\delta^2(p_\alpha^\perp - p_\beta^\perp)$$

denotes a covariant normalization factor, and $\hat{Q}_a^5(0)$ designates an axial-charge operator defined by integration on the null plane $x^+ = 0$. Specifically,

$$\hat{Q}_a^5(x^+) = \int d^2x^\perp \int dx^- A^+(x)\ , \qquad (2.8)$$

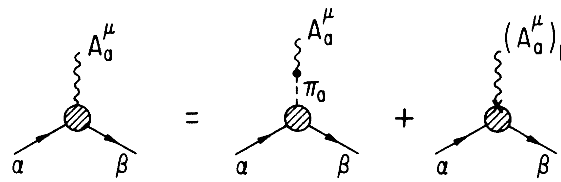

FIG. 1. Separation of the pion-pole term in matrix elements of the axial-vector current.

where the order of integration is essential and corresponds to the order of limits of Eq. (2.6). Note that the definition of the null-plane generator is not ambiguous in the physical world with $m_\pi \neq 0$, as can be seen simply by replacing $(q^2)^{-1}$ in Eq. (2.4) by $(q^2+m_\pi^2)^{-1}$ and following the arguments leading to (2.8). The definition (2.8) given for $\hat{Q}_5$ in the chiral-symmetric world thus corresponds to a smooth limit as $m_\pi \to 0$. (See Appendix A for a detailed discussion.)

Equation (2.7) establishes the importance of the null-plane charges for the study of pionic transitions. Measurement of the amplitudes for such processes specifies various matrix elements of $\hat{Q}_a^5$ or, equivalently, indicates how the various states transform under this operator. As mentioned above, if the states are to transform in any simple fashion under the $\hat{Q}_a^5$, it is essential that these operators annihilate the vacuum,

$$\hat{Q}_a^5(x^+)|0\rangle = 0 \ . \qquad (2.9)$$

This is indeed the case, as a simple exercise in momentum space will demonstrate.[14] The definition (2.8) implies that the $\hat{Q}_a^5$ carry momenta $q^\perp = q^+ = 0$. Consequently, they can create from the vacuum only states of mass $q^2 = 0$.[15] Although massless particles (pions) do exist in our theory, the $\hat{Q}_a^5$ are defined not to excite them, so they do annihilate the vacuum.

The insensitivity of the $\hat{Q}_a^5$ to the chiral structure of the vacuum allows for the possibility that physical states do transform simply under these charges. To investigate this important question, we must first inquire as to the algebraic properties of the $\hat{Q}_a^5$. A ready answer is provided by the work of Weinberg[13] on the pion-coupling matrices,

$$[\chi_a]_{\beta\alpha} = \frac{\langle \beta | \hat{Q}_a^5(0) | \alpha\rangle}{\langle p_\beta \| p_\alpha \rangle} \ . \qquad (2.10)$$

Making use of chiral symmetry, Regge behavior, and the narrow-resonance approximation, Weinberg has shown that the χ_a matrices form, together with matrix elements of the SU(3) generators, an algebra SU(3)×SU(3). In terms of the null-plane charges, we anticipate therefore the relation[16]

$$[\hat{Q}_a^5(x^+), \hat{Q}_b^5(x^+)] = if_{abc}\,\hat{Q}_c(x^+) \ . \qquad (2.11)$$

The right-hand side of this equation involves the vector charges

$$\hat{Q}_a(x^+) = \int d^4y\, \delta(x^+ - y^+)\, V_a^+(y) \ . \qquad (2.12)$$

Since these charges are not coupled to any massless particles, they are in fact identical with the corresponding static charges,

$$\hat{Q}_a(x^+) = Q_a \ . \qquad (2.13)$$

Alternatively, one may derive Eq. (2.11) as an operator relation from the null-plane algebra approach. This has been carried out in the gluon and σ models for the case $m_\pi \neq 0$ (when there are none of the complications which arise from a massless pion).[17] The result is identical to (2.11). This relation is independent of m_π and hence remains valid in the limit $m_\pi \to 0$. We will confirm this statement in Sec. III by directly calculating this commutator in the chiral-symmetric σ model.

From Eq. (2.11), we see that $\hat{Q}_a$ and $\hat{Q}_a^5$ form an algebra SU(3)×SU(3). The problem of pionic transitions has thus been reduced to a question of how physical states are classified under this chiral algebra on the null plane.[18] The question of chiral representation mixing can now be approached through the transformation properties of canonical fields under the $\hat{Q}_a^5$. This task will be carried out for the σ model in Sec. III.

First we should emphasize the important distinctions between the operators $\hat{Q}_a^5$ and Q_a^5. Although both obey an SU(3)×SU(3) algebra, they are not equal. The $\hat{Q}_a^5$ annihilate the vacuum while the Q_a^5 do not. The Q_a^5 are conserved, and consequently they cannot connect states with different 4-momenta, i.e.,

$$\langle \beta | Q_a^5 | \alpha \rangle = 0 \quad \text{if } m_\beta \neq m_\alpha \ . \qquad (2.14)$$

This means that the Q_a^5 have no relevance for the pionic decays of hadrons. As demonstrated in Appendix A, the conservation of Q_a^5 is achieved by explicit cancellation of the pole and nonpole terms of A_a^μ. Since $\hat{Q}_a^5$ contains no pion pole, it is not conserved; and, as shown by Eq. (2.7), it is indeed relevant for *arbitrary* pionic transitions. The properties of the two sets of charges are summarized in Table I.

For the reader versed in the history of this subject, a brief further comment is necessary on our distinction of $\hat{Q}_a^5$ from Q_a^5. In the conventional approach[19] one deals with matrix elements of a *nonconserved* ($m_\pi \neq 0$) charge Q_a^5 in the infinite-

TABLE I. Properties of the static and null-plane axial charges.

	Q_a^5	$\hat{Q}_a^5$
Algebra	SU(3) × SU(3)	SU(3) × SU(3)
Pion pole?	Yes	No
Conserved?	Yes	No
Vacuum annihilating?	No	Yes
Invariant under	Rotations	Boosts along x^3 direction

momentum frame. Nonconservation of Q_a^5 implies a frame dependence such that in the infinite-momentum limit the pion-pole contribution is lost. The infinite-momentum limit of Q_a^5 is in this case equivalent to the $\hat{Q}_a^5$ (evaluated in some frame of finite momentum). In our approach there is always exact chiral symmetry; the charges Q_a^5 are conserved, satisfy Eq. (2.14) exactly, and are distinct from $\hat{Q}_a^5$ in *any* frame.[20] Stated differently, in the transition from the physical world ($m_\pi \neq 0$) to the chiral world ($m_\pi = 0$), $\hat{Q}_a^5$ changes smoothly while Q_a^5 (evaluated in the infinite-momentum frame) does not.

III. σ MODEL

The properties of the null-plane charges and their utility in the classification of states can be explicitly illustrated in the σ model. This renormalizable model[21] is unique in exhibiting explicitly the spontaneous breakdown of chiral symmetry. For simplicity we confine our attention to the SU(2) version of the model. [All our results are easily generalized to the SU(3) case.] The Lagrangian is

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi - G\bar{\psi}(\sigma + i\vec{\pi}\cdot\vec{\tau}\gamma_5)\psi - \tfrac{1}{2}[(\partial_\mu\sigma)^2 + (\partial_\mu\vec{\pi})^2] - B(\sigma^2 + \vec{\pi}^2 - C)^2 . \qquad (3.1)$$

It is invariant under the infinitesimal transformations:

$$\begin{aligned} &\psi \to \psi + i\delta\vec{\alpha}\cdot\tfrac{1}{2}\vec{\tau}\psi ,\\ &\sigma \to \sigma ,\\ &\vec{\pi} \to \vec{\pi} - \delta\vec{\alpha}\times\vec{\pi} \end{aligned} \qquad (3.2)$$

and

$$\begin{aligned} &\psi \to \psi + i\delta\vec{\beta}\cdot\tfrac{1}{2}\vec{\tau}\gamma_5\psi ,\\ &\sigma \to \sigma + \delta\vec{\beta}\cdot\vec{\pi} ,\\ &\vec{\pi} \to \vec{\pi} - \delta\vec{\beta}\sigma . \end{aligned} \qquad (3.3)$$

(Note that a fermion mass term $-m_0\bar{\psi}\psi$ would spoil this invariance.) The vector currents

$$V_a^\mu(x) = \bar{\psi}(x)\gamma^\mu \frac{\tau_a}{2}\psi(x) - \epsilon_{abc}\pi_b\partial^\mu\pi_c \qquad (3.4)$$

and axial-vector currents

$$A_a^\mu(x) = \bar{\psi}(x)\gamma^\mu\gamma_5\frac{\tau_a}{2}\psi(x) - \sigma(x)\overleftrightarrow{\partial}^\mu\pi_a(x) \qquad (3.5)$$

are both conserved. The associated static charges Q_a and Q_a^5 [Eq. (2.1)] are just the generators of transformations (3.2) and (3.3). Consequently,

$$\begin{aligned} &[\psi(x), Q_a] = \tfrac{1}{2}\tau_a\psi(x) ,\\ &[\pi_b(x), Q_a] = i\epsilon_{bac}\pi_c(x) ,\\ &[\sigma(x), Q_a] = 0 \end{aligned} \qquad (3.6)$$

and

$$\begin{aligned} &[\psi(x), Q_a^5] = \tfrac{1}{2}\tau_a\gamma_5\psi(x) ,\\ &[\pi_b(x), Q_a^5] = i\delta_{ab}\sigma(x) ,\\ &[\sigma(x), Q_a^5] = -i\pi_a(x) . \end{aligned} \qquad (3.7)$$

We see that the σ and $\vec{\pi}$ fields transform as the representation $(\frac{1}{2},\frac{1}{2})$ of the SU(2)×SU(2) chiral group generated by Q_a and Q_a^5. Likewise, the right-handed and left-handed fermion fields,

$$\psi_{R,L}(x) = \tfrac{1}{2}(1\pm\gamma_5)\psi(x) , \qquad (3.8)$$

transform as the irreducible representations $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$, respectively. This clear distinction between field components with opposite handedness is, of course, characteristic of fermions with zero mass.

The Nambu-Goldstone mechanism comes into play when the constant C in the Lagrangian is positive. The classical potential function is then stable around the point $\sigma = \sqrt{C}$, $\vec{\pi} = 0$ (instead of $\sigma = \vec{\pi} = 0$). The σ field therefore acquires a vacuum expectation value $\langle\sigma\rangle$, and the chiral symmetry of the ground state is spontaneously broken. Rewriting the Lagrangian (3.1) in terms of the displaced σ field $\phi(x) = \sigma(x) - \langle\sigma\rangle$, one obtains

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi - G\bar{\psi}(\phi + i\vec{\pi}\cdot\vec{\tau}\gamma_5)\psi - \tfrac{1}{2}(\partial_\mu\vec{\pi})^2 - \tfrac{1}{2}[(\partial_\mu\phi)^2 + \mu_\sigma^2\phi^2] - 4B\sqrt{C}\,\phi(\phi^2 + \vec{\pi}^2) - B(\phi^2 + \vec{\pi}^2)^2 . \qquad (3.9)$$

The following points are immediately apparent:

(i) The pion becomes massless, assuming the role of a Nambu-Goldstone boson;

(ii) the σ mass is shifted to $\mu_\sigma^2 = 8BC$;

(iii) the fermion acquires a nonzero mass

$$m = G\langle\sigma\rangle . \qquad (3.10)$$

With respect to $\{Q_a; Q_a^5\}$ the ϕ and $\vec{\pi}$ fields no longer transform as a pure $(\frac{1}{2},\frac{1}{2})$ representation. Furthermore, for massive fermion fields, the distinction between right-handed and left-handed components becomes a frame-dependent concept. It is clear, therefore, that the chiral properties of the fields must be completely reexamined.

As emphasized in Sec. II the physically relevant operators in this situation are the null-plane charges $\hat{Q}_a$ and $\hat{Q}_a^5$. The vector charges are defined as in Eq. (2.12). For the axial charges one can use Eq. (2.8). A more precise definition is possible, however, in the σ model because the *canonical* π field allows an explicit pole subtraction:

$$\hat{Q}_a^5(x^+) = \int d^4y\, \delta(x^+ - y^+)\,[A_a^+(y) - f_\pi \partial^+ \pi_a(y)] \ . \tag{3.11}$$

It is simple to verify that matrix elements of (3.11) and (2.8) are equal and that both are equal to the $m_\pi = 0$ limit of a null-plane charge defined for $m_\pi \neq 0$. With the pole term explicitly removed, there is no longer an ambiguity in the order of integration; this allows the unambiguous calculation of commutators of $\hat{Q}_a^5$ directly from the expression (3.11). The constant f_π is the pion decay constant,

$$\langle 0 | A_a^\mu(0) | \pi_b(q) \rangle = i f_\pi \delta_{ab} q^\mu \ . \tag{3.12}$$

In the classical (tree) approximation f_π effectively measures the strength of the average σ field, $f_\pi = -\langle \sigma \rangle$.

The commutation relations of the $\hat{Q}_a^5$ can now be obtained by straightforward algebraic manipulations. Since these charges are not conserved, however, these calculations must be done on the null-surface $x^+ = \text{const}$. For completeness we summarize the relevant rules for null-plane quantization in Appendix B. With the aid of these rules one readily verifies that the null-plane charges $\hat{Q}_a$ and $\hat{Q}_a^5$ do indeed satisfy an SU(2)×SU(2) algebra (see Appendix C), i.e.,

$$\begin{aligned} &[\hat{Q}_a, \hat{Q}_b] = i\epsilon_{abc} \hat{Q}_c \ , \\ &[\hat{Q}_a^5(x^+), \hat{Q}_b] = i\epsilon_{abc} \hat{Q}_c^5(x^+) \ , \\ &[\hat{Q}_a^5(x^+), \hat{Q}_b^5(x^+)] = i\epsilon_{abc} \hat{Q}_c \ . \end{aligned} \tag{3.13}$$

This confirms the results obtained from other approaches as discussed in Sec. II. We may also remark that had we chosen anything other than the canonical field to make the pole subtraction in the definition of $\hat{Q}_a^5$ in Eq. (3.11), this SU(2)×SU(2) algebra would be spoiled.

Let us now consider how the basic fields transform under this algebra. Straightforward algebra yields

$$\begin{aligned} &[\phi(x), \hat{Q}_a^5(x^+)] = -i\pi_a(x) + \tfrac{1}{4} i[\pi_a(\infty) + \pi_a(-\infty)] \ , \\ &[\pi_b(x), \hat{Q}_a^5(x^+)] = i\delta_{ab}\,\phi(x) - \tfrac{1}{4} i\delta_{ab}[\phi(\infty) + \phi(-\infty)] \ , \end{aligned} \tag{3.14}$$

where again $\phi(x) = \sigma(x) - \langle\sigma\rangle$ is the displaced σ field and $\pi(\pm\infty)$ and $\phi(\pm\infty)$ denote the values of these fields on the surfaces $(x^+, x^- = \pm\infty, \vec{x}_\perp)$. These "surface terms" vanish if we restrict attention to states *localizable in* x^-. Equations (3.14) then reduce to a form directly comparable with (3.7). In fact $(\phi, \vec{\pi})$ transform as a $(\frac{1}{2}, \frac{1}{2})$ representation under $\frac{1}{2}(\hat{Q}_a \pm \hat{Q}_a^5)$. Details of the calculation leading to (3.14) may be found in Appendix C.

Of much more interest is the transformation property of the basic fermion field. As is well known,[22] on the null plane the Dirac spinor field ψ can be separated into dynamically independent components ψ_+ and dependent components ψ_-, where

$$\psi_\pm(x) = \tfrac{1}{2}(1 \pm \alpha_3)\,\psi(x) \ . \tag{3.15}$$

The dependent components ψ_- are determined by the constraint equation

$$\partial_-\psi_-(x) = -\tfrac{1}{2}\gamma^+[\gamma^i\partial_i + iG(\sigma + i\vec{\pi}\cdot\vec{\tau}\gamma_5)]\,\psi_+(x) \ , \tag{3.16}$$

which can be rewritten in terms of the displaced σ field $\phi(x)$

$$\partial_-\psi_-(x) = -\tfrac{1}{2}\gamma^+[\gamma^i\partial_i + im + iG(\phi + i\vec{\pi}\cdot\vec{\tau}\gamma_5)]\,\psi_+(x) \ . \tag{3.17}$$

Note how the physical mass of the fermion $m = G\langle\sigma\rangle$ explicitly enters the expression.

The commutator of ψ_+ with $\hat{Q}_a^5$ can be directly obtained from the canonical quantization rules,

$$[\psi_+(x), \hat{Q}_a^5(x^+)] = \tfrac{1}{2}\tau_a\gamma_5\psi_+(x) \ . \tag{3.18}$$

The corresponding commutator of ψ_- may be evaluated using Eqs. (3.14), (3.17), and (3.18), with the result

$$\begin{aligned} [\partial_-\psi_-(x), \hat{Q}_a^5(x^+)] &= \tfrac{1}{2}(\tau_a/2)\gamma_5\gamma^+(im)\psi_+(x) - \tfrac{1}{2}(\tau_a/2)\gamma_5\gamma^+[\gamma^i\partial_i + iG(\phi + i\vec{\pi}\cdot\vec{\tau}\gamma_5)]\,\psi_+(x) \\ &= \tfrac{1}{2}\tau_a\gamma_5\partial_-\psi_-(x) + \tfrac{1}{2} im\,\tau_a\gamma_5\gamma^+\psi_+(x) \ . \end{aligned} \tag{3.19}$$

Upon integrating with respect to the x^- variable and combining with (3.18), we obtain the final result for the transformation property of the quark field under $\hat{Q}_a^5$:

$$[\psi(x), \hat{Q}_a^5(x^+)] = \tfrac{1}{2}\tau_a\gamma_5\psi(x) + \tfrac{1}{2} im(\tfrac{1}{2}\tau_a)\gamma_5\gamma^+ \int dy^-\,\epsilon(x^- - y^-)\,\psi(x^+, y^-, \vec{x}_\perp) \ . \tag{3.20}$$

We see that the second term on the right-hand side induces a mixing of the right- and left-handed quark fields. *With respect to* $\{\hat{Q}_a; \hat{Q}_a^5\}$, *both* $\psi_R(x)$ *and* $\psi_L(x)$ *transform as the reducible representation* $(\frac{1}{2}, 0)\oplus(0, \frac{1}{2})$. This representation mixing of the quark fields is a direct consequence of the spontaneously generated fermion mass. In the absence of spontaneous symmetry breaking [$C \leq 0$ in the Lagrangian (3.1)], the fermion mass is zero ($m = G\langle\sigma\rangle = 0$) and there is no mixing.

IV. GENERALIZATIONS

We have shown that the chiral representation mixing of the fermion fields in the σ model results directly from the spontaneously generated fermion mass. Physically, this means that the null-plane charges $\hat{Q}_a^5$ are aware of this mass, a point implicit in their nonconserved nature. Since the phenomenon of spontaneous mass generation occurs in any fermion model where chiral symmetry is realized in the Nambu-Goldstone mode, it is natural to assume that *the transformation law (3.20) is generally valid*. We shall adopt this assumption for the quark model without trying to specify the interaction in any further detail.

The representation mixing of the quark fields implies of course that composite operators of the form $\psi^\dagger\psi$ or $\psi\psi\psi$ are also a mixture of chiral representations. For example, the operator $\psi^\dagger(\frac{1}{2}\lambda_a)\psi$ will transform as a mixed representation $[(1, 8) + (8, 1)]\oplus[(3, \overline{3}) + (\overline{3}, 3)]$ under the chiral algebra $\{\hat{Q}_a; \hat{Q}_a^5\}$. Under the static charges this same operator transforms as a pure $[(1, 8) + (8, 1)]$ representation.

Algebraically, our mixing is identical with that which would result in a model of free massive quarks. Defining the axial-vector current in that case as

$$A_a^\mu(x) = \overline{\psi}(x)\,\gamma^\mu\gamma_5\,\frac{\lambda_a}{2}\,\psi(x)\ , \qquad (4.1)$$

and using the Dirac equation to relate ψ_- to ψ_+, one readily obtains the formula (3.20). This equivalence forms a bridge between the present work and that which Melosh[10] carried out in the free-quark model. One notes further that while the formula (3.20) mixes the right- and left-handed components of the quark field, it does not change the *number* of quarks. Again, this feature is one that plays an important role in all applications of Melosh's work.[11]

To go from these general considerations to the actual computation of matrix elements of $\hat{Q}_a^5$, we must specify the null-plane wave functions of the hadronic states. The precise form of these wave functions is of course unknown in the absence of complete dynamical information. Nonetheless, a reliable hint of quark structure is provided by the observed SU(6)×O(3) multiplets of the hadron spectrum. Stated in terms of the field-theory model adopted here, the physical mesons are generated by interpolating fields $\Phi_\alpha \sim \psi^\dagger\psi$ and baryons by $\Phi_\alpha \sim \psi\psi\psi$. Transformation properties of these states under $\hat{Q}_a^5$ can be obtained by noting that

$$\begin{aligned}\hat{Q}_a^5\,|\,\alpha\rangle &= \hat{Q}_a^5\,\Phi_\alpha^\dagger\,|\,0\rangle\\ &= [\hat{Q}_a^5, \Phi_\alpha^\dagger]\,|\,0\rangle\ ,\end{aligned} \qquad (4.2)$$

where the vacuum-annihilation property of $\hat{Q}_a^5$ [Eq. (2.9)] is used. As observed earlier, the transformation properties of the interacting fields (with respect to $\hat{Q}_a^5$) are formally identical to those of a free massive quark model. Equation (4.2) then implies that the algebraic properties of the matrix elements $\langle\beta\,|\,\hat{Q}_a^5\,|\,\alpha\rangle$ can be reliably extracted from an effective free-particle representation of the hadron wave functions.

This simplifying feature suggests that a momentum-space construction of the wave function is most convenient. We do this in Sec. V. Using this "quasiparticle" representation, we study pionic decays of various meson and baryon resonances via the PCAC relation (2.7). In a similar spirit, one can apply this approximation to the magnetic-moment operator and the electromagnetic transitions it generates. The limitations of this simple picture and the possibility of probing more deeply into the details of hadronic structure will be discussed in Secs. VII and VIII.

V. SU(6) WAVE FUNCTIONS

Let us now construct the quasiparticle wave functions on the null plane. We make use of the empirical classification of hadron resonance states by adopting the assumption that mesons are composed of quark-antiquark pairs and baryons of three quarks. The only ambiguities in this specification are the choice of a spin basis for the quarks and the definition of the O(3) generators L_1, L_2, and L_3.

What restrictions can be placed on possible spin bases for the quarks? We are interested particularly (through PCAC) in matrix elements of the null-plane charges $\hat{Q}_a^5$. These charges leave invariant the momentum components p^+ and $\vec{p}_\perp$ of any state on which they act. Because of their nonconserved nature, however, they may change p^- and hence the mass. We are thus interested in matrix elements of states with different velocities in the x^3 direction. For this reason we shall demand that the SU(6) specification of a state be invariant under boosts along this direction.

An arbitrary boost-invariant basis may be written in the following form[23]:

$$|p^+,\vec{p}_\perp;s\rangle = e^{-i\chi K_3}\, e^{-i\xi\hat{p}_\perp\cdot\vec{K}_\perp}\, e^{-i\Theta\hat{n}_\perp\cdot\vec{J}_\perp}\,|m/\sqrt{2},\vec{0};s\rangle\ , \tag{5.1}$$

where $|m/\sqrt{2},\vec{0};s\rangle$ denotes a reference rest state ($p^+=m/\sqrt{2}$) with spin projection s along x^3, and the parameters Θ and $\hat{n}_\perp$ are independent of p^+. The parameters χ and ξ are related to the momentum components by

$$\begin{aligned} p^\pm &= (m/\sqrt{2})\cosh\xi\, e^{\pm\chi}\ ,\\ p^\perp &= m\sinh\xi\ . \end{aligned} \tag{5.2}$$

A further constraint on the choice of spin basis is the requirement that it describe states with definite transformation properties under parity. The relevant operator which leaves the null plane $x^+=$const invariant is

$$Y = P e^{-i\pi J_2}\ . \tag{5.3}$$

(This is simply a reflection in the x^1-x^3 plane.) Applying Y to the state (5.1) we demand the spin index s to change sign (like J_3):

$$Y|p^+,\vec{p}_\perp;s\rangle = \pm|p^+,\vec{p}_\perp\,';-s\rangle\ , \tag{5.4}$$

where $\vec{p}_\perp\,'$ is the Y reflection of $\vec{p}_\perp$. This requirement dictates that we choose $\hat{n}_\perp=\hat{e}_3\times\hat{p}_\perp$, where $\hat{e}_3$ is a unit vector in the x^3 direction. In this case we have

$$\begin{aligned} Y\exp(-i\Theta\hat{n}_\perp\cdot\vec{J}_\perp)\,Y^{-1} &= Y\exp[-i\Theta(\hat{p}_\perp\times\vec{J}_\perp)]\,Y^{-1}\\ &= \exp[-i\Theta(\hat{p}_\perp\,'\times\vec{J}_\perp)]\\ &= \exp(-i\Theta\hat{n}_\perp\,'\cdot\vec{J}_\perp)\ , \end{aligned} \tag{5.5}$$

consistent with Eq. (5.4).[24]

The final form for the basis states is

$$\begin{aligned} |p^+,\vec{p}_\perp;s\rangle_\Theta = {}& \exp(-i\chi K_3)\exp(-i\xi\hat{p}_\perp\cdot\vec{K}_\perp)\\ &\times|m/\sqrt{2},\vec{0};r\rangle\,R(\varphi,\Theta,-\varphi)^r_s\ , \end{aligned} \tag{5.6}$$

where χ and ξ are specified by Eqs. (5.2), $\varphi=\tan^{-1}(p_1/p_2)$, and R is the rotation (about $\hat{n}_\perp$) matrix for spin $\frac{1}{2}$. Any given choice of the function $\Theta(p_\perp,m)$ specifies a basis Θ suitable for the construction of boost-invariant SU(6) wave functions. These wave funtions involve, in general, certain linear combinations of products of the quasiparticle creation operators $a^\dagger$ and $b^\dagger$. At fixed x^+ we expand the quark field in its usual momentum-space decomposition:

$$\begin{aligned} \psi^i(x) = \int dp\sum_s\,[&a^i(p,s)\,u(p,s)\,e^{ip\cdot x}\\ &+b^{i\dagger}(p,s)\,v(p,s)e^{-ip\cdot x}]\ , \end{aligned} \tag{5.7}$$

where i is an SU(3) index, u and v denote 4-component Dirac spinors for particles and antiparticles of mass m, and $dp=d^2p_\perp dp^+/2p^+(2\pi)^3$. Note that since the spin index s is summed in this expression, the choice of a spin basis for the a's and b's (and hence the u's and v's) is arbitrary.

It is useful to label the creation and annihilation operators with a spinor index α in place of the spin s. Thus we define the annihilation operators

$$\begin{aligned} a^{(\alpha,i)}(p) &= a^i(p,\alpha)\ ,\\ b_{(\alpha,i)}(p) &= (-1)^{\alpha+1/2}\,b_i(p,-\alpha) \end{aligned} \tag{5.8}$$

and the creation operators

$$\begin{aligned} a^\dagger_{(\alpha,i)}(p) &= [a^{(\alpha,i)}(p)]^\dagger\ ,\\ b^{(\alpha,i)\dagger}(p) &= [b_{(\alpha,i)}(p)]^\dagger\ . \end{aligned} \tag{5.9}$$

The index pair (α,i) may be interpreted as an SU(6) index, with the quarks and antiquarks belonging to the SU(6) representations $\underline{6}$ and $\underline{\bar{6}}$, respectively.

To complete our specification of the wave functions we must construct representations of the orbital angular momentum part O(3). Having chosen a boost-invariant description it is natural to focus attention on the boost-invariant subgroup of O(3), namely the O(2) group generated by the operator L_3. [The difficult problem of obtaining explicit representations of the full O(3) group will be discussed in Sec. VII.] We must now construct the operator $L_3(\Theta)$ relevant for use with the Θ basis. Note first the form of the generator of quark spin rotations:

$$W^3(\Theta) = \int dp\left[a^\dagger_{\Theta}(p)\,\frac{\sigma^3}{2}\,a_{\Theta}(p) - b^\dagger_{\Theta}(p)\,\frac{\sigma^3}{2}\,b_{\Theta}(p)\right]\ , \tag{5.10}$$

where we have suppressed the spin and SU(3) labels of the operators a and b. It is easy to see that the states $|p^+,\vec{p}_\perp;s\rangle_\Theta$ of Eq. (5.6) are eigenstates of the operator (5.10) with eigenvalue s.

The orbital component L^3 may now be defined by the relation

$$L^3(\Theta) = J^3 - W^3(\Theta)\ . \tag{5.11}$$

The requirement $[L^3(\Theta), W^3(\Theta)]=0$ is automatically satisfied as one can verify from the explicit form of J^3,

$$J^3=\sqrt{2}\int d^2x_\perp dx^- \psi_+^\dagger(x)\left(ix^2\frac{\partial}{\partial x^1}-ix^1\frac{\partial}{\partial x^2}+\frac{\sigma^3}{2}\right)\psi_+(x)$$

$$=\int dp\left[a^\dagger(p)\left(-i\frac{\overleftrightarrow{\partial}}{\partial\varphi}+\frac{\sigma^3}{2}\right)a(p)-b^\dagger(p)\left(-i\frac{\overleftrightarrow{\partial}}{\partial\varphi}+\frac{\sigma^3}{2}\right)b(p)\right]\,. \tag{5.12}$$

with $\varphi=\tan^{-1}(p_1/p_2)$. Comparing (5.12) and (5.11) one obtains the explicit expression for L^3,

$$L^3(\Theta)=\int dp\left[a_\Theta^\dagger(p)\frac{1}{i}\frac{\overleftrightarrow{\partial}}{\partial\varphi}a_\Theta(p)-b_\Theta^\dagger(p)\frac{1}{i}\frac{\overleftrightarrow{\partial}}{\partial\varphi}b_\Theta(p)\right]\,. \tag{5.13}$$

Using the operators W^3 and L^3 it is now a simple task to construct quark-antiquark combinations which correspond to the standard SU(6)×O(2) classification scheme.[25] The pion octet, for example, is described by a wave function

$$\Pi_a(p^+)=\int\frac{dq^+}{2q^+}\frac{dq^{+\prime}}{2q^{+\prime}}q_\perp dq_\perp d\varphi\,\delta(p^+-q^+-q^{+\prime})f(q^+,q_\perp)a_{(\alpha,i)}^\dagger(q^{+\prime},\vec{q}_\perp)[\lambda_a]^i_j\,b^{(\alpha,j)\dagger}(q^+,-\vec{q}_\perp)|0\rangle\,, \tag{5.14}$$

and the vector nonet by the expression

$$V_a^k(p^+)=\int\frac{dq^+}{2q^+}\frac{dq^{+\prime}}{2q^{\prime +}}q_\perp dq_\perp d\varphi\,\delta(p^+-q^+-q^{\prime +})f(q^+,q_\perp)a_{(\alpha,i)}^\dagger(q^{\prime +},\vec{q}_\perp)[\sigma^k]^\alpha_\beta[\lambda_a]^i_j\,b^{(\beta,j)\dagger}(q^+,-\vec{q}_\perp)|0\rangle\,. \tag{5.15}$$

It is a straightforward matter to construct similar expressions for L-excited meson states. A slight complication is introduced by the fact that we have explicitly represented only the O(2) subgroup (L_3) of the angular momentum group O(3). This means, for example, that for the SU(6)×O(3) multiplet $L=1$ (with mesons of $J^{PC}=0^{++},1^{++},1^{+-},2^{++}$) there will be two arbitrary functions: One will describe the $L_3=\pm 1$ components of the wave function and the other the $L_3=0$ component.

For baryons a similar construction can also be effected. The octet part of the 56, $L=0$ multiplet, for example, may be written in the form

$$\begin{aligned}B_a^s(p^+)=&\int d^4q\,d^4q'\,d^4q''\,\delta(q^2+m^2)\delta(q'^2+m^2)\delta(q''^2+m^2)\delta(p^+-q^+-q'^+-q''^+)\delta^2(\vec{q}_\perp+\vec{q}_\perp'+\vec{q}_\perp'')\\&\times h(q,q',q'')a_{(\beta,i)}^\dagger(q)a_{(\gamma,j)}^\dagger(q')a_{(\delta,k)}^\dagger(q'')\\&\times[\epsilon^{ijl}(\lambda_a)^k_l\epsilon^{\beta\gamma}\chi^\delta_{(s)}+\epsilon^{jkl}(\lambda_a)^i_l\epsilon^{\gamma\delta}\chi^\beta_{(s)}+\epsilon^{kil}(\lambda_a)^j_l\epsilon^{\delta\beta}\chi^\gamma_{(s)}]\,,\end{aligned} \tag{5.16}$$

where ϵ^{ijk} and $\epsilon^{\alpha\beta}$ denote antisymmetric symbols in three and two dimensions, respectively, and $\chi_{(s)}$ is a two-component spinor. The general baryonic expression is of the same typical form, but with some different SU(3)- and spin-dependent expression in the brackets.

VI. STATIC APPLICATIONS

Using the wave functions developed in Sec. V it is now a straightforward matter to evaluate (in the quasiparticle approximation) the matrix elements of various operators. We need only express these operators in terms of the quasiparticle creation and annihilation operators and proceed by invoking the canonical commutation relations of these objects. In this section we discuss the matrix elements of the axial charge operator $\hat{Q}_a^5$ and the magnetic-moment operator $D_a^{1\pm i2}$. These are relevant for pionic and electromagnetic transitions of hadron resonances. We will reserve for Sec. VII a discussion of the limits of the quasiparticle approximation and the possible extension of our approach to other, more complex, operators.

Let us then examine the appropriate quasiparticle expression for $\hat{Q}_a^5$. Using the momentum-space decomposition (5.7) for the (massive) quark fields $\psi(x)$, we obtain the following expression for these null-plane charges[26]:

$$\begin{aligned}\hat{Q}_a^5(0)=\int dp\sum[&a_{(\alpha,i)}^\dagger(p)\,\Gamma(p)^\alpha_\beta(\tfrac{1}{2}\lambda_a)^i_j\,a^{(\beta,j)}(p)\\&-b^{(\beta,j)\dagger}(p)\,\overline{\Gamma}(p)^\alpha_\beta(\tfrac{1}{2}\lambda_a)^i_j\,b_{(\alpha,i)}(p)],\end{aligned} \tag{6.1}$$

where

$$\begin{aligned}\Gamma(p)^\alpha_\beta&=u_+^\dagger(p,\alpha)\gamma^5u_+(p,\beta)\,,\\\overline{\Gamma}(p)^\alpha_\beta&=(-1)^{\alpha-\beta}v_+^\dagger(p,-\alpha)\gamma^5v_+(p,-\beta)\,.\end{aligned} \tag{6.2}$$

The explicit form for these matrices depends on the choice of spin basis (which is arbitrary) in (6.1). Of particular relevance for the calculation of matrix elements of the $\hat{Q}_a^5$ is the choice of the Θ basis, Eq. (5.6). Thus, suppressing the SU(3) and spin indices we have the expression

$$\hat{Q}_a^5 = \int dp\left[a_\Theta^\dagger(p)\Gamma(\Theta,\vec{p}_\perp,m)\frac{\lambda_a}{2}a_\Theta(p) - b_\Theta^\dagger(p)\Gamma^*(\Theta,\vec{p}_\perp,m)\frac{\lambda_a^*}{2}b_\Theta(p)\right], \quad (6.3)$$

where the 2×2 matrix has the explicit form

$$\Gamma(\Theta,\vec{p}_\perp,m) = \left(\frac{p_\perp}{\omega}\cos\Theta - \frac{m}{\omega}\sin\Theta\right)\hat{p}_\perp\cdot\vec{\sigma} + \left[\frac{m}{\omega}\cos\Theta + \frac{p_\perp}{\omega}\sin\Theta\right]\sigma^3, \quad (6.4)$$

where $\omega = (p_\perp^2 + m^2)^{1/2}$.

This matrix effectively specifies the transformation properties of $\hat{Q}_a^5$ under the classification algebra SU(6)×O(3). Consider, in particular, the SU(3)×SU(3)×O(2) subalgebra of that algebra, with elements

$$W_a(\Theta) = \int dp\left[a_\Theta^\dagger(p)\frac{\lambda_a}{2}a_\Theta(p) - b_\Theta^\dagger(p)\frac{\lambda_a^*}{2}b_\Theta(p)\right], \quad (6.5)$$

$$W_a{}^3(\Theta) = \int dp\left[a_\Theta^\dagger(p)\sigma^3\frac{\lambda_a}{2}a_\Theta(p) - b_\Theta^\dagger(p)\sigma^3\frac{\lambda_a^*}{2}b_\Theta(p)\right], \quad (6.6)$$

and L_3. Under this algebra $\hat{Q}_a^5$ will transform as a mixture of representations. The piece $\hat{p}_\perp\cdot\vec{\sigma}$ transforms as a representation $[(3,\bar{3})-(\bar{3},3)]_{L_3=\pm 1}$; the σ^3 term transforms as $[(8,1)-(1,8)]_{L_3=0}$.

Note that the algebraic properties of $\hat{Q}_a^5$ are independent of the form of the function Θ. Exceptions could occur only if

$$\Theta = \Theta_L \equiv \tan^{-1}(p_\perp/m), \quad (6.7)$$

or

$$\Theta = \Theta_0 \equiv -\tan^{-1}(m/p_\perp), \quad (6.8)$$

when one of the terms of Eq. (6.4) would be absent. These special cases are, however, incompatible with our over-all physical picture, as we will demonstrate at the end of this section. Thus, we can exclude the possibility of a trivial mixing scheme.[27]

To make the relation of this mixing and the "Melosh transformation" more apparent, let us rewrite $\hat{Q}_a^5$ in another basis—the so-called lightlike spin basis[23] (L) defined by

$$|p^+,\vec{p}_\perp;s\rangle_L = \exp(-i\zeta K_3)\exp(-i\eta\hat{p}_\perp\cdot\vec{E}_\perp)\times|m/\sqrt{2},\vec{0};s\rangle, \quad (6.9)$$

where

$$E^i = \sqrt{2}M^{+i} = K^i + \epsilon^{ij}J_j, \quad (6.10)$$

$\eta = p_\perp/m$, and $\zeta = \ln(\sqrt{2}p^+/m)$. Equation (6.3) then simplifies to[28]

$$\hat{Q}_a^5(0) = \int dp\left[a_L^\dagger(p)\sigma^3\frac{\lambda_a}{2}a_L(p) - b_L^\dagger(p)\sigma^3\frac{\lambda_a^*}{2}b_L(p)\right], \quad (6.11)$$

indicating that the L basis is the natural one for describing these null-plane charges. This simple form is equivalent to Eqs. (6.3) and (6.4) because the spin bases L and Θ are related by a rotation. Indeed, by comparing (6.9) and (5.6) one can show

$$|p^+,\vec{p}_\perp;s\rangle_L = |p^+,\vec{p}_\perp;s\rangle_{\Theta=\Theta_L}, \quad \Theta_L = \tan^{-1}(p_\perp/m). \quad (6.12)$$

Hence, in Eq. (6.4),

$$\Gamma(\Theta,p_\perp,m) = R^{-1}(\varphi,\Theta-\Theta_L,-\varphi)\sigma_3 R(\varphi,\Theta-\Theta_L,-\varphi). \quad (6.13)$$

From this viewpoint, the representation mixing results from the nontrivial[27] transformation between the spin basis natural for describing the $\hat{Q}_a^5$ operators (L) and the one used for classifying states (Θ). The "Melosh transformation," which was originally formulated in the coordinate space, corresponds in momentum space to the special case $\Theta = 0$ and hence to the rotation

$$R(\varphi,-\Theta_L,-\varphi) = \left(\frac{\omega+m}{2\omega}\right)^{1/2} - i\left(\frac{\omega-m}{2\omega}\right)^{1/2}\hat{p}_\perp\times\vec{\sigma}_\perp. \quad (6.14)$$

We have emphasized that the algebraic structure of $\hat{Q}_a^5$ is the same for any value of Θ.[27] It is this structure which has been exploited in applications of the Melosh transformation to pionic decays,[11] so we can be confident of recovering all of these results. The algebraic properties here refer to the transformation of the operators $\hat{Q}_a^5$ with respect to the algebra $\{W_a; W_a^3; L_3\}$. One could equivalently reverse this language and speak of the transformation properties of the *states* under the operators $\hat{Q}_a$ and $\hat{Q}_a^5$.

Consider, for example, a meson wave function M of the form [cf. Eq. (5.15)],

$$M \sim a_\Theta^\dagger\ \Sigma\lambda b_\Theta^\dagger. \quad (6.15)$$

Commutation with $\hat{Q}_a^5$ yields [with the aid of Eq. (6.3)] the result

$$[M, \hat{Q}_a^5] \sim \frac{1}{\omega}\left(\cos\Theta - \frac{m}{p_\perp}\sin\Theta\right) a_\Theta^\dagger [\vec{p}_\perp \cdot \vec{\sigma}_\perp \lambda_a, \Sigma\lambda] b_\Theta^\dagger + \frac{1}{\omega}(m\cos\Theta + p_\perp \sin\Theta) a_\Theta^\dagger [\sigma^3 \lambda_a, \Sigma\lambda] b_\Theta^\dagger . \tag{6.16}$$

In particular, for pseudoscalar mesons Π_b, $\Sigma\lambda \sim \lambda_b$, and we find

$$[\Pi_b, \hat{Q}_a^5] \sim \frac{2}{\omega}\left(\cos\Theta - \frac{m}{p_\perp}\sin\Theta\right) d_{abc} a_\Theta^\dagger \vec{p}_\perp \cdot \vec{\sigma}_\perp \lambda_a b_\Theta^\dagger + \frac{i}{\omega}(m\cos\Theta + p_\perp \sin\Theta) f_{abc} a_\Theta^\dagger \sigma^3 b_\Theta^\dagger . \tag{6.17}$$

These two terms indicate pieces of the pion wave function transform as $[(3,\bar{3})-(\bar{3},3)]$ and $[(1,8)-(8,1)]$, respectively. In a similar fashion one can calculate the chiral representation mixing of all the usual states of the quark model.

As another illustration of our procedure for calculating matrix elements, let us examine the magnetic-dipole transition operator. The relevant null-plane form is

$$D_a^{1\pm i2}(x^+) = \frac{1}{\sqrt{2}} \int d^2x_\perp dx^- (x^1 \pm ix^2) V_a^+(x) . \tag{6.18}$$

In the quasiparticle approximation it is again convenient to express this in a momentum-space representation. As was the case with $\hat{Q}_a^5$, this expression is particularly transparent in the L basis, viz.

$$D_a^{1\pm i2}(x^+) = -\frac{i}{\sqrt{2}} \nabla_{(q)}^{1\mp i2} \int dp [a_L^\dagger(p)\lambda_a a_L(p+q) - b_L^\dagger(p+q)\lambda_a^* b_L(p)]_{q=0} . \tag{6.19}$$

When studying the algebraic properties of matrix elements of this operator between states constructed in the Θ basis, we rewrite (6.19) in terms of field operators of that basis. Thus performing the rotation $R(\varphi, \Theta-\Theta_L, -\varphi)$ which connects these bases and carrying out the differentiation in (6.19), we obtain

$$D_a^{1\pm i2}(x^+) = \frac{-i}{\sqrt{2}} \int dp [a_\Theta^\dagger \lambda_a \Gamma_D^\pm a_\Theta - b_\Theta^\dagger \lambda_a^* \Gamma_D^{\pm *} b_\Theta] , \tag{6.20}$$

where, if we use the variables $p_\perp e^{\pm i\varphi} = p^1 \pm ip^2$,

$$\Gamma_D^\pm = \pm e^{\pm i\varphi}\Gamma_1 + \Gamma_2 e^{\pm i\varphi}\sigma^3 + \Gamma_3 \sigma^\pm + \Gamma_4 e^{\pm 2i\varphi}\sigma^\mp , \tag{6.21}$$

and the Γ_i are functions of $p_\perp$ and Θ only. These functions are derived and listed in Appendix D. From the explicit display of φ and spin dependences given in (6.21), it is clear that under the algebra $\{W_a; W_a^3; L_3\}$, the four terms of (6.21) transform as

$$[(8,1)+(1,8)]_{L_3=\pm 1},$$

$$[(8,1)-(1,8)]_{L_3=\pm 1},$$

$$[(3,\bar{3})\oplus(\bar{3},3)]_{L_3=0},$$

and

$$[(3,\bar{3})\oplus(\bar{3},3)]_{L_3=\pm 2},$$

respectively. If, for example, we now sandwich Eq. (6.20) between states of the <u>56</u>, $L=0$ baryons, only the third term will contribute, and one obtains, among other things, the famous ratio $\mu_n/\mu_p = -\frac{2}{3}$. The numerous other relations for the electromagnetic excitation of hadronic resonances have been extensively explored[29] and found to agree with available data.

To conclude this section we must comment on two exceptional values of Θ in our basis specification (5.6) which would lead to trivial representation mixing in Eq. (6.3). If, on the one hand, $\Theta = \Theta_0 = \tan^{-1}(m/p_\perp)$, then

$$\hat{Q}_a^5 \sim \hat{p}_\perp \cdot \vec{\sigma}_\perp \lambda_a / 2 \tag{6.22}$$

would transform purely as $[(3,\bar{3})-(\bar{3},3)]$ under the algebra $\{W_a; W_a^3\}$. This is unacceptable when we consider the static limit $p_\perp/m \to 0$. One expects in this limit that the $\hat{Q}_a^5$ should belong to a static SU(6) algebra,

$$\hat{Q}_a^5 \underset{p_\perp/m \to 0}{\sim} \sigma_3 \lambda_a / 2 . \tag{6.23}$$

This is clearly incompatible with Eq. (6.22).

If, on the other hand, $\Theta = \Theta_L = -\tan^{-1}(p_\perp/m)$, the L and Θ bases coincide and Eq. (6.23) is trivially satisfied, but there is no representation mixing. Aside from being phenomenologically untenable, this situation is totally incompatible with the spirit of our approach. Since m is a measure of spontaneous chiral-symmetry breaking, in the limit $m \to 0$ we should recover a chiral-symmetric world with degenerate SU(3)×SU(3) multiplets. In particular, the pion must become degenerate with a $J^P = 0^+$, $G = +$ state. Within the quark model, such a state can only have $I = 0$. Therefore, the pion would lie in a $(3,\bar{3})\oplus(\bar{3},3)$ representation. However, a glance at Eq. (6.17) shows that with $\Theta = \Theta_L$ this limiting behavior would not be possible. The exclusion of the angles Θ_0 and Θ_L indicates that for finite quark mass m, there is always some nontrivial mixing.

The algebraic properties of various operators (e.g., $\hat{Q}_a^5$ and $D_a^{1\pm i2}$) can consequently be discussed independently of any specific choice of Θ. It is of course still interesting to ask if any specific values of Θ are preferred. This is really a dynamical question. For simplicity, however, one might prefer $\Theta = 0$. This choice satisfies all the requirements we have posed and corresponds, in fact, to the original form of the Melosh transformation.[10]

VII. DISCUSSION

In the previous sections we have utilized extensively a quasiparticle approximation for the hadronic wave functions on the null plane. We will now try to clarify this approximation scheme and point the way for probing more deeply into the precise structure of the wave functions. Let us begin by looking at the SU(6)×O(3) classification group. One notes that while we have assumed the existence of an O(3) group, we have constructed explicitly only the generator of its O(2) subgroup. The reason for this is fundamental in the null-plane approach: The angular momentum components J_1 and J_2 are—like the Hamiltonian—dynamical quantities.[30,31] There is, consequently, no possibility of specifying J_1 and J_2 (and hence L_1 and L_2) in terms of the quasiparticles alone, since—like the mass—these quantities depend on details of the force which binds the quasiparticles together.

In the free-quark model, the dynamics are trivial, and J_1 and J_2 may be explicitly written down. An SU(6)×O(3) classification algebra has thus been constructed by Melosh[32] for this case. The approach is not without drawbacks, however. First of all, there is in the free-quark model no PCAC relation, so there are no definite predictions for pionic transitions. Secondly, the requirements of boost invariance and satisfactory angular momentum properties require that the representation mixing involve states with different numbers of quarks,[33] a significant departure from the original scheme. This poses no difficulty for matrix elements of states of the same mass, but it spoils the results obtained for arbitrary mass transitions. Additional terms in Eqs. (6.3) and (6.20) vitiate the results obtained here.

For these reasons it seems clear that the assumption of free-quark dynamics is too severe for a realistic description of physical processes. Thus we adhere to a more general picture of quasiparticles bound by some (possibly complex) interaction. One should emphasize, of course, that the resulting algebraic structure is still the same as that which has been employed in the successful phenomenological analyses.

An alternative approach to the angular properties—short of a complete dynamical solution of the theory—is to study the chiral transformation properties of the relevant Poincaré generators. In this spirit[34] Weinberg and others have investigated the commutation relations of the chiral charges $\hat{Q}_a^5$ with the mass and helicity-raising operators. We anticipate that within the present framework these relations will provide significant constraints on the mass and spin properties of the states.

The essential feature underlying our model is the spontaneous breakdown of chiral symmetry. As emphasized by Nambu and co-workers, analogous systems exist in the form of, say, ferromagnets (where rotational symmetry is broken) or superconductors (where gauge invariance is broken). Our primary emphasis so far has been on the spontaneously generated fermion mass (analogous to the energy gap of a superconductor) and the associated quasiparticle excitations. Another striking feature of systems of this type is the degenerate, or asymmetric, ground state (vacuum). This ground state is not structureless: In the BCS ground state of a superconductor for example, there are an infinite number of electron pairs. The analogous vacuum structure in chiral models is manifested by the nonvanishing expectation value of σ in the σ model or of $\bar{\psi}\psi$ in the Nambu–Jona-Lasinio model. Loosely speaking, the vacuum is described as being comprised of σ mesons or of quark-antiquark pairs in these two models.

The structure of this chiral sea is exhibited in the Nambu–Jona-Lasinio model by a transformation (the analog of the "Bogoliubov transformation") from the asymmetric physical vacuum to a fictitious chiral-symmetric vacuum. In the null-plane formulation of the σ model adopted in Sec. III, a similar construction is possible. As the "dressing" of the vacuum results from the acquisition of a vacuum expectation value by the σ field, one anticipates that the dressing operator will, at fixed x^+, be closely related to the translation operator for the σ field, $e^{iN(x^+)}$, where

$$N(x^+) = 2\langle\sigma\rangle \int d^4y\, \delta(x^+ - y^+)\partial^+\sigma(y)\,. \tag{7.1}$$

In fact, one can show that the null-plane operator

$$\overline{Q}_a^5 = e^{-iN(x^+)}\hat{Q}_a^5(x^+)e^{iN(x^+)} \tag{7.2}$$

has the following properties: (i) It is conserved, i.e., $(d/dx^+)\overline{Q}_a^5 = 0$; (ii) the canonical fields σ, π, and ψ obey the same commutation relations with $\overline{Q}_a^5$ as with Q_a^5. Thus the $\overline{Q}_a^5$, just like the static charges Q_a^5, can serve as the generators of chiral symmetry.[35] If we now define a fictitious vacuum state[36]

$$|\Omega(x^+)\rangle = e^{iN(x^+)}|0\rangle \tag{7.3}$$

then

$$\overline{Q}_a^5|\Omega(x^+)\rangle = 0\,, \tag{7.4}$$

i.e., $|\Omega(x^+)\rangle\rangle$ is a chiral scalar. Equation (7.3) thus demonstrates that $e^{-iN(x^+)}$ specifies the structure of the chiral sea.

We expect the construction (7.3) to be generally valid. In a quark model (such as the Nambu–Jona-Lasinio model) the chiral sea is comprised of quark-antiquark pairs, with each pair being roughly equivalent to some effective σ field. Any detailed distinctions will, in fact, be irrelevant until one begins to probe the short-distance structure of the chiral sea. In order to understand how this may be done, let us consider what happens when a fermion excitation is present in the sea. Interaction between the fermion and the sea will produce a local deformation or polarization of the sea. In other words, the effective field $\sigma(x)$ will become c-number function $f(x)$ which is asymptotically constant, but is locally distorted by the fermion. This situation can be represented semiclassically by the state vector[37]

$$|\psi\rangle = a^\dagger \exp\left[i2\int d^4y\, \delta(x^+ - y^+) f(y)\partial^+\sigma(y)\right] |\Omega(x^+)\rangle\rangle \,. \tag{7.5}$$

In a similar fashion one can represent quark-antiquark and three-quark states. The expression corresponding to (7.5) is then complicated by the wave function describing the positions of the various quarks.

In the discussions of Secs. V and VI we have considered the effects of interactions only insofar as they generate the quark's mass. This is justified for the calculation of matrix elements of $\hat{Q}_a^5$ by the arguments of Sec. IV, whereby chiral representation mixing is seen as a direct consequence of the spontaneously generated mass. The same approximation has been used to calculate matrix elements of the first moment of the electromagnetic current and has been found to be phenomenologically viable. For higher moments of the currents, which probe in greater detail the distribution of charge within a state, we expect this approximation to become less reliable.[38] In deep-inelastic scattering, where the bilocal structure of the states is directly probed, the approximation will become completely invalid. This process thus permits a direct probe of the interactions which bind the quasiparticles and polarize the sea around them.

VIII. DEEP-INELASTIC SCATTERING

We have already mentioned that deep-inelastic scattering provides a sensitive probe of hadronic wave functions. In this section we will expand on this remark briefly and discuss the relevance of the present approach for this problem. We recall that inelastic lepton-hadron cross sections are specified by the absorptive parts of the current-hadron forward scattering amplitudes

$$W_{ab}^{\mu\nu}(q,p) = \int d^4x\, e^{iqx} \langle p|[J_a^\mu(x), J_b^\nu(0)]|p\rangle \,, \tag{8.1}$$

where J_a^μ denotes some linear combination of the vector and axial-vector currents. It is well known that in the deep-inelastic limit ($q^2 \to \infty$ with $\xi = -q^2/2q\cdot p$ fixed) the x integral of Eq. (8.1) is dominated by contributions from the null plane, $x^+=0$. Causality further restricts these contributions to the light cone, $x^2=0$. Thus, assuming a canonical (scaling) structure for the light-cone commutators, we obtain the expression

$$W_{ab}^{\mu\nu}(q,p) \underset{\substack{q^2\to\infty \\ \xi \text{ fixed}}}{\longrightarrow} \sum_n \int d^4x\, e^{iqx} C_{ab}^{\mu\nu,n}(x^2) \times \langle p|J_n(x,0)|p\rangle_{x^2=0} \,. \tag{8.2}$$

Here $C_{ab}^{\mu\nu,n}(x^2)$ denote (c-number) functions with canonically prescribed singularities at $x^2=0$ and $J_n(x,0)$ are regular bilocal operators. The various "structure functions" $F_i(\xi)$ are simply Fourier transforms of the dominant bilocals.

In principle one can calculate the structure functions directly in terms of the null-plane wave function of the target state $|p\rangle$. However, since the operators J_a^μ do not annihilate the vacuum, and since we lack for these operators any simple analog of the mixing equation (3.20), we can no longer justify the use of the quasiparticle wave functions of Sec. V. Stated differently, the operators J_a^μ (unlike, say, the $\hat{Q}_a^5$) *are* sensitive to details of the the interactions between the quasiparticles and to the structure of the chiral sea in which the quasiparticles reside.

For a rough orientation let us briefly ignore these complications (as if the dynamics of the free-quark model were valid). In that case we would obtain[40] the result

$$F_{2n}(\xi)/F_{2p}(\xi) = \tfrac{2}{3} \tag{8.3}$$

for arbitrary values of ξ. This prediction is, of course, contradicted by experimental data[41] which show a ratio decreasing from a value close to 1 at $\xi=0$ to about $\frac{1}{4}$ at $\xi=1$.

To obtain an accurate description of the structure functions it is clearly necessary to incorporate into this naive picture further details of the quasiparticle interactions. This may be done in several ways. One is to study the moments of the structure functions, $\int d\xi\, \xi^n F_i(\xi)$. These moments, as is well known, are related to matrix elements of the local operators $O_n^{\mu_1\cdots\mu_n}(0)$ generated in the Taylor expansion of the bilocal

$J_n(x,0)$. One might search for some analog of the mixing equation (3.20) which would describe the properties of the $O_n^{\mu_1\cdots\mu_n}$ simply in terms of the quasiparticles. Alternatively, one can focus attention on the wave functions and investigate the relations of the quasiparticle picture to the standard quark-parton description.[42] These questions are presently under study.

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APPENDIX A: THE AXIAL CHARGES

The various charge operators associated with the axial-vector current in the case of Nambu-Goldstone realization of chiral symmetry must be defined with care. In this appendix we discuss this problem and derive some of the important properties of these charges.

One starts with the well-defined matrix elements of the axial-vector current $\langle\beta|A_a^\mu(0)|\alpha\rangle$ given by Eq. (2.4) of the text. The matrix elements of the static charges defined by Eq. (2.1) are related to the above by

$$\langle\beta|Q_a^5|\alpha\rangle=\frac{\langle p_\beta\|p_\alpha\rangle}{(p_\alpha+p_\beta)^0}\lim_{\vec{q}\to 0}\langle\beta|A_a^0|\alpha\rangle\,. \tag{A1}$$

Similarly, the matrix elements of the null-plane charges (2.8) are

$$\langle\beta|\hat{Q}_a^5|\alpha\rangle=\frac{\langle p_\beta\|p_\alpha\rangle}{(p_\alpha+p_\beta)^+}\lim_{q^+,\vec{q}_\perp\to 0}\langle\beta|A_a^+|\alpha\rangle\,. \tag{A2}$$

Because of the pion-pole term in $\langle\beta|A_a^\mu|\alpha\rangle$, however, both of the limits (A1) and (A2) are path-dependent and hence not well-defined.

The principle of PCAC furnishes the means to resolve this ambiguity: We should start with $m_\pi\neq 0$ where no ambiguity arises and take the smooth $m_\pi\to 0$ limit. Let us now rewrite Eq. (2.4) as

$$\langle\beta|A_a^\mu|\alpha\rangle_{m_\pi}=if_\pi\langle\beta,\pi_a|\alpha\rangle\frac{q^\mu}{q^2+m_\pi^2}+\langle\beta|A_a^\mu|\alpha\rangle_N\,. \tag{A3}$$

Using the PCAC relation $\partial_\mu A_a^\mu=m_\pi^2 f_\pi\pi_a$ we can easily rederive Eq. (2.5),

$$\langle\beta,\pi_a|\alpha\rangle=-(if_\pi)^{-1}q_\mu\langle\beta|A_a^\mu|\alpha\rangle_N\,. \tag{A4}$$

Substituting (A4) into (A3), we get

$$\langle\beta|A_a^\mu|\alpha\rangle_{m_\pi}=\left(g^\mu{}_\nu-\frac{q^\mu q_\nu}{q^2+m_\pi^2}\right)\langle\beta|A_a^\nu|\alpha\rangle_N\,. \tag{A5}$$

This expression can now be used in (A1) and (A2) with the unambiguous results:

$$\frac{\langle\beta|Q_a^5|\alpha\rangle_{m_\pi}}{\langle p_\beta\|p_\alpha\rangle}=\left[\frac{1}{p_\alpha^0+p_\beta^0}\frac{m_\pi^2}{-q_0^2+m_\pi^2}\langle\beta|A_a^0|\alpha\rangle_N\right]_{\vec{q}=0} \tag{A6}$$

and

$$\frac{\langle\beta|\hat{Q}_a^5|\alpha\rangle_{m_\pi}}{\langle p_\beta\|p_\alpha\rangle}=\left[\frac{1}{p_\alpha^++p_\beta^+}\langle\beta|A_a^+|\alpha\rangle\right]_{q^+=\vec{q}_\perp=0} \tag{A7}$$

The correct definition for these charges in the chiral-symmetric world must correspond to the $m_\pi\to 0$ limit of the above.

Let us first consider the static charges. Using (A6), we can conclude[43]

$$\frac{\langle\beta|Q_a^5|\alpha\rangle}{\langle p_\beta\|p_\alpha\rangle}=\lim_{m_\pi\to 0}\frac{\langle\beta|Q_a^5|\alpha\rangle_{m_\pi}}{\langle p_\beta\|p_\alpha\rangle}=\begin{cases}0 & \text{when } m_\alpha\neq m_\beta\\ \dfrac{1}{2p_\alpha^0}\langle\beta|A_a^0|\alpha\rangle_N\Big|_{q^\mu=0} & \text{when } m_\alpha=m_\beta\,.\end{cases} \tag{A8}$$

Note the vanishing of this matrix element when $m_\alpha\neq m_\beta$ is a result of cancellation between the pole and nonpole terms in (A5), whereas the nonzero result for the case $m_\alpha=m_\beta$ comes from the nonpole term alone. The conservation of the static charges is derived from substituting (A8) in the simple relation

$$\left\langle\beta\left|\frac{d}{dx^0}Q_a^5\right|\alpha\right\rangle=-iq^0\langle\beta|Q_a^5|\alpha\rangle\,. \tag{A9}$$

One thus obtains the result,

$$\frac{d}{dx^0}Q_a^5=0\,. \tag{A10}$$

It is trivial to establish that Q_a^5 are invariant under rotations. In contrast to the case of conventional symmetries (Wigner type), however, these charges are not boost-invariant (hence they are not Lorentz scalars). To see this, we observe that the right-hand side of (A8) would be boost-invariant only if the 4-vector $\langle\beta|A_a^\mu|\alpha\rangle_N$ were proportional to p_α^μ. The fact that this is not the case for Nambu-Goldstone realization of the symmetry can be seen from a simple example: When both α and β are single nucleon states,

$$\begin{aligned}\frac{1}{2p^0}\langle p,s|A^0|pr\rangle_N&=\frac{1}{2p^0}u_s^\dagger(p)\gamma_5 u_r(p)\\&=\frac{1}{p^0}(\vec{\sigma}\cdot\vec{p})_{sr}\\&=(\vec{\sigma}\cdot\vec{v})_{sr}\,,\end{aligned} \tag{A11}$$

where $\vec{v}$ is the velocity of the nucleon. When the nucleon mass is nonzero, this is clearly not boost-invariant. On the other hand, if $m=0$ and $|v|=1$, the symmetry is realized in the conventional (Wigner) mode.

We turn next to the null-plane charges. Since the right-hand side of (A7) is independent of m_π (the pion pole being automatically absent), we have simply

$$\frac{\langle\beta|\hat{Q}_a^5|\alpha\rangle}{\langle p_\beta\|p_\alpha\rangle}=\frac{1}{p_\alpha^+ + p_\beta^+}\langle\beta|A_a^+|\alpha\rangle_N\Big|_{q^+=\vec{q}_\perp=0}. \tag{A12}$$

This expression is manifestly boost-invariant in the x^3 direction. These charges are *not* conserved. This can be seen as follows:

$$\left\langle\beta\left|\frac{d}{dx^+}\hat{Q}_a^5\right|\alpha\right\rangle=-iq^-\langle\beta|\hat{Q}_a^5|\alpha\rangle = i\frac{m_\alpha^2-m_\beta^2}{(p_\alpha^+ + p_\beta^+)^2}\langle\beta|A_a^+|\alpha\rangle_N\langle p_\beta\|p_\alpha\rangle. \tag{A13}$$

APPENDIX B: NULL-PLANE QUANTIZATION

We collect here null-plane quantization rules which are useful for understanding results in the text. Details on this procedure can be found in Ref. 22.

The null-plane components for 4-vectors are

$$x^\pm=(x^0\pm x^3)/\sqrt{2}, \quad \vec{x}^\perp=(x^1,x^2). \tag{B1}$$

The scalar product is

$$x\circ y=\vec{x}_\perp\cdot\vec{y}_\perp - x^-y^+ - x^+y^- \equiv g_{\mu\nu}x^\mu y^\nu, \tag{B2}$$

which also defines the metric tensor $g_{\mu\nu}$.

In the canonical formulation of null-plane dynamics, the variable x^+ plays the role of "time." The "Hamiltonian" is $P^-=(P^0-P^3)/\sqrt{2}$. The "equations of motion" involve $\partial_+=\partial/\partial x^+=-\partial^-$, whereas the "equations of constraint" only involve $\partial_-=\partial/\partial x^-=-\partial^+$ and $\vec{\partial}^\perp=\partial/\partial\vec{x}^\perp$.

In the σ model, the dynamically independent fields are $\psi_+(x)=\frac{1}{2}(1+\alpha_3)\psi(x)$, $\sigma(x)$, and $\vec{\pi}(x)$. The canonical conjugates for these are $\psi_+^\dagger(x)$, $\partial_-\sigma(x)$, and $\partial_-\vec{\pi}(x)$, respectively. The canonical commutation relations are

$$\{\psi_+(x),\psi_+^\dagger(y)\}\delta(x^+-y^+)=\frac{1}{2\sqrt{2}}(1+\alpha_3)\delta^4(x-y), \tag{B3}$$

$$[\sigma(x),\partial_-\sigma(y)]\delta(x^+-y^+)=\tfrac{1}{2}i\delta^4(x-y), \tag{B4}$$

$$[\pi_a(x),\partial_-\pi_b(y)]\delta(x^+-y^+)=\tfrac{1}{2}i\delta_{ab}\delta^4(x-y). \tag{B5}$$

All other commutators (anticommutators) vanish. The fermion field components $\psi_-(x)=\frac{1}{2}(1-\alpha_3)\psi(x)$ are dependent fields related to $\psi_+(x)$ by an equation of constraint. Its commutation relations with other fields are hence determined from those for ψ_+. In the σ model

$$\psi_-(x)=\frac{i}{2\sqrt{2}}\int dy^-\epsilon(x^--y^-)[i\vec{\alpha}_\perp\cdot\vec{\partial}_\perp+G\beta(\sigma+i\vec{\pi}\circ\vec{\tau}\gamma_5)] \times\psi_+(x^+,y^-,\vec{x}^\perp). \tag{B6}$$

Note also Eqs. (B4) and (B5) can be integrated in the variable x^- to give

$$[\sigma(x),\sigma(y)]_{x^+=y^+}=-\tfrac{1}{4}i\epsilon(x^--y^-)\delta^2(\vec{x}^\perp-\vec{y}^\perp), \tag{B7}$$

$$[\pi_a(x),\pi_b(y)]_{x^+=y^+}=-\tfrac{1}{4}i\delta_{ab}\epsilon(x^--y^-)\delta^2(\vec{x}^\perp-\vec{y}^\perp). \tag{B8}$$

The choice of the Green's function $\epsilon(x^--y^-)$ in Eqs. (B6), (B7), and (B8) is determined by the requirement that the Poincaré generators (derived from the Lagrangian by the usual variation procedure) have the correct Hermiticity properties and satisfy the Poincaré algebra.

APPENDIX C: ALGEBRAIC PROPERTIES OF $\hat{Q}_a^5$

Using the definitions (3.5) and (3.11), in the σ model we have

$$\hat{Q}_a^5(x^+)=\int d^2x^\perp dx^-\left[\psi_+^\dagger(x)\gamma_5\frac{\tau_a}{2}\psi_+(x)+\phi(x)\overleftrightarrow{\partial}_-\pi_a(x)\right], \tag{C1}$$

where $\phi(x)=\sigma(x)-\langle\sigma\rangle$ satisfies the same canonical commutation relations as $\sigma(x)$ given in Appendix B. Now, $\phi(x)$ and $\pi_a(x)$ commute with the first term in (C1) and

$$[\phi(x),\phi(y)\overleftrightarrow{\partial}_-\pi_a(y)]_{x^+=y^+}=-i\pi_a(x)\delta^2(\vec{x}^\perp-\vec{y}^\perp)\delta(x^--y^-)-\tfrac{1}{4}i\delta^2(\vec{x}^\perp-\vec{y}^\perp)\partial_-^{(y)}[\pi_a(y)\epsilon(x^--y^-)],$$

$$[\pi_b(x),\phi(y)\overleftrightarrow{\partial}_-\pi_a(y)]_{x^+=y^+}=i\phi(x)\delta_{ab}\delta^2(\vec{x}^\perp-\vec{y}^\perp)\delta(x^--y^-)+\tfrac{1}{4}i\delta_{ab}\delta^2(\vec{x}^\perp-\vec{y}^\perp)\partial_-^{(y)}[\phi(y)\epsilon(x^--y^-)]. \tag{C2}$$

Upon integrating these equations in the variable y^-, we arrive at the commutators $[\phi(x),\hat{Q}_a^5]$ and $[\pi_b(x),\hat{Q}_a^5]$

given in Eq. (3.14). It is to be noted that canonical null-plane quantization rules in the σ model already give rise to "Schwinger terms" in (C2) which result in the surface terms in (3.14). These surface terms do not contribute to matrix elements between localizable states.

To study the algebra satisfied by $\hat{Q}_a$ and $\hat{Q}_a^5$, we observe that $\hat{Q}_a = Q_a$ are just generators for the ordinary isospin. Hence,

$$[\hat{Q}_a, \hat{Q}_b] = i\epsilon_{abc}\hat{Q}_c \,. \tag{C3}$$

Furthermore, since each term in the integrand of $\hat{Q}_a^5$, Eq. (C1), is manifestly an isovector, it follows that

$$[\hat{Q}_a^5, \hat{Q}_b] = i\epsilon_{abc}\hat{Q}_c^5 \,. \tag{C4}$$

The only nontrivial commutator is therefore $[\hat{Q}_a^5, \hat{Q}_b^5]$. Using definition (C1) and the quantization rules of Appendix B, it is a straightforward exercise to obtain

$$\begin{aligned}[\hat{Q}_a^5(x^+), \hat{Q}_b^5(x^+)] = i\epsilon_{abc}\hat{Q}_c - \tfrac{1}{2}i\delta_{ab}\langle\sigma\rangle \int d^2x^\perp dx^- d^2y^\perp dy^- \,\delta^2(\vec{x}^\perp - \vec{y}^\perp)\\ \times[\partial_-^{(y)}\sigma(y)\delta(x^- - y^-) + \sigma(y)\delta'(x^- - y^-)\\ + \partial_-^{(x)}\sigma(x)\delta(x^- - y^-) + \sigma(x)\delta'(x^- - y^-) + f_\pi\delta'(x^- - y^-)] \,.\end{aligned} \tag{C5}$$

The second term vanishes upon change of integration variables $x \leftrightarrow y$.

APPENDIX D: MAGNETIC-DIPOLE TRANSITION OPERATOR

We indicate the derivation of (6.20) from (6.19). For clarity, we consider the case of $1 + i2$ and focus our attention on the $a_L^\dagger \lambda_a a_L$ term only.

Using new variables $p_\perp, \varphi$, related to p_1 and p_2 by

$$p_\perp e^{\pm i\varphi} = p^1 \pm ip^2 \,, \tag{D1}$$

we get

$$\frac{\partial}{\partial p_1} + i\frac{\partial}{\partial p_2} = e^{i\varphi}\left(\frac{\partial}{\partial p_\perp} + \frac{i}{p_\perp}\frac{\partial}{\partial\varphi}\right). \tag{D2}$$

From (5.6) and (6.12) we see

$$a_L^\dagger(p, s) = a_\Theta^\dagger(p, r) R(\varphi, \Theta_L - \Theta, -\varphi)_s^r \tag{D3}$$

and

$$a_L(p, s) = R^{-1}(\varphi, \Theta_L - \Theta, -\varphi)_r^s a_L(p, r) \,. \tag{D4}$$

Consequently,

$$a_L^\dagger(p, s)\lambda_a \frac{\partial}{\partial p_\perp} a_L(p, s) = \sum_r a_\Theta^\dagger(p, r)\lambda_a \left\{e^{-i\varphi\sigma_3/2}\left[\tfrac{1}{2}i\sigma_2 \frac{\partial}{\partial p_\perp}(\Theta_L - \Theta)\right] e^{i\varphi\sigma_3/2}\right\}_s^r a_\Theta(p, s) + a_\Theta^\dagger(p, s)\lambda_a \frac{\partial}{\partial p_\perp} a_\Theta(p, s) \tag{D5}$$

and

$$a_L^\dagger(p, s)\lambda_a \frac{\partial}{\partial\varphi} a_L(p, s) = \sum_r a_\Theta^\dagger(p, r)\lambda_a \{R(-\tfrac{1}{2}i\sigma_3)R^{-1} + \tfrac{1}{2}i\sigma_3\}_s^r a_\Theta(p, s) + a_\Theta^\dagger(p, s)\lambda_a \frac{\partial}{\partial\varphi} a_\Theta(p, s) \,. \tag{D6}$$

Evaluating the curly brackets in (D5) and (D6) and combining these results, we obtain

$$a_L^\dagger(p, s)\lambda_a e^{i\varphi}\left(\frac{\partial}{\partial p_\perp} + \frac{i}{p_\perp}\frac{\partial}{\partial\varphi}\right) a_L(p, s) = a_\Theta^\dagger(p, r)\lambda_a (e^{i\varphi}\Gamma_1 + \Gamma_2\sigma_3 e^{i\varphi} + \Gamma_3\sigma_+ + \Gamma_4\sigma_- e^{2i\varphi})_s^r a_\Theta(p, s) \,, \tag{D7}$$

where

$$\begin{aligned}\Gamma_1 &= \frac{\partial}{\partial p_\perp} + \frac{i}{p_\perp}\frac{\partial}{\partial\varphi}, \qquad \Gamma_2 = -\frac{1}{p_\perp}\sin^2\tfrac{1}{2}(\Theta_L - \Theta), \\ \Gamma_3 &= \tfrac{1}{2}\left(\frac{\sin(\Theta_L - \Theta)}{p_\perp} + \frac{m}{p_\perp^2 + m^2} - \frac{\partial\Theta}{\partial p_\perp}\right), \qquad \Gamma_4 = \tfrac{1}{2}\left(\frac{\sin(\Theta_L - \Theta)}{p_\perp} - \frac{m}{p_\perp^2 + m^2} + \frac{\partial\Theta}{\partial p_\perp}\right).\end{aligned} \tag{D8}$$

The derivations for the $b_L^\dagger \lambda_a b$ term and for the case of $1-i2$ are identical. These results together imply Eq. (6.20).

It is to be noted that the derivative terms in Γ_1 do not affect the total transverse momentum of the states on which $D^{1\pm i2}$ operates; they affect only the internal wave function of the quarks. To be specific, consider the action of $a_\Theta^\dagger(p)(\partial/\partial p_\perp)\, a_\Theta(p)$ on the state (5.14). Concentrating only on the transverse degrees of freedom,

$$a_\Theta^\dagger(\vec{p}_\perp)\frac{\partial}{\partial p_\perp}a_\Theta(\vec{p}_\perp)\int d^2q_\perp f(q_\perp)a_\Theta^\dagger(\vec{q}_\perp)b_\Theta^\dagger(-\vec{q}_\perp)|0\rangle = f(p_\perp)a_\Theta^\dagger(\vec{p}_\perp)\frac{\partial}{\partial p_\perp}b_\Theta^\dagger(-\vec{p}_\perp)|0\rangle + f'(p_\perp)a_\Theta^\dagger(\vec{p}_\perp)b_\Theta^\dagger(-\vec{p}_\perp)|0\rangle \tag{D9}$$

when this state is contracted with some other state [with wave function $g(q_\perp)$, say] to form the transition matrix element, the derivative on $b^\dagger$ above will get transferred to g as well. The algebraic properties of these operators can therefore be inferred as if the derivatives were not there.

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[15] It is also possible for the null-plane charge to create states of finite mass and infinite momentum q^-. The vacuum annihilation property (2.9) holds insofar as such states may be ignored.

[16] This relation is rigorously true on the space of single-particle states, precisely the sector of the complete Hilbert space on which all our applications will be made.

[17] F. L. Feinberg, Phys. Rev. D 7, 540 (1973).

[18] Note that the $\hat{Q}_a^5$ are not conserved and hence the multiplets of the null-plane algebra are not mass-degenerate. The arguments leading to Eq. (2.11) can in fact be used to show that each multiplet involves physical states with an infinite range of masses. See Ref. 7.

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[24] Were we to choose $\hat{n}_\perp = \hat{p}_\perp$ we would instead obtain

$$\begin{aligned} Y|p^+, \vec{p}_\perp; s\rangle &= e^{-i\chi K_3} e^{-i\xi \hat{p}'_\perp \cdot \vec{K}_\perp} e^{i\Theta \hat{p}'_\perp \cdot \vec{J}_\perp} |m/\sqrt{2}, \vec{0}; -s\rangle \\ &= [e^{-i\chi K_3} e^{-i\xi \hat{p}'_\perp \cdot \vec{K}_\perp} e^{-i\Theta \hat{p}'_\perp \cdot \vec{J}}] e^{2i\Theta \hat{p}'_\perp \cdot \vec{J}} |m/\sqrt{2}, \vec{0}; -s\rangle \\ &= |p^+, \vec{p}'_\perp; r\rangle R_{\hat{p}'_\perp}(2\Theta)^r_{-s}. \end{aligned}$$

One sees that an extra rotation $\exp[2i\Theta \hat{p}'_\perp \cdot \vec{J}]$ spoils the relation (5.4) by introducing a mixture of terms involving all values of the spin index.

[25] See, for example, R. Carlitz and M. Kislinger, Phys. Rev. D 2, 336 (1970).

[26] The absence of a term $a^\dagger(p,s)\, b^\dagger(-p,r)$ is due to the kinematical constraint of the x^- integration. This is just the vacuum annihilation property discussed in Sec. II.

[27] This overcomes one of the objections raised to the transformation of Melosh's thesis (Ref. 10). See Eichten, Feinberg, and Willemson, Ref. 23.

[28] This is precisely the same as the form of W_a^3 in the

basis [see Eq. (6.6)]. It is easy to understand why $\hat{Q}_a^5$ is diagonal in the L basis: $\hat{Q}_a^5$ commutes with the boost generators K_3 and $\vec{E}_\perp$.

[29] F. J. Gilman and I. Karliner, Ref. 11.

[30] Physically this is easy to understand. A wave function at $x^+=0$ includes components with arbitrary values of virtual momentum p^-. These components carry arbitrary amounts of angular momentum along the orthogonal directions J_1 and J_2.

[31] Here we differ with the philosophy of Melosh [H. J. Melosh, Phys. Rev. D 9, 1095 (1974)] which led him to reject the transformation derived in his thesis (Ref. 10). See S. P. de Alwis and J. Stern, CERN Report No. CERN-TH-1783 (unpublished).

[32] See H. J. Melosh, Ref. 31.

[33] H. Osborn, Nucl. Phys. B80, 113 (1974).

[34] S. Weinberg, Phys. Rev. Lett. 22, 1023 (1969); S. Fubini and G. Furlan, Nuovo Cimento Lett. 3, 168 (1970).

[35] Since neither Q_a^5 nor $\bar{Q}_a^5$ annihilate the vacuum, they are not, strictly speaking, well-defined operators on the Hilbert space of normalizable states. This pathology means that while we cannot claim Q_a^5 and $\bar{Q}_a^5$ to be equal, we can utilize either as a chiral-symmetry generator.

[36] Note that $|\Omega(x^+)\rangle$ has $p^+=\vec{p}_\perp=0$ but $p^-=\infty$. This is an exact analog of the Nambu–Jona-Lasinio model, where the fictitious ground state lies infinitely higher in energy than the true ground state.

[37] T. D. Lee and G. C. Wick, Phys. Rev. D 9, 2291 (1974); W. A. Bardeen *et al.*, *ibid.* 11, 1094 (1975).

[38] Some evidence for this is found in a study of the second moment of the electromagnetic current. Considering only quasiparticle terms one obtains a prediction that the neutron's charge radius, $\langle r_n^2\rangle$ should vanish. Experimentally, its value is roughly 20% that of a proton, a deviation rather more severe than those found in the magnetic-moment predictions.

[39] R. Jackiw, R. Van Royen, and G. B. West, Phys. Rev. D 2, 2473 (1970).

[40] S. P. de Alwis, Nucl. Phys. B55, 427 (1973).

[41] See, for instance, *Proceedings of the Sixth International Symposium on Electron and Photon Interactions at High Energies, Bonn, W. Germany*, edited by H. Rollnik and W. Pfeil (North-Holland, Amsterdam, 1974), p. 236.

[42] R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, New York, 1972).

[43] We assume that as $m_\pi \to 0$ the masses m_α and m_β will not become equal if $m_\alpha \neq m_\beta$ to begin with. If this were not the case, the symmetry would be of the conventional type (Wigner mode).