PHYSICAL REVIEW D VOLUME 11, NUMBER 5 1 MARCH 1975

Anomalous lepton-hadron interactions and gauge models

Jogesh C. Pati*
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742 and International Centre for Theoretical Physics, Trieste, Italy

Abdus Salam
International Centre for Theoretical Physics, Trieste, Italy and Imperial College, London, England

(Received 5 September 1974)

Parameters for anomalous lepton-hadron interactions (such as their signs, V and A character, and allowed and forbidden nature of certain transitions) are abstracted from the class of guage models proposed earlier by the authors. This information is used to determine the strength of the anomalous interactions by fitting e^-e^+-annihilation data. We then make quantitative estimates of the energy dependence of this cross section, the deviation of the ratio of (e^+p/e^-p) cross sections from unity at high q^2, and (apparent) deviations from scaling in ep scattering. Also discussed are consequences of anomalous interactions (with the restrictions mentioned above) on enhanced lepton production in hadronic collisions, hyperfine structure splitting in hydrogen, and leptonic decay modes of π^0 and η. On the theoretical side, we discuss variants of the basic gauge model which allow the anomalous lepton-hadron interactions to be relevant at present energies. An analysis of the data involving $e^\pm$ alone inclines one to the view that the electron is likely to be "strange" if its interaction with hadrons is "anomalous" at present energies. Further data are needed to test this possibility and also whether the muon and the electron neutrino are anomalous and whether parity is conserved in these anomalous lepton-hadron interactions. It may, of course, be that leptons do possess anomalously strong interactions but only at high energies proposed in our basic gauge model, in which case such interactions are irrelevant for SLAC energies. Several intriguing consequences of the maximal $SU(16)_L \times SU(16)_R$ local symmetry are mentioned.

I. INTRODUCTION

In attempting to unify[1] baryons and leptons within a gauge-theory context, we postulated[2] in 1973 a new class of lepton-hadron interactions which eventually must acquire the same strength as hadron-hadron interactions. For the so-called basic model of leptons and hadrons, which was examined in detail in an earlier paper, there appeared theoretical limitations, so it was estimated that the new anomalous interaction would manifest itself for energies in the region of 10^4 GeV.

Experimentally, however, the CEA-SPEAR enhancement[3] of $e^+ + e^- \to$ hadrons might possibly be indicative of the fact that the mechanism suggested by us may already have become operative at much lower energies, and this suggestion was advanced in a letter.[4] It was pointed out in this letter that

(i) experimental studies involving energy dependence and the magnitude of $\sigma(e^-e^+ \to \text{hadrons})$,
(ii) deviations of the ratios of (e^+p/e^-p) and (μ^+p/μ^-p) total cross sections from unity at high q^2, and
(iii) scaling behavior of $e+p \to e+$hadrons and apparent deviations therefrom

may provide further information on the existence and nature of such anomalous interactions.

In this paper we give quantitative estimates of the above effects and discuss other possible tests including enhancement of lepton production in hadron collisions.[5] In making these estimates, we rely heavily on gauge models for obtaining the basic parameters of the anomalous interaction (like coupling strengths, their signs, V and A character of these interactions, and the allowed and forbidden nature of certain transitions). On the dynamical side we make use of the parton-model hypothesis in order to get a feeling for the magnitude of these effects. We are prompted to make such estimates by the fact that some of the experiments relevant for testing our ideas are in progress and some are already completed (in particular the ratio of e^+p/e^-p total cross sections[6] for q^2 up to -15 GeV^2, while similar data on μ^+p/μ^-p are expected[7] to be available in the near future. Our chief conclusion is that the electron is likely to be a "strange" particle and its neutrino "charmed" in the sense of our gauge models (if its interactions with hadrons are to be anomalous at present energies), and this would imply an enhanced production of φ^0 and η^0 and possibly also $K\bar{K}$ as SLAC energy increases. In Appendix A we discuss explicit variants of our original model, to show that limitations on the relevant energies of the basic model can be relaxed so that anomalous lepton-hadron interaction can begin to manifest itself with requisite strength at the present low SLAC energies. However, we consider the new variant to be of a forced model and would hope that, if the electron does prove to be strange, a somewhat more attractive version of it emerges.

II. ANOMALOUS LEPTON-HADRON INTERACTIONS

The anomalous interactions of the charged leptons (e and μ) with quarks, which arise in the gauge-theory context of Ref. 2 and which are further discussed in Appendix A of this paper, are given by

$$\mathcal{L}_X = f_e^V(\bar{e}\gamma_\alpha q_e)X_{e\alpha}^V + f_e^A(\bar{e}i\gamma_\alpha\gamma_5 q_e)X_{e\alpha}^A + (e\to\mu) + \text{H.c.}, \qquad (1)$$

where q_e denotes the specific quark (in our case it is *either* $\mathfrak{N}$ *or* λ) coupled to the electron via the X's; similarly for q_μ. Since there is a triplet of X's corresponding to three baryonic colors, a summation over the color index of the quarks and of the X's is implied. In general, the vector fields $X_{e,\mu}^V$ may not be identical with the axial-vector fields $X_{e,\mu}^A$ and parity is conserved. [One may, however, also consider the case $X^V = X^A$ which will lead to parity-violating X interactions. Since there is no *a priori* reason (theoretically[8] or experimentally[9]) for the interactions in the X sector to be parity-conserving, we allow this possibility also.]

We wish to emphasize that in renormalizable gauge theories there are restrictions on the choices of q_e and q_μ owing to the interplay between the weak and strong gauge groups. For example, if one assumes that the gauge group has the structure $G = G_w \times G_s$, where G_w and G_s denote respectively the "weak" and nonchiral "strong" gauge groups commuting with each other (as considered in Ref. 2) and that the X's belong to the set of gauge mesons generated by G_s, then e^- can be coupled via X to either the $\mathfrak{N}$ quark or the λ quark, but not to both (see Appendix A). We refer to these two situations (e^- coupled to $\mathfrak{N}$ or λ) *as situations where the electron is "nonstrange" or "strange," respectively*. Also (if μ and e belong to the same fermion multiplet) μ will couple to the λ quark, where e^- couples with $\mathfrak{N}$ and vice versa. (In other words, if e is "strange," μ is not.) If, on the other hand, e and μ belong to different fermionic multiplets (as is the case of the variant model of Appendix A), both e and μ may be "strange." *But in no case can* e^- *and* μ^- *couple via the X's to the proton quark or the charmed quark* without conflicting with the charge and isospin assignments in the class[10] of gauge models mentioned above. In the sequel, though our discussion is phenomenological, we abstract the features of the X interaction from gauge-theory considerations. As will be seen, this leads to important experimental differences from other models[11] recently proposed in the literature. In summary, we consider the following three possibilities for the choices of q_e and q_μ:

(i) $(q_e, q_\mu) = (\mathfrak{N}, \lambda)$; e^- nonstrange, μ^- strange.
(ii) $(q_e, q_\mu) = (\lambda, \mathfrak{N})$; μ^- nonstrange, e^- strange.
(iii) $(q_e, q_\mu) = (\lambda, \lambda')$ (such a possibility arises in the so-called prodigal model— see Appendix A). Both e^- and μ^- are strange.

A. Effective 4-fermion interaction; heavy-X case

In addition to quark mass, two important parameters in the model are the square of the coupling constant (f^2) and m_X^2. Two typical cases arise: (1) $f^2/4\pi$ is small (perhaps as small as $\approx 10^{-2}$); in this case, in order to account for the e^-e^+-annihilation data, X ought to be "light" ($M_X \approx 15$–30 GeV, say). The m_X^2 may be exceeded in energy by the next generation of experiments. We refer to this as the *light-X case* ($m_X^2 \sim s$). (2) f^2 is large ($f^2/4\pi \sim 1$); in this case X ought to be heavy ($m_X \simeq 100$ GeV) to account for the e^-e^+-annihilation data (see estimates later). In this case s (and t) $\ll m_X^2$. We refer to this as the *heavy-X case*. Most of this paper is concerned with the (simpler) heavy-X case, although in Sec. III B we briefly consider the case of a light-X mass. (From the gauge-theory point of view, both cases may be permissible; see the remark in Appendix A.)

For s and $t \ll m_X^2$, one may treat the effective current-current interaction mediated by the X's as a local 4-fermion interaction, which for the sequence of cases mentioned above reads as follows, after a Fierz reshuffle has been effected.

(I) Parity-conserving case. $(X_i^V)_{i=e,\mu}$ and $(X_i^A)_{i=e,\mu}$ are distinct fields with $f_i^V = \pm f_i^A \equiv f_i$:

$$\mathcal{L}_X^{\text{eff}} = \left(\frac{f}{m_{X^V}}\right)_e^2 (\tfrac{1}{4})(-4S_e S_{q_e} + 2V_e V_{q_e} + 2A_e A_{q_e} - 4P_e P_{q_e}) + \left(\frac{f}{m_{X^A}}\right)_e^2 (\tfrac{1}{4})(4S_e S_{q_e} + 2V_e V_{q_e} + 2A_e A_{q_e} + 4P_e P_{q_e}) + (e\to\mu). \qquad (2)$$

(II) Parity-nonconserving case. X_i^V and X_i^A are identical fields[12] with $f_i^V = \pm f_i^A \equiv f_i$:

$$\mathcal{L}_X'^{\text{eff}} = \left(\frac{f_e^2}{m_{X_e}^2}\right)[V_e V_{q_e} + A_e A_{q_e} \pm (V_e A_{q_e} + A_e V_{q_e})] + (e\to\mu). \qquad (3)$$

Here S_e, V_e, A_e, and P_e denote the bilinear lepton covariants $\{\bar{e}(X)\Gamma_i e(X)\}$ with $\Gamma_i = \underline{1}$, γ_μ, $i\gamma_\mu\gamma_5$, and γ_5, respectively; similarly for the quark covariants S_{q_e}, V_{q_e}, A_{q_e}, and P_{q_e}. The equality relations between $f^V_{e,\mu}$ and $f^A_{e,\mu}$ for I and II are suggested by various variants of gauge models. Note that the over-all sign of the above effective interaction is fixed in gauge models, since it is a consequence of a basic vector-type Yukawa interaction. This will result in the sign of the interference term between the X and the photon-mediated amplitudes (in e^-e^+ annihilation and $e^\pm p$ scatterings, for example) being fully determined.

Under (I), we consider two typical possibilities:

(IA) The mass of the axial-vector meson X^A is much larger than that of the vector meson X^V, so that we may drop the axial contribution. Alternatively, vector mass much exceeds axial mass so that we may drop the vector contribution. The two cases lead to identical results for all our considerations, since they differ only in the signs of the $S_e S_{q_e}$ and $P_e P_{q_e}$ terms, and these terms do not interfere with any other contributions. We refer to the two cases as "vector-X" and "axial-X" interactions, respectively.

(IB) Vector and axial masses are equal; in this case one has an effective $(VV+AA)$ interaction before and after Fierz reshuffle with no net S and P terms. (Consequences of the intermediate situation of m_{XA} being comparable to m_{XV} can, of course, be worked out from the formulas appearing in the text.) In summary, then we have three cases to consider:

(IA) "vector-X" or "axial-X," with effective interaction $\frac{1}{2}(VV+AA)\pm(SS+PP)$;

(IB) $(VV+AA)$ effective interaction, and

(II) $(V\pm A)(V\pm A)$ effective interaction.

For all three cases, only one X mass is relevant for "low"-energy X interactions involving the electron and similarly the muon and the superscripts V and A may therefore be dropped. The strength of such interactions may thus be characterized by the two positive parameters $\epsilon_{e,\mu}$ defined by

$$a_{X_i} \equiv (f_i^2/4\pi)(1/m_{X_i}^2) \equiv (\alpha\epsilon_i)/(\mathrm{GeV})^2 \quad (i=e,\mu)\,, \tag{4}$$

where $\alpha = e^2/4\pi = 1/137$. From now on we drop the subscript i also, as we consider processes involving the electron only; muon processes can be obtained by simple substitutions $\epsilon_e \to \epsilon_\mu$, $q_e \to q_\mu$.

III. ELECTRON-POSITRON ANNIHILATION

A. Heavy-X case

The contributions of one photon and X interactions (for $s \ll m_X^2$) to the cross section for e^-e^+ annihilation into hadrons are in general given by

$$\sigma_h(s) = \frac{4\pi s}{3}\left[\frac{\alpha^2\rho_{\gamma\gamma}(s)}{s^2} + \frac{2\alpha a_X \rho_{\gamma X}(s)}{s} + a_X^2\rho_{XX}(s)\right], \tag{5}$$

where $\rho_{\gamma\gamma}(s)$ and $\rho_{\gamma X}(s)$ represent the hadronic tensors for the current correlations $(V_\mu^{\mathrm{em}} V_\nu^{\mathrm{em}})$ and $\frac{1}{2}(V_\mu^{\mathrm{em}} V_\nu^X + V_\mu^X V_\nu^{\mathrm{em}})$, respectively, with $V_\mu^X = \frac{1}{2}(V_{q_e})_\mu$ for case (IA) and (V_{q_e}) for cases (IB) and (II). The function $\rho_{XX}(s)$ represents the sum of contributions from the correlations $(S_{q_e}S_{q_e})$, $(P_{q_e}P_{q_e})$, $(V_{q_e}V_{q_e})$, and $(A_{q_e}A_{q_e})$ with appropriate coefficients, which may be worked out for the three cases from Eqs. (2) and (3).

If s is in the asymptotic region, dimensional considerations and the scaling hypothesis suggest that all three functions $\rho_{\gamma\gamma}(s)$, $\rho_{\gamma X}(s)$, and $\rho_{XX}(s)$ are essentially "constants." If, in addition, we assume the validity of the light-cone hypothesis or parton-model considerations, we may evaluate these constants for a given model[13] using the general formulas in Appendix B. For the case where the proton quark is not involved,[14] we obtain

$$\begin{aligned} &\rho_{\gamma X}(s) = -1\,, \quad \rho_{XX}(s) = 6\,, \quad \text{(IA)}\\ &\rho_{\gamma X}(s) = -2\,, \quad \rho_{XX}(s) = 6\,, \quad \text{(IB)}\\ &\rho_{\gamma X}(s) = -2\,, \quad \rho_{XX}(s) = 12\,, \quad \text{(II)}\end{aligned} \tag{6}$$

while[15]

$$\rho_{\gamma\gamma}(s) = \sum_i Q_i^2$$

and $|e|Q_i$ denotes the electric charge of the ith-type quark. Collecting the formulas (4), (5), and (6), we obtain[16] (for the heavy-X case)

$$\sigma_h(s) = \frac{4\pi\alpha^2}{s}\left[\frac{\sum Q_i^2}{s} - \begin{pmatrix}1\\2\\2\end{pmatrix}\epsilon + \begin{pmatrix}6\\6\\12\end{pmatrix}\epsilon^2 s\right], \tag{7}$$

with the three rows corresponding to the cases (IA), (IB), and (II), respectively. Note the destructive interference between electromagnetism and the vector part of the X interaction. This comes about because $\mathcal{L}_X^{\mathrm{eff}}$ or $\mathcal{L}_X'^{\mathrm{eff}}$ were derived from a basic vector-type Yukawa interaction together with the fact that the electric charge of both $\mathfrak{N}$ and λ quarks is[14] $-\frac{1}{3}|e|$. The sign of the interference term is important in determining the magnitude of ϵ from the annihilation data.

We find that the reported data[3] for $\sigma_h(s)$, with s varying from 9 to 25 GeV^2, can be fitted reasonably well in all three cases (IA, IB, and II) with values of ϵ given in Table I and two typical values[17] for $\sum Q_i^2$:

$$\begin{aligned}\sum Q_i^2 &= 2 \quad \text{(3 triplets of fractional charges)}\\ &= \tfrac{10}{3} \quad \text{(3 quartets of fractional charges)}.\end{aligned} \tag{8}$$

These values of $\sum Q_i^2$ are still compatible with

TABLE I. Values of ϵ from annihilation data.

	"Vector X" (IA)	$(VV+AA)_{\text{eff}}$ (IB)	$(V\pm A)(V\pm A)_{\text{eff}}$ (II)
$\sum Q_i^2=2$	$\frac{1}{30}$	$\frac{1}{25}$	$\frac{1}{40}$
$\sum Q_i^2=\frac{10}{3}$	$\frac{1}{50}$	$\frac{1}{40}$ to $\frac{1}{50}$	$\frac{1}{50}$ to $\frac{1}{60}$

three quartets of integer-charge quarks, if we remark that when neither color nor charm is excited at SPEAR energies, then $\sum Q_i^2=2$, while if charm is excited but not color, then $\sum Q_i^2=\frac{10}{3}$. (Note that the values of ϵ given in Table I are in the same range as suggested in Ref. 4.)

B. Light-X case

For a lighter X mass ($m_X \simeq 10-30$ GeV), the local 4-fermion approximation to the effective interaction would be completely inadequate as available center-of-mass (energy)^2 s exceeds 50 GeV^2. In this situation, the s (and t) dependence of $e^-e^+ \to$ hadrons will be strongly dependent on the "structure" of the matrix element and the particular final state considered and one may no longer use the simple formula (7), which is valid exclusively for the heavy-X case ($m_X^2 \gg s, t$). Given the fact that we are dealing with a renormalizable theory, we remark that the s dependence of the cross section is not expected to be as steeply rising as for Eq. (7) when s approaches or exceeds m_X^2. [In fact, a variety of different complexions may arise depending upon the precise value of $\sqrt{s}$ in relation to m_X, m_q (quark mass) and possibly other masses[18] in the theory.] *In summary, the lack of linear rise of the cross section with s (at high s) is not to be taken as evidence against the possibility that the X mechanism provides an explanation for the known e^-e^+-annihilation data.*

From now on, we shall confine our discussions to the (simpler) heavy-X case, since it allows us to make definite quantitative predictions. It should be remarked, however, that if the s dependence is not as steep for the light-X case, as it is for the heavy-X case, the deviations of the (e^+p/e^-p) ratio from unity and departures from scaling in ep scattering would in general be less pronounced for a light X than for the heavy X for a given high s $(=q^2)$.

IV. COMPARISON OF (e^+p) VERSUS (e^-p) AND (μ^+p) VERSUS (μ^-p) IN THE DEEP-INELASTIC REGION

It was stressed in Ref. 4 that the interference between the vector (which arises from the electromagnetic as well as the X interaction) and the axial-vector interaction (originating from the X interaction only) should in general lead to a measurable difference between (e^+p) and (e^-p) cross sections, especially for large $q^2 \gtrsim 1/\epsilon$. Below, we make a quantitative estimate of this difference for the heavy-X case if we assume $q_e=\mathcal{N}$ or λ and make free use of the parton-model hypothesis. The relevant formula for deep-inelastic $e^{\pm}p \to e^{\pm}+H$ cross sections for a general 4-fermion interaction containing the covariants SS, PP, VV, AA, VA, and AV is given in Appendix B. The ratio of (e^+p) and (e^-p) cross sections for given values of incident lepton energy E, scattering angle θ, and momentum transfer squared q^2 is given by

$$\frac{d\sigma^{e^+}(E,\theta,q^2)}{d\sigma^{e^-}(E,\theta,q^2)}=\frac{X_+}{X_-}\ , \tag{9}$$

where $X_\pm$ for our three cases (IA, IB, and II) are given by

$$X_\pm=\left\{Q_{\mathcal{P}}^2 f_{\mathcal{P}}(x)+Q_{\mathcal{N}}^2 f_{\mathcal{N}}(x)+f_{q_e}(x)\left[-\frac{2}{3}\begin{pmatrix}\frac{1}{2}\\ 1\\ 1\end{pmatrix}\epsilon q^2+\epsilon^2 q^4\begin{pmatrix}\frac{1}{2}\\ 2\\ 4\end{pmatrix}\right]\right\}\times(1-y+\tfrac{1}{2}y^2)$$
$$+\left[f_{q_e}(x)\epsilon^2 q^4\begin{pmatrix}\frac{1}{2}\\ 0\\ 0\end{pmatrix}\right]y^2\pm\xi_{q_e}f_{q_e}(x)\left[-\frac{1}{3}\begin{pmatrix}\frac{1}{2}\\ 1\\ 1\end{pmatrix}\epsilon q^2+\epsilon^2 q^4\begin{pmatrix}\frac{1}{4}\\ 1\\ 2\end{pmatrix}\right]y(2-y)\,. \tag{10}$$

Here x and y are familiar kinematic variables defined in Appendix B; $f_{\mathcal{P},\mathcal{N},\lambda}(x)$ denote the quark momentum distribution functions within the proton; $\xi_{q_e}=+1$ for quarks and -1 for antiquarks. In (10) we have not exhibited the antiquark contribution or the λ-quark contribution in $\sum Q_i^2 f_i(x)$, since the antiquark and λ-quark distribution functions are negligible[19] (less than 5%) compared with those of the $\mathcal{P}$ and $\mathcal{N}$ quarks at $x>0.2$ (i.e., $\omega<5$). Note, however, that the antiquark contributions systematically tend to reduce the difference between (e^+p) and (e^-p) scatterings. This is because, on the one hand, they increase the symmetric term (given by the first two brackets) and, on the other, they decrease the asymmetric term [the last term in Eq. (10)] due to the ξ_{q_e} factor.

Following the same reasoning, we note that the ratio (X_+/X_-) *is expected to stay near unity for all values of* q^2 *in the event the electron is strange* ($i.e.$, $q_e=\lambda$), since in this case, if we assume that $f_\lambda(x)=f_{\bar\lambda}(x)$ within the proton for all x, the contribution of the λ quark to the last term in Eq. (10) is canceled by that of the $\bar\lambda$ quark.

On the other hand, *if the electron is nonstrange* ($i.e.$, $q_e=\mathfrak{N}$), *the ratio* (X_+/X_-) *would differ from unity for all* $q^2\neq 0$, since the neutron-quark and antineutron-quark distributions within the proton are very different from each other. The precise deviation from unity, however, depends sensitively on the ratio $f_{\mathcal{P}}(x)/f_{\mathfrak{N}}(x)$. The kinematic region of interest at which deviations from unity would be appreciable corresponds to high $|q^2|\gtrsim 10$ GeV^2 and, therefore, low $\omega\lesssim 5$ given that the energy E of the incident lepton at which the SLAC experiment[6] is performed is 13.9 GeV. For such low ω [especially for $\omega<2$, which is appropriate for $q^2\simeq-15$ GeV^2], the functions of $f_{\mathcal{P}}(x)$ and $f_{\mathfrak{N}}(x)$ as well as their ratio vary rapidly, leaving room for considerable uncertainty. Available information on $f_{\mathcal{P}}(x)/f_{\mathfrak{N}}(x)$ is given by the curves in Ref. 19, which are based on (ep) and (en) data as well as the fitting of the known sum rules. However, due to their rapidly varying nature, the precise numerical estimates of $f_{\mathcal{P}}(x)/f_{\mathfrak{N}}(x)$ for low ω based on these curves (and therefore the estimates of the deviations of e^+p/e^-p from unity) should be treated with some caution; only the qualitative trend may still be trusted. Be that as it may, we present in Table II values of (e^+p/e^-p) for the case where the electron is nonstrange (i.e., $q_e=\mathfrak{N}$) at several points relevant to the SLAC experiment, taking $(f_{\mathcal{P}}/f_{\mathfrak{N}})$ from Ref. 19.

Preliminary experimental measurements[6] seem to indicate that the ratio (e^+p/e^-p) is unity within $\pm$ 10% for q^2 varying between 0 and -15 GeV^2. This appears to exclude possibilities (IB) and (II) for the case where the electron is nonstrange ($q_e=\mathfrak{N}$). Case (IA) may still be acceptable, if we allow for the uncertainty in parton-model calculations together with the uncertainty in $f_{\mathcal{P}}(\omega)/f_{\mathfrak{N}}(\omega)$ for low values of ω, and allow for the antiquark contributions as well as the modifications due to the 2-photon contribution to $e^\pm p$ scatterings. However, if the experiments preserve the present trend for high $|q^2|$ ($\simeq 25$ GeV^2) and $\omega>1.5$, even case (IA) (with $q_e=\mathfrak{N}$) may be ruled out. *This would then leave us with the only possibility that electron is strange* ($i.e.$, $q_e=\lambda$).

Our remarks for the ratio (e^+p/e^-p) apply equally well to the ratio (μ^+p/μ^-p) if we substitute $\epsilon_e\to\epsilon_\mu$ and $q_e\to q_\mu$. Thus, a comparison of e^+p versus e^-p and (μ^+p) versus (μ^-p) at high $|q^2|$ would be most helpful in deciding if either the electron or the muon may be coupled to the neutron quark[20] via X with an effective strength[21] of order $2\times10^{-2}\alpha$ GeV^{-2}.

V. APPARENT DEVIATIONS FROM SCALING IN $(e^\pm p)$ AND $(\mu^\pm p)$ SCATTERINGS

As mentioned in Ref. 4, the replacement of the $1/q^2$ photonlike propagator by a constant $(-1/m_X{}^2)$ in the X contribution to the scattering amplitude would reflect itself in an apparent violation of scaling in deep-inelastic ep (or μp) scattering, even though intrinsic scaling may hold in the structure functions involving quark densities [such as $(V^{em}V^{em})$, $(V_{q_e}V_{q_e})$, $(A_{q_e}A_{q_e})$, $(V^{em}A_{q_e})$, $(V_{q_e}A_{q_e})$, and $(S_{q_e}S_{q_e})$, etc.]. The theoretical formula for the cross section $d^2\sigma/dxdy$ for ep scattering in the presence of axial-vector and vector interactions is given by

$$\frac{d^2\sigma}{dxdy}=\frac{4\pi\alpha^2 s}{q^4}\left[(1-y)\nu W_2(q^2,\nu)+xy^2m_pW_1(q^2,\nu)-xy(1-y/2)\nu W_3(q^2,\nu)\right], \tag{11}$$

where $s=(p+k)^2$, $x=1/\omega=-(q^2/2m_N\nu)$, $y=(p\cdot q/p\cdot k)$, and $\nu=(p\cdot q)/m_N=(E-E')$. The quantities p and k denote the 4-momenta of the incoming nucleon and lepton, respectively, while q is the 4-momentum transfer between the incoming and the outgoing lepton. Note the appearance of the νW_3 term due to (vector-axial-vector) density correlation (analogous to the case of neutrino-nucleon scattering). The parton-model formulas for the functions $\nu W_{2,3}$ and M_NW_1 for our three cases (IA, IB, and II) are given by

$$\nu W_2(x,q^2)=x\left\{\sum_i Q_i{}^2f_i(x)+f_{q_e}(x)\left[q^4\begin{pmatrix}\epsilon^2/4\\ 2\epsilon^2\\ 4\epsilon^2\end{pmatrix}+q^2(Q_{q_e})\begin{pmatrix}\epsilon\\ 2\epsilon\\ 2\epsilon\end{pmatrix}\right]\right\}$$

$$\equiv x\left[\sum_i Q_i{}^2f_i(x)+\Delta_2(q^2,\omega)\right],$$

$$M W_1(x,q^2)=\frac{1}{2}\left[\sum_i Q_i^2 f_i(x)+f_{q_e}(x)q^4\begin{pmatrix}\epsilon^2\\0\\0\end{pmatrix}\right]\equiv\frac{1}{2}\left[\sum_i Q_i^2 f_i(x)+\Delta_1(q^2,\omega)\right], \quad (12)$$

$$\nu W_3(x,q^2)=\pm 2 f_{q_e}(x)\left[Q_{q_e}q^2\begin{pmatrix}\epsilon/2\\ \epsilon\\ \epsilon\end{pmatrix}+q^4\begin{pmatrix}\epsilon^2/4\\ \epsilon^2\\ 2\epsilon^2\end{pmatrix}\right]\xi_{q_e}\,.$$

The three rows correspond to the cases (IA), (IB), and (II), respectively. The sum i runs over $\mathcal{P}$, $\mathfrak{N}$, and λ; $|e|Q_i$ denotes the electric charge of the ith quark and $f_i(x)$, as before, denotes the i-type quark momentum distribution function inside the proton. The factor ξ_{q_e} is $+1$ for a quark-parton contribution and it is -1 for an antiquark-parton contribution. The $\pm$ signs in Eq. (12) correspond to $e^{\pm}p$ scatterings. Note that the new terms, which arise in the presence of X interaction, are proportional to $f_{q_e}(x)$, where $q_e(x)$ is λ or $\mathfrak{N}$, depending upon whether the electron is strange or nonstrange. These terms depend upon q^2 and q^4 and thus provide the scale-noninvariant contributions to νW_2 and MW_1. The measure of the deviations from scaling in these two functions is given by

$$D_{1,2}(q^2,\omega)\equiv\frac{\Delta_{1,2}(q^2,\omega)}{\sum Q_i^2 f_i(x)}\,, \quad (13)$$

where $\Delta_{1,2}$ are defined through Eq. (12).

Since ϵ is small ($\approx\frac{1}{50}$), it follows that $D_{1,2}(q^2,\omega)$ will be appreciable only for large $|q^2|\gtrsim 15$ GeV^2. This, however, corresponds to small ω (<2.5) at SLAC energies. For such values of ω, it is easy to see that $D_{1,2}(q^2,\omega)$ *would be less than or of order 5% for* $|q^2|<25$ GeV^2 *in the event the electron is strange* ($q_e=\lambda$). This is because the functions $D_{1,2}(q^2,\omega)$ are proportional to $f_{q_e}(x)$ and the strange-quark content function $f_\lambda(x)$ is at least an order of magnitude smaller than the non-strange-quark content functions $f_{\mathcal{P}}(x)$ and $f_{\mathfrak{N}}(x)$ within the proton for $\omega<4$. These remarks hold for all three cases (IA), (IB), and (II).

If, on the other hand, the electron is nonstrange (i.e., $q_e=\mathfrak{N}$), one would in general expect to see significant violation of scaling for large $|q^2|\gtrsim 20$ GeV^2. The precise value of such violation depends sensitively on the ratio $f_{\mathcal{P}}(x)/f_{\mathfrak{N}}(x)$. As remarked earlier in connection with the comparison of (e^+p) versus (e^-p), this ratio does not seem to be well known for small values of $\omega<2$. This is the value which is relevant for large $|q^2|>20$ at SLAC energies. Once again, we estimate the degree of violation for different values of q^2 by taking $f_{\mathcal{P}}(x)/f_{\mathfrak{N}}(x)$ from Ref. 19 and listing the corresponding numbers in Table III. We should stress that the precise numerical estimates may not be taken seriously, although the qualitative trend of increasing deviations from scaling with increasing $|q^2|$ and the effects for the νW_2 function being large for cases (II) and (IB) compared with (IA) can be trusted.

To summarize:

(1) Despite the presence of scalar and pseudoscalar interactions for case (IA) (corresponding to either "vector X" or "axial X"), the deviations

TABLE II. $\sigma^{e^+p}(E=13.9,q^2,\theta=50°)/\sigma^{e^-p}(E=13.9,q^2,\theta=50°)$ for the heavy-X case ($m_X^2\gg s,t$) with the electron being nonstrange. (If the electron is strange, the above ratio is expected to be unity for all values of q^2.) The entries shown are for $\epsilon=\frac{1}{50}$. Values of q^2 and E are in units of GeV.

	$q^2=-5$	$q^2=-10$	$q^2=-15$	$q^2=-20$
ω	$\simeq 5$	$\simeq 2.42$	$\simeq 1.54$	$\simeq 1.18$
$f_p(\omega)/f_n(\omega)$	1.8	2	3.6	6
IA ($q_e=\mathfrak{N}$) "vector X"	$e^+/e^-=1.08$	1.16	1.18	1.15
IB ($q_e=\mathfrak{N}$) $(VV+AA)_{\rm eff}$	$e^+/e^-=1.19$	1.42	1.43	1.42
II ($q_e=\mathfrak{N}$) $(V\pm A)(V\pm A)_{\rm eff}$	$e^+/e^-=1.23$	1.57	1.63	1.65

TABLE III. Scaling violations for the case where the electron is nonstrange ($q_e = \mathfrak{N}$), with some typical values of ω and $\epsilon = \frac{1}{50}$. (If e^- is strange, violation of scaling is much less than 5% for $|q^2| < 25$ GeV^2 and $\omega < 4$.)

			Vector X (IA) ($q_e = \mathfrak{N}$)		$(VV+AA)_{\text{eff}}$ (IB) ($q_e = \mathfrak{N}$)		$(V \pm A)(V \pm A)_{\text{eff}}$ (II) ($q_e = \mathfrak{N}$)	
	ω	$f_{\mathcal{P}}(\omega)/f_{\mathfrak{N}}(\omega)$	D_1	D_2	D_1	D_2	D_1	D_2
$q^2 = -15$	2	2.6	0.07	0.09	0	0.28	0	0.40
$q^2 = -25$	1.5	3.6	0.14	0.13	0	0.50	0	0.80

from scaling in $MW_1(q^2, \nu)$ are tolerable in the presently available kinematic region for ep scattering. This differs from the conclusion drawn in Ref. 11, where large deviations from scaling are noted in the presence of scalar interactions. Such a difference is in large part due to the fact that all three quarks ($\mathcal{P}$, $\mathfrak{N}$, and λ) are assumed to share the anomalous interaction with equal strength in Ref. 11, while only the $\mathfrak{N}$ quark or the λ quark is involved in the anomalous interaction for our case.

(2) While the above estimates are given for (ep) scattering, the deviations from scaling will be enhanced considerably for (en) scattering compared with (ep) scattering in the event that electron is nonstrange ($q_e = \mathfrak{N}$). This is because of the larger $\mathfrak{N}$-quark content compared with the $\mathcal{P}$-quark content within the neutron relative to the proton. Of course, if the electron is strange, the effects are equally suppressed for (en) scattering, as is the case for (ep) scattering.

(3) A characteristic feature of Eq. (12) is that the functions νW_2, νW_3 (for e^-), and MW_1 must increase with $|q^2|$ for all q^2 and fixed ω. This is because the interference terms in νW_2 and νW_3 [being proportional to $(q^2 Q_{q_e})$] are necessarily constructive for spacelike $q^2 < 0$ and $Q_{q_e} = -\frac{1}{3}$ corresponding[14] to the electron being coupled to either the $\mathfrak{N}$ or the λ quark via X.

(4) The entries in Table III indicate that deviations from scaling in the νW_2 function are excessive for case (II) and case (IB) with $q_e = \mathfrak{N}$. This appears to be inconsistent with the (ep) data which seem to assert that scaling holds within 10–15%. However, it may not be easy to draw clear-cut conclusions unless one reanalyzes[22] the data in terms of W_1, W_2, *and* W_3 functions [Eq. (11)]. We feel that such an analysis of the data is worthwhile not only in view of testing the possibility of additional interactions as considered here, but also to estimate[23] the 2-photon contribution.

(5) *If the electron is strange* ($q_e = \lambda$), the contribution of the νW_3 term vanishes if $f_\lambda(x) = f_{\bar{\lambda}}(x)$ for all x. This is because the λ-quark contribution to the νW_3 term is canceled by the anti-λ-quark contribution due to the ξ_{q_e} factor in Eq. (11). In this case the data may be analyzed only in terms of the νW_2 and MW_1 functions. The deviations from scaling for these two functions are small (less than 5%) in the presently available kinematic region, since $f_\lambda(x)/f_{\mathcal{P},\mathfrak{N}}(x)$ is small for low ω (<3). However, such deviations should increase with ω (see Ref. 24) for a fixed high $|q|^2 \gtrsim 20$ GeV^2 as ω increases between 3 and 10, since the ratio $f_\lambda(x)/f_{\mathcal{P},\mathfrak{N}}(x)$ rises[19] rapidly in this region.

(6) The remarks made here with regard to (ep) scattering apply also to (μp)-scattering experiments being carried out at Fermilab with the substitutions $\epsilon_e \to \epsilon_\mu$ and $q_e \to q_\mu$. It is worth noting that the higher ω values (together with high $|q^2|$ values) available in this case (due to high incident energies) are useful to avoid uncertainty in theoretical estimates stemming from the uncertainty in $f_{\mathcal{P}}(\omega)/f_{\mathfrak{N}}(\omega)$. This applies both to the comparison of $(\mu^+ p)$ versus $(\mu^- p)$ (Sec. IV) and to deviations from scaling in (μp) scattering. In view of remark (5), it is especially interesting to verify whether deviations from scaling, if any, rise in this case with increasing ω for a fixed high $|q^2|$. If this effect is observed, it may be inferred that the muon is "strange."

VI. HYPERFINE STRUCTURE

Due to the presence of axial-vector interactions of the form $(A_e A_{q_e})$ in all three cases (IA, IB, and II), the X interaction will contribute[25] to hyperfine splitting in hydrogen, which is given by

$$\frac{\Delta\nu_{\text{hfs}}}{\nu_{\text{hfs}}} = 1000 \begin{pmatrix} \epsilon/2 \\ \epsilon \\ \epsilon \end{pmatrix} g^A_{q_e} \text{ parts per million}, \tag{14}$$

where

$$g^A_{q_e} \bar{u}_p i\gamma_\mu\gamma_5 u_p \equiv \langle P | \bar{q}_e i\gamma_\mu\gamma_5 q_e | P\rangle . \tag{15}$$

If the electron is strange ($q_e = \lambda$), we expect $g^A_{q_e} \simeq 0$, so that the above contribution is well within the theoretical uncertainty[26] of about 4 to 6 parts per million (in magnitude).

If the electron is nonstrange ($q_e = \mathfrak{N}$), the extent to which the above contribution may pose a restriction depends on the magnitude of $g^A_{q_e}$ with $q_e = \mathfrak{N}$. If one accepts the value $g^A_{q_e} = (1 - g^\beta_A) = (1 - 1.2) = -0.2$, for $q_e = \mathfrak{N}$, as suggested in Ref. 9, we obtain (setting $\epsilon \simeq \frac{1}{50}$)

$$\frac{\Delta \nu_{\rm hfs}}{\nu_{\rm hfs}} = -\begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} \text{parts per million}, \tag{16}$$

which is still compatible with the theoretical uncertainty mentioned above. Thus hfs considerations do not, at present experimental and theoretical accuracy, rule out the possibility of a nonstrange electron. (For the strange electron, of course, there never was any problem.)

VII. $\pi^0 \to e^+e^-$, $\eta \to \mu^+\mu^-$, AND $\eta \to e^+e^-$ DECAYS

The X interaction will in general contribute to $\pi^0 \to e^+e^-$, $\eta \to \mu^+\mu^-$, and $\eta \to e^+e^-$ decays. Even though the X contribution is expected to be of the same order as the 2-photon contribution [since $\alpha\epsilon \simeq O(\alpha^2)$], the rate of $\pi^0 \to e^+e^-$ decay[27] for case (IA) (with "vector X") is in general expected to be enhanced compared with the cases (IB) and (II) as well as the 2-photon case by a factor $(m_\pi/m_e)^2$, assuming that the matrix elements are of the same order in all cases. This is due to the presence of pseudoscalar effective interaction for case (IA), which is absent in all other cases. Below we estimate the rates.

$\pi^0 \to e^+e^-$ decay. If the electron is strange ($q_e = \lambda$), the X contribution to $\pi^0 \to e^+e^-$ decay is suppressed in all three cases (IA, IB, and II), since $(\bar\lambda\lambda)$ is isoscalar while π^0 is isovector.

Thus the X contribution to $\pi^0 \to e^+e^-$ decay would be significant *only provided that the electron is nonstrange* ($q_e = \mathfrak{N}$). The rates for the X contribution in the different cases are given by

Case (IA) [*"vector X"* $(\frac{1}{2}(VV + AA) - (SS + PP))$ *effective interaction*]:

$$\Gamma(\pi^0 \to e^+e^-) = \frac{(f^2/m_X{}^2)^2}{16\pi m_\pi{}^3} (f_\pi{}^2 m_e{}^2 + h_\pi{}^2 m_\pi{}^2)(m_\pi{}^2 - 4m_e{}^2)^2 \tag{17a}$$

$$\simeq \frac{(f^2/m_X{}^2)^2}{16\pi} \left(\frac{h_\pi}{m_\pi}\right)^2 m_\pi{}^5 . \tag{17b}$$

Cases (IB) and (II) [$(VV + AA)$ *effective and* $(V \pm A)$ *chiral*]:

$$\Gamma(\pi^0 \to e^+e^-) = \frac{(f^2/m_X{}^2)^2}{4\pi m_\pi{}^3} (f_\pi{}^2 m_e{}^2)(m_\pi{}^2 - 4m_e{}^2)^2$$

$$\simeq \frac{(f^2/m_X{}^2)^2}{4\pi} m_\pi{}^3 m_e{}^2 , \tag{18}$$

where the constants f_π and h_π are defined by

$$\langle 0 | \bar q_e i \gamma_\mu \gamma_5 q_e | \pi^0 \rangle \equiv i f_\pi p_{\pi\mu} \frac{1}{(2\pi)^{3/2}} (2E_\pi)^{-1/2}$$

and

$$\langle 0 | \bar q_e \gamma_5 q_e | \pi^0 \rangle \equiv i h_\pi m_\pi \frac{1}{(2\pi)^{3/2}} (2E_\pi)^{-1/2} . \tag{19}$$

In going from Eq. (17a) to Eq. (17b), we have dropped the $m_e{}^2$ term. We have also assumed for both (17b) and (18) that f_π is of the same order as the $\pi \to \mu\nu$ decay constant, i.e., $f_\pi \sim m_\pi$ for $q_e = \mathfrak{N}$, which seems reasonable. Substituting $(f^2/4\pi) m_X{}^{-2} \simeq (\alpha/50)\ \text{GeV}^{-2}$ in the above formulas, we obtain the following branching ratios:

$$\frac{\Gamma(\pi^0 \to e^+e^-)}{\Gamma(\pi^0 \to 2\gamma)} \simeq \left(\frac{h_\pi}{m_\pi}\right)^2 \times (5 \times 10^{-4}) \quad \text{(IA)}$$

$$\simeq 2.5 \times 10^{-8} \quad \text{(IB and II)} \tag{20}$$

for the case of nonstrange electron. A recent review of the data[28] appears to set an upper limit of 8×10^{-6} for the above branching ratio at 90% confidence level. This is certainly consistent with cases (IB) and (II); but it excludes case (IA) (vector X) with the nonstrange electron, if $h_\pi \sim f_\pi \sim m_\pi$. This is presumed to be the case by many authors (see, for example, Refs. 24 and 28). However, the constant h_π need not be as large as f_π. For example, if one uses the field equations to equate the pseudoscalar quark density P_q with $(1/2m_q)(\partial_\mu A^q_\mu)$ where m_q is the quark mass, then one obtains $h_\pi = (m_\pi/2m_q) f_\pi$, which may comfortably be of order $(f_\pi/10)$ for even a moderately heavy quark. Because of this uncertainty in the estimate of h_π, we conclude that $\pi^0 \to e^+e^-$ decay does not as yet yield decisive information to choose between the cases (IA), (IB), and (II) even for the nonstrange-quark case, although lowering of the branching ratio to the level 10^{-7} should disfavor case (IA) (with $q_e = \mathfrak{N}$). If the electron is strange, $\pi^0 \to e^+e^-$ decay is not sensitive to the anomalous interaction in any case.

$\eta \to l+\bar{l}$ *decays* ($l=e, \mu$). In contrast to π^0 decay, where ($\lambda\bar{\lambda}$) density does not contribute, for η decay such densities are important. Thus η decay is sensitive to both strange and nonstrange lepton possibilities. Of course the absolute rate is again suppressed for cases (IB) and (II) compared with case (IA) (if the matrix elements are of the same order) just as for $\pi^0 \to e^+e^-$ decay. The contributions to $\eta \to l\bar{l}$ decay ($l=e$ or μ) (using the formulas for π^0 decay with the substitution $\pi \to \eta$) are given by[29]

$$\Gamma(\eta \to e^+e^-) \simeq (h_\eta/m_\eta)^2 \times (3\text{ eV}) \quad \text{(IA)}$$
$$\simeq (f_\eta/m_\pi)^2 \times (2\times 10^{-7}\text{ eV}) \quad \text{(IB and II)}, \tag{21}$$

$$\Gamma(\eta \to \mu^+\mu^-) \simeq (h_\eta/m_\eta)^2 \times (2.1\text{ eV}) \quad \text{(IA)}$$
$$\simeq (f_\eta/m_\pi)^2 \times (6\times 10^{-3}\text{ eV}) \quad \text{(IB and II)}. \tag{22}$$

The constants f_η and h_η are defined in the same manner as f_π and h_π with the substitution $\pi \to \eta$ in Eq. (19). One may expect (barring selection rules) that f_η is nearly equal to f_π, which in turn is of the same order as the $\pi \to \mu + \nu$ decay constant; thus $f_\eta \sim m_\pi$ (within a factor of 2 or 3). On the experimental side,[30] there is no number quoted for the $\eta \to e^+e^-$ decay, while $\Gamma_{\text{exp}}(\eta \to \mu^+\mu^-) \simeq 0.057$ eV. This latter number is certainly consistent with cases (IB) and (II) (for either strange or nonstrange leptons). The consistency for case (IA) depends upon whether (h_η/m_η) may be as small as $(\frac{1}{10})$ or not. [Note that, if we replace the pseudoscalar quark density P_q by $(1/2m_q)(\partial_\mu A_\mu^q)$, we obtain $(h_\eta/m_\eta) \simeq (m_\pi/2m_q)$.]

In summary, the leptonic decay modes of π^0 and η are compatible with cases (IB) and (II) (with either strange or nonstrange leptons) and with case (IA) for strange leptons; the compatibility with case (IA) for nonstrange leptons is not easy to judge (under the present theoretical and experimental accuracy) due to uncertainty in the estimates of h_π and h_η. In view of the possible existence of the anomalous interactions, a search for $\pi^0 \to e^+e^-$ and $\eta \to e^+e^-$ decays at a level much higher than the 2-photon contribution would be helpful. (We should also urge a search for $\pi^0 \to \bar{\mu}e$ and $\pi^0 \to \mu\bar{e}$ decays, as these decays arise in certain variants of our gauge models and would decide whether electrons and muons have the same or different "colors.")

VIII. MASSIVE LEPTON-PAIR PRODUCTION IN $p+p$ COLLISIONS AND $p+p \to l+H$

The X interaction will, in general, affect[31] the production of lepton pairs in hadron-hadron collisions in a manner very similar to the production of hadrons in $(e^- + e^+)$ annihilation. The general dependence of the cross section on the invariant lepton pair (mass)^2 $M_{l\bar{l}}^2$ for the case of heavy X ($M_{l\bar{l}}^2 \ll m_X^2$) is given by

$$\frac{d\sigma^{\text{"photon}+X\text{"}}}{d\sigma^{\text{"photon"}}} = 1 + O(\epsilon)M_{l\bar{l}}^2 + O(\epsilon^2)M_{l\bar{l}}^4,$$

where the terms of $O(\epsilon)$ and $O(\epsilon)^2$ can be determined[32] in a parton-model framework.

In view of the fact that recent experiments on $p+p \to l^+ + l^- + H$ carried out[33] at BNL and $p+p \to l+H$ being carried out[5] at Fermilab and at the CERN ISR seem to indicate that lepton production is as much as one to two orders of magnitude higher than what is expected on the basis of 1-photon diagrams and parton-model formulas, it is tempting to suppose that the same mechanism which is responsible for the anomalous behavior of e^-e^+ hadrons may also be responsible for the anomalously large production of lepton pairs. (Note that in the experiments which so far study $p+p \to l+H$, one does not yet know whether the observed lepton is associated with its antilepton. However, if the above explanation is to apply, this must be the case.)

Fitting of the data for some specific cases has been made recently by Soni.[24] We may add the following remarks:

(1) If the produced lepton (e^- or μ^-) is strange, the cross section will be modified significantly compared with the 1-photon cross section only in a region which involves high $q^2 = M_{l\bar{l}}^2$ and high ω. [What is needed is that $f_\lambda(\omega) f_{\bar{\lambda}}(s/M_{l\bar{l}}^2\omega)$ be large, where s = invariant (energy)^2 for the p-p system.]

(2) The lepton pair produced via the X interaction can, of course, be distinguished from that produced via the decay of vector mesons through the characteristic mass plot. (This latter mechanism has been suggested by many authors[34] as a possible explanation of the data.)

(3) We mention a third explanation. Assume that quarks carry integer charges and are not too heavy ($m_q \approx$ a few GeV) and that they decay into (leptons + pions) with lifetimes of order 10^{-12} to 10^{-10} sec violating baryon- and lepton-number conservation. Such a possibility could arise as a limiting case within our gauge scheme,[2] without conflicting with the known stability of the proton. In this case, the supposedly large production of leptons may be attributed to the production of real $(q+\bar{q})$ pairs with cross section of order 10^{-4} com-

pared with pion production [sufficiently above $(q\bar{q})$ threshold] followed by decay of the quark to (lepton + pions); similarly for the antilepton. With this mechanism, the production of a lepton need not always be associated with that of its antilepton. This provides a distinction from the other explanations. Furthermore, we expect (see Ref. 2) the quark decays to be parity-violating, i.e., the lepton to carry net helicity. Of course, for this explanation to hold, it is necessary that there is a threshold[35] associated with $q\bar{q}$ production.

IX. CONCLUSIONS

The purpose of this paper is twofold: (A) First, we wish to abstract, from the class of gauge models proposed[1,2] to unify leptonic and baryonic phenomena, information about the types of allowed and forbidden couplings and their signs and to utilize this information in making predictions about magnitude and energy dependence of $e^+ + e^- \rightarrow$ hadrons, deviations of (e^+p/e^-p) total cross sections from unity, and apparent deviations from scaling in $(e^{\pm}p)$ experiments. (B) Second, we wish to show (and this is done in Appendix A) that the severe limitation on characteristic energy at which anomalous lepton-hadron interactions would manifest themselves imposed by our original basic gauge model—and which would have excluded SLAC energies as being low—can be relaxed and the masses of the exotic X particles responsible for the anomalous interactions can be lowered, from being superheavy ($> 10^4$ GeV) as in the basic model to being just heavy ($\approx 10^2$ GeV) or even light ($\simeq$ 15–30 GeV).

With respect to (A), the most severe limitations which our gauge models impose is that X-mediated anomalous interactions *never* permit a coupling of electrons to the $\mathcal{P}$ quarks or to the charmed quarks, but only to $\mathfrak{N}$ quarks or to the λ quarks (referred to as the cases of "nonstrange" or "strange" electron, respectively). Identical restrictions apply to the anomalous coupling of the muon. This has significant consequences for all processes considered and leads to important quantitative differences between our predictions and those of other authors.

Further, we allow for the possibility of the X particles being light (15–30 GeV) and remark that this may have the effect that the anomalous cross sections for $e^+ + e^- \rightarrow$ hadrons do not rise so steeply with energy as is the case for the heavy X particles ($\simeq$ 100 GeV). An analysis of present data (summarized in Table IV) with these points in mind inclines one to the view that even though we cannot yet exclude the possibility of the electron being nonstrange (particularly with a light X), the trend of the data is toward a "strange" character for the electron (and toward its neutrino being "charmed"). The strangeness attribute of the electron has experimental consequences—for example, one may predict a predominant production of ϕ^0's and η^0's and possibly also $(K\bar{K})$ in future $e^+ + e^-$ experiments at higher SLAC energies.

On the theoretical side with respect to constructing variant models which should permit a heavy or light X (instead of a superheavy X with mass $> 10^4$ GeV, which would be irrelevant for SPEAR energies), we have succeeded (Appendix A), but at the unattractive price of doubling the number of fermions (including quarks) in the new models. In view of the theoretical difficulties of constructing an attractive gauge model, we wonder if it is not the basic gauge model—with its superheavy or heavy X—which is, after all, the model likely to be correct and that at SLAC energies the anomalous lepton-hadron interaction which we predicted is really inoperative. Future SLAC, Fermilab, and ISR experiments involving both $e^{\pm}$ and $\mu^{\pm}$ may help confirm or remove such reservations.

ACKNOWLEDGMENTS

We thank E. Bloom, J. Ellis, G. Furlan, Riazuddin, and especially C. H. Llewellyn Smith for innumerable helpful discussions. One of us (J.C.P.) wishes to thank the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

APPENDIX A

In this appendix we consider some of the gauge models of Ref. 2 and the pattern of anomalous lepton-hadron interactions they give rise to. As remarked in Ref. 2, it i an inescapable conclusion of our gauge models which unite leptons and hadrons that these anomalous interactions must eventually become as strong as hadron-hadron interactions. However, for the basic gauge model of Ref. 2, the energies at which these strong effects begin to manifest themselves are unreasonably large[36] $[s > (10^4 \text{ GeV})^2]$ and thus irrelevant for SPEAR. In Ref. 2 we postulated a number of variants of the basic model which, though they are not as elegant as the basic model, do permit the lowering of this energy. Some of these models have other limitations; there is one, however, the "prodigal model," which appears a "possible" candidate and which we further examine in this appendix in this regard. In summary, it appears that the severest restrictions on gauge models—

TABLE IV. Summary of the main results[a] (for the heavy-X case with $\epsilon = \frac{1}{50}$).

	IA ("Vector"-X) $q_e = \mathfrak{N}$	$q_e = \lambda$	IB [$(VV+AA)$ eff. int.] $q_e = \mathfrak{N}$	$q_e = \lambda$	II [$(V\pm A)(V\pm A)$ eff. int.] $q_e = \mathfrak{N}$	$q_e = \lambda$
$\frac{e^+p}{e^-p}$ ($q^2=-15, E=13.9, \theta=50°$)	1.18	1.0	1.43	1.0	1.63	1.0
"Violation of scaling" ($q^2=-25$, $\omega=1.5$)						
in MW_1	14%	<5%	0	0	0	0
in νW_2	13%	<5%	50%	<5%	80%	<5%
Hfs splitting (parts per million)	-2	≈ 0	-4	$\simeq 0$	-4	$\simeq 0$
$\pi^0 \to e^+e^-$ (branching ratio)	$(h_\pi/m_\pi)^2(5\times 10^{-4})$	$\simeq 0$	2.5×10^{-8}	$\simeq 0$	2.5×10^{-8}	$\simeq 0$
Anomalous mag. mom. of e and μ	Need M_q to be small[b]		Suppressed (see Ref. 41)		Suppressed (see Ref. 41)	

[a] For details and necessary qualifications, see text. Values of q^2 and E are in units of GeV.
[b] See Footnote 8 of Ref. 4.

if we wish to lower the energies at which electron-hadron interactions become effectively strong—arise from the apparent absence of anomalous ν-hadron couplings at low energies. This, in turn, leads to the conclusion that the electron must be strange—a conclusion consistent with the picture which appears to emerge from the phenomenological analysis of the text and which has strong implications for future experiments (see, however, the "addendum").

1. The basic gauge model and its problems

The basic model assigns the twelve quarks and the four (4-component) leptons to the fermionic multiplets:

$$F_{L,R} = \begin{vmatrix} \mathcal{P}_a & \mathcal{P}_b & \mathcal{P}_c & \nu \\ \mathfrak{N}_a & \mathfrak{N}_b & \mathfrak{N}_c & e^- \\ \lambda_a & \lambda_b & \lambda_c & \mu^- \\ \chi_a & \chi_b & \chi_c & \nu' \end{vmatrix}_{L,R}, \tag{A1}$$

with the symmetry group $SU_L(4)\times SU_R(4) \times SU_{L+R}(4')$. For purposes of this appendix, the $SU_L(4)$ and $SU_R(4)$ groups—which after gauging of their $SU_L(2)\times SU_R(2)$ subgroup, give rise to weak interactions—are basically irrelevant. The anomalous lepton-hadron interactions of concern to us in this paper arise from gauging the color group SU(4′). In the basic model, freedom from anomalies dictates that these interactions be purely vector. There are seven gauge-mesons which give rise to leptonic and semileptonic interactions; in the notation of the second paper of Ref. 2, these are S^0; the exotics X^0, X^-, $X^{-\prime}$; and the anti-exotics $\bar{X}^0$, X^+, $X^{+\prime}$. The relevant interactions are

$$\frac{f}{(24)^{1/2}} S^0 \Big[\sum_{abc} (\bar{\mathcal{P}}_a \mathcal{P}_a + \bar{\mathfrak{N}}_a \mathfrak{N}_a + \bar{\lambda}_a \lambda_a + \bar{\chi}_a \chi_a) - 3(\bar{\nu}\nu + \bar{e}e + \bar{\mu}\mu + \bar{\nu}'\nu') \Big]$$

plus the exotic interactions

$$\begin{aligned} &f X^0(\bar{\nu}\mathcal{P}_a + \bar{e}\mathfrak{N}_a + \bar{\mu}\lambda_a + \bar{\nu}'\chi_a)\,, \\ &+ f X^-(\bar{\nu}\mathcal{P}_b + \bar{e}\mathfrak{N}_b + \bar{\mu}\lambda_b + \bar{\nu}'\chi_b)\,, \\ &+ f X^{-\prime}(\bar{\nu}\mathcal{P}_c + \bar{e}\mathfrak{N}_c + \bar{\mu}\lambda_c + \bar{\nu}'\chi_c)\,. \end{aligned} \tag{A2}$$

The lower limits on the masses of S^0 and the X particles are given by the following relations:

(1) The effective S^0 coupling of ν-hadronic interactions is $f^2/(m_S)^2$ at low energy. Since this must be much smaller than G_F (in order that there are no unconventional weak-interaction effects at low energies), we conclude that

$$(m_S)^2 > G_F^{-1} f^2\,.$$

With $f^2/4\pi \sim 1$, this implies $m_S \gtrsim 1000$ GeV.

(2) The X particles induce

$$K^0 \to \bar{\lambda} + \mathfrak{N} \to \mu^- + (\bar{X}+X) + e^+ \to \mu^- + e^+\,,$$

with the effective strength $\approx f^2/m_X{}^2$. Since $K_L \to \mu^+ + \mu^-$ amplitude $\approx G_F\alpha^2$, and no events of the variety $K_L \to \mu^\mp + e^\pm$ have yet been observed, we must have $f^2/m_X{}^2 \ll G_F \alpha^{-2}$, i.e., $m_X{}^2 \gg G_F{}^{-1}\alpha^{-2}$ ($m_X > 3\times 10^4$ GeV), for $f^2/4\pi \sim 1$.

To summarize, if we assume that $f^2/4\pi \sim 1$, the X's in the basic gauge model must be superheavy ($>3\times 10^4$ GeV) in order to suppress the $K_L \to e + \mu$ transition. The X-mediated anomalous lepton-hadron interactions would then become effective only for energies in excess of 10^4 GeV and the model, as it stands, would be irrelevant to SLAC energies so far as its purely gauge interactions[37] are concerned. If the model could be modified so that $K \to e + \mu$ is rigorously forbidden, the severe limitation on X mass might be relaxed. Stated quantitatively, the e^-e^+-annihilation data require $\epsilon \sim \frac{1}{50}$ (see text). With $(f^2/4\pi)(1/m_X{}^2) \simeq \epsilon\alpha$ GeV^{-2}, this implies a mass $m_X \sim 100$ GeV (*heavy-X-case*) if $f^2/4\pi \approx 1$. If, on the other hand, $f^2/4\pi$ is of the order[38,39] $\frac{1}{100}$ to $\frac{1}{10}$, we need m_X to be as low as 10 to 30 GeV (*light-X-case*). In the next section we study how to forbid $K \to e + \mu$ and thus bring the X mass down from being "superheavy" to just "heavy" or even "light."

One independent remark: In view of the fact that the rates of $K_L \to \mu e$ decays set the scale of energy at which the new class of interactions mediated by the X particles become important in the basic gauge model, we especially wish to urge a search for this decay mode. (Could it in fact be true that the rates of these decays are much larger than what are thought to be the upper limits for these decays?)

2. The prodigal model

To forbid the $K \to e + \mu$ transition, one possibility is to make a distinction between the muon and the electron "colors" L_μ and L_e. In the prodigal model we assume that the muon is, as it were, the news-bearer of the existence of a new heavier fermionic multiplet with "new" quarks and new muonic leptons M^0 and M^-. Thus we work with two basic multiplets:

$$F_e = \begin{vmatrix} \mathcal{P}_a & \mathcal{P}_b & \mathcal{P}_c & E^0 \\ \mathfrak{N}_a & \mathfrak{N}_b & \mathfrak{N}_c & E^- \\ \lambda_a & \lambda_b & \lambda_c & e^- \\ \chi_a & \chi_b & \chi_c & \nu \end{vmatrix}_{L,R} ,$$

$$F_\mu = \begin{vmatrix} \mathcal{P}'_a & \mathcal{P}'_b & \mathcal{P}'_c & M^0 \\ \mathfrak{N}'_a & \mathfrak{N}'_b & \mathfrak{N}'_c & M^- \\ \lambda'_a & \lambda'_b & \lambda'_c & \mu^- \\ \chi'_a & \chi'_b & \chi'_c & \nu' \end{vmatrix}_{L,R} . \qquad \text{(A3)}$$

Here E^0, E^- and M^0, M^- are heavy leptons[40] with $L_e = 1$ and $L_\mu = 1$, respectively, while the primed particles are new quarks. We assume that the normal hadrons are made up of quarks in F_e (see remarks later). The ratio of masses of primed and unprimed quarks (and M^Ω to E^Ω leptons) may be $\approx m_\mu/m_e \approx \alpha^{-1}$ (possibly as a consequence of a "natural" symmetry-breaking mechanism where the masses for the F_e multiplet arise from "radiative" corrections of order α to the masses of the F_μ multiplet).

In order to gauge, we consider the local symmetries

$$SU_L(2)\times SU_R(2)\times SU_e(4')\times SU_\mu(4')$$

and the following interaction[41]:

$$g_L(\overline{F}_{eL} W_L F_{eL} + \overline{F}_{\mu L} W_L F_{\mu L}) + (L \to R) + f_1 V_1(\overline{F}_e F_e) + f_2 V_2(\overline{F}_\mu F_\mu) . \qquad \text{(A4)}$$

Both V_1 and V_2 are distinct vector particles corresponding to the color gauge groups $SU_e(4')$ and $SU_\mu(4')$, respectively. The lighter e-type quarks and the very massive μ-type quarks have no mutual interaction, except the weak and the electromagnetic, thus guaranteeing that normal hadrons may be considered as made up of e-type quarks only. (If one wished to minimize the mixing of V_1 and V_2 in the Lagrangian after spontaneous symmetry breaking, one simple assumption would be to take all quarks q_e and q_μ to be fractionally charged.[42] Another amusing possibility is that e-type quarks are integrally charged and μ-type quarks are fractionally charged. In either case it is only the singlet fields S_1^0 and S_2^0 contained in V_1 and V_2 which need to be mixed[43] to generate the massless photon through the Higgs mechanism.)

With this preparation and writing X_1 and X_2 interactions for F_e and F_μ in analogy to (A2), one can now easily see the following features.

(1) The model forbids $K^0 \to e^- + \mu^+$, and also neutral decays of the type $K \to e + E$ provided $m_E > m_K$.

(2) Since ν_e is charmed and normal hadrons are not, the X mechanism does not affect neutrino interactions $\nu + H \to \nu + H$. The S^0 particles also do not lead to any anomalous enhancement of the neutrino interactions if one assumes that they are sufficiently massive.[44]

(3) To forbid the enhancement of $K \to \bar{e}e$ as well as $K^+ \to e^+ + \bar{\nu}_e$ through X mediation, the Cabibbo rotations must be made for the $(\mathfrak{N}, \lambda)$ quarks with leptons (e, E) [and possibly also (μ, M) and $(\mathfrak{N}', \lambda')$] rotated in the same manner.[45] The unconventional rotation of leptons has important (though not easily measurable) consequences for the sequence of weak-interaction constants. The weak Lagrangian now reads

$$W_L[\bar{\mathcal{P}}(\mathcal{N}\cos\theta+\lambda\sin\theta)+\bar{\nu}_e(e\cos\theta+E\sin\theta)+\bar{\nu}_\mu(\mu\cos\theta+M\sin\theta)]\,.$$

Thus β-decay versus μ-decay constants have the ratio 1 : 1 rather than $\cos\theta : 1$, though K-decay versus π-decay constants still exhibit the ratio $\tan\theta$.

(4) The charmed character of ν_e implies that its doublet partner for $SU_L(2)$, i.e., the electron, is strange. With X_1 mass arranged (through the Higgs mechanism) to be around 100 GeV, we obtain the desired enhancement of $e^+ + e^- \to$ hadrons at SPEAR energies, though no anomalous $\mu^+ + \mu^- \to$ hadron interaction is expected because the muon is *not* color-coupled to normal hadrons which are assumed to be e-type quark composites. (For the model where e-type quarks are integrally charged and μ-type quarks are fractionally charged or vice versa, normal hadrons could contain contributions from both quark types and $\mu^+ + \mu^- \to$ hadrons could also be anomalous.)

(5) The "strange" character of the electron implies that ϕ's, η^0's would be predominantly produced[46] as the SLAC energy goes up. In proton-antiproton annihilation there will be no anomalous production of $e^+ + e^-$ pairs in the kinematic region where the $\lambda + \bar{\lambda}$ quark amplitude is not significant. Likewise, for a strange electron, the ratio $(e^+ + p \to e^+ + H)/(e^- + p \to e^- + H)$ would not be affected appreciably by the X mechanism.

To conclude, for the prodigal model (with new heavy leptons, with a "strange" electron, and with two various types of quarks) the exotic gauge particle mediation can manifest itself as enhancing $e^+ + e^- \to$ hadrons at SPEAR energies. Even though this model provides a natural "niche" for the muon, the fact that we had to double the number of fermions makes the model somewhat unattractive. We ourselves prefer the basic gauge model where X particles are more massive than 10^4 GeV (if $f^2/4\pi \sim 1$) and the electron is nonstrange. But then who can dictate to Nature?

Addendum. It is possible to avoid $K \to \bar{\mu}e$ decays without putting the electron and the muon in different fermionic multiplets, if we introduce an extended gauge group[48] as follows:

Assume that the gauge group is $SU(16)_L \times SU(16)_R$, so that the 16-fold set of F_L and the 16-fold set of F_R in the basic model [see Eq. (A1)] transform as (16, 1) and (1, 16), respectively, under this gauge group. (Such a gauge group would in any case be desirable from the point of view of complete unification of all forces and would involve only one basic coupling constant if we assume that the theory possesses left $\leftrightarrow$ right discrete symmetry in the gauge sector.) Clearly, this extended group can contain the gauge group $SU(2)_L \times SU(2)_R \times SU(3')_{L+R}$ of the basic gauge model. The important new feature, however, is that spontaneous symmetry breaking may allow the four sets of X particles carrying different valencies (in this case) to remain unmixed and chiral with their couplings given by

$$f\{(X_p^0)_L(\bar{\nu}p_a)_L+(X_n^0)_L(\bar{e}n_a)_L+(X_\lambda^0)_L(\bar{\mu}\lambda_a)_L+(X_\chi^0)_L(\bar{\nu}'\chi_a)_L\}+f\{(X_p^-)_L(\bar{\nu}p_b)_L+(X_n^-)_L(\bar{e}n_b)_L+(X_\lambda^-)_L(\bar{\mu}\lambda_b)_L+(X_\chi^-)_L(\bar{\nu}'\chi_b)_L\}$$
$$+f\{(X_p')_L(\bar{\nu}p_c)_L+(X_n'^-)(\bar{e}n_c)_L+(X_\lambda'^-)_L(\bar{\mu}\lambda_c)_L+(X_\chi'^-)_L(\bar{\nu}'\chi_c)_L\}+(L\to R)+\text{H.c.}\,, \quad \text{(A5)}$$

where we assume that $X_P \neq X_n \neq X_\lambda \neq X_\chi$ and the masses of all these particles are different. [Note that the valency quantum numbers ($\mathcal{P}$, $\mathcal{N}$, λ, χ) of the fermions and, therefore, of the currents are fixed essentially by the "observed" weak interactions. In writing the above we have set the electron to be "nonstrange" with its neutrino ν "uncharmed" and the muon to be "strange" with its neutrino ν' "charmed," although *a priori the opposite choice is equally permissible*. Note that Eq. (A5) would reduce to Eq. (A2) if we set $X_p = X_n = X_\lambda = X_\chi$].

With $X_n \neq X_\lambda$ (unlike the basic gauge model), the X interactions may no longer induce $K \to \bar{\mu}e$ decays. They do not, of course, induce $K \to \bar{e}e$ or $K \to \bar{\mu}\mu$ decays, if we assume that both sets of $(\lambda, \mathcal{N})$ and (μ, e) are Cabibbo-rotated with the same angle. [This is analogous to $(\lambda, \mathcal{N})$ and (e, E^-) being Cabibbo-rotated in the same manner in the prodigal model.]

Furthermore, with $X_{\mathcal{P}} \neq X_{\mathcal{N}}$ and $X_{\mathcal{P}} \neq X_\lambda$, the X interactions cannot induce β decays $(n \to p + e^- + \bar{\nu}_e)$ and $K^+ \to \mu^+ + \nu_e$ decays. This again is in contrast to the basic gauge model.

The major restriction on the strength of $X_{\mathcal{P}}$ interactions arises from considerations of the semileptonic "neutral-current" processes (involving "uncharmed" neutrinos), i.e., $\nu + N \to \nu + H$, etc. Now, if the spontaneous symmetry-breaking mechanism [which must preserve $SU(3')_{L+R}$ as a good symmetry] at the same time forces the masses of the X particles to be approximately equal, i.e., $M_{X_{\mathcal{P}}} \simeq M_{X_{\mathcal{N}}} \simeq M_{X_\lambda} \simeq M_{X_\chi}$, then the fact that the observed strength of these "neutral-current"-neutrino interactions is of order G_{Fermi} would imply that the X mechanism will not be relevant for the observed enhancement of electron-hadron interactions at SPEAR energies. We have not yet investigated by an actual construction of the symmetry-breaking terms if this is indeed the case. But even if it is, there still remains one interesting exception, which we mention below.

Assume that the X_L's and the X_R's, which are coupled to left- and right-handed currents, respectively, are eigenstates of the mass matrix (rather than their linear combinations) and that it is the X_R's which are light and relevant for the enhanced electron-hadron interaction with

$$M_{X_L} \gg M_{X_R}.$$

In this case, one should expect enhanced "diagonal" interactions of only the right-handed neutrinos and left-handed antineutrinos with hadrons; but the interactions of hadrons with left-handed neutrinos and right-handed antineutrinos (which the basic model contains and which on account of the assumed chirality[41] of the interactions in the present case can also be massless) would still be suppressed. As far as one knows, experimentally, it is at least permissible to assume that the available neutrino beams in the laboratory consist predominantly of neutrinos of the latter variety (i.e., ν_L's and $\bar{\nu}_R$'s). Thus the observed strength of order G_{Fermi} for reactions of the variety $\nu + N \to \nu + H$) (where the neutrinos are predominantly ν_L's and $\bar{\nu}_R$'s) does not exclude the possibility that the effective strength of the X_R-mediated interactions is of order $(\alpha/50)\ \text{GeV}^{-2}$. One may therefore attribute the CEA-SPEAR enhancement to the interactions mediated by these X_R particles, which are coupled to the right-handed currents. If this explanation is to apply, one would predict (i) *large parity violation* in $e^-e^+ \to$ hadrons and other related processes at SPEAR energies (and similarly for the muon-induced reactions), and (ii) *enhanced interactions of neutrinos of* the unfamiliar helicities (i.e., ν_R's and $\bar{\nu}_L$'s) with hadrons at presently available energies (even though neutrinos of the familiar helicities (i.e., ν_L's and $\bar{\nu}_R$'s) may interact with a strength of order G_{Fermi}). This would manifest itself in enhanced rates for decays of the type $\eta \to \pi^0 + \nu_R + \bar{\nu}_L$. [Note that the neutrinos in question must be "uncharmed" (i.e., those which couple to the $\mathcal{P}$ quarks via X. These may be *either* ν_e's *or* ν_μ's, depending on the details of the model. Rough estimates with $f^2/m_{X_{\mathcal{P}}}{}^2 \approx 10^{-3}$ indicate that $\Gamma(\eta \to \pi^0 + \nu_R + \bar{\nu}_L)/\Gamma(\eta \to \pi^0 + \gamma + \gamma)$ may be of order 1–10%, which is about four orders of magnitude higher than what would be given by an effective interaction strength of G_{Fermi}.

In summary, the chiral nature of the color-gauge interactions and the assumption of the distinctions of $X_{\mathcal{P}}$, $X_{\mathfrak{N}}$, X_λ, and X_χ from each other (something which is permissible within the extended gauge structure, but not in the basic gauge model) leads to a number of new experimental possibilities including the lowering of the energy at which the anomalous electron-hadron interactions mediated via the X particles become effective. Thus in contrast to the prodigal model, we have the following two conclusions:

(1) *Both* the electron and the muon may exhibit anomalous interactions with hadrons at present energies with either one of them being "strange" and the other "nonstrange."

(2) With the condition $M_{X_L} \gg M_{X_R}$, which provides one likely solution[49] for the model to be relevant to SPEAR results in the first place, one should expect to see large parity violation in $e^-e^+ \to$ hadrons and other related processes as well as enhanced interactions of the right-handed ("unfamiliar" helicity) neutrinos permitted by the basic model (either ν_e or ν_μ) with hadrons, even though the left-handed ("familiar" helicity) neutrinos couple with an effective strength $\sim G_{\text{Fermi}}$. The question of parity nonconservation may be tested by starting with polarized e^- and e^+ beams and looking for possible $\langle\vec{\sigma}\rangle\cdot\vec{p}$-type correlation (where $\vec{p}$ is the momentum of a given outgoing hadron).

On the theoretical side, one needs to examine, with this extended gauge structure, whether an allowed pattern of spontaneous symmetry breaking[48] with Higgs-Kibble multiplets would lead to the desired solutions; in particular, it must leave the X particles of different valencies unmixed and must ensure the emergence of a global (or local) SU(3′) color symmetry commuting with the familiar global SU(3) symmetry. The model as it stands possesses Adler-Bell-Jackiw anomalies, the resolution of which [as long as one assumes a gauge group of the type $\text{SU}(n)_L \times \text{SU}(n)_R$ with $n \geq 3$] would have to involve the unattractive introduction of a new set of fermions F' (the two sets F and F' must then couple with opposite chiralities to the same set of gauge bosons and F' would have to be associated with new heavier quarks and leptons). Finally, the model contains a whole host of new currents, which change both color as well as valency quantum numbers. The corresponding gauge mesons are presumably superheavy and ineffective for the interactions considered in this paper in the low- and intermediate-energy range. Notwithstanding these theoretical problems, if SLAC experiments reveal large parity-violating effects and also if both electron and muon exhibit anomalous interactions with hadrons at presently available energies, one will have to entertain this model very seriously.

APPENDIX B

A general electron-lepton 4-fermion interaction, relevant to our discussions in the text, is given by

$$\mathcal{L}=\sum_i [g_S^i(\bar{e}e)(\bar{q}_iq_i)+g_P^i(\bar{e}\gamma_5e)(\bar{q}_i\gamma_5q_i)+g_V^i(\bar{e}\gamma_\mu e)(\bar{q}_i\gamma_\mu q_i)+g_A^i(\bar{e}i\gamma_\mu\gamma_5e)(\bar{q}_ii\gamma_\mu\gamma_5q_i)$$
$$+g_{VA}^i(\bar{e}\gamma_\mu e)(\bar{q}_ii\gamma_\mu\gamma_5q_i)+g_{AV}^i(\bar{e}i\gamma_\mu\gamma_5e)(\bar{q}_i\gamma_\mu q)]\,. \tag{B1}$$

For the cases (IA, IB, and II) discussed in Sec. III A, the values of the coupling constants introduced above are as follows. [We give below *only those constants which are nonvanishing* for the case where the electron is coupled to the $\mathfrak{N}$ quark (nonstrange electron). The constants for the strange-electron case are obtained by the substitution $\mathfrak{N}\to\lambda$.]

Case (IA) ["Vector-X"; $(\frac{1}{2}(VV+AA)-(SS+PP))$ effective interaction]: $g_V^{\mathfrak{N}}/4\pi\alpha=g_A^{\mathfrak{N}}/4\pi\alpha=\epsilon/2$,

$$g_S^{\mathfrak{N}}/4\pi\alpha=g_P^{\mathfrak{N}}/4\pi\alpha=-\epsilon\,, \tag{B2}$$

where ϵ is defined by Eq. (4).

Case (IB) [$(VV+AA)$ effective interaction]: $g_V^{\mathfrak{N}}/4\pi\alpha=g_A^{\mathfrak{N}}/4\pi\alpha=\epsilon$. (B3)

Case (II) [$(V\pm A)(V\pm A)$ effective interaction]: $g_V^{\mathfrak{N}}/4\pi\alpha=g_A^{\mathfrak{N}}/4\pi\alpha=\epsilon$,

$$g_{VA}^{\mathfrak{N}}/4\pi\alpha=g_{AV}^{\mathfrak{N}}/4\pi\alpha=\mp\epsilon\,. \tag{B4}$$

If s is in the asymptotic region (so that parton-model considerations may be applied) but not high enough to invalidate the local 4-fermion interaction approximation given by Eq. (B1), the cross section for $e^-e^+\to$ hadrons is given for the *heavy X case* by

$$\sigma(e^+e^-\to\text{hadrons})=\frac{4\pi\alpha^2}{3s}\left(\sum_i Q_i^2\right)+\frac{s}{16\pi}\left\{\sum\left[(g_S^i)^2+(g_P^i)^2\right]\right\}+\frac{s}{12\pi}\sum_i\left[(g_V^i)^2+(g_A^i)^2+(g_{VA}^i)^2+(g_{AV}^i)^2+\tfrac{2}{3}\,\alpha\left(\sum_i Q_ig_i^V\right)\right], \tag{B5}$$

where $|e|Q_i$ denotes the charge of the ith quark and s is the center-of-mass (energy)^2.

$e^\pm p\to e^\pm+H$. The ratio of (e^+p) and (e^-p) cross sections for given values of incident lepton energy E, scattering angle θ, and momentum transfer square q^2, is given by

$$\frac{d\sigma^{e^+}(E,\theta,q^2)}{d\sigma^{e^-}(E,\theta,q^2)}=\frac{X_+}{X_-}\,, \tag{B6}$$

where, with the interaction (B1) (and the parton-model hypothesis), $X_\pm$ are given by[47]

$$X_\pm=\sum_i\left\{f_i(x)\left[\left(\frac{Q_i}{q^2}+\frac{g_V^i}{e^2}\right)^2+\left(\frac{g_A^i}{e^2}\right)^2+\left(\frac{g_{VA}^i}{e^2}\right)^2+\left(\frac{g_{AV}^i}{e^2}\right)^2\right](1-y+\tfrac{1}{2}y^2)+[(g_S^i/e^2)^2+(g_P^i/e^2)^2]\,y^2/4\right.$$
$$\left.\pm\xi_i\left[\left(\frac{Q_i}{q^2}+\frac{g_V^i}{e^2}\right)\left(\frac{g_A^i}{e^2}\right)+\frac{g_{VA}^ig_{AV}^i}{e^4}\right]y(2-y)\right\}, \tag{B7}$$

where $x=1/\omega=-(q^2/2M_N\nu)$, $y=(p\cdot q/p\cdot k)$, and $\nu=(p\cdot q)/M_N=E-E'$. The quantities p and k denote the 4-momenta of the incoming nucleon and lepton, respectively, while q is the 4-momentum transfer between the incoming and outgoing leptons. The factor ξ_i is +1 for the ith quark and −1 for the ith antiquark. The function $f_i(x)$ denotes the ith-type quark momentum distribution within the proton.

Structure functions. The general formulas for the functions νW_2, MW_1, and νW_3 defined by Eq. (11) are given by

$$\nu W_2(q^2,\nu)=x\left\{\sum_i Q_i^2f_i(x)+q^4\sum_i\left[\left(\frac{g_V^i}{e^2}\right)^2+\left(\frac{g_A^i}{e^2}\right)^2+\left(\frac{g_{VA}^i}{e^2}\right)^2+\left(\frac{g_{AV}^i}{e^2}\right)^2\right]f_i(x)+2q^2\sum_i\left(\frac{g_V^i}{e^2}\right)Q_if_i(x)\right\},$$

$$M_NW_1(q^2,\nu)=\tfrac{1}{2}\sum_i Q_i^2f_i(x)+\frac{q^4}{4}\sum_i\left[\left(\frac{g_S^i}{e^2}\right)^2+\left(\frac{g_P^i}{e^2}\right)^2\right]f_i(x)\,,$$

$$\nu W_3(q^2,\nu)=\pm\sum_i\left\{\left[Q_i q^2+\left(\frac{g_V^i}{e^2}\right)q^4\right]\left(\frac{g_A^i}{e^2}\right)+\left(\frac{g_{VA}^i}{e^2}\right)\left(\frac{g_{AV}^i}{e^2}\right)\right\}2f_i(x)\,, \quad \text{(B8)}$$

where the ± signs are for $e^{\pm}p$ scatterings.

---

*Work supported in part by the National Science Foundation under Grant No. GP 43662-X.

[1]J. C. Pati and Abdus Salam, cited in a review lecture by Professor J. D. Bjorken, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 2, p. 304; J. C. Pati and Abdus Salam, Phys. Rev. D 8, 1240 (1973).

[2]J. C. Pati and Abdus Salam, Phys. Rev. Lett. 31, 661 (1973); Phys. Rev. D 10, 275 (1974).

[3]B. Richter (report on SPEAR data): Irvine Conference on Lepton-Induced Reactions, Irvine, California, 1973 (unpublished); in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974,* edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. IV-37.

[4]J. C. Pati and Abdus Salam, remark at the Irvine Conference on Lepton-Induced Reactions, Irvine, California, 1973 (unpublished); Phys. Rev. Lett. 32, 1083 (1974).

[5]See L. Lederman, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974,* edited by J. R. Smith (see Ref. 3), p. V-55.

[6]Preliminary SLAC data reported by E. D. Bloom at the Topical Meeting on the Physics of Colliding Beams, Trieste, Italy, 1974 (unpublished), and by R. Taylor in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974,* edited by J. R. Smith (see Ref. 3), p. IV-60.

[7]L. Hand (private communication).

[8]Quite possibly, all interactions (including strong) may start as chiral gauge interactions with $(V-A)$ and $(V+A)$ currents coupled to distinct massless gauge-mesons X_L and X_R with equal coupling strengths f_L and f_R. Parity conservation would hold only provided that spontaneous symmetry breaking arranges itself to lead to $(X_L\pm X_R)/\sqrt{2}$ as the eigenstates of the gauge-meson mass matrix. These, as well as alternative possibilities where X_L and X_R are eigenstates, may be realized in the context of the prodigal model discussed in Appendix A.

[9]Some relevant tests have been suggested in a recent note by M. A. B. Bég and G. Feinberg, Phys. Rev. Lett. 33, 606 (1974).

[10]In the Addendum we briefly consider the possibility of gauging the extended group $SU_L(16)\times SU_R(16)$.

[11]For example, I. I. Y. Bigi and J. D. Bjorken in a recent paper [Phys. Rev. D 10, 3697 (1974)] have made the assumption for a number of their considerations that all quarks are involved in anomalous lepton-hadron interactions with the same strength. This may appear difficult to arrange in a renormalizable gauge theory, and in any case it is not permissible in our scheme. Owing to this difference, there are significant quantitative differences with regard to deviations from scaling with or without scalar four-fermion interactions. (See remarks later.)

[12]In general, in this case one may allow $(V+A)$ as well as $(V-A)$ interactions with different strengths.

[13]A phenomenologically minded reader may have reservations about the precise values of these constants, dependent as they are on parton-model considerations.

[14]Since *only* the $\mathfrak{N}$ or the λ quark is coupled to e^- via X, one obtains the same result for $q_e=\mathfrak{N}$ or λ. Note also that the result, in this case, is the same for the fractionally charged quark (i.e., electric charge $-|e|/3$ for all three colors) or the integrally charged quark [i.e., electric charges $(-1,0,0)|e|$ for the three colors (a,b,c)].

[15]In models with integrally charged quarks, the color octet of gluons carry electric charges (see Ref. 1), which should also contribute to $\rho_{\gamma\gamma}(s)$ above the threshold.

[16]Formula (7) will need modification if new channels involving charm and/or color open at some intermediate energy. This will modify the 1-photon contribution to $\sigma_h(s)$, leading to a threshold behavior over a range of energy followed by an increase in $\sum Q_i^2$ sufficiently above the threshold.

[17]A still higher value of $\sum Q_i^2$ like 6 corresponding to three quartets of integrally charged quarks with both color and charm having been excited appears not to give a good fit to the e^-e^+-annihilation data at lower values of center-of-mass (energy)^2 like $s=7$ to 10 GeV^2.

[18]These would include masses of Higgs particles of the light variety (5–10 GeV), which do arise in our theory (see remarks later, Ref. 37). To study the energy dependence for the light-X case, we have considered a simple example of an X-mediated box diagram for $e^+ + e^- \to \sigma+\sigma$, where the σ's are spin-zero objects, in the region $m_X{}^2 \lesssim s \lesssim m_q{}^2$, where m_q is quark mass, and find a dependence of the type $\frac{4}{3}\pi(\alpha^2\sum Q_i{}^2/s+\delta+\delta+\delta'\sqrt{s})$, i.e., less steeply rising than (7).

[19]See, for example, J. D. Bjorken, in proceedings of the Second International Conference on Elementary Particles, Aix-en-Provence, 1973 [J. Phys. (Paris) Suppl. 34, C1-385 (1973)].

[20]If the muon is coupled to the λ quark, the ratio of (μ^+N/μ^-N) cross sections is expected to remain unity for all q^2 and ω if $f_\lambda(x)=f_{\bar\lambda}(x)$ within the nucleon for all x.

[21]We should emphasize that there is no compelling reason (in the absence of an experiment of the type $\mu^- + \mu^+ \to$ hadrons) to assume that the muon is involved in the anomalous interaction with the same strength as the electron.

[22]The contribution of the νW_3 term may be eliminated by combining (e^+p) with (e^-p) data.

[23]We thank C. H. Llewellyn Smith for emphasizing this point of us.

[24]This point has been independently noted by A. Soni, Columbia University Report No. C0-2271-38, 1974 (unpublished).

[25]This was kindly pointed out to us by G. Feinberg (private communication), and has been stressed in

a recent paper by M. A. B. Bêg and G. Feinberg (Ref. 9). As discussed in the text, the conclusion drawn in this paper appears to be overstated in the context of our models. This is because in our gauge models the electron is never coupled to the $\mathcal{P}$ quark—a restriction which Bêg and Feinberg do not impose.

[26]For a recent review see B. E. Lautrup, A. Peterman, and E. de Rafael, Phys. Rep. 3C, 193 (1972), and a review talk by N. Kroll, 3rd International Conference on Atomic Physics, Boulder, Colorado, 1972 (unpublished).

[27]The possible significance of $\pi^0 \to e^+e^-$ decay for our considerations has been emphasized to us by M. Gell-Mann and C. H. Llewellyn Smith and has been discussed in two recent papers (Ref. 24 and Ref. 28; see below). There appears to be an incorrect statement in Ref. 24 with regard to contributions from $(V+A)$ and $(S-P)$ covariants in the starting Yukawa interaction.

[28]J. D. Davies, J. G. Guy, and R. P. K. Zia, Nuovo Cimento 24A, 324 (1974); also C. H. Llewellyn Smith (private communications).

[29]$\eta \to e^+e^-$ decay is discussed in Ref. 11. See, however, remarks on the value of h_η in the text.

[30]Particle Data Group, Rev. Mod. Phys. 45, S1 (1973).

[31]This has also been noted independently by H. S. Mani (private communications), A. Soni (Ref. 24), and Bigi and Bjorken (Ref. 11).

[32]See Ref. 24 for evaluation of these coefficients in some specific cases.

[33]J. H. Christenson *et al.*, Phys. Rev. D 8, 2016 (1973).

[34]See, for example, a remark by J. D. Bjorken at the 17th International Conference on High Energy Physics, London, 1974 (unpublished).

[35]We thank Professor L. Lederman for this remark.

[36]This is, provided f^2 is large (i.e., $f^2/4\pi \sim 1$). See, however, remarks later.

[37]Of course, even in the basic model, it is possible that Higgs scalars may provide the desired anomalous lepton-hadron interactions. Some specific possibility of this kind (involving s-channel exchanges in $e^-e^+ \to$ hadrons) has been suggested by T. Goldman and P. Vinciarelli, Phys. Rev. Lett. 33, 246 (1974). If experiments establish a predominantly scalar-pseudoscalar interaction (possibly through polarization measurement mentioned by these authors and in Ref. 11), it is worth remarking that the pseudo-Goldstone particles of masses 5–10 GeV or their composites with each other or with the X's in our basic model could be the objects which are the relevant ones. Some of these particles would have the quantum numbers of X particles. For a discussion of the pseudo-Goldstone particles in our basic model see D. A. Ross, Phys. Rev. D 11, 911 (1975).

[38]Such a low effective coupling in the X subsector together with (perhaps) a larger effective coupling in the SU(3′) sector (giving rise to low-energy "strong" interactions) may well arise due to finite renormalization effects following spontaneous symmetry breaking. As pointed out in a general context in Ref. 1, the effective coupling of X mesons need not be identical to the coupling of the SU(3′) color octet gauge mesons, even though these latter particles belong to the same 15-fold set of SU(4′) as the X's. This is because (finite) renormalization effects following spontaneous symmetry breaking are likely to affect these various particles differently. We plan to investigate this question in detail in a subsequent note. As regards the coupling in the SU(3′) sector, a recent estimate, though crude, suggests that the effective constant $f^2(\mu)/4\pi$ renormalized at the mass $\mu = 2$ GeV may be as small as $\frac{1}{10}$ [H. D. Politzer (unpublished)]. These estimates apply to our scheme.

[39]That all effective constants may approach the value $\simeq \frac{1}{137}$ at sufficiently high energies in a theory with universality of coupling constants is an idea suggested recently by a number of authors in the interest of complete unification of all interactions; see, for example, H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974); H. Fritzsch and P. Minkowski, Caltech report 1974 (unpublished).

[40]Note that the heavy-lepton search based on ν_μ-induced reactions [see, for example, B. C. Barish *et al.*, Phys. Rev. Lett. 32, 1387 (1974)] applies to the heavy lepton M^- introduced here only to the extent of the Cabibbo rotation (θ_μ) in the $(\mathfrak{N}', \lambda')$ and (μ, M) spaces. The amplitude for $\nu_\mu \to M^- + W^+$ is proportional to $\sin\theta_\mu$ (see remarks later).

[41]A second anomaly-free interaction could be written down with the form $f V_1(\overline{F}_{eL}F_{eL} + \overline{F}_{\mu R}F_{\mu R}) + f V_2(\overline{F}_{\mu L}F_{\mu L} + \overline{F}_{eR}F_{eR})$. Here, spontaneous symmetry breaking must be arranged to guarantee that it is $(V_1 + V_2)$ and $(V_1 - V_2)$ which are the physical particles and are vector and axial-vector, respectively. [Strictly speaking, one needs to arrange that parity is conserved at least in the SU(3′) sector, i.e., $V_1(8) \pm V_2(8)$ are the physical particles, leaving the possibility that in the X sector the interactions are still chiral and parity-violating with X_1 and X_2 as the eigenstates. We do not exhibit this here, but have verified that such patterns of mixing are obtainable through the Higgs-Kibble mechanism.] One distinct advantage of this version is that it is γ_5-invariant in the X sector (in contrast to our basic model or the prodigal model [Eq. (A.4)]). This may help preserve the masslessness of the 4-component neutrino (ν_e and ν_μ) without a necessity for introducing the ζ fermions (see the second paper of Ref. 2, Sec. 5.2). Furthermore (due to γ_5 invariance), it also depresses contributions to the anomalous magnetic moments of e and μ from lighter-mass X exchanges. (See the remark in Ref. 4 footnote 8.) This fact is reflected in Table IV. In this version the two types of quarks (both necessarily integrally or fractionally charged for parity conservation) mutually interact through (at least) $[V_1(\underline{8}) + V_2(8)]$ and $[V_1(\underline{8}) - V_2(\underline{8})]$ fields. Normal hadrons may still be assumed to be predominantly e-quark composites with μ-quark composites lying higher due to the heavier mass of the μ quarks. Small admixtures of $(\overline{\lambda}'\lambda')$ with $(\overline{\lambda}\lambda)$ (for example) are of course harmless.

[42]If both types of quarks q_e and q_μ are integrally charged, the charge formula receives symmetric contributions from SU(4), SU(4′)$_e$, and SU(4′)$_\mu$ generators. The SU(3′) octet of gluons $V_e(\underline{8})$ must mix with $V_\mu(\underline{8})$ to generate the photon. Remarks made earlier (see end of previous footnote) with regard to the composition of normal low-lying hadrons would apply here as well.

[43]Once again, we have verified that such a mixing can be realized through the Higgs-Kibble mechanism. The scalar multiplets necessary for this purpose and their potential are simple generalizations of those presented

in Ref. 2 for the case of the prodigal model with two different SU(4′) groups. These may be presented in detail elsewhere.

[44]Note that the mass of the singlet S^0 can be made as large as desired compared with the masses of the exotics m_X by introducing reducible Higgs multiplets of the type $(1,1,4\times4\times4,1)$ to generate V_1 masses and $(1,1,1,4\times4\times4)$ to generate V_2 masses. This could then ensure the possibility that neutrino interactions mediated by S^0 are not unduly enhanced at present energies, while electron (and possibly muon) interactions with hadrons mediated by X's are enhanced to the extent observed at SPEAR. As noted in Ref. 2 (second paper), Sec. 4.5, large reducible multiplets such as mentioned above are also nedded, if one desires to give masses to the SU(3′) color octet of gluons in a model with fractionally charged quarks.

[45]Of course, in general, one may also allow Cabibbo rotations for F_e and F_μ to be different, which will lead to the coupling

$$W_L[\overline{\mathcal{P}}(\mathfrak{N}\cos\theta_e+\lambda\sin\theta_e)+\overline{\nu}_e(e\cos\theta_e+E\sin\theta_e)+\overline{\nu}_\mu(\mu\cos\theta_\mu+M\sin\theta_\mu)].$$

This will lead to μ-decay versus β-decay constants having the ratio $\cos\theta_\mu:\cos\theta_e$, while $K\to e\nu$ versus $\pi\to e\nu$ constants would have the ratio $\tan\theta_e$, etc. Note that such rotation of leptons (with e and μ belonging to different colors) does not affect the rate of $\mu\to e+\gamma$ decay and the 2-neutrino experiment.

[46]If we note that η^0 primarily decays through neutral modes, this may provide an ingredient to explain the so-called energy crisis. See, for example, C. H. Llewellyn Smith, CERN Report No. TH. 1849, 1974 (unpublished). Note that [since the $(\underline{\overline{\lambda}\lambda})$ density relevant to the case of the strange electron is isoscalar] the physical ϕ and η^0 production in $e^-e^+\to$ hadrons at higher energies must be accompanied by at least 2-pion production or other multiparticle states to balance isospin and energy-momentum conservation.

[47]This formula, in this generality, is due to C. H. Llewellyn Smith.

[48]Details of this extended gauge model, in particular the scheme of spontaneous symmetry breaking for this model, are being studied in collaboration with Professor R. N. Mohapatra.

[49]A second interesting possibility within this extended gauge model is worth noting. It arises even if X_R is not lighter than X_L and also even if X interactions are parity conserving (so that $(X_L\pm X_R)/\sqrt{2}$ are the eigenstates of the mass matrix). Since only the "uncharmed" neutrinos (which are coupled to the $\mathcal{P}$ quarks via $X_{\mathcal{P}}$) can exhibit their anomalous interactions with normal hadrons if the electron neutrinos are "uncharmed" (and therefore the muon neutrinos are "charmed"), the X interactions can induce reactions of the type

$$\nu_e+N\xrightarrow{X_{\mathcal{P}}}\nu_e+H,$$

but not reactions of the type

$$\nu_\mu+N\xrightarrow{X}\nu_\mu+H.$$

On the other hand, the *available* neutrino beams contain primarily muon neutrinos (with less than 2% contamination of electron neutrinos). Thus the observed strength of order G_{Fermi} for the reactions $\nu+N\to\nu+H$ can still allow for the X-induced *"diagonal"* interactions of electron neutrinos with hadrons to possess anomalous strength (as large as 5 to 10 times bigger than G_{Fermi}), even though muon neutrinos may interact with hadrons with normal strength (order G_{Fermi}). A test of this possibility with beams designed to contain a large fraction of electron neutrinos would be worthwhile. For this possibility to be compatible with SPEAR results, one must of course assume $(f^2/m_{X_{\mathfrak{N}}}{}^2)$ to be least an order of magnitude bigger than $(f^2/m_{X_{\mathcal{P}}}{}^2)$. Also worth noting is that if $X_{\mathcal{P}}$ interactions are parity conserving (i.e., if $(X_{\mathcal{P}_L}\pm X_{\mathcal{P}_R})/\sqrt{2}$ are the eigenstates) the anomalous ν_e interactions leading to $\nu_e+N\to\nu_e+H$ should possess, in general, large scalar, pseudoscalar, vector, and axial-vector interactions after Fierz reshuffling (see Sec. II). The presence of scalar and pseudoscalar terms may be desirable if the new Argonne data on pion production in "neutral current" processes are sustained [see S. Adler, Phys. Rev. Lett. 33, 1511 (1974)]. If the effect is confirmed, one may attribute it in this extended model primarily to vector or axial-vector X-induced neutrino interactions (either ν_e's or ν_μ's).