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Determination of the Δ^{++}-Δ^0 mass difference*

James S. Ball and Robert L. Goble†
Department of Physics, University of Utah, Salt Lake City, Utah 84112
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The 3-3 phase shift obtained from π^+p and π^-p scattering is used to determine the positions of the poles in the S matrix corresponding to the Δ^{++} and the Δ^0. The mass difference is then given by the separation of these poles.

INTRODUCTION

Electromagnetic mass differences have long been a challenge to our understanding of hadron physics; the oldest and most famous example is the still-to-be-understood proton-neutron mass difference. Until now the only unambiguous experimental determinations of mass differences have been for stable particles. This is understandable, since unstable particles have lifetimes very much shorter than 1/mass differences, and care must be taken in defining the mass of a resonance. It is also regrettable, since common to all of our present-day understanding of hadrons is the belief that there is no fundamental difference between stable and unstable particles; they are all members of quark algebra multiplets, or members of Regge trajectories, or whatever.

Recently Ball *et al.*[1] have shown that present π^+p scattering data are sufficiently reliable to permit a determination of the location of the pole in the 3-3 scattering amplitude to an accuracy better than the expected electromagnetic mass differences. The important feature of their analysis is the observation that the location of the pole is essentially independent of which resonance formula was used to fit the data, even though the parameters (M and Γ) in each formula range over several MeV, depending on which formula was used. A properly designed resonance formula has correct analytic properties in the physical region and a nearby pole on the second sheet which gives the rapid variation characterizing the resonance. The differences between resonance formulas are differences in the treatment of distant singularities which one hopes would not affect the pole parameters very much. It is gratifying that this seems to be the case in π-nucleon scattering as the pole position is the fundamental quantity: The position and residue of the pole define the resonance from the S-matrix viewpoint; they would be the expected output of a Lagrangian field theory.

The primary goal of this paper is to obtain the experimental $\Delta^{++}(1236)$, $\Delta^0(1236)$ mass difference in a model-independent manner. We hope this will help show that precision π-nucleon scattering experiments are of value, and that their significance does not depend on making "internal Coulomb corrections" which cannot be done accurately in the absence of a specific model for hadronic interactions.

In this paper we extend the work of Ball *et al.*[1] in three ways. First, the formulas they use do not, strictly speaking, have the correct analytic structure in the physical region, because they do not treat exactly the essential singularity at threshold caused by Coulomb scattering. The strength of the singularity is only of order α, the fine structure constant, but we are interested in determining masses to sufficient accuracy to see electromagnetic mass differences of that order. Following Wong and Noyes,[2] and Hamilton *et al.*,[3] we give formulas of the correct analytic form and show that with the present experimental accuracy this singularity is not important, provided one uses the so-called "nuclear" scattering amplitudes, as was done in Ref. 1. Second, we use the new data and phase-shift analysis of Carter *et al.*[4] Third, we perform a similar extrapolation for π^-p scattering, again using the data and analysis of Carter *et al.*[4] In making the extrapolation for π^-p scattering, we have been guided by the rule that our formula should have the correct analytic structure on the right. We thus explicitly take into account the difference between the π^0n channel and the π^-p channel thresholds, the existence of Coulomb scattering in the π^-p channel, and the inelastic process $\pi^-p\rightarrow\gamma n$. Comparing the pole positions we find for π^+p and π^-p scattering gives, we believe, the first reliable determination of the Δ^{++}, Δ^0 mass difference.

In their analysis Carter *et al.*[4] make use of a number of simplifications which are well supported by their data to their present level of accuracy and which we also assume. Specifically, we neglect magnetic moment terms and spin flip in treating Coulomb amplitudes; we take the π^+p inelasticity and likewise any purely hadronic π^-p inelasticity to be zero; and we assume the isospin-breaking amplitude to be of order α, which means that the imaginary part of the 3-3 π^-p amplitude comes from 3-3 intermediate states plus γn states, since the photoproduction amplitude is of order $\sqrt{\alpha}$. There are, however, two respects in which we treat the data differently from Carter *et al.*[4] (in addition, of course, to the major difference that we make an extrapolation in the complex plane). We have eliminated their calculated "Coulomb corrections" from their data, since we are looking for the electromagnetic mass difference, and if those corrections really were done correctly they would presumably not have seen any difference between the two charge states. We have also chosen not to use form factors in going from the total amplitude to the "nuclear" amplitude since they simply correspond to adding extra left-hand singularities and our analysis eschews a consideration of distant singularities.

EXTRAPOLATION PROCEDURE

We will begin our discussion with the simplest case, π^+p scattering, which has essentially only one channel. We define a function with simple analytic properties by considering the total p-wave scattering amplitude S,

$$S = S - S_C + S_C\,, \tag{1}$$

where S_C is the p-wave partial-wave amplitude for pure Coulomb scattering,

$$S_C = e^{2i\sigma}\,, \tag{2}$$

where σ is the argument of $\Gamma(2+i\eta)$ and $\eta = \alpha/v$, where v is the lab relative velocity. Then

$$\frac{S-1}{2i} \equiv T = \frac{S-S_C}{2i} + \frac{S_C-1}{2i} = S_C T_N + T_C\,, \tag{3}$$

where T_N is the "nuclear" scattering amplitude and satisfies an elastic unitarity condition. As a first approximation we could neglect the fact that T_N still contains a residual effect of the Coulomb scattering in the form of an essential singularity at threshold, and define a function with good analytic properties,

$$M = \frac{T_N}{q^3}\,, \tag{4}$$

where q is the c.m. momentum. The analyticity of M can be expressed in

$$M^{-1} = K - iq^3\,, \tag{5}$$

where (while we still neglect the residual essential singularity of K at $q^2=0$) K is a real analytic function in a neighborhood of the physical region. We exploit the good analytic behavior of K by making an effective-range approximation; we fit the data by using a third-order polynomial for K and then extrapolate the corresponding M^{-1} onto the second sheet to find its zero. This was essentially the procedure of Ref. 1. We check the neglect of the essential singularity in K by defining a function free of it[2,3]:

$$\tilde{M}^{-1} = C^2(1+\eta^2)M^{-1}\,, \tag{6}$$

$$C^2 = 2\pi\eta/(e^{2\pi\eta}-1)\,. \tag{7}$$

Then the analytic properties of M are given by the expression

$$\tilde{M}^{-1} = \tilde{K} - Q - iq^3C^2(1+\eta^2)\,, \tag{8}$$

where $\tilde{K}$ is analytic in a neighborhood of the physical region, even with the effects of Coulomb scattering, and

$$Q = q^3\eta\big(\psi(i\eta)+\psi(-i\eta) - 2\ln\eta\big)(1+\eta^2)\,, \tag{9}$$

where ψ is the digamma function. We use this analyticity by fitting $\tilde{K}$ to a third-order polynomial and find a zero in $\tilde{M}^{-1}$ at the point on the second sheet,

$$M_{\Delta^{++}} = \sqrt{s} = (1211.5 - 50.1i)\ \text{MeV}\,. \tag{10}$$

Our fit to $\tilde{K}$ is

$$\tilde{K} = A_0 + A_1x + A_2x^2 + A_3x^3\,, \tag{11}$$

where $x = W/140\ \text{MeV} - 8.5$, W is the total c.m. energy, and our determination of the parameters is $A_0 = 2.608$, $A_1 = -5.873$, $A_2 = -5.646$, and $A_3 = -2.550$. We have used the quoted errors of Carter *et al.*,[4] χ^2 was 1 per degree of freedom, and the pole position in Eq. (10) was determined with an error of 0.6 MeV absolute magnitude in the complex plane. To calculate the error in determining the pole position we used the standard formulas discussed by Cziffra and Moravcsik[5]; however, we have no way of accounting for the fact that in the analysis of Carter *et al.*[4] the errors for each data point may not really be independent. We find that within this experimental error it makes no difference whether K or $\tilde{K}$ is used for the extrapolation, though this could become important if there were an improvement by a factor of 3 in experimental accuracy or if lower-energy data points were used.

The analysis of π^-p scattering is more complicated in two ways. First, there are three relevant channels, only one of which has an asymptotic Coulomb interaction; second, the two hadronic channels have a sizable difference in their threshold energies, which violates isospin invariance. For this reason we must work with charge states rather than isospin states. Hence, we take the partial-wave S matrix to be a 3×3 matrix operating in the states

$$\begin{pmatrix} \pi^- p \\ \pi^0 n \\ \gamma n \end{pmatrix}.$$

As before, we separate out a "nuclear" scattering matrix T_N,

$$T = S_C{}^{1/2} T_N S_C{}^{1/2} + T_C\,, \tag{12}$$

where S_C is now the matrix for attractive pure p-wave Coulomb scattering in the $\pi^- p$ channel,

$$S_C = \begin{pmatrix} e^{-2i\sigma} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{13}$$

We define a matrix

$$M = \rho^{-1/2} T_N \rho^{-1/2}\,, \tag{14}$$

$$\rho = \begin{pmatrix} q_-{}^3 & 0 & 0 \\ 0 & q_0{}^3 & 0 \\ 0 & 0 & q_\gamma{}^3 \end{pmatrix}, \tag{15}$$

where q_-, q_0, and q_γ are the c.m. momenta in the $\pi^- p$, $\pi^0 n$, and γn channels, respectively. Again, neglecting the essential singularity at the $\pi^- p$ threshold, M has simple analytic properties:

$$M^{-1} = K - i\rho\,, \tag{16}$$

with K analytic in the neighborhood of the physical region. It is convenient to perform an isospin rotation on M using the states

$$\begin{pmatrix} I = \frac{3}{2} \\ I = \frac{1}{2} \\ \gamma n \end{pmatrix}$$

as a basis:

$$M_I = RMR^{-1}\,, \tag{17}$$

where

$$R = \begin{pmatrix} (\frac{1}{3})^{1/2} & (\frac{2}{3})^{1/2} & 0 \\ -(\frac{2}{3})^{1/2} & (\frac{1}{3})^{1/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{18}$$

M_I has the property that the only terms of order $\sqrt{\alpha}$ or bigger are M_{11}, M_{33}, and $M_{3\gamma}$, and the pole in M appears with a large residue in M_{33}. This makes it convenient to fit $M_{33}{}^{-1}$,

$$M_{33}{}^{-1} = K_1 - i(\tfrac{1}{3} q_-{}^3 + \tfrac{2}{3} q_0{}^3) - i q_\gamma{}^3 K_2\,, \tag{19}$$

where K_1 and K_2 are meromorphic on the right (still neglecting that essential singularity) and the other contributions to the imaginary part of $M_{33}{}^{-1}$ are smaller by a factor of α. [Here we use the fact that in the energy range of the experiments $(q_- - q_0)/q_0 \sim \alpha$.] We have fitted K_1 to a third-order polynomial and K_2 to a constant and have then extrapolated the resulting M_{33} in the second sheet to find its pole. We can again test the neglect of the essential singularity by defining a matrix which is free of it[3]:

$$\tilde{M}^{-1} = \phi^{1/2} M^{-1} \phi^{1/2}\,, \tag{20}$$

where

$$\phi = \begin{pmatrix} C_-{}^2(1+\eta^2) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{21}$$

and

$$C_-{}^2 = -2\pi\eta/(e^{-2\pi\eta} - 1)\,. \tag{22}$$

Then $\tilde{M}$ has simple analytic properties

$$\tilde{M}^{-1} = \tilde{K} + \tilde{Q} - i\phi^{1/2}\rho\phi^{1/2}\,, \tag{23}$$

where $\tilde{K}$ is a real matrix analytic in a neighborhood of the physical region,

$$\tilde{Q} = \begin{pmatrix} Q & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{24}$$

with Q defined by Eq. (9). Again, it is useful to rotate in isotopic spin,

$$\tilde{M}_I = R\tilde{M}R^{-1}\,, \tag{25}$$

and again, to first order in α, the pole can be located by writing

$$\tilde{M}_{33}{}^{-1} = \tilde{K}_1 + \tfrac{1}{3} Q - i[\tfrac{1}{3} q_-{}^3 C_-{}^2 (1+\eta^2) + \tfrac{2}{3} q_0{}^3 + q_\gamma{}^3 K_2]\,. \tag{26}$$

We have fitted $\tilde{K}_1$ to a third-order polynomial, $\tilde{K}_2$ to a constant, and when $\tilde{M}_{33}$ is continued into the second sheet, we find a pole located at

$$M_{\Delta^0} = \sqrt{s} = (1211.6 - 53.0i)\text{ MeV}\,. \tag{27}$$

$\tilde{K}_1$ was written

$$\tilde{K}_1 = A_0' + A_1' x + A_2' x^2 + A_3' x^3\,, \tag{28}$$

with x as before and the parameters of our fit were $A_0' = 2.426$, $A_1' = -6.172$, $A_2' = -6.171$, $A_3' = -2.189$, and $K_2 = 0.0182$. It is difficult for us to estimate the error of our pole determination for this case because of the way the data were analyzed. In their analysis Carter *et al.*[4] imposed an "effective inelasticity" that came partly from the data on $\pi^- p \to \gamma n$,[6] and partly was to account for the difference in thresholds. We have made two assumptions for our fits: One was to assume a uniform error of 1% in all data points; the second, which corresponds slightly better to the quoted errors, was to assume an error of 0.01 in $\cot\delta$. Both assumptions gave comparable fits, both with acceptable χ^2. From the quoted errors in the phase shift we would estimate that the error in this determination is somewhat greater than for the Δ^{++}.

RESULTS AND CONCLUSION

Our final result is

$$M_{\Delta^{++}} - M_{\Delta^0} = 0 + 3i \text{ MeV}. \tag{29}$$

It is interesting that this mass difference is small and, to the accuracy of the data, entirely imaginary. This would never happen in conventional field-theoretic calculations such as those of Socolow[7] in which the tacit assumption is made that the particles are stable, so all mass differences are real. It will clearly be necessary to take into account the fact that there are open channels in explaining this result.

The residue of the poles is in principle also an important quantity and can be calculated from the parameters of our fits, Eqs. (11) and (28). However, it is necessary to decide in the case of the Δ^0 precisely for which amplitude the residue is to be calculated when one is considering differences of the order of electromagnetic effects.

We wish once again to emphasize that more precise data can substantially improve the determination in Eq. (29). Such data can and should be analyzed in a model-independent way.

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†Present address: Department of Physics, Montana State University, Bozeman, Montana 59715.

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[7]For a survey of such calculations see R. H. Socolow, Ph.D. thesis, Harvard University, 1964 (unpublished); also S. Coleman and R. H. Socolow, Phys. Rev. 135, B1451 (1964). It is interesting that in Socolow's calculation the tadpole contribution is of opposite sign to the other terms, so he does predict a small real part for this mass difference.