PHYSICAL REVIEW D VOLUME 11, NUMBER 5 1 MARCH 1975

ρNN and $\rho N\Delta$ couplings from Reggeon duality

Paul Hoyer*

Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11794

(Received 3 December 1974)

We show how the cancellation between the nucleon and $\Delta(1232)$ contributions to finite-energy sum rules for $\pi N \to \pi N$ amplitudes can be used to determine the πNN and $\pi N\Delta$ coupling constants. We then apply the same method to finite-energy sum rules for the $\rho N \to \pi N$ and $\rho N \to \rho N$ Reggeon amplitudes. A unique solution is found, for which the ρNN nonflip coupling vanishes and the $\rho N\Delta$ coupling is of the $M1$ type. The relative size of the ρNN flip and $\rho N\Delta$ $M1$ couplings is also in agreement with experiment.

I. INTRODUCTION

Phenomenological studies of two-body reaction amplitudes[1] have revealed a number of remarkable correlations between direct-channel resonance contributions and crossed-channel Regge exchanges. These similarities are usually described in the framework of finite-energy sum rules (FESR's). They imply a semilocal equivalence between the low-energy contributions and the extrapolated Regge term. One of the best-known examples of this phenomenon is the zero structure of the peripheral N^* resonances in πN elastic scattering.[2] For all resonances, the positions of the zeros in their Legendre functions are at roughly the same values of t, and are in agreement with the zeros in the Regge terms. This fact comes about because the dominant resonances in πN scattering have the "correct" relation between their masses, spins, and parities.

It has recently been shown[3,4] that standard FESR's can be derived also for Reggeon-particle amplitudes. One would therefore expect these amplitudes to have duality features analogous to those of ordinary two-body amplitudes. As an example, consider the $\rho N \to \pi N$ amplitude (where ρ is a Reggeon) shown in Fig. 1(a). The direct-channel N^* resonances [Fig. 1(b)] are the same as those in πN elastic scattering. This makes an extension of the duality features seen in $\pi N \to \pi N$ to this amplitude seem fairly straightforward.

In this paper we want to discuss how duality requirements on Reggeon amplitudes constrain the ρNN^* couplings. We shall only consider the two lowest N^* contributions: the nucleon $N(938)$ and $\Delta(1232)$. We believe that a study of these states can be particularly fruitful for several reasons:

(i) General arguments (given in Sec. II) and the specific example of $\pi N \to \pi N$ suggest that the duality connection between N and Δ is particularly strong.

(ii) The low spin of N and Δ makes a straightforward evaluation of their contributions technically feasible.

(iii) The ρNN and $\rho N\Delta$ Regge couplings have been thoroughly studied both experimentally and phenomenologically. On the other hand, there are unfortunately very few data on the ρNN^* couplings for higher-mass N^*'s.

In addition, any predictions for the N and Δ couplings can be compared to those previously derived in other dynamical schemes, such as in the static model.[5] It is interesting to observe (see Sec. V) that many of our results are quite similar to those obtained in the static model. It should be emphasized, however, that restricting ourselves to the N and Δ contributions is a matter of choice rather than necessity. In fact, since duality connects all the N^*'s to each other, it should be relatively straightforward to generalize the results of this paper to the heavier N^*'s.

In deriving the ρNN and $\rho N\Delta$ couplings we shall take full advantage of the available duality constraints. These consist of FESR's for two different Reggeon amplitudes. The first is the $\rho N \to \pi N$ amplitude shown in Fig. 1(a). Counting helicity and isospin, there are eight distinct amplitudes to which we can apply the FESR. The $\rho N \to \rho N$ reaction of Fig. 2(a) provides us with another eight amplitudes. These latter amplitudes are in general unmeasurable, being obtained by factorization from a $3 \to 3$ reaction. However, the N and Δ contributions [Fig. 2(b)] can be *calculated* in terms of the usual ρNN and $\rho N\Delta$ couplings.

In the forward direction the discontinuity of the amplitude in Fig. 2(a) is proportional[6] to the inclusive cross section for $\pi^{\mp} p \to \pi^0 X$. This was why the Reggeon FESR's [commonly called FMSR's (finite-mass sum rules)] were first derived[3] for the forward elastic Reggeon-particle amplitude of Fig. 2. Here we shall use the duality constraint also away from the forward direction. The derivation of the FESR in this more general case is discussed in the Appendix

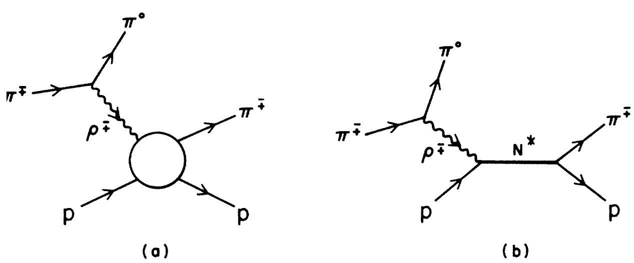

FIG. 1. (a) The Regge limit in which the $\pi^{\pm}p \to \pi^0 p^{\pm}p$ amplitude is proportional to the $\rho^{\pm}p \to \pi^{\pm}p$ Reggeon amplitude. (b) The N^* intermediate states in the $\rho N \to \pi N$ amplitude.

We should stress that while this is the first time that all duality constraints have been imposed simultaneously, there have of course been many previous applications of Reggeon FESR's. In fact, it is the success of these studies that inspires the present calculation. The inclusive FMSR has proved[7] to be a valuable tool both for obtaining triple-Regge couplings and for estimating relative magnitudes of quasi-two-body processes. The nonforward FESR[4] was recently applied[8] to study ρ and f resonance production in $\pi^- p \to \pi^-\pi^+ n$. The resonance couplings to both the π and A_2 Reggeons were found to be in good agreement with semilocal duality.

From the point of view of the present work, an especially interesting application of the FMSR is one by Finkelstein,[9] who studied the N^* contributions to the $\rho N \to \rho N$ forward amplitude [Fig. 2(a)]. On the basis of the observed smallness of the ρNN helicity-nonflip coupling he argued that the N and Δ had to cancel in the FMSR. Since we shall make no assumption about the ρNN coupling in the present calculation, we do not impose this condition on the N and the Δ. Rather, the cancellation turns out (see Sec. IV) to be a *prediction* of our scheme, as is indeed required for over-all consistency (since we also predict that the ρNN nonflip coupling is small). The full power of the duality constraints becomes particularly apparent when one compares the FESR's we shall use to the FMSR. While the FMSR applies[9] to *one* (nonflip) amplitude in the *forward direction*, we shall consider *eight* amplitudes as a *function of the momentum transfer* between the nucleons.

In Sec. II we briefly review the N^* contributions to FESR's for $\pi N \to \pi N$ amplitudes. On the basis of this example and on general arguments we deduce an empirical rule that relates the N and Δ residues in the Reggeon amplitudes. The formalism for dealing with the Reggeon amplitudes is developed in Sec. III. In Sec. IV we study the constraints imposed on the ρNN and $\rho N\Delta$ couplings by the empirical rule found in Sec. II. Both Secs. III and IV are rather technical and may be skipped in a first reading. We summarize our results and discuss their comparison with data in Sec. V. We also comment on the relation of our results to those obtained in the static bootstrap model.

II. THE N-$\Delta(1232)$ CANCELLATION

The first purpose of this section is to review the cancellation between the N and $\Delta(1232)$ contributions to FESR's for $\pi N \to \pi N$. We then proceed to give general arguments as to the reason why the cancellation is expected to occur also in other amplitudes. In the approximation where the N and the Δ masses are equal and the pion mass is neglected, we show that the cancellation in $\pi N \to \pi N$ can be simply understood analytically and imposes a constraint on the πNN and $\pi N\Delta$ coupling constants. Finally, on the basis of the πN example, we formulate the cancellation constraint that we shall impose on the Reggeon amplitudes.

The FESR's can be written down only for $\pi N \to \pi N$ amplitudes that are odd under $s \leftrightarrow u$ crossing, i.e., odd functions of ν,

$$\nu \equiv \tfrac{1}{2}(s-u) = s - m^2 + \tfrac{1}{2}(t - 2\mu^2)\,, \tag{1}$$

where m (μ) are the nucleon (pion) masses. The standard way to ensure this is to form amplitudes of given t-channel isospin:

$$T^{(\pm)}(\pi N \to \pi N) \equiv \tfrac{1}{2}[T(\pi^- p \to \pi^- p) \pm T(\pi^+ p \to \pi^+ p)]\,. \tag{2}$$

The odd-crossing amplitudes are then, in terms of the usual t-channel helicity-nonflip and helicity-flip amplitudes A' and B: $A'^{(-)}$, $B^{(+)}$, $\nu A'^{(+)}$, $\nu B^{(-)}$, etc. Each additional factor of ν in front of the amplitude has the effect of suppressing the low-mass N, Δ contributions to the FESR while enhancing the contributions of the higher-mass N^*'s. Thus, for the nucleon, $\nu = \nu_N$, where

$$\nu_N = \tfrac{1}{2}(t - 2\mu^2) \tag{3}$$

is a small number at small $|t|$ (in this paper we shall restrict ourselves to the region $-0.6 \leq t \leq 0$).

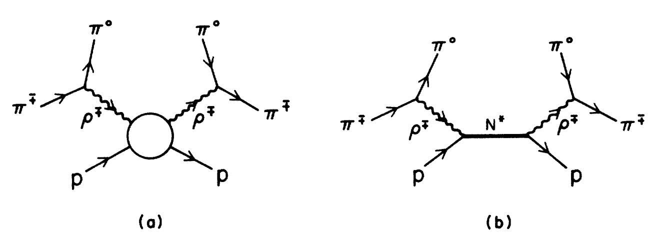

FIG. 2. (a) The $\rho^{\pm}p \to \rho^{\pm}p$ Reggeon amplitude as obtained from a high-energy limit of the $\pi^{\pm}\pi^0 p \to \pi^{\pm}\pi^0 p$ amplitude. (b) The N^* intermediate states of the $\rho N \to \rho N$ amplitude.

It follows, then, that for the FESR to efficiently constrain the N and Δ contributions we should consider only the amplitudes $A'^{(-)}$ and $B^{(+)}$, which do not have additional factors of ν.

We shall now briefly discuss the resonance contributions to the FESR's for the $A'^{(-)}$ and $B^{(+)}$ amplitudes. The phenomenon we want to point out, namely the mutual cancellation of N and Δ, has of course been observed several times before in previous studies of FESR's (see, e.g., Ref. 2). The reason we discuss the cancellation here is to emphasize its usefulness for determining coupling constants.

In Fig. 3(a) we show the contributions to $\mathrm{Im}B^{(+)}(t=0)$ of all the N^* resonances listed in the Particle Data Group tables.[10] The nucleon δ-function contribution is represented by the area of the box. The dominance of the N and Δ over the higher-mass contributions, as well as over the extrapolated Regge term[11] (dashed line), can be clearly seen. Hence the validity of the FESR, even with a cutoff around $\nu/2m \simeq 2$ GeV, requires the N and the Δ to cancel rather accurately. This is seen even more clearly in Fig. 3(b), where the same contributions are shown at $t=-0.6$ GeV^2. At this negative value of t, the higher-mass N^*'s and the Regge term, having a steep t dependence, have become very small. The N and the Δ, on the other hand, have only a weak t dependence.

The resonance contributions[10] to the $A'^{(-)}$ amplitude at $t=0$ are shown in Fig. 4(a). The situation is now very different: The N and the Δ are comparable or smaller than the heavier N^*'s. Hence there need not be any, and in fact there is no cancellation between N and Δ. The reason for the small magnitude of the N contribution is readily understood. The A' amplitude can be expressed in terms of the usual invariant amplitudes A and B as

$$A' = A + \frac{2m\nu}{4m^2 - t} B \,. \qquad (4)$$

It is well known that the nucleon Born term contributes only to the B, not to the A amplitude. Hence the nucleon contribution to A' is suppressed by a factor ν. According to Eq. (3), $\nu_N = -\mu^2$

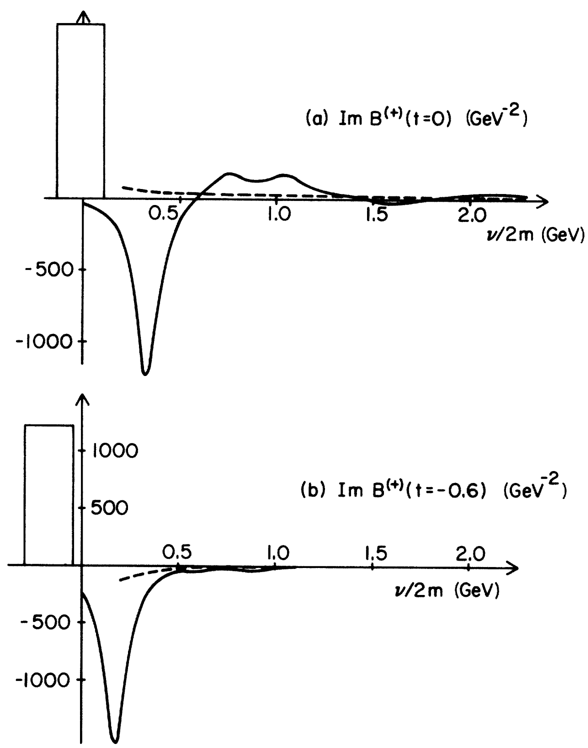

FIG. 3. The N^* contributions to the $\pi N \rightarrow \pi N$ amplitude $B^{(+)}$ at (a) $t=0$ and (b) $t=-0.6$ GeV^2. The nucleon Born-term contribution is given by the area of the box. All established (Ref. 10) N^* resonances are included with a Bright-Wigner shape and the normalization determined using the narrow-width approximation. The dashed line represents the $P' + P''$ Regge-pole terms of Ref. 11.

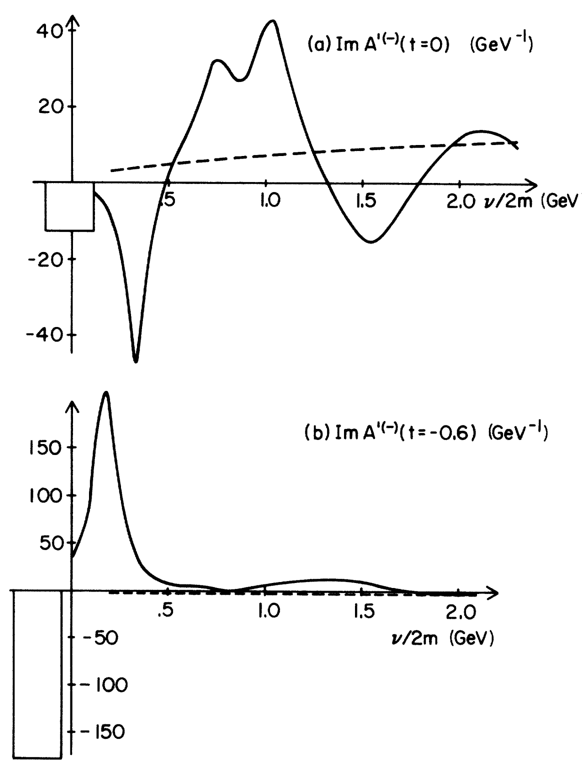

FIG. 4. The N^* contributions to the $\pi N \rightarrow \pi N$ amplitude $A'^{(-)}$ at (a) $t=0$ and (b) $t=-0.6$ GeV^2. The resonance contributions were calculated as in Fig. 3. The dashed line represents the $\rho + \rho'$ Regge-pole terms of Ref. 11.

$\simeq -0.02$ at $t=0$. However, we also see from Eq. (3) that $|\nu_N|$, and thus the whole nucleon contribution, grows very rapidly as t becomes negative. At the same time the heavier N^* contributions are expected to decrease. Thus the only way the FESR can remain valid is for the Δ to start canceling the N as t becomes negative. That this indeed is what happens is seen in Fig. 4(b), where $t=-0.6$ GeV^2.

As indicated above, the cancellation between N and Δ is a natural consequence of the validity of the FESR and the following two facts:

(a) The nucleon and $\Delta(1232)$ contributions are considerably larger than those of the higher-mass N^*'s, except in amplitudes where suppressing factors of ν occur.

(b) The spins of the N and Δ are low, implying a weak dependence on t (apart from the t dependence coming from factors of ν). On the other hand, the N^* resonances that dominate at larger masses have higher spin and thus steeper t dependence. Likewise the Regge contribution falls off quickly with increasing $-t$.

Both (a) and (b) should be valid for many other reactions besides $\pi N\to\pi N$. We would then expect an analogous cancellation to occur also in these reactions. In particular, for the Reggeon amplitudes $\rho N\to\pi N$ and $\rho N\to\rho N$ that we shall consider here the contribution of the heavier N^*'s should be relatively *smaller* than in $\pi N\to\pi N$. This can be seen, e.g., from the FMSR for $\rho N\to\rho N$ forward scattering. On the average, the resonance contribution at a mass $\mathfrak{M}$ should equal the Regge term

$$\frac{d\sigma_{\rm res}}{dt\,d\mathfrak{M}^2}\propto(\mathfrak{M}^2)^{\alpha(0)-2\alpha_\rho(t)},$$

where $\alpha(0)=\frac{1}{2}$. Since $\alpha_\rho(t)\geqslant 0$ for $t\geqslant -0.6$ the exponent of $\mathfrak{M}^2$ is smaller than $\frac{1}{2}$, implying relatively smaller contributions of heavy N^*'s in $\rho N\to\rho N$ than in $\pi N\to\pi N$. The available data[12] are consistent with this duality prediction.

As an illustration of how the cancellation between N and Δ can be used to determine coupling constants, let us consider the N, Δ contributions to the $A'^{(-)}$ and $B^{(+)}$ amplitudes in $\pi N\to\pi N$. The exact result for the Δ contribution is a rather lengthy expression involving t and the Δ, N, and π masses. However, since we in any case want to impose only an approximate N-Δ cancellation, say to within 20%, we can use approximate expressions for the masses. The natural choice is to assume

$$M=m=1\ \text{GeV},\quad \mu=0 \tag{5}$$

where M is the Δ mass. We have checked that this approximation generally agrees with the exact results to within 20%.

Using the approximation (5) the N and Δ contributions to the amplitudes are (all units are in GeV)

$$\begin{aligned}
A_N'^{(-)}&=\frac{G^2}{m^2-s}\;\frac{t}{4-t}\,,\\
A_\Delta'^{(-)}&=-\frac{G^{*2}}{M^2-s-iM\Gamma}\;\frac{t}{4-t}\times\tfrac{8}{9}(1-\tfrac{3}{16}t)\,,\\
B_N^{(+)}&=\frac{G^2}{m^2-s}\,,\\
B_\Delta^{(+)}&=-\frac{G^{*2}}{M^2-s-iM\Gamma}\times\tfrac{8}{9}(1-\tfrac{3}{8}t)\,.
\end{aligned}\tag{6}$$

Above, G and G^* are the π^0pp and $\pi^+p\Delta^{++}$ coupling constants,[13] respectively. The dominant constant term in Im$B^{(+)}$ cancels (in the narrow-width approximation) provided

$$G^{*2}/G^2=\tfrac{9}{8}\,. \tag{7}$$

The same condition (7) also ensures that the coefficient of t in Im$A'^{(-)}$ is equal and opposite for N and Δ. Using $G^2/4\pi=14.6$ and the experimental[14] mass and width of the Δ, the ratio (7) is 1.30 ± 0.10, quite close to $\frac{9}{8}=1.125$.

The above example shows that the FESR constraint on the N, Δ contributions to the $A'^{(-)}$ and $B^{(+)}$ amplitudes is particularly simple in the approximation (5). The dominant terms of the N and Δ contributions have to be equal and opposite in both amplitudes. In Sec. IV we shall impose this constraint on the crossing-odd t-channel helicity Reggeon-particle amplitudes. We find that the constraint should be generalized in the following natural way:

(i) The dominant terms in the N, Δ contributions may vanish separately for the N and the Δ, rather than cancel each other (as in our $\pi N\to\pi N$ example). It is clear that this is an equally good way of satisfying the FESR's.

(ii) Terms that are proportional to the Reggeon "masses" are treated as small and need not cancel. On the other hand, the momentum transfer between the nucleons in Figs. 1(a) and 2(a) is related to the N^* spin as discussed under fact (b) above. Terms proportional to this momentum transfer have to cancel to leading order (as in the $A'^{(-)}$ amplitude above).

With this extension our cancellation constraint on the Reggeon amplitudes is completely defined.

III. THE REGGEON AMPLITUDES

A. Invariant and helicity amplitudes for $\rho N\to\pi N$

The $\rho N\to\pi N$ Reggeon amplitude is obtained by factorization from a $\pi N\to\pi\pi N$ amplitude at high energy, as shown in Fig. 1(a). The t-channel

($=p\bar{p}$) view of the same process is shown in Fig. 5. Because we want to discuss t-channel helicity amplitudes we shall generally work in this frame. In analogy to what is usually done for the $\pi N \to \pi N$ amplitudes we first define a set of invariant amplitudes for the process $p\bar{p} \to 3\pi$ in Fig. 5. These have the advantage of being free of kinematic singularities. We then express the helicity amplitudes in terms of the invariant amplitudes.

There are four independent invariant amplitudes that describe the process $p\bar{p} \to 3\pi$. We have taken them to be P, Q, R, and S defined through[15]

$$T_{\lambda_3\lambda_4}(p\bar{p} \to 3\pi) = \bar{v}(p_4\lambda_4)\gamma_5\left[P + \not{p}_{56}Q + (\not{p}_1 - \not{p}_2)\frac{1}{s_{23}}R + i\sigma_{\mu\nu}p_1^\mu p_2^\nu \frac{1}{s_{23}}S\right]u(p_3\lambda_3). \tag{8}$$

The momenta are defined in Fig. 5. The extra factors of $1/s_{23}$ have been introduced so that all amplitudes P, Q, R, and S have the same Reggeon propagator. We follow the standard convention of Ref. 16 for the Dirac spinors and matrices, except for an extra phase factor $(-1)^{\lambda_4+1/2}$ in $\bar{v}(p_4\lambda_4)$ (corresponding to the Jacob and Wick "particle 2" phase convention[17] for helicity amplitudes).

It is straightforward to evaluate the expressions for the helicity amplitudes from Eq. (8). Unlike the case in πN elastic scattering both natural- and unnatural-parity exchange contributes in the t channel. These correspond to the combinations of helicity amplitudes

$$\begin{aligned} N_{\lambda_3\lambda_4} &= T_{\lambda_3\lambda_4} + (-1)^{\lambda_3-\lambda_4}T_{-\lambda_3-\lambda_4}, \\ U_{\lambda_3\lambda_4} &= T_{\lambda_3\lambda_4} - (-1)^{\lambda_3-\lambda_4}T_{-\lambda_3-\lambda_4}, \end{aligned} \tag{9}$$

where N and U refer to natural and unnatural parity, respectively. After some algebra one finds for the natural-parity amplitudes

$$N_{++} = \frac{-2i}{s_{23}m[(s_{34}-4m^2)^{1/2}]}\epsilon_{\mu\nu\rho\sigma}p_1{}^\mu p_2{}^\nu p_3{}^\rho p_4{}^\sigma S, \tag{10a}$$

$$N_{+-} = \frac{4i\epsilon_{\mu\nu\rho\sigma}p_1{}^\mu p_2{}^\nu p_3{}^\rho p_4{}^\sigma}{s_{23}m(s_{34})^{1/2}(s_{34}-4m^2)q_{56}\sin\theta_{56}} \times[(4m^2-s_{34})R + \tfrac{1}{2}m(s_{123}-s_{124})S], \tag{10b}$$

where $N_{+\pm} \equiv N_{1/2,\pm 1/2}$ and q_{56}, θ_{56} are the momentum and angle of 56 in the t-channel c.m. (with $\vec{p}_3$ along the $+z$ axis). The unnatural-parity amplitudes are, to leading order in the high-energy limit of Fig. 5,

$$U_{++} = \frac{1}{m(s_{34})^{1/2}}\left\{s_{34}P + m(s_{34}-s_{12}+\mu^2)Q + 2m\frac{s_{156}}{s_{23}}R + \frac{1}{4}\left[\left(1-\frac{s_{24}}{s_{23}}\right)(s_{34}+s_{12}-\mu^2) + \frac{s_{156}}{s_{23}}(s_{123}-s_{124})\right]S\right\}, \tag{10c}$$

$$\begin{aligned} U_{+-} = \frac{1}{ms_{34}[(s_{34}-4m^2)^{1/2}]q_{56}\sin\theta_{56}} \Bigg\{ &[s_{34}s_{123}s_{124} - s_{34}(m^2-s_{12})(m^2-\mu^2) - m^2(\mu^2-s_{12})^2]Q \\ &- \tfrac{1}{2}\left[s_{34}(s_{123}-s_{124})\left(1-\frac{s_{24}}{s_{23}}\right) - (4m^2-s_{34})(s_{34}+s_{12}-\mu^2)\frac{s_{156}}{s_{23}}\right]R \\ &+ \tfrac{1}{4}m\left[[s_{34}(s_{34}-2s_{12}-2\mu^2) + (\mu^2-s_{12})^2]\left(1-\frac{s_{24}}{s_{23}}\right)\right. \\ &\left.+ (s_{123}-s_{124})(s_{34}+s_{12}-\mu^2)\frac{s_{156}}{s_{23}}\right]S\Bigg\}. \end{aligned} \tag{10d}$$

B. Invariant and helicity amplitudes for $\rho N \to \rho N$

The t-channel process $p\bar{p} \to 4\pi$ is shown in Fig. 6. Because of the additional pion the kinematics of this process is in general considerably more complicated than that of $p\bar{p} \to 3\pi$. It is therefore essential to take advantage of the fact that we only need to consider the special case when

$$\frac{s_{156}}{s_{23}} = \frac{s_{126}}{s_{45}} = \frac{s_{16}}{s_{23}s_{45}} = 0 + O\left(\frac{1}{s_{23}}, \frac{1}{s_{45}}\right). \tag{11}$$

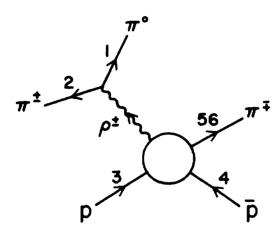

FIG. 5. The (t-channel) process $p\bar{p} \to \pi^\pm\pi^0\pi^\mp$.

As is shown in the Appendix, the FESR for the $\rho N \to \rho N$ amplitude is valid only under the condition (11). Thus we lose none of the duality constraints by restricting ourselves to the manifold where Eq. (11) is valid.

It is important to verify that the three conditions (11) are compatible with the four-dimensionality of space-time. Among the nine linearly independent variables s_{ij} that we use to describe the six-point function in Fig. 6 there is one kinematic constraint. Thus the value of the last ratio in Eq. (11) is completely determined by the first two ratios (and the other independent variables). It turns out that the value 0 is indeed one of the possible values for the last ratio (given that the two first ratios vanish). We can see this most simply by constructing explicitly the momentum vectors of all particles in Fig. 6:

$$\begin{aligned} p_3 &= (\tfrac{1}{2}(s_{34})^{1/2}, 0, 0, p),\\ p_4 &= (\tfrac{1}{2}(s_{34})^{1/2}, 0, 0, -p),\\ p_{12} &\equiv p_1 + p_2\\ &= \left(\frac{s_{34}+s_{12}-s_{56}}{2(s_{34})^{1/2}}, q\sin\theta, 0, q\cos\theta\right),\\ p_{56} &\equiv p_5 + p_6\\ &= \left(\frac{s_{34}-s_{12}+s_{56}}{2(s_{34})^{1/2}}, -q\sin\theta, 0, -q\cos\theta\right),\\ p_0 &\equiv p_1/s_{23}\\ &= -p_2/s_{23}\\ &= p_5/s_{45}\\ &= -p_6/s_{45}\\ &= \frac{1}{2p}\left(0, \cot\theta, \frac{-i}{\sin\theta}, -1\right) + O\left(\frac{1}{s_{23}}, \frac{1}{s_{45}}\right), \end{aligned} \tag{12}$$

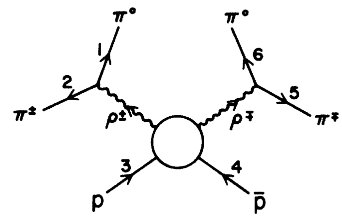

FIG. 6. The (t-channel) process $p\bar{p} \to \pi^\pm\pi^0\pi^0\pi^\mp$.

where

$$\begin{aligned} p &= \tfrac{1}{2}(s_{34} - 4m^2)^{1/2},\\ q &= \frac{1}{2(s_{34})^{1/2}}[(s_{34}-s_{12}-s_{56})^2 - 4s_{12}s_{56}]^{1/2}, \\ 2pq\cos\theta &= \tfrac{1}{2}(s_{123}-s_{124}) \equiv \nu. \end{aligned} \tag{13}$$

It is readily verified that Eq. (11) is indeed satisfied by this choice of momenta. All the remaining six variables are, however, unconstrained in the representation (12). It is particularly important to notice that Eq. (11) implies that all the momenta p_1, p_2, p_5, and p_6 are proportional to a single vector p_0. This makes the kinematics for the process $p\bar{p} \to 4\pi$ no more complicated than that for $p\bar{p} \to 3\pi$ discussed above.

We can now define the invariant amplitudes $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$ analogously to Eq. (8),

$$\mathcal{T}_{\lambda_3\lambda_4}(p\bar{p}\to 4\pi) = i\bar{v}(p_4\lambda_4)[\mathcal{A} + \tfrac{1}{2}(\not{p}_{56} - \not{p}_{12})\mathcal{B} + \not{p}_0\mathcal{C} + \not{p}_0(\not{p}_{56} - \not{p}_{12})\mathcal{D}]u(p_3\lambda_3). \tag{14}$$

We use script symbols to distinguish amplitudes describing $p\bar{p} \to 4\pi$ from those of $p\bar{p} \to 3\pi$. The helicity amplitudes with given t-channel naturality are

$$\mathcal{N}_{++} = -\frac{i}{2mp}[(4m^2 - s_{34})\mathcal{A} + m(s_{123}-s_{124})\mathcal{B} + 2m\mathcal{C} + (s_{56}-s_{12})\mathcal{D}], \tag{15a}$$

$$\begin{aligned} \mathcal{N}_{+-} = \frac{-i}{8m(s_{34})^{1/2}p^2 q\sin\theta}\big(&\tfrac{1}{2}\{s_{34}(s_{123}-s_{124})^2 + (4m^2 - s_{34})[(s_{34}-s_{12}-s_{56})^2 - 4s_{12}s_{56}]\}\mathcal{B}\\ &+ s_{34}(s_{123}-s_{124})\mathcal{C} + 2m(s_{56}-s_{12})(s_{123}-s_{124})\mathcal{D}\big), \end{aligned} \tag{15b}$$

$$\mathcal{U}_{++} = -i\frac{2q}{m}\mathcal{D}, \tag{15c}$$

$$\mathcal{U}_{+-} = -\frac{i}{2m(s_{34})^{1/2}p\sin\theta}[s_{34}\mathcal{C} + 2m(s_{56}-s_{12})\mathcal{D}]. \tag{15d}$$

C. Crossing symmetry

In Sec. II we saw that the cancellation between N and Δ is expected to occur only in the crossing-odd amplitudes. It is therefore essential to form combinations of s- and u-channel amplitudes that have a definite symmetry under crossing. In analogy to Eq. (2) for $\pi N \to \pi N$ scattering, this means that we consider the combination

$$T^{(\tau)}(\pi N \to \pi\pi N) \equiv \tfrac{1}{2}[T(\pi^- p \to \pi^0\pi^- p) + \tau T(\pi^+ p \to \pi^0 \pi^+ p)] \quad (\tau = \pm) \tag{16}$$

with a similar relation for the six-point amplitude.

Figure 7(a) shows the combination (16) in the t channel. Let us call the invariant amplitudes of the first term in Eq. (16) $P(3,4)$, $Q(3,4)$, etc. We would then expect the amplitudes of the second term to be, up to a sign, $P(4,3)$, $Q(4,3)$, etc. Here $P(4,3)$ is obtained from $P(3,4)$ through the analytic continuation implied by $p_3 \leftrightarrow p_4$, i.e., $s_{123} \leftrightarrow s_{124}$, $s_{23} \leftrightarrow s_{24}$, and so on. To see how this comes about, we first observe that the second term in Fig. 7(a) becomes, under C conjugation, the amplitude of

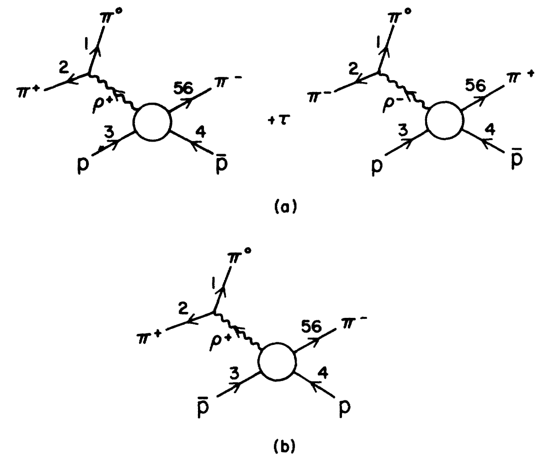

FIG. 7. (a) The combinations of $p\bar{p} \to 3\pi$ amplitudes ($\tau = \pm 1$) that have definite isospin in the $p\bar{p}$ channel. (b) The amplitude related by charge conjugation to the second amplitude of Fig. 7(a).

Fig. 7(b). Now the amplitude in Fig. 7(b) is identical to the first term of Fig. 7(a), provided we make the replacement $p_3, \lambda_3 \leftrightarrow p_4, \lambda_4$. Hence by Eq. (8) we have

$$T(7(\mathrm{b})) = \bar{v}(p_3\lambda_3)\gamma_5 \left[P(4,3) + \not{p}_{56} Q(4,3) + (\not{p}_1 - \not{p}_2)\frac{1}{s_{24}} R(4,3) + i\sigma_{\mu\nu} p_1^\mu p_2^\nu \frac{1}{s_{24}} S(4,3)\right] u(p_4\lambda_4). \tag{17}$$

The symmetrized amplitude (16) can now be constructed by C conjugating Eq. (17) and adding it to the amplitude of Eq. (8). Taking into account the C-conjugation properties[16] of the various couplings in Eq. (17), one gets

$$\begin{aligned} T^{(\tau)}(p\bar{p} \to 3\pi) = \bar{v}(p_4\lambda_4)\gamma_5 \tfrac{1}{2}\Big[&[P(3,4) + \tau P(4,3)] + \not{p}_{56}[Q(3,4) + \tau Q(4,3)] \\ &+ (\not{p}_1 - \not{p}_2)\left(\frac{1}{s_{23}} R(3,4) + \tau\frac{1}{s_{24}} R(4,3)\right) + i\sigma_{\mu\nu}p_1^\mu p_2^\nu\left(\frac{1}{s_{23}} S(3,4) - \tau \frac{1}{s_{24}} S(4,3)\right)\Big] u(p_3\lambda_3). \end{aligned} \tag{18}$$

The crossing symmetry of the Reggeon amplitudes becomes especially simple when

$$\frac{s_{156}}{s_{23}} = 0 \quad \text{(to leading order in } s_{23}). \tag{19}$$

This is the condition, analogous to Eq. (11), that must be satisfied[4] for FESR duality to be valid. Equation (19) further implies

$$s_{23} = -s_{24} \tag{20}$$

so that $s_{23} \to -s_{23}$ under $p_3 \leftrightarrow p_4$ crossing.

The behavior under crossing of the invariant amplitudes in Eq. (18) is the result of two separate analytic continuations: (a) the $s_{23} \to -s_{23}$ continuation [we assume Eq. (19) to hold], and (b) the $\nu \equiv \frac{1}{2}(s_{123} - s_{124}) \to -\nu$ continuation. In discussing FESR's for the Reggeon amplitudes only the behavior under $\nu \to -\nu$ is relevant. We must therefore explicitly take into account the effect of $s_{23} \to -s_{23}$ in Eq. (18). This is readily done since the s_{23} dependence enters only in the ρ Reggeon propagator. The signature of the ρ trajectory being odd, $s_{23} \to -s_{23}$ implies a minus sign in the crossing behavior of the full amplitude.

We can now write down the isospin combinations of the invariant amplitudes in Eq. (18) that have given symmetry under $\nu \to -\nu$ crossing. Denoting

$$P^{(\tau)} \equiv \tfrac{1}{2}[P(3,4) + \tau P(4,3)]$$

and similarly for Q, R, and S, the following amplitudes are odd under $\nu \to -\nu$:

$$P^{(+)},\ Q^{(+)},\ R^{(-)},\ \text{and } S^{(+)}. \tag{21}$$

Finally we note that the combinations of invariant amplitudes that correspond to a given t-channel helicity [see Eq. (10)] also have definite symmetry under $\nu \to -\nu$.

The crossing symmetry properties of the $\rho N \to \rho N$ amplitudes in Eq. (14) can be found in an analogous way. The combination of physical amplitudes is shown in Fig. 8. There are now two ρ propagators, so that no extra sign is introduced by $s_{23} \to -s_{23}$ and $s_{45} \to -s_{45}$. The amplitudes that are odd under $\nu \to -\nu$ are

$$\mathcal{A}^{(-)},\ \mathcal{B}^{(+)},\ \mathcal{C}^{(-)},\ \text{and } \mathcal{D}^{(-)}. \tag{22}$$

D. The N and $\Delta(1232)$ contributions

In calculating the diagrams of Figs. 1(b) and 2(b) with N and Δ intermediate states we need to know the N and Δ propagators and their couplings to πN and ρN. The nucleon propagator and the πNN coupling is standard.[16] For the Δ propagator we have[18]

$$\sum_\lambda u_\mu(p,\lambda)\bar{u}_\nu(p,\lambda) = \frac{(\not{p}+M)}{6M}\left[\frac{2}{M^2}p_\mu p_\nu - 3g_{\mu\nu} + \gamma_\mu\gamma_\nu + \frac{1}{M}(\gamma_\mu p_\nu - \gamma_\nu p_\mu)\right], \tag{23}$$

where M is the Δ mass and u_μ is a spin-$\frac{3}{2}$ spinor normalized to -1. The $\Delta^{++}(\lambda_1) \to p(\lambda_2) + \pi^+$ vertex

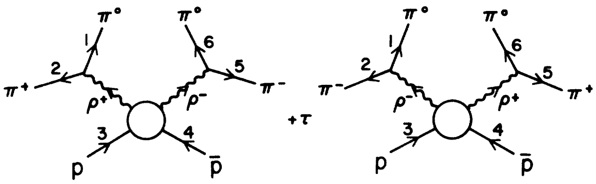

FIG. 8. The combinations of $p\bar{p} \to 4\pi$ amplitudes ($\tau = \pm 1$) that have definite isospin in the $p\bar{p}$ channel.

is

$$-G^*\bar{u}(p_2\lambda_2)p_2^\mu u_\mu(p_1\lambda_1), \tag{24}$$

where p_2 is the proton momentum.

The ρNN coupling is determined by the high-energy behavior of the $\pi^- p \to \pi^0 n$ amplitude [Fig. 9(a)]

$$T_{\lambda_4\lambda_3}(\pi^- p \to \pi^0 n) = i\bar{u}(p_4\lambda_4)[A + \tfrac{1}{2}(\not{p}_1 + \not{p}_2)B] \times u(p_3\lambda_3). \tag{25}$$

In the t channel, B corresponds to pure helicity flip and A' [defined in Eq. (4)] corresponds to pure helicity nonflip at the ρNN vertex. It is convenient to define a parameter β such that

$$s_{23}B = (\beta - 2m)A, \tag{26}$$

where β is (asymptotically) only a function of $t \equiv s_{12}$. A pure helicity-flip ρNN coupling ($A' = 0$) then implies $\beta = t/2m$.

We define a complete set of $\rho N\Delta$ couplings analogously by expressing the $\pi^+ p \to \pi^0\Delta^{++}$ amplitude [Fig. 9(b)] in terms of the invariant amplitudes $g_1, \ldots, g_4$:

$$T_{\lambda_4\lambda_3}(\pi^+ p \to \pi^0\Delta^{++}) = i\bar{u}_\mu(p_4\lambda_4)\left[p_3^\mu g_1 + \frac{1}{s_{23}}(p_1+p_2)^\mu g_2 + \frac{1}{s_{23}}p_3^\mu(\not{p}_1+\not{p}_2)g_3 + \frac{1}{s_{23}^{\,2}}(p_1+p_2)^\mu(\not{p}_1+\not{p}_2)g_4\right]\gamma_5 u(p_3\lambda_3). \tag{27}$$

Finally, we shall give the relation between the couplings $g_1, \ldots, g_4$ in Eq. (27) and the $M1$ or Stodolsky-Sakurai[19] coupling for the $\rho N\Delta$ vertex. If we write the $M1$ coupling in the form[20]

$$T^{(M1)}_{\lambda_4\lambda_3}(\pi^+ p \to \pi^0\Delta^{++}) = \bar{u}^\mu(p_4\lambda_4)u(p_3\lambda_3)\epsilon_{\mu\nu\rho\sigma}(p_3+p_4)^\nu(p_2-p_1)^\rho(p_2+p_1)^\sigma\frac{1}{s_{23}}E, \tag{28}$$

we have

$$\begin{aligned} g_1 &= -2E, \\ g_2 &= [(M+m)^2 - s_{12}]E, \\ g_3 &= 2ME, \\ g_4 &= 0. \end{aligned} \tag{29}$$

It is now straightforward to calculate the contribution of the N and the Δ to each of the invariant amplitudes in $\rho N \to \pi N$ and $\rho N \to \rho N$. We shall only give the result in the approximation (5) of equal

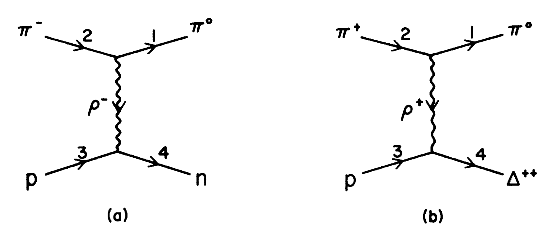

FIG. 9. Amplitudes for the processes (a) $\pi^- p \to \pi^0 n$ and (b) $\pi^+ p \to \pi^0\Delta^{++}$ at high energy. Note that the definition of the momenta p_2 and p_4 is different from that in the previous figures.

N and Δ masses and with the π mass set equal to zero.

The nucleon contributes equally to both isospin combinations ($\tau = \pm$) in Figs. 7(a) and 8. In terms of the $\pi^- p \to \pi^0 n$ amplitude A and the parameter β of Eq. (26) the nucleon Born term in $p\bar{p} \to 3\pi$ [Fig. 1(b)] is (in units of GeV)

$$\begin{aligned} P_N^{(\tau)} &= \frac{1}{\sqrt{2}} \frac{GA}{(m^2 - s_{123})} (\beta - 2), \\ Q_N^{(\tau)} &= \frac{1}{\sqrt{2}} \frac{GA}{(m^2 - s_{123})}, \\ R_N^{(\tau)} &= 0, \\ S_N^{(\tau)} &= -P_N^{(\tau)}. \end{aligned} \tag{30}$$

To find the nucleon Born term in the $p\bar{p} \to 4\pi$ amplitude of Fig. 2, let $A_1 \equiv A(s_{23}, s_{12})$ and $A_2 \equiv A(s_{45}, s_{56})$ be the $\pi^- p \to \pi^0 n$ amplitudes in the "initial" and "final" states of Fig. 2(b). Similarly, let $\beta_1 \equiv \beta(s_{12})$ and $\beta_2 \equiv \beta(s_{56})$. Then the Born-term contribution to the invariant amplitudes is

$$\begin{aligned} \mathcal{A}_N^{(\tau)} &= \frac{1}{4} \frac{A_1 A_2}{(m^2 - s_{123})} (\beta_1 + \beta_2), \\ \mathcal{B}_N^{(\tau)} &= \frac{1}{2} \frac{A_1 A_2}{(m^2 - s_{123})}, \\ \mathcal{C}_N^{(\tau)} &= \frac{1}{2} \frac{A_1 A_2}{(m^2 - s_{123})} (\beta_1 \beta_2 - \beta_1 - \beta_2), \\ \mathcal{D}_N^{(\tau)} &= \frac{1}{4} \frac{A_1 A_2}{(m^2 - s_{123})} (\beta_2 - \beta_1). \end{aligned} \tag{31}$$

Above and in the following we treat the amplitudes $A_{1,2}$ as being real. They have a well-defined phase from the ρ signature factor, which can be taken out as a common factor in all equations.

By isospin, the Δ contribution to the $\pi^- p \to \pi^0 \pi^- p$ amplitude in Fig. 7(a) is $-\frac{1}{3}$ of its contribution to the $\pi^+ p \to \pi^0 \pi^+ p$ amplitude (note the extra minus sign, compared to $\pi N \to \pi N$, coming from the $\rho\pi\pi$ vertex). The isospin coefficient of the Δ in the combination (16), relative to the $\pi^+ p \to \pi^0 \pi^+ p$ amplitude, is thus d_τ, where

$$d_\tau = \begin{cases} \frac{1}{3}, & \tau = + \\ -\frac{2}{3}, & \tau = - \end{cases} \tag{32}$$

We shall denote

$$G_i = \frac{1}{3} \frac{g_i G^*}{M^2 - s_{123} - iM\Gamma} \quad (i = 1, \ldots, 4), \tag{33}$$

where the g_i are the $\rho N \Delta$ couplings of Eq. (27), G^* is the $\pi N \Delta$ coupling [Eq. (24)], and Γ is the Δ width. Then in the approximation (5),

$$\begin{aligned} P_\Delta^{(\tau)} &= d_\tau[-4(G_2 + 2G_3 + G_4) + 3s_{34}(G_1 + G_3) - s_{12}(3G_1 + G_3)], \\ Q_\Delta^{(\tau)} &= d_\tau[2(G_2 + 2G_3) - \tfrac{3}{2} s_{34} G_1 + \tfrac{1}{2} s_{12} G_1], \\ R_\Delta^{(\tau)} &= -3d_\tau G_3 (s_{34} - s_{12}), \\ S_\Delta^{(\tau)} &= d_\tau[4(G_2 + 2G_3 + G_4) - 3s_{34} G_3 + s_{12} G_3]. \end{aligned} \tag{34}$$

In deriving Eq. (34) we used the duality constraint[4] $s_{156}/s_{23} = 0$ [see Eq. (19)].

The isospin coefficient e_τ of the Δ in the $p\bar{p} \to 4\pi$ reaction (Fig. 8) is the same as in $\pi N \to \pi N$:

$$e_\tau = \begin{cases} \frac{2}{3}, & \tau = + \\ -\frac{1}{3}, & \tau = -. \end{cases} \tag{35}$$

Let

$$G_{ij} = \frac{1}{3} \frac{g_i(s_{12}) g_j(s_{56})}{M^2 - s_{123} - iM\Gamma} \quad (i, j = 1, \ldots, 4). \tag{36}$$

Then the Δ contribution to the invariant amplitudes of the $p\bar{p} \to 4\pi$ reaction is, in the approximation (5) and assuming $G_{ij} = G_{ji}$,

$$\begin{aligned} \mathcal{A}_\Delta^{(\tau)} &= e_\tau[-4(G_{24} - G_{34}) - 3s_{34} G_{13} + (s_{12} + s_{56})(-G_{33} + \tfrac{1}{2} G_{12} + 4G_{13} + G_{14} + G_{23}) - s_{12} s_{56} (\tfrac{1}{2} G_{11} + G_{13})], \\ \mathcal{B}_\Delta^{(\tau)} &= e_\tau[2G_{22} - 4G_{33} - 4G_{23} + \tfrac{3}{2} s_{34} G_{11} + (s_{12} + s_{56})(-\tfrac{3}{2} G_{11} - G_{12} + G_{13}) + \tfrac{1}{2} s_{12} s_{56} G_{11}], \\ \mathcal{C}_\Delta^{(\tau)} &= e_\tau[8(G_{44} + G_{24} - G_{34}) + 6s_{34}(G_{33} + G_{13}) - (s_{12} + s_{56})(4G_{33} + G_{12} + 8G_{13} + 4G_{14} + 2G_{23} + 4G_{34}) + 2s_{12} s_{56} (G_{33} + 2G_{13})], \\ \mathcal{D}_\Delta^{(\tau)} &= e_\tau (s_{56} - s_{12})(-G_{33} + \tfrac{1}{2} G_{12} + G_{13} - G_{14} + G_{23}). \end{aligned} \tag{37}$$

IV. DERIVATION OF THE ρNN AND $\rho N\Delta$ COUPLINGS

Using the formulas given in Sec. III it is now straightforward to investigate what constraints the requirement of N, Δ cancellation, as formulated in Sec. II, imposes on the ρNN and $\rho N\Delta$ couplings. We shall treat the four $\rho N\Delta$ couplings $g_1, \ldots, g_4$ [Eq. (27)] and the relative size of the

ρNN helicity-flip and -nonflip couplings [parametrized by β in Eq. (26)] as unknown parameters, to be determined by the cancellation constraints. The (arbitrary) over-all normalization is fixed by the $\pi^- p \to \pi^0 n$ amplitude A. There are altogether 16 helicity amplitudes for the two processes $\rho N \to \pi N$ and $\rho N \to \rho N$, half of which are crossing odd. Hence we have an overconstrained situation: The N and Δ have to cancel in eight helicity amplitudes, while there are only five unknowns.

We shall start by considering the $\rho N \to \pi N$ helicity amplitude U_{++} given in Eq. (10c). Remember that we always work in the approximation (5), $M = m = 1$, with $\mu = 0$, and that the duality condition (19) must be satisfied. According to Eq. (21) we have to choose the isospin combination $\tau = +$ in Fig. 7(a) for the amplitude to be odd under $\nu \to -\nu$ crossing. Since the cancellation has to work for all values of s_{34} the coefficients of both s_{34} and s_{12} in Eq. (10c) have to vanish:

$$\sum_{N,\Delta} (P^{(+)} + Q^{(+)} + \tfrac{1}{2} S^{(+)}) = 0 , \tag{38a}$$

$$\sum_{N,\Delta} (Q^{(+)} - \tfrac{1}{2} S^{(+)}) = 0 . \tag{38b}$$

Using Eqs. (30) and (34) one finds that both equations give the same constraint:

$$g_4 = \frac{9\sqrt{2}}{8} \beta \frac{G}{G^*} A . \tag{39}$$

The U_{+-} amplitude of Eq. (10d) turns out to give the constraint (38b) once more.

Next consider the N_{++} and N_{+-} amplitudes, Eqs. (10a) and (10b). Cancellation of the constant term in the N and Δ contributions to $S^{(+)}$ requires

$$g_2 + 2g_3 = -\frac{9\sqrt{2}}{4} \frac{G}{G^*} A . \tag{40}$$

The N_{+-} amplitude is similar to the A' amplitude of $\pi N \to \pi N$ scattering [see Eq. (6)] in that neither N nor Δ contribute a constant term (independent of s_{34} and s_{12}) to the brackets in Eq. (10b). According to our discussion in Sec. II we should then require the linear term in s_{34} to cancel. This gives

$$g_2 - 4g_3 = -\frac{9\sqrt{2}}{16} (3\beta - 2) \frac{G}{G^*} A . \tag{41}$$

Solving Eqs. (40) and (41) for g_2 and g_3 we find

$$g_2 = -\frac{9\sqrt{2}}{16} (\beta + 2) \frac{G}{G^*} A ,$$
$$g_3 = \frac{9\sqrt{2}}{32} (\beta - 2) \frac{G}{G^*} A . \tag{42}$$

We have now satisfied all the cancellation constraints for the $\rho N \to \pi N$ helicity amplitudes. Note that the ρNN helicity-nonflip/helicity-flip ratio, as measured by β, is still unconstrained. Considering next the $\mathfrak{N}_{++}$ amplitude of the $\rho N \to \rho N$ reaction [Eq. (15a)], we see from Eq. (22) that we should choose $\tau = -$ in Fig. 8 to have a crossing-odd amplitude. From the N and Δ contributions in Eqs. (31) and (37) it is readily seen that cancellation of the constant term requires

$$A_1 A_2 \beta_1 \beta_2 = \tfrac{16}{9} g_4(s_{12}) g_4(s_{56}) = 4 A_1 A_2 \beta_1 \beta_2 , \tag{43}$$

where we used Eqs. (7) and (39) in the latter equality. The only nontrivial solution to Eq. (43) is

$$\beta = 0 \ \ [+ O(t)] . \tag{44}$$

From the definition of β in Eq. (26) this implies that the A' amplitude is small compared to B in $\pi^- p \to \pi^0 n$ at high energies. The size of "small" here is measured by the accuracy of the N, Δ cancellation and the approximation (5). Both are expected to be better than 20%. Note also that as indicated in Eq. (44), there is no constraint on a possible linear t dependence in $\beta(t)$.

By Eq. (39), $\beta = 0$ implies $g_4 = 0$. Neither the N nor the Δ then contributes a constant term to $\mathfrak{N}_{++}$. Hence, here as for the $A'^{(-)}$ amplitude (cf. Sec. II) we should require that the term linear in s_{34} cancels. It can be seen from Eqs. (37) that the Δ contribution is determined by the couplings g_2 and g_3, whose values are already fixed by Eq. (42). Thus we have a consistency check. Using the value (7) for the G^*/G ratio we find that the linear term in s_{34} indeed cancels exactly.

It can further be seen from Eqs. (31) and (37) that the terms in $\mathfrak{N}_{++}$ that are linear in s_{12} and s_{56} also cancel. In the forward direction ($s_{34} = 0$, $s_{12} = s_{56} = t$) this cancellation implies, through the optical theorem,[6]

$$\frac{d\sigma}{dt} (\pi^- p \to \pi^0 n) = \frac{2}{3} \frac{d\sigma}{dt} (\pi^+ p \to \pi^0 \Delta^{++}) \tag{45}$$

for all (moderate) values of t. This relation was previously derived by Finkelstein,[9] who used the smallness of the ρNN helicity-nonflip coupling as as an argument for requiring the N and the Δ to cancel in the inclusive FMSR. It is noteworthy that this cancellation is "accidental" from the point of view of our cancellation constraint, which only restricts the coefficients of s_{34} [cf. generalization (ii) in Sec. II]. However since we already predicted that ρNN helicity-nonflip is small, overall consistency requires the relation (45), through Finkelstein's argument. It is very encouraging to see that all constraints seem to lead to a consistent scheme, although the reason for this is rather unclear at present.

The leading contribution to the $\mathfrak{N}_{+-}$ amplitude [Eq. (15b)] comes from the term proportional to $\mathfrak{B}$. It is again straightforward to verify that the N and Δ contributions to $\mathfrak{B}^{(+)}$ cancel (to leading order) when the couplings g_2 and g_3 are given by Eq. (42).

The $\mathfrak{U}_{++}$ amplitude automatically satisfies the cancellation rule, as neither the N nor the Δ contributes a constant or s_{34}-dependent term. The same is true for the N contribution to $\mathfrak{U}_{+-}$ (with $\beta=0$). However, the Δ does contribute an s_{34}-dependent term to $\mathfrak{U}_{+-}$ unless

$$g_1 = -g_3 \,. \tag{46}$$

With Eq. (46) all $\rho N\Delta$ couplings are determined. At the same time all the $\rho N \to \rho N$ amplitudes have been shown to satisfy the cancellation constraint. We still have to go back and check those $\rho N \to \pi N$ amplitudes for consistency in which the constant term of the N (or Δ) contribution vanishes because $\beta=0$. According to our rules we must then demand that the leading s_{34} dependence cancels. For the U_{++} amplitude, the absence of terms of order ${s_{34}}^2$ in the brackets of Eq. (10c) is ensured by Eq. (46). Similarly for U_{+-}, Eq. (46) is the condition for no ${s_{34}}^3$ terms.

Finally we observe that the relative size of the $\rho N\Delta$ couplings of our solution is, according to Eqs. (39), (42), (44), and (46),

$$g_1 : g_2 : g_3 : g_4 = -1 : 2 : 1 : 0 \,.$$

Comparing this with Eq. (29), we see that the unique $\rho N\Delta$ coupling which satisfies all our constraints [in the approximation (5)] in the $M1$-type coupling.

V. DISCUSSION

We shall start by summarizing our results for the couplings and discussing their comparison with data. It should be remembered that all our predictions can be trusted only to within 20%, due to the approximations we have made. In particular, we have treated the momentum transfers as "small," effectively ignoring t-dependent terms in the relative magnitudes of the Reggeon couplings.

(i) The relative size of the πNN and $\pi N\Delta$ coupling constants is determined by Eq. (7). This result followed directly from the FESR constraints on the $\pi N \to \pi N$ amplitudes, but was also needed for consistency between the constraints on the $\rho N \to \pi N$ and $\rho N \to \rho N$ Reggeon amplitudes. Equation (7) agrees within 15% with the $\pi N\Delta$ coupling determined from the narrow-width approximation[14] and $G^2/4\pi = 14.6$.

(ii) We predict [Eqs. (26) and (44)] that the ρNN helicity-nonflip coupling vanishes so that the ρNN coupling is dominantly helicity-flip. This is in agreement with standard phenomenological results. For example, a recent Regge-pole fit[21] to the $\pi^- p \to \pi^0 n$ reaction obtains $A'^{(-)}/sB^{(-)} \simeq 0.04$ at $t=0$.

(iii) The only $\rho N\Delta$ coupling that satisfies the FESR constraints is the $M1$-type coupling first suggested by Stodolsky and Sakurai.[19] This coupling is known to describe the data well (cf. p. 270 of Ref. 1).

(iv) The relative size of the ρNN helicity-flip coupling and the $\rho N\Delta$ $M1$ coupling leads to the prediction (45). The agreement with data is again very good—see Fig. 3 of Ref. 9.

According to (i)–(iv), the FESR's applied to the reactions $\pi N \to \pi N$, $\rho N \to \pi N$, and $\rho N \to \rho N$ determine the relative magnitudes of all relevant couplings to the N and Δ intermediate states. Because the equations are linear, the absolute normalization is arbitrary for each reaction. The reason that one can obtain such strong predictions is that the N and Δ intermediate states completely dominate the imaginary part of certain amplitudes, as evidenced by Figs. 3 and 4. We do not claim to know the reason for this dominance—we have merely taken advantage of it to be able to constrain the N and Δ parameters independently of those of the heavier N^*'s.

It should be clear that the FESR's provide many more constraints than those we have exploited here. For example, one may look at amplitudes like $\nu B^{(-)}$ in which the N and Δ do not cancel. Because of the extra factor of ν, the heavier N^*'s are relatively much more important in $\nu B^{(-)}$ than in $B^{(+)}$. Thus one should be able to connect the contributions of the heavier N^*'s to those of N and Δ.

The success of the present scheme strongly suggests that our assumption of N, Δ dominance in the crossing-odd $\rho N \to \pi N$ and $\rho N \to \rho N$ amplitudes is correct. Thus duality in Reggeon amplitudes, as expressed by the Reggeon FESR's, seems to be closely analogous to duality in ordinary particle reactions. This is, of course, in agreement with previous experience.[7] It is also interesting to observe that the conditions (11) and (19) were essential in order to obtain our results. These conditions were derived from analyticity requirements on the Reggeon amplitudes (see Ref. 4 and the Appendix). There are no corresponding conditions for particle amplitudes.

In our way of imposing the FESR constraints, the existence of a large number of seemingly independent but mutually consistent constraints is rather mysterious. We first encountered this phenomenon in our $\pi N\to\pi N$ example of Sec. II. Both the $A'^{(-)}$ and the $B^{(+)}$ amplitude gave the same relation (7) between the πNN and $\pi N\Delta$ coupling constants. In Sec. IV we found that there were many more FESR constraints than adjustable couplings; however, the ρNN helicity-flip and $\rho N\Delta$ $M1$ couplings satisfied all conditions. Another intriguing consistency was the "accidental" cancellation between N and Δ in a "small" term ($\propto$ Reggeon masses) of the forward $\rho N\to\rho N$ amplitude, which led to Eq. (45). Although the N and Δ may not dominate the heavier N^* contributions to this term, the cancellation can be understood[9] as a consequence of another of our predictions: the smallness of the ρNN helicity-nonflip coupling. This prediction requires, through semilocal duality, that all N^*'s cancel semilocally.

It is clearly very encouraging to see that all constraints lead to a unique and consistent set of couplings. It would be important to better understand the reason for this consistency, and to further enlarge the set of related couplings. The couplings that we have found here may have important implications for many other processes. For example, it has been suggested,[22] that the suppression of exotic double-particle exchange amplitudes is related to the small value of the ρNN helicity-nonflip coupling. Since we have implicitly neglected exotic exchanges, this is another indication of the over-all consistency of our approach.

We believe that the t-channel view we have adopted can give valuable insights into the ρNN^* couplings. An indication of this can already be seen from the ρNN and $\rho N\Delta$ couplings that we discussed in this paper. In the reactions $\rho N\to\pi N$ and $\rho N\to\rho N$ both natural- and unnatural-parity exchanges are allowed. The leading unnatural-parity trajectories $\pi, A_1, h, \ldots$ have intercepts close to 0 and are thus predominantly real at small momentum transfers. Hence one would expect the sum over the N^* resonances to be small in *all* unnatural-parity exchange amplitudes, independently of the exchanged isospin and the (helicity-flip or -nonflip) coupling at the $N\bar{N}$ vertex. This is different from natural-parity exchange amplitudes, which may be small for one coupling (say, ρNN helicity-nonflip) but are large for the other coupling (ρNN helicity flip). It implies that the cancellation mechanism between the N^* resonances must be different for unnatural- and natural-parity amplitudes. In $\pi N\to\pi N$, the cancellation of N and Δ in $B^{(+)}$ implies a noncancellation in $\nu B^{(-)}$. This is desirable, because the ρNN helicity-flip coupling is large. For the $\rho N\to\pi N$ unnatural-parity amplitudes $U^{(\tau)}_{+\frac{1}{2}}$ [Eqs. (10c) and (10d)], however, we want a small $N+\Delta$ contribution in both $U^{(+)}_{+\frac{1}{2}}$ and $\nu U^{(-)}_{+\frac{1}{2}}$. The only way this can be achieved is by having the N and Δ vanish *separately,* rather than by cancellation, in $U^{(\tau)}_{+\frac{1}{2}}$. It can be readily seen that the self-consistent couplings of our solution, namely, ρNN helicity flip and $\rho N\Delta$ $M1$, are precisely such as to ensure the separate vanishing of N and Δ in all unnatural-parity amplitudes of $\rho N\to\pi N$ and $\rho N\to\rho N$. In fact, we could have derived the dominance of the ρNN helicity-flip coupling simply from the requirement that the N should decouple from unnatural-parity exchange amplitudes.

It is very interesting to observe the close relation between our results and those of the static model.[5,23] Although we use a rather different formalism (relativistically invariant t-channel helicity amplitudes) it is clear that the basic assumptions and approximations are quite similar. Thus we make use of the near degeneracy in mass between the N and the Δ, and neglect the π mass and momentum transfers in comparison with the baryon mass. Furthermore, we neglect the contributions of the heavier N^*'s to the amplitudes we consider.

The relation (7) between the πNN and $\pi N\Delta$ couplings is the same as that originally obtained in the static model.[24] Dashen and Frautschi[25] have applied the static model to amplitudes of the type $R+N\to\pi+N$, where R is a Reggeon. Their approach is different from ours insofar as they do not also consider the $R+N\to R+N$ type amplitudes. Instead, they regard R as a member of an SU(3) multiplet and demand consistency under SU(3) for all couplings. They predict[25,26] the $\rho N\Delta$ coupling to be $M1$, and find the same ratio (45) between the ρNN helicity-flip and $\rho N\Delta$ couplings. Their ρNN helicity-nonflip coupling does not vanish, but the predicted F/D ratio[25] makes it rather small.

There seems, thus, to be an intimate relation between our approach and the static model. The solutions of the static bootstrap model have been elegantly classified[27] in terms of the eigenvalues of a crossing matrix. The model has been applied also to matrix elements for weak and electromagnetic currents.[27] Our formulation has the advantage of being relativistically invariant. Moreover, as we have indicated above, because we discuss the phenomena from the t-channel point of view it should be possible to extend our arguments to heavier N^*'s. The t-channel exchange description indeed becomes more natural at higher N^* masses. This can be seen, e.g.. in the zero structure[2] of the heavier N^* resonances in $\pi N\to\pi N$ that we mentioned in the Introduction. Finally, because duality arguments apply equally well to mesonic as to baryonic systems, arguments similar to ours can be applied[7,8] to meson resonances. This is, of course, desirable for a unified picture of hadronic amplitudes.

ACKNOWLEDGMENTS

I wish to thank R. Dashen, J. Finkelstein, G. Kane, M. Olsson, C. Quigg, and D. P. Roy for helpful and inspiring discussions. My particular thanks go to H. Thacker for some illuminating advice. I am also grateful for a grant from the University of Helsinki.

APPENDIX

The purpose of this Appendix is to briefly discuss the validity of FESR's for amplitudes of the type $R+a\to R+b$, where R is a Reggeon (cf. Fig. 6). It has previously been shown[3] that the FMSR is valid for the forward $3\to 3$ amplitude (where $p_1=-p_6$, $p_2=-p_5$, $p_3=-p_4$ in Fig. 6). We want to discuss the generalization of this to the nonforward amplitude. Our treatment will be closely analogous to that of Ref. 4, where an FESR was derived for the $R+a\to b+c$ amplitude (Fig. 5).

Probably the most novel feature of the Reggeon amplitudes is the necessity to impose constraints like Eqs. (11) and (19). We shall start by recalling the reasons[4] for the constraint (19) in the case of the $R+a\to b+c$ amplitude. Consider a $2\to 3$ amplitude in the limit of Fig. 5. The Regge-pole behavior implies

$$T(2\to 3)=s_{23}^{\alpha_{12}}\,T_R\left(s_{123},\ s_{34},\ s_{12},\ \frac{s_{156}}{s_{23}}\right), \tag{A1}$$

where $\alpha_{12}\equiv\alpha(s_{12})$ is the exchanged trajectory. Equation (A1) explicitly displays the singularities of T in s_{23} (a cut from 0 to $-\infty$). By symmetry, T must have a similar cut in s_{156}. This means that the Reggeon amplitude T_R has singularities in the variable s_{156}/s_{23}. In deriving the FESR, we need a simple analytic structure of T_R in the variable s_{123}, *keeping* κ *fixed,* where

$$\kappa\equiv s_{123}s_{156}/s_{23}. \tag{A2}$$

If κ were not held fixed, the amplitude T would not approach the double-Regge limit as $|s_{123}|\to\infty$. Through Eq. (A2), the singularities of T_R in s_{156}/s_{23} get reflected into the s_{123} plane, giving unknown contributions to the FESR. The only exception is when $\kappa=0$ [Eq. (19)], in which case $s_{156}/s_{23}=0$ does not depend on s_{123}. The fact that $\kappa=0$ is a singular point of T_R does not affect the FESR, because s_{123} and s_{156} do not have overlapping singularities.

The validity of the above argument was verified[4] in the dual resonance model (B_5) and a ϕ^3 ladder-diagram model. Both were found to have a very similar structure. For simplicity we shall only consider the B_6 model in discussing the $R+a\to R+b$ amplitude.

In the Regge limit of Fig. 6, the B_6 amplitude becomes[28]

$$\begin{aligned}B_6=&\int_0^1 du\,u^{-\alpha_{123}-1}(1-u)^{-\alpha_{34}-1}\langle AB\rangle^{\alpha_{12}}\langle BC\rangle^{\alpha_{56}}\\ &\times\int_0^\infty dz_1dz_2z_1^{-\alpha_{12}-1}z_2^{-\alpha_{56}-1}\\ &\times\exp\left(-z_1-z_2-\frac{\langle ABC\rangle}{\langle AB\rangle\langle BC\rangle}z_1z_2\right),\end{aligned} \tag{A3}$$

where

$$\langle AB\rangle=-\alpha_{156}u-\alpha_{23}(1-u),$$

$$\langle BC\rangle=-\alpha_{126}u-\alpha_{45}(1-u),$$

$$\langle ABC\rangle=-\alpha_{16}u.$$

In the triple-Regge limit ($s_{123}\to\infty$ in Fig. 6), the ratios

$$\alpha_{123}\frac{\alpha_{156}}{\alpha_{23}},\quad \alpha_{123}\frac{\alpha_{126}}{\alpha_{45}},\quad (\alpha_{123})^2\frac{\alpha_{16}}{\alpha_{23}\alpha_{45}} \tag{A4}$$

are all finite. Regarded as a function of α_{123}, with the ratios (A4) held fixed, B_6 in Eq. (A3) has cuts in addition to the ordinary resonance poles. This implies, through the dispersion relations, that the sum over the resonances in general is not proportional to a triple-Regge term, there being additional contributions to the FESR.

Let us now consider the special case of Eq. (11), when all three ratios in Eq. (A4) vanish. This is singular point for the full B_6 amplitude. However, all resonance poles in α_{123} have residues that are polynomials, hence nonsingular, in the ratios (A4). Thus it is clear that the function obtained by naively imposing Eq. (11) on Eq. (A3),

$$\begin{aligned}\tilde B_6=&\Gamma(-\alpha_{12})\Gamma(-\alpha_{56})(-\alpha_{23})^{\alpha_{12}}(-\alpha_{45})^{\alpha_{56}}\\ &\times B_4(-\alpha_{123},\ -\alpha_{34}+\alpha_{12}+\alpha_{56}),\end{aligned} \tag{A5}$$

has exactly the same pole residues as the full B_6 [under the condition (11)]. But $\tilde B_6$ in Eq. (A5) is simply proportional to a B_4 function, hence the sum of its resonances gives a Regge term in the standard FESR sense. This shows that ordinary duality is valid also for B_6 when Eq. (11) is satisfied.

*Research supported by the National Science Foundation under Grant No. GP-32998X and by a grant from the University of Helsinki.

[1]For a recent review see, e.g., G. C. Fox and C. Quigg, Annu. Rev. Nucl. Sci. 23, 219 (1973).

[2]R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968).

[3]J. Kwieciński, Nuovo Cimento Lett. 3, 619 (1972); A. I. Sanda, Phys. Rev. D 6, 280 (1972); M. B. Einhorn, J. E. Ellis, and J. Finkelstein, Phys. Rev. D 5, 2063 (1972).

[4]P. Hoyer and J. Kwieciński, Nucl. Phys. B60, 26 (1973).

[5]G. F. Chew and F. E. Low, Phys. Rev. 101, 1570 (1956).

[6]A. H. Mueller, Phys. Rev. D 2, 2963 (1970).

[7]For a review see, e.g., P. Hoyer, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974,* edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. I–158.

[8]P. Hoyer, P. Estabrooks, and A. D. Martin, Phys. Rev. D 10, 80 (1974).

[9]J. Finkelstein, Phys. Rev. D 6, 931 (1972).

[10]Particle Data Group, Rev. Mod. Phys. 45, S1(1973).

[11]V. Barger and R. J. N. Phillips. Phys. Rev. 187, 2210 (1969).

[12]Compare, e.g., Fig. 2 of Ref. 9 with our Fig. 4a. Further examples of this phenomenon can be found in P. Hoyer, Stony Brook Report No. ITP-SB-74-11 (unpublished) [talk presented at the Symposium on Production of Heavy Excited Hadrons, ANL, 1974 (unpublished)].

[13]The definition of G* is given in Eq. (24).

[14]Particle Data Group, Phys. Lett. 50B, 1 (1974).

[15]We use the standard notation for the invariants: $s_{ij}=(p_i+p_j)^2$, $s_{ijk}=(p_i+p_j+p_k)^2$, etc. Note, however, that incoming momenta in Fig. 5 enter with a minus sign: $s_{23}\equiv s_{2\bar{3}}=(p_2-p_3)^2$, etc.

[16]J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964); and *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

[17]M. Jacob and G. C. Wick, Ann. Phys. (N.Y.) 7, 404 (1959).

[18]See, e.g., H. Pilkuhn *et al.*, Nucl. Phys. B65, 460 (1973).

[19]L. Stodolsky and J. J. Sakurai, Phys. Rev. Lett. 11, 90 (1963).

[20]G. H. Renninger and K. V. L. Sarma, Phys. Rev. 178, 2201 (1969).

[21]V. Barger and R. J. N. Phillips, Phys. Lett. 53B, 195 (1974). This is an update of Ref. 11, using data that have become recently available.

[22]F. S. Henyey, G. L. Kane, and J. J. G. Scanio, Phys. Lett. 27, 350 (1971).

[23]I am grateful to Professor R. Dashen for pointing this out to me.

[24]G. F. Chew, Phys. Rev. Lett. 9, 233 (1962).

[25]R. Dashen and S. Frautschi, Phys. Rev. 152, 1450 (1966).

[26]These results were also obtained in the static model by Y. Hara, Phys. Rev. 140, B178 (1965).

[27]R. F. Dashen and S. C. Frautschi, Phys. Rev. 143, 1171 (1966).

[28]See, e.g., Eq. (12) in Chan Hong-Mo, P. Hoyer, and P. V. Ruuskanen, Nucl. Phys. B 38, 125 (1972)