Vacuum stability and the Cabibbo angle

Heinz Pagels

Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11794*
and Rockefeller University,† New York, New York 10021

(Received 13 November 1974)

It is shown that vacuum stability determines the Cabibbo angle θ_C. From a simple example we obtain $\tan^2\theta_C \simeq f_\pi\mu_\pi^2/f_K\mu_K^2$ and a nonelectromagnetic $\Delta I = 1$ interaction. The effect of charm is included. We also discuss the superweak model of CP violation as an isotopic rotation through an angle θ_{CP}.

I. VACUUM-STABILITY CONDITIONS

By way of introduction consider the classical Heisenberg ferromagnet.[1,2] This is an infinite magnet with a spin density $\vec{S}(x)$. The Hamiltonian, $H = H_0 + \epsilon H'$, is the sum of a term H_0 which is rotationally O(3) invariant plus a term $\epsilon H' = -\mu\vec{B}\cdot\vec{S}(x)$ which breaks the O(3) invariance. Here μ is a positive constant and $\vec{B}$ is an applied constant external magnetic field. $\epsilon H'$ is rotationally O(1) invariant about the axis defined by $\vec{B}$.

The ground state of the ferromagnet, in the absence of an external field, is a threefold infinitely degenerate vacuum $|\vec{M}\rangle$, with the direction $\vec{M}$, the magnetization, defining the different vacua. Each vacuum $|\vec{M}\rangle$ is sufficiently rich to build up a complete Hilbert space to describe the states of the ferromagnet.

The vacuum energy of this system is given by

$$F(\theta) = \langle\vec{M}|[H_0 - \mu\vec{B}\cdot\vec{S}(x)]|\vec{M}\rangle$$
$$= \text{const} - \mu|\vec{B}||\vec{M}|\cos\theta ,$$

where $\vec{M} = \langle\vec{M}|\vec{S}(x)|\vec{M}\rangle$ and θ is the angle between $\vec{M}$ and $\vec{B}$. Requiring that $F(\theta)$ be a minimum, $F'(\theta) = 0$, $F''(\theta) > 0$ implies $\theta = 0$, so that $\vec{B}$ and $\vec{M}$ are parallel.

As we show below, these same ideas applied to the problem of symmetry breaking in he strong and weak interactions can determine the Cabibbo angle, θ_C. The result we find requiring vacuum stability of an effective Hamiltonian is

$$\tan^2\theta_C \simeq f_\pi\mu_\pi^2/f_K\mu_K^2 . \qquad (1)$$

This result is in agreement with the experimental value[3] $\theta_C \simeq 15°$. Equation (1) with $f_\pi/f_K = 1.26 \pm 0.02$ has $\theta_C \simeq 17°$. Before establishing (1) we briefly review Dashen's theorems which serve the purpose, if satisfied, of guaranteeing vacuum stability.

Suppose in the ferromagnet we had $\vec{B}$ and $\vec{M}$ not parallel. Then one finds that the spin waves, corresponding to Goldstone excitations, have complex frequencies, implying vacuum instability. So vacuum stability requires that the external field $\vec{B}$ point in the same direction specified by that O(1) subgroup of O(3) that leaves the vacuum invariant.

The vacuum-stability condition is implemented by requiring that Dashen's theorem be satisfied. Let $|\vec{M}\rangle_{\text{phys}}$ be the physical ground state, with $\vec{B} \neq 0$, and

$$|\vec{M}\rangle = \lim_{B\to 0} |\vec{M}\rangle_{\text{phys}} .$$

Then Dashen's theorem 2 of Ref. 1 requires the explicit breaking Hamiltonian to be such that

$$F(\vec{\theta}) = \langle\vec{M}|e^{i\vec{\theta}\cdot\vec{J}}\epsilon H' e^{-i\vec{\theta}\cdot\vec{J}}|\vec{M}\rangle$$

have a local minimum at $\vec{\theta} = 0$. Here $\vec{J} = \int\vec{S}(x)\,d^3x$ is the angular momentum.[4] It is easy to see that the requirements $\partial F(\vec{\theta})/\partial\vec{\theta} = 0$ and $\partial^2 F(\vec{\theta})/\partial\vec{\theta}\,\partial\vec{\theta} > 0$ are met provided $\vec{B}$ and $\vec{M}$ are parallel. Once the invariance group of the ground state is specified the invariance group of the explicit symmetry breaking is restricted and corresponds to that subgroup that leaves the vacuum invariant.

Let us next consider the internal-symmetry group SU(3)×SU(3) with generators $\vec{Q}$ and the Hamiltonian $H = H_0 + \epsilon H'$. Here H_0 is SU(3)×SU(3) invariant and $\epsilon H'$ breaks this symmetry. Then we have Dashen's theorems:

Theorem 1. Let $|\text{vac}\rangle$ be the ground state for H. Then

$$F(\vec{\omega}) = \langle\text{vac}|e^{i\vec{\omega}\cdot\vec{Q}}\epsilon H' e^{-i\vec{\omega}\cdot\vec{Q}}|\text{vac}\rangle$$

has a minimum at $\vec{\omega} = 0$. The proof is elementary but will not be given here. From theorem 1 follows

Theorem 2. Let $|0\rangle$ be the ground state of H_0 defined as $|0\rangle = \lim_{\epsilon\to 0}|\text{vac}\rangle$. Then

$$F_1(\vec{\omega}) = \langle 0|e^{i\vec{\omega}\cdot\vec{Q}}\epsilon H' e^{-i\vec{\omega}\cdot\vec{Q}}|0\rangle$$

has at least a local minimum at $\vec{\omega} = 0$.

These theorems imply that if the symmetry of $|0\rangle$ and H_0 are not identical, so that there are Goldstone excitations, then these Goldstone bosons, when the degeneracy is lifted, will have real masses corresponding to a stable vacuum. These theorems also imply the domain structure of chiral-symmetry breaking described by Mathur and Okubo.[5]

Dashen's theorems, when applied to the symmetry of the strong interactions, constrain the form of strong symmetry breaking as was shown by him. If we include weak interactions another theorem will be useful for our purposes of establishing θ_C.

Theorem 3. Let $|\text{vac}\rangle$, the physical vacuum, be the ground state of $\hat{H} = U_C H U_C^{-1}$, $U_C = e^{iQ_7 2\theta_C}$ with *H independent* of θ_C. Then

$$F_C(\theta_C) \equiv \langle \text{vac}|\hat{H}|\text{vac}\rangle = \langle \text{vac}|U_C H U_C^{-1}|\text{vac}\rangle \quad (2)$$

satisfies

$$F_C'(\theta_C) = 0, \quad F_C''(\theta_C) > 0. \quad (3)$$

This theorem follows from the observation that $\hat{H}$ satisfies theorem 1. Consequently,

$$F(\theta) = \langle \text{vac}|e^{i2\theta Q_7}\hat{H}\, e^{-i2\theta Q_7}|\text{vac}\rangle = F_C(\theta + \theta_C)$$

satisfies $F'(0) = 0$, $F''(0) > 0$. This implies Eqs. (3).

The vacuum-stability conditions (3) can determine θ_C. The rule of this game is to pick an invariance group for $|\text{vac}\rangle$, pick an H (independent of θ_C), and calculate $F_C(\theta_C)$. The minimum of $F_C(\theta_C)$ fixes θ_C. The stability conditions (3) just guarantee that the Hamiltonian is diagonal with respect to the correct ground state. $\hat{H}$ then satisfies vacuum stability and $|\text{vac}\rangle$ is its ground state.

The conditions of theorem III are quite universal and the same ideas can be applied to other problems. For example, we will describe a superweak model for which CP violation is a rotation in the internal space in the same way that strangeness violation is implemented by a universal rotation.

II. EXAMPLES

Our procedure is best seen by explicit examples. To construct examples we assume that we can describe the strong, weak, and superweak interactions by an effective Hamiltonian at least to lowest order in the weak interactions. No attempt will be made to construct a complete renormalizable field theory of the weak and strong interactions (such as a natural gauge theory) or to show how the effective Hamiltonian can be obtained from a complete theory in some approximation. A satisfactory complete theory does not exist. Interestingly, our results can be obtained from an effective Hamiltonian. This effective Hamiltonian is constructed in accord with existing experimental phenomenology so that, hopefully, a future complete theory will reduce to this effective Hamiltonian in a suitable approximation.

The effective Hamiltonian we consider is

$$H = H_0 + H_{SB} + H_W(0) + H_{EM}. \quad (4)$$

H_0 is SU(3)×SU(3) invariant and it is assumed that in the absence of the additional terms in (4) the vacuum symmetry is SU(3) invariant. This implies that the SU(3)×SU(3) symmetry of H_0 is realized by degenerate SU(3) multiplets and an octet of pseudoscalar Goldstone excitations identified with the physical π, K, η. The SU(3)×SU(3) symmetry of H_0 is principally broken by H_{SB}. We assume that this term transforms like the neutral members of the $(\bar{3}, 3) + (3, \bar{3})$ representation of SU(3)×SU(3). Other pure representations may be considered, but we do not do this here. The $\bar{3}3 + 3\bar{3}$ model of Glashow and Weinberg[6] and Gell-Mann, Oakes, and Renner[7] adopted here is in reasonable agreement with existing experiments.[2]

The effective weak-interaction Hamiltonian (in the absence of strangeness violation) is taken to be

$$H_W(0) = (G/\sqrt{2})J_\mu^+ J_\mu^- + \text{H.c.}, \quad (5)$$

where

$$\vec{J}_\mu = \bar{N}_L\gamma_\mu(\tfrac{1}{2}\vec{\tau})N_L + \bar{l}_L^e\gamma_\mu(\tfrac{1}{2}\vec{\tau})l_L^e + \bar{l}_L^\mu\gamma_\mu(\tfrac{1}{2}\vec{\tau})l_L^\mu,$$

$$N = \begin{pmatrix}\mathcal{P}\\ \mathcal{N}\end{pmatrix}, \quad l^e = \begin{pmatrix}\nu_e\\ e^-\end{pmatrix}, \quad l^\mu = \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix},$$

$$N_L = \tfrac{1}{2}(1+\gamma_5)N, \quad l_L = \tfrac{1}{2}(1+\gamma_5)l.$$

Here N_L transforms like an $SU_L(2)$ doublet. For convenience we use the usual $(\mathcal{P}, \mathcal{N}, \lambda)$ SU(3) quark triplet, q. Charm can also be introduced.

The electromagnetic interaction $H_{EM} = eJ_\mu^{em}A_\mu$ plays no role in what follows, so we ignore it.

A. An elementary example

Let us assume H_{SB} in the $\bar{3}3 + 3\bar{3}$ representation conserves electric charge, parity, CP invariance and that H is specified as

$$H = H_0 + a\bar{\lambda}\lambda + b(\bar{\lambda}\mathcal{N} + \bar{\mathcal{N}}\lambda) + H_W(0). \quad (6)$$

Consequently,

$$\begin{aligned}\hat{H} = {} & U_C H U_C^{-1}\\ = {} & H_0 + (a\cos^2\theta_C + 2b\cos\theta_C\sin\theta_C)\bar{\lambda}\lambda\\ & + (a\sin^2\theta_C - 2b\cos\theta_C\sin\theta_C)\bar{\mathcal{N}}\mathcal{N}\\ & + [a\cos\theta_C\sin\theta_C + b(\cos^2\theta_C - \sin^2\theta_C)](\bar{\lambda}\mathcal{N} + \bar{\mathcal{N}}\lambda)\\ & + H_W(\theta_C),\end{aligned} \quad (7)$$

with $H_W(\theta_C)= U_C H_W(0) U_C^{-1}$. To compute the vacuum energy we must specify the invariance group of the physical ground state $|\text{vac}\rangle$. This physical $|\text{vac}\rangle$ we assume, in accord with observation, conserves the third component of isospin and hypercharge to $O(G)$. Consequently,

$$\langle \text{vac}|H_W(\theta_C)|\text{vac}\rangle = O(G)$$

and

$$\langle \text{vac}|(\bar{\lambda}\mathfrak{N} + \bar{\mathfrak{N}}\lambda)|\text{vac}\rangle = O(G).$$

From (7) one obtains for the vacuum energy correct up to $O(G)$

$$F_C(\theta_C) = \langle \text{vac}|\hat{H}|\text{vac}\rangle = \text{const} + (\sqrt{3}\langle U_8\rangle_0 - \langle U_3\rangle_0) \times[\tfrac{1}{4}a(\sin^2\theta_C - \cos^2\theta_C) - b\cos\theta_C\sin\theta_C] \tag{8}$$

where

$$U^a = \bar{q}\lambda^a q$$

and

$$\langle U^a\rangle_0 = \langle \text{vac}|U^a|\text{vac}\rangle .$$

Including the terms of $O(G)$ can only alter our conclusion by a very small amount.

The vacuum-stability condition, theorem 3, requires that $F'(\theta_C) = 0$ or

$$(\sqrt{3}\langle U_8\rangle_0 - \langle U_3\rangle_0) \times[a\sin\theta_C\cos\theta_C - b(\cos^2\theta_C - \sin^2\theta_C)] = 0 . \tag{9}$$

Since SU(3) is broken in the physical vacuum much more than isospin,

$$\sqrt{3}\langle U_8\rangle_0 - \langle U_3\rangle_0 \neq 0 ,$$

and (9) implies

$$\tan 2\theta_C = 2b/a . \tag{10}$$

Note that if the vacuum were SU(3) invariant, $\langle U_8\rangle_0 = \langle U_3\rangle_0 = 0$, so (9) is trivial and θ_C is undefined, corresponding to the well-known fact that SU(3) must be broken to define θ_C.[8]

Substituting (10) into (7) the Hamiltonian is

$$\hat{H} = H_0 + m_\lambda(\bar{\lambda}\lambda - \tan^2\theta_C\,\bar{\mathfrak{N}}\mathfrak{N}) + H_W(\theta_C), \tag{11}$$

where $m_\lambda = a + b = a(1 + \frac{1}{2}\tan 2\theta_C)$ is the mass of the λ quark. Note that (11) has the $\mathfrak{N}$-quark mass negative relative to the λ-quark mass. This implies $\mu_\pi^2/\mu_K^2 < 0$ and is the signal that (11) does not yet satisfy Dashen's theorems. To satisfy Dashen's theorems we transform H by $U_T = e^{i\mathfrak{N}_R^\dagger\mathfrak{N}_R\gamma}$ with $\mathfrak{N}_R = \frac{1}{2}(1-\gamma_5)\mathfrak{N}$ and γ an angle to be determined. So

$$\hat{H}' = U_T\hat{H}U_T^{-1} = H_0 + m_\lambda(\bar{\lambda}\lambda - \tan^2\theta_C\cos\gamma\,\bar{\mathfrak{N}}\mathfrak{N}) + H_W(\theta_C).$$

With the convention $\langle\bar{\lambda}\lambda\rangle_0 < 0$ and using the fact that vacuum SU(3)× SU(3) breaking is much larger than SU(3) breaking, the vacuum value of $\hat{H}'$ is a minimum for $\gamma = \pi$. (This corresponds to $\mathfrak{N}_R \to -\mathfrak{N}_R$.)

Finally we have the Hamiltonian that satisfies vacuum stability with respect to $|\text{vac}\rangle$:

$$\hat{H}' = H_0 + m_\lambda(\bar{\lambda}\lambda + \tan^2\theta_C\bar{\mathfrak{N}}\mathfrak{N}) + H_W(\theta_C). \tag{12}$$

This implies

$$\tan^2\theta_C = m_{\mathfrak{N}}/m_\lambda, \quad m_{\mathcal{P}} = 0 . \tag{13}$$

In terms of the parameters ϵ_i of the Gell-Mann–Oakes–Renner model[7]

$$H' = H_0 + \epsilon_0 U_0 + \epsilon_8 U_8 + \epsilon_3 U_3 + H_W(\theta_C). \tag{14}$$

The quark mechanical masses are

$$m_\lambda = (\tfrac{2}{3})^{1/2}\epsilon_0 - (2/\sqrt{3})\epsilon_8 ,$$

$$m_{\mathfrak{N}} = (\tfrac{2}{3})^{1/2}\epsilon_0 + \epsilon_8/\sqrt{3} - \epsilon_3 ,$$

$$m_{\mathcal{P}} = (\tfrac{2}{3})^{1/2}\epsilon_0 + \epsilon_8/\sqrt{3} + \epsilon_3 ,$$

and the relations (13) imply

$$\tan^2\theta_C = \frac{2(\sqrt{2}\,\epsilon_0 + \epsilon_8)}{\sqrt{2}\,\epsilon_0 - 2\epsilon_8}, \quad \frac{\sqrt{3}\,\epsilon_3}{\sqrt{2}\,\epsilon_0 + \epsilon_8} = -1 . \tag{15}$$

The almost-Goldstone-boson masses are calculated as

$$\frac{\mu_\pi^2 f_\pi}{\mu_K^2 f_K} = \frac{\epsilon_0\sqrt{2} + \epsilon_8}{\epsilon_0\sqrt{2} - \frac{1}{2}\epsilon_8}, \tag{16}$$

so using $\mu_\pi^2 f_\pi/\mu_K^2 f_K \ll 1$, Eqs. (15) and (16) imply

$$\tan^2\theta_C \simeq f_\pi\mu_\pi^2/f_K\mu_K^2 .$$

The relation is (1), which is good.

The Hamiltonian (12) also implies the presence of a nonelectromagnetic $|\Delta I| = 1$ interaction. Often this term is estimated from the observed K^+-K^0 mass difference. However, the purely electromagnetic contribution to this level shift cannot be calculated reliably,[9] and it competes with the $\epsilon_3 U_3$ term. But conventional electromagnetism, because of Sutherland's theorem,[10] contributes only a tiny faction of the observed $\eta\to 3\pi$ rate. So the observed rate is good evidence for the presence of a new nonelectromagnetic $|\Delta I| = 1$ interaction such as $\epsilon_3 U_3$ to which Sutherland's theorem does not apply. From the observed $\eta\to 3\pi$ rate one finds[11]

$$\left|\frac{\sqrt{3}\,\epsilon_3}{\sqrt{2}\,\epsilon_0 + \epsilon_8}\right| \simeq 0.7 , \tag{17}$$

with errors in excess of 50%. From (16) one has

$$\left|\frac{\sqrt{3}\,\epsilon_3}{\sqrt{2}\,\epsilon_0 + \epsilon_8}\right| = 1 , \tag{18}$$

in agreement with (17). The $\epsilon_3 U_3$ term also contributes to $\Delta I = 1$ baryon and meson mass shifts with the right sign, but using (17) it is about a factor of 3 too large[11] in magnitude.

These relations (18) and (15) have appeared since the early literature on this problem. Oakes[12] assumed an initial Hamiltonian of the form $H = H_0 + a\bar{\lambda}\lambda + H_W(0)$, so the strong interaction was SU(2)×SU(2) invariant. Upon rotation $H \to U_C H U_C^{-1}$ and throwing away a large strangeness-violating U_6 term Oakes obtains (18) and (15). Although this procedure is not justified, it suggests the relations among the various small parameters given by (18) and (15). Similar results are obtained in the programs of Cabibbo and Maiani[13] and Gatto, Sartori, and Tonin.[14] These approaches find the origin of the weak angle in the dynamics of higher-order weak interactions (which have no explicit part in our approach).

Our results are a consequence of our choice (6) of the Hamiltonian H and the invariance group of the physical ground state. H appears to violate strangeness because of the large $b(\bar{\lambda}\mathfrak{N} + \bar{\mathfrak{N}}\lambda)$ term. Since the physical ground state is strangeness-conserving up to order G, the vacuum-stability condition requires that we rotate away the b term. Then the strong interaction is strangeness-conserving but the weak interactions are not. The rotation angle, in our example, is completely specified in terms of symmetry-breaking parameters $\epsilon_{0,8,3}$ that appear in the diagonalized Hamiltonian $\hat{H}'$ that satisfies all vacuum-stability conditions. As these parameters are connected to experimental numbers we may calculate θ_C.

We may consider a more general example than (6). Let H_{SB} conserve Q, P, CP and have $|\Delta I| \leq \frac{1}{2}$. Then the Hamiltonian is uniquely specified as

$$H = H_0 + a\bar{\lambda}\lambda + b(\bar{\lambda}\mathfrak{N} + \bar{\mathfrak{N}}\lambda) + c(\bar{\mathcal{P}}\mathcal{P} + \bar{\mathfrak{N}}\mathfrak{N}) + H_W(0)\,. \tag{19}$$

Application of theorem 3 yields as a solution to the vacuum-stability problem the Hamiltonian (14) with

$$\tan^2\theta_C = \left|\frac{m_{\mathcal{P}} - m_{\mathfrak{N}}}{m_\lambda - m_{\mathcal{P}}}\right| = \left|\frac{2\epsilon_3}{\sqrt{3}\,\epsilon_8 - \epsilon_3}\right|\,, \tag{20}$$

so that θ_C is related to the $\eta \to 3\pi$ rate. Using (16) and (17) one finds from (20) $\theta_C \approx 15°$. Of course, there are large uncertainties in this estimate.

Suppose we relax the condition $|\Delta I| \leq \frac{1}{2}$ for the Hamiltonian given by (19). Then we can add a term $d(\bar{\mathcal{P}}\mathcal{P} - \bar{\mathfrak{N}}\mathfrak{N})$ to (19). Going through the vacuum stability exercise one finds θ_C specified in terms of the unknown parameters a, b, c, d of H but it is not given purely in terms of the known parameters $\epsilon_{0,8,3}$ of $\hat{H}'$. So the angle θ_C is fixed and nontrivial if $b \neq 0$, but it is not computable in terms of parameters we can control.

It is just this problem that prevents a calculation of θ_C in gauge models.[15] In gauge models of the weak interactions, with the strong interactions included, which are renormalizable and natural (in the technical sense) the vacuum-stability conditions can fix θ_C. However, typically, θ_C is specified and nontrivially given in terms of Lagrangian parameters that are not known experimentally. As is well known, parameters abound if we allow all couplings consistent with gauge invariance, renormalizability, and continuous and discrete symmetries, and one typically finds in the effective Hamiltonian a term like $\bar{\mathfrak{N}}\mathfrak{N}$ that prohibits a calculation of θ_C. Within the context of a satisfactory renormalizable field theory the problem of computing and getting the observed answer for θ_C (rather than simply fixing the angle in terms of unknown parameters) is completely unsolved. However, should such a field theory yield the effective Hamiltonian (19), then the problem is solved utilizing vacuum stability.

B. Spontaneous violation of strangeness

The Hamiltonian H given in (6) can be the consequence of spontaneous violation of "strangeness". This is accomplished by writing

$$H = H_0 + a\bar{\lambda}\lambda + b(\bar{\lambda}\mathfrak{N}\phi + \text{H.c.}) + H_W(0) + H(\phi, W_\mu)\,, \tag{21}$$

where ϕ is a complex scalar field with "S" = 1 so that (21) is "strangeness"-conserving. $H(\phi, W_\mu)$ is the Hamiltonian for the ϕ field and W_μ is a gauge vector field introduced as a compensating field for the local transformations $\lambda \to e^{i\theta(x)}\lambda$, $\phi \to e^{i\theta(x)}\phi$. $H(\phi, W_\mu)$ is arranged so that $\langle\phi\rangle_0 \neq 0$. Then W_μ acquires a mass by the Higgs mechanism. The remaining real fields ϕ' and W_μ can be made massive, so (21) with the displaced field $\phi' = \phi - \langle\phi\rangle_0$ has the same effective structure as (6). We return this example when we consider CP violation.

This example is completely unrealistic and is given for illustrative purposes only. It is not difficult to construct realistic renormalizable gauge models of the weak interactions which implement spontaneous strangeness violation (including charm). However, all such realistic, renormalizable, and natural models known to the author at most fix θ_C and do not allow a computation in terms of known parameters.

C. Charm

If one assumes that the weak charges belonging to SU(3) obey an SU(2) algebra, as is required if we are to have a gauge theory, and that they have both strangeness-violating and strangeness-conserving components, then the semileptonic currents have $|\Delta S|=1$ neutral components. This is associated with the fact that SU(3) does not have two commuting SU(2) subgroups. If one goes to SU(4), there are two commuting SU(2) subgroups and by arranging these properly one avoids neutral $|\Delta S|=1$ currents while retaining the desired currents. This Glashow-Iliopoulos-Maiani mechanism[16] offers symptomatic relief for the problem of no observed $|\Delta S|=1$ neutral, semileptonic currents comparable to $|\Delta S|=1$ charged currents. If it is really the solution for this problem, then we will see charmed states.[17]

To include charm we introduce the SU(2) doublets

$$N^s=\begin{pmatrix}\mathcal{P}'\\ \lambda\end{pmatrix},\quad N=\begin{pmatrix}\mathcal{P}\\ \mathfrak{N}\end{pmatrix}, \tag{22}$$

where $\mathcal{P}$, $\mathfrak{N}$, λ are as before and $\mathcal{P}'$, the charmed quark, is a strange partner of the $\mathcal{P}$. The $\mathcal{P}'$ mass should be larger than the masses of the other quarks but not too large.[18] The weak current is

$$\vec{J}_\mu=\overline{N}^s_L(\tfrac{1}{2}\vec{\tau})\gamma_\mu N^s_L+\overline{N}_L(\tfrac{1}{2}\vec{\tau})\gamma_\mu N_L+\overline{l}_L(\tfrac{1}{2}\vec{\tau})\gamma_\mu l_L \tag{23}$$

and the effective weak Hamiltonian

$$H_W(0)=(G/\sqrt{2})(\vec{J}_\mu\cdot\vec{J}_\mu+\text{H.c.}). \tag{24}$$

There is, of course, an additional U(1) current required to construct the electromagnetic current. Our discussion remains unaffected by its inclusion.

There are now two rotations $U_\lambda(\theta_\lambda)$ and $U_{\mathcal{P}'}(\theta_{\mathcal{P}'})$:

$$\begin{aligned}
&\lambda\to\lambda\cos\theta_\lambda-\mathfrak{N}\sin\theta_\lambda,\\
&\mathfrak{N}\to\mathfrak{N}\cos\theta_\lambda+\lambda\sin\theta_\lambda,\\
&\mathcal{P}'\to\mathcal{P}'\cos\theta_{\mathcal{P}'}-\mathcal{P}\sin\theta_{\mathcal{P}'},\\
&\mathcal{P}\to\mathcal{P}\cos\theta_{\mathcal{P}'}+\mathcal{P}'\sin\theta_{\mathcal{P}'}.
\end{aligned} \tag{25}$$

The effective Hamiltonian is assumed to be

$$H=H_0+a_{\mathcal{P}'}\overline{\mathcal{P}}'\mathcal{P}'+a_\lambda\overline{\lambda}\lambda+b(\overline{\mathcal{P}}'\mathcal{P}+\overline{\lambda}\mathfrak{N}+\text{H.c.})+H_W(0) \tag{26}$$

with H_0 SU(4)×SU(4) invariant and $a_{\mathcal{P}'}\neq a_\lambda$ positive constants.

If one minimizes the vacuum energy

$$F(\theta_\lambda,\theta_{\mathcal{P}'})=\langle\text{vac}|U_\lambda U_{\mathcal{P}'}HU_{\mathcal{P}'}^{-1}U_\lambda^{-1}|\text{vac}\rangle \tag{27}$$

with respect to θ_λ and $\theta_{\mathcal{P}'}$, carrying out the same exercise as for our previous example, one obtains for the Hamiltonian that satisfies vacuum stability

$$\hat{H}=H_0+m_{\mathcal{P}'}(\overline{\mathcal{P}}'\mathcal{P}'+\tan^2\theta_{\mathcal{P}'}\overline{\mathcal{P}}\mathcal{P})+m_\lambda(\overline{\lambda}\lambda+\tan^2\theta_\lambda\overline{\mathfrak{N}}\mathfrak{N})+H_W(\theta_C), \tag{28}$$

where the charged current in $H_W(\theta_C)$ is

$$J^-_\mu=\overline{\mathcal{P}}_L\gamma_\mu\mathfrak{N}_L\cos\theta_C+\overline{\mathcal{P}}_L\gamma_\mu\lambda_L\sin\theta_C+\overline{\mathcal{P}}'_L\gamma_\mu\lambda_L\cos\theta_C-\overline{\mathcal{P}}'_L\gamma_\mu\mathfrak{N}_L\sin\theta_C, \tag{29}$$

with

$$\theta_C=\theta_\lambda-\theta_{\mathcal{P}'}. \tag{30}$$

It is important that the λ and $\mathcal{P}'$ not be degenerate in (26) with $a_{\mathcal{P}'}=a_\lambda$. If this is the case, $\theta_C=\theta_\lambda-\theta_{\mathcal{P}'}=0$ and there is no strangeness violation.

From (28) we have

$$\tan^2\theta_{\mathcal{P}'}=m_{\mathcal{P}}/m_{\mathcal{P}'},\quad \tan^2\theta_\lambda=m_{\mathfrak{N}}/m_\lambda. \tag{31}$$

Since we have assumed equal $\overline{\mathcal{P}}'\mathcal{P}$ and $\overline{\lambda}\mathfrak{N}$ mixing in (26) we obtain an additional relation,

$$m_{\mathcal{P}'}m_{\mathcal{P}}(1-m_{\mathfrak{N}}/m_\lambda)^2=m_\lambda m_{\mathfrak{N}}(1-m_{\mathcal{P}}/m_{\mathcal{P}'})^2. \tag{32}$$

Assuming that the strange quarks are more massive than the nonstrange, $m_{\mathfrak{N}}/m_\lambda\ll 1$ and $m_{\mathcal{P}}/m_{\mathcal{P}'}\ll 1$, and (32) implies

$$m_\lambda/m_{\mathcal{P}'}=m_{\mathcal{P}}/m_{\mathfrak{N}}. \tag{33}$$

Since the charmed quark is the most massive $m_{\mathcal{P}'}>m_\lambda$, and (33) implies $m_{\mathfrak{N}}>m_{\mathcal{P}}$. This means that the nonelectromagnetic isospin-violating term ϵ_3U_3 has the correct sign to agree with observed $\Delta I=1$ mass shifts. These inequalities also imply $m_{\mathfrak{N}}/m_\lambda>m_{\mathcal{P}}/m_{\mathcal{P}'}$, so (31) implies $|\tan\theta_\lambda|>|\tan\theta_{\mathcal{P}'}|$.

Assuming, consistent with the above and existing spectroscopy, $m_{\mathfrak{N}}\gg m_{\mathcal{P}}$, $m_{\mathcal{P}'}>m_\lambda$, (31) implies $|\theta_\lambda|\gg|\theta_{\mathcal{P}'}|$, so that $\theta_C\simeq\theta_\lambda$. Using this, one has

$$\tan^2\theta_C\simeq\frac{m_{\mathfrak{N}}}{m_\lambda}=\frac{f_\pi\mu_\pi^2}{2f_K\mu_K^2}\left(1-\frac{\sqrt{3}\,\epsilon_3}{\sqrt{2}\,\epsilon_0+\epsilon_8}\right). \tag{34}$$

With the crude estimate from $\eta\to3\pi$, $-\sqrt{3}\,\epsilon_3\simeq(0.7)(\sqrt{2}\,\epsilon_0+\epsilon_8)$ as in (19), one obtains

$$\tan^2\theta_C\simeq(0.8)f_\pi\mu_\pi^2/f_K\mu_K^2. \tag{35}$$

So the result for the Cabibbo angle remains essentially intact.

If we allow unequal mixings of $\overline{\mathcal{P}}'\mathcal{P}$ and $\overline{\lambda}\mathfrak{N}$, then (32) and (33) are lost but (31) is retained. If we include terms like $c\overline{\mathfrak{N}}\mathfrak{N}$ or $d\overline{\mathcal{P}}\mathcal{P}$ in H, then (31) is lost and we cannot relate the angle θ_C to experimental numbers.

III. SUPERWEAK *CP* VIOLATION AS A ROTATION

Phenomenologically, the only observed $|\Delta S|=2$ weak effect is of order G^2 in the K_L-K_S mass difference. So such a $|\Delta S|=2$ term should be present in the effective Hamiltonian. We write

$$H=H+H_{SB}+H_W(0)+H_{EM}+H_{SW}, \tag{36}$$

where

$$H_{SW}=fG^2(U_6U_6-U_7U_7) \tag{37}$$

is $|\Delta S| = 2$ and f is a constant ≈ 1. For H_{SB} we take, as in (21),

$$H_{SB} = a\bar{\lambda}\lambda + b(\bar{\lambda}\mathfrak{N}\phi + \text{H.c.}) \quad (38)$$

with a and b real constants and ϕ a scalar field. As in Lee's discussion of CP violation[19] we allow the vacuum value of ϕ to be complex,

$$\langle\phi\rangle_0 = \rho e^{i\gamma}. \quad (39)$$

Assuming ϕ to have large mass the net effect of (39) is to have for the effective H_{SB} the following:

$$H_{SB} = a\bar{\lambda}\lambda + b\rho(\cos\gamma(\bar{\lambda}\mathfrak{N} + \bar{\mathfrak{N}}\lambda) + i\sin\gamma(\bar{\lambda}\mathfrak{N} - \bar{\mathfrak{N}}\lambda)). \quad (40)$$

We could also have simply taken (40) as our starting point.

Now the "CP" violating term in (40) is proportional to $b\rho\sin\gamma$, and this need not be particularly small. But the vacuum-stability conditions, assuming the physical vacuum has only an extremely small CP violation which can be dropped in evaluating the vacuum energy, will take care of this.

The solution to the vacuum-stability problem is achieved by the rotation

$$V = U_T U_C U_3, \quad (41)$$

where U_T and U_C are as before and $U_3 = e^{iQ_3 2\theta_{CP}}$, with Q_3 the generator of rotations about the third component of isospin. Minimizing the vacuum energy

$$F(\theta_C, \theta_{CP}) = \langle \text{vac}|VHV^{-1}|\text{vac}\rangle$$

with respect to the angles θ_C and θ_{CP} one obtains, as before,

$$\tan^2\theta_C = \frac{m_{\mathfrak{N}}}{m_\lambda} \simeq \frac{f_\pi \mu_\pi^2}{f_K \mu_K^2}, \quad m_\varphi = 0 \quad (42)$$

and

$$\theta_{CP} = \gamma. \quad (43)$$

The Hamiltonian satisfying the vacuum-stability conditions is

$$\hat{H} = VHV^{-1} = H_0 + m_\lambda(\bar{\lambda}\lambda + \tan^2\theta_C \bar{\mathfrak{N}}\mathfrak{N}) + H_W(\theta_C) + H_{EM} + H_{SW}(\theta_C, \theta_{CP}), \quad (44)$$

with

$$\begin{aligned} H_{SW}(\theta_C, \theta_{CP}) = fG^2(&\cos 2\theta_{CP}[(\cos^4\theta_C + \sin^4\theta_C)(V_7^2 - V_6^2) - \tfrac{1}{2}\sin^2(2\theta_C)(V_7^2 + V_6^2)] \\ &+ \sin^2\theta_{CP}\cos 2\theta_C(V_6V_7 + V_7V_6) + 2\sin 2\theta_{CP}\sin 2\theta_C\, V_6[2(\tfrac{2}{3})^{1/2}U_0 - (1/\sqrt{3})U_8 - U_3] \\ &+ \cos 2\theta_{CP}\sin 2\theta_C[2(\tfrac{2}{3})^{1/2}U_0 - (1/\sqrt{3})U_8 - U_3]\{2\cos 2\theta_C V_7 + \sin^2\theta_C[2(\tfrac{2}{3})^{1/2}U_0 - (1/\sqrt{3})U_8 - U_3]\}), \end{aligned} \quad (45)$$

where $V_a = i\bar{q}\gamma_5\lambda^a q$, $U_a = \bar{q}\lambda^a q$.

Accordingly, one has real CP violation that is $|\Delta S| = 2$ and $|\Delta S| = 1$ of order G^2. The new superweak $|\Delta S| = 1$ piece, proportional to $G^2\sin 2\theta_{CP}\sin 2\theta_C$, has no presently observable consequences. What remains is Wolfenstein's superweak model[20] with the mixing mass determined from (45):

$$\frac{\delta K_{LS}^2}{K_L^2 - K_S^2} = \tfrac{1}{2}\tan 2\theta_{CP}\frac{\cos 2\theta_C}{1 + \tfrac{1}{2}\sin^2 2\theta_C} \cong \tfrac{1}{2}\tan 2\theta_{CP}. \quad (46)$$

As is well known, this model is in agreement with all observed CP-violating effects.[21] From the observed $K_L \to 2\pi$ rate one obtains from (46)

$$\theta_{CP} \simeq 3\times 10^{-3}. \quad (47)$$

We see that superweak CP violation is a small rotation about Q_3 as strangeness violation is a rotation about Q_7. The fundamental angle of this rotation, θ_{CP}, is not determined in this approach, unlike the angle θ_C. Our point is that whatever is responsible for CP violation, if its primary effect is to induce a term $i(\bar{\lambda}\mathfrak{N} - \bar{\mathfrak{N}}\lambda)$ in the effective Hamiltonian, then this corresponds to the superweak model,[20] provided there is a $|\Delta S| = 2$ CP-conserving term in H. In a fundamental theory one would want to determine $\tan\theta_{CP}$, which is just the relative strength of the $i(\bar{\lambda}\mathfrak{N} - \bar{\mathfrak{N}}\lambda)$ "CP"-violating term to the $(\bar{\lambda}\mathfrak{N} + \bar{\mathfrak{N}}\lambda)$ "strangeness"-violating term in H. This suggests that CP and strangeness violation may have the same origin. Perhaps such terms are induced as a quantum effect[22] due to weak and electromagnetic interactions and the conjecture[23] $\theta_{CP} \sim \alpha/\pi$ is realized. One may include charm in this picture of CP violation, but it does not modify these ideas in an essential way.

IV. CONCLUSIONS

What this work shows is that the principle of vacuum stability expressed in theorems 1, 2, and 3 can determine the weak-interaction angle. Some elementary phenomenological examples yield realistic results. What this work leaves unanswered is the origin of strangeness and *CP* violations as we have expressed it in our effective Hamiltonians. Granted that these symmetries can be broken spontaneously, but why are they broken at all? In gauge theories of the weak interaction the structure of spontaneous breaking in the tree approximation is assumed from the beginning; so there are no surprises. This problem is quite unsolved. Possibly it would be instructive to examine vacuum stability on the quantum (loop) level and strangeness violation as well as *CP* violation as a quantum effect.

There may be other possible applications of these ideas. One that comes to mind is constraining the Weinberg angle[24] θ_W. It could be that this angle is fixed by embedding the $SU_L(2)\times U(1)$ group into a larger simple or semisimple group,[25] which is an alternative to these ideas of vacuum stability. A fanciful example of another application is worked out in the Appendix.

ACKNOWLEDGMENTS

I would like to thank Dr. Paul Langacker for discussions on an earlier version of this work. Most of this work was done at the Institute for Theoretical Physics at SUNY, Stony Brook, and I wish to thank Professor C. N. Yang and other members of the Institute for a pleasant visit. After completing this work I learned that similar conclusions regarding the Cabibbo angle had been independently reached by M. A. B. Bég and A. Zee (unpublished).

APPENDIX

Consider a pseudoscalar octet Π^a interacting with a baryon octet N_a. The SU(3)-invariant interaction is

$$I = g\Gamma_{abc}\overline{N}_a i\gamma_5 N_b \Pi_c, \quad \Gamma_{abc} = \alpha d_{abc} + i(1-\alpha)f_{abc}\,.$$

To lowest order in g^2 the vacuum energy is given as a function of α according to

$$V(\alpha) \propto g^2\Gamma_{abc}\Gamma^*_{abc} = g^2[\alpha^2 d_{abc}d_{abc} + (1-\alpha)^2 f_{abc}f_{abc}] = 8g^2[\tfrac{5}{3}\alpha^2 + 3(1-\alpha)^2]\,.$$

$V(\alpha)$ has a minimum at $\alpha = \frac{9}{14} = \frac{2}{3}\times\frac{27}{28} \simeq 0.64$, which is the experimental value. Since there is no reason to believe terms of $O(g^4)$ and higher will not completely change this result, it is probably fortuitous. However, this serves as an example of how variational principles might determine parameters, such as the f/d ratio.

---

*Visitor, September–October, 1974.

†Permanent address.

[1] R. Dashen, Phys. Rev. D 3, 1879 (1971). Our discussion, in part, follows this reference.

[2] These ideas are also reviewed in H. Pagels, Phys. Rep. 16C, 219 (1975), Chap. V.

[3] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963).

[4] The angular momentum $\vec{J}$ does not exist but its commutators do, and that is all that is actually required.

[5] V. Mathur and S. Okubo, Phys. Rev. D 1, 2046 (1970); 1, 3468 (1970).

[6] S. Glashow and S. Weinberg, Phys. Rev. Lett. 20, 224 (1968).

[7] M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[8] N. Cabibbo, in *Proceedings of the Fifth Coral Gables Conference on Symmetry Principles at High Energy, 1968*, edited by A. Perlmutter, C. A. Hurst, and B. Kurşunoğlu (Benjamin, New York, 1968), p. 339.

[9] P. Langacker and H. Pagels, Phys. Rev. D 8, 4620 (1973).

[10] D. G. Sutherland, Phys. Lett. 23, 384 (1966), J. S. Bell and D. G. Sutherland, Nucl. Phys. B4, 315 (1968).

[11] P. Langacker and H. Pagels, Phys. Rev. D 10, 2904 (1974).

[12] R. J. Oakes, Phys. Lett. 29B, 683 (1969).

[13] N. Cabibbo and L. Maiani, Phys. Lett. 28B, 131 (1968); Phys. Rev. D 1, 707 (1970).

[14] R. Gatto, G. Sartori, and M. Tonin, Phys. Lett. 28B, 128 (1967).

[15] Some attempts to determine θ_C in the context of gauge theories are the following: A. Zee, Phys. Rev. D 9, 1772 (1974); B. de Wit, Nucl. Phys. B 51, 237 (1973); H. Georgi and A. Pais, Phys. Rev. D 10, 539 (1974); Sec. II B; W. F. Palmer, *ibid.* 6, 2897 (1972).

[16] S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[17] For implications of charm see Ref. 16 and G. Snow, Nucl. Phys. B55, 445 (1973); J. Smith and Z. Uy, Phys. Rev. D 7, 2738 (1973); F. Bogomolny *et al.*, *ibid.* 8, 3237 (1973).

[18] Limits on the charmed mass are discussed in B. W. Lee and M. H. Gaillard, Phys. Rev. D 10, 897 (1974) and references therein.

[19] T. D. Lee, Phys. Rep. 9C, 145 (1973).

[20] L. Wolfenstein, Phys. Rev. Lett. 13, 180 (1964).

[21] This is discussed in M.A.B. Bég, Ann. Phys. (N.Y.) 52, 577 (1969).

[22] A. Zee [Phys. Rev. D 9, 1772 (1974)] and R. N. Mohapatra [*ibid.* 9, 3461 (1974)] have suggested that *CP* violation is a quantum effect. This is further discussed in H. Georgi and A. Pais, Phys. Rev. D 10, 1246 (1974).

[23] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. 139, B1650 (1965).

[24] S. Weinberg, Phys. Rev. Lett. 29, 388 (1972).

[25] H. Georgi and S. Glashow, Phys. Rev. Lett. 32, 438 (1974).