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Role of complex poles in a field-theoretical model for πN scattering*

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The "uncrossed meson line" approximation for the nucleon propagator is appled to the pion-nucleon S matrix. The resulting simplification resembles the Born approximation, but with the dressed nucleon propagator of Brown, Puff, and Wilets (BPW), which contains a pair of conjugate complex poles. The complex poles appear naturally in the consistent solution of the factorized BPW equations, and are interpreted according to Lee and Wick as states of indefinite metric. Inclusion of these poles produces considerably closer agreement with experimental results. The Adler-Weinberg consistency condition on pion-nucleon scattering is now satisfied to within 20%, and this is reflected in the near-threshold suppression of the cross sections and amplitudes (nucleon-antinucleon pair suppression). The $(3, 3^+)$ resonance is absent in the model. The model is most appropriate for calculating the two-pion contribution to the nucleon-nucleon potential, where subthreshold behavior is more important than the scattering resonances.

I. INTRODUCTION

The simplest model of πN scattering available is the Born approximation (BA) where just two graphs (see Fig. 1) contribute to the S matrix. When treated relativistically with pseudoscalar coupling, it encounters the difficulties that it produces very large S-wave πN amplitudes which lead to unphysical behavior in the total cross section near threshold, and it does not reproduce the resonances observed in πN scattering. The approximation discussed in this paper is an "augmented Born approximation" (ABA) with which calculations may be performed quite easily and which partially alleviates the discrepancies which are so glaring in the Born approximation. The present work is based upon the Green's function formulation of Brown, Puff, and Wilets (BPW).[1]

The assumption of a partially conserved axial-vector current mediating the πN interaction leads to a consistency condition due to Adler[2] and Weinberg.[3] ABA and BA are tested against this "soft" pion limit, and the comparison is used as one check on the validity of the approximation.

The importance of these soft-pion constraints has been demonstrated by Brown and Durso.[4] Calculations of the two-pion exchange contribution to the nucleon-nucleon potential via dispersion relations rely heavily on the subthreshold behavior of the πN amplitudes. (For a recent example, see Cottingham *et al.*[5]) The subthreshold region contains the physical t channel, which is the most important channel in the calculation of the two-pion contribution to the nucleon-nucleon potential. The two-pion contribution is given by integrating over the pion four-momenta of two πN amplitudes in the t channel. Because of the emphasis on the low-t values in the nucleon-nucleon potential, the effect of the s-channel resonances in the amplitudes is quite small. Thus the $N^*(1780)$ resonance predicted by this model really has little effect on the nucleon-nucleon potential; nor would the inclusion of a $(3, 3^+)$ resonance produce a large change in the resulting potential.

II. THEORY

The S matrix for πN scattering is given by the Lehmann-Symanzik-Zimmermann (LSZ) reduction in terms of the vacuum expectation value of the time-ordered product of field operators,

$$S_{fi} = \delta_{fi} + Z_2^{-1}\bar{u}_{\lambda_f p'} \int d^4x\, d^4y\, d^4z\, d^4v \exp[i(q'\cdot y + p\cdot v - p'\cdot z - q\cdot x)]$$
$$\times(-\partial_x^2+\mu^2)(-i\gamma_\mu\partial_z^\mu + M)\langle 0|T(\phi(x)\phi^\dagger(y)\psi(z)\bar{\psi}(v))|0\rangle(-\overleftarrow{\partial}_y^2+\mu^2)(i\tilde{\gamma}_\mu\overleftarrow{\partial}_v^\mu + M)u_{\lambda_i p}, \quad (1)$$

where Z_2 is the nucleon renormalization constant for the nucleon propagators, $u_{\lambda p}$ is the usual Dirac positive-energy spinor, ϕ is the pion field operator, and ψ is the nucleon field operator. The tilde denotes a transpose, and the arrows over the operators indicate the direction in which they operate. The time-ordered product includes a sign change for odd permutations of the nucleon operators.

To reduce the vacuum expectation value to a more tractable form, a dynamical theory is required. The Green's function formulation of BPW[1] provides the necessary dynamics. The field equations are

$$(\gamma \cdot p + M_0)_{\xi\xi'} \psi_{\xi'}(x) = -\Omega^j_{\xi\xi'} \psi_{\xi'}(x)\phi^j(x),$$
$$(-\partial^2 + \mu_{0j}{}^2)\phi^j(x) = -\tfrac{1}{2}[\bar{\psi}_\xi(x), \Omega^j_{\xi\xi'} \psi_{\xi'}(x)]. \qquad (2)$$

The ξ implies both the spin (α) and isospin (β). The coupling matrix $\Omega^j_{\xi\xi'}$ for pions is $g_{0\pi}\gamma^5_{\alpha\alpha'}\tau^j_{\beta\beta'}$, where τ is the nuclear isospin. The μ_{0j} is the bare mass of the jth isospin component of the Hermitian pion field and $g_{0\pi}$ is the unrenormalized pion-nucleon coupling constant.

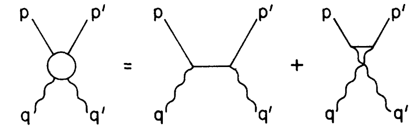

FIG. 1. The Born approximation to the S matrix.

The generalized Green's function of Schwinger is defined by

$$G_{(n,\,\nu/2)}(x_1 \cdots x_n, x_1' \cdots x_n', \xi_1 \cdots \xi_\nu) \equiv i^{n+[\nu/2]}\langle 0 | T(\psi(x_1)\cdots\psi(x_n)\bar{\psi}(x_n')\cdots\bar{\psi}(x_1')\phi(\xi_1)\cdots\phi(\xi_\nu))|0\rangle, \qquad (3)$$

where $[\nu/2]$ is the largest integer contained in the fraction.

This mixed Green's function satisfies two recursion relations, only one of which is useful here:

$$G_{(n,\,\nu/2)}(x_1 \cdots x_n, x_1' \cdots x_n', \xi_1 \cdots \xi_\nu)$$
$$= i^{-\nu^2}\Omega^{j\nu}_{\xi_{n+1}\xi'_{n+1}} \mathcal{G}_0^{j\nu}(\xi_\nu - x_{n+1})\delta(x_{n+1} - x'_{n+1})G_{(n+1,\,\nu/2-1)}(x_1\cdots x_n, x_1'\cdots x_n', x'_{n+1}, \xi_1\cdots\xi_{\nu-1})$$
$$+ \sum_{l=1}^{\nu} \mathcal{G}_0^{j\nu}(\xi_l - \xi_\nu)G_{(n,\,\nu/2-1)}(x_1\cdots x_n, x_1'\cdots x_n', \text{omit } \xi_l \text{ and } \xi_\nu), \qquad (4)$$

where $\mathcal{G}_0^{j\nu}$ is the bare pion propagator.

Applying this recursion relation twice reduces the mixed Green's function $G_{(1,\,1)}$ to a nucleon Green's function of higher order:

$$\langle 0 | T(\phi(x)\phi^\dagger(y)\psi(z)\bar{\psi}(v))|0\rangle$$
$$= i\Omega^{j_2}_{\xi_2\xi_2'}\Omega^{j_1}_{\xi_3\xi_3'}\delta(x_3 - x_3')\delta(x_2 - x_2')\mathcal{G}_0^{j_2}(y - x_2')\mathcal{G}_0^{j_1}(x - x_3')G_3(zx_2x_3; vx_2'x_3') + 2G_1(z, v)\mathcal{G}_0(x - y), \qquad (5)$$

where G_3 is the three-body Green's function. The last term will not contribute due to the $\gamma \cdot p + M$ factors. The superscript(s) for this last $\mathcal{G}_0$ depend on the superscripts of the pion field operators, which are not displayed here.

The simplest factorization of the G_3 is six antisymmetrized products of three one-nucleon Green's functions. This is equivalent to setting the correlating potential due to meson exchange to zero. Since the mixed Green's function is sandwiched between the positive-energy Dirac spinors, the relationship

$$(-i\gamma^\mu\partial_\mu + M)G_{\xi\xi'}(1\,1') = Z_2\delta(1\,1')\delta_{\xi\xi'} \qquad (6)$$

removes two one-nucleon Green's functions in each term. The resulting S matrix is given by Brown[6]:

$$S_{fi} = \delta_{fi} + \frac{i}{(2\pi)^2}\delta^4(p' + q' - p - q)g_\pi{}^2\bar{u}_{\lambda_f p'}\gamma^5$$
$$\times[\tau_i\tau_f G_R(p - q') + \tau_f\tau_i G_R(p + q)]\gamma^5 u_{\lambda_i p}. \qquad (7)$$

Notice that the matrix element is given in terms of renormalized quantities only and that if one makes the substitution $G_R(p) \to (\gamma\cdot p + M)^{-1}$ one would have the usual Born approximation. The term "augmented Born approximation" is used to describe the approximation of Eq. (7) with G_R given by the BPW Green's function. The approximation represents a summation of a subset of graphs appearing in the Chew-Low model.[7,8] The S matrix is not derived from an integral equation and this has a larger effect on the isospin-$\frac{3}{2}$ channel since some of the graphs which contribute strongly to this channel are missing. The particular set summed is determined by the approximation to the nucleon Green's function and is represented pictorially by Fig. 2. Note that no meson crossings are allowed in the BPW approximation. The resulting approximation to the S matrix is repre-

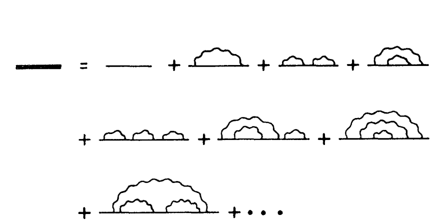

FIG. 2. The BPW Green's function expanded in perturbation series.

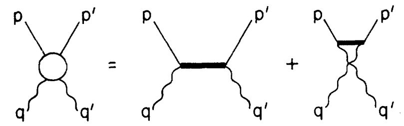

FIG. 3. The augmented Born approximation to the S matrix.

sented in Fig. 3.

The Green's function is usually represented in terms of a spectral function via the Lehmann-Källén representation,

$$G_R(p) = \int \frac{A_R(K)dK}{\gamma \cdot p + K}, \tag{8}$$

where $A_R(K)$ has a Dirac δ function of unit weight at the physical nucleon mass and a continuum beginning at $|K| > M + \mu$. (See Fig. 4.) This spectral representation with K real and A_R positive assures that G_R has no singularities off the real axis (in the complex variable p_0). In BPW, however, the integral in Eq. (8) must be interpreted as an integral over the real variable K plus the discrete sum of a pair of conjugate complex poles (A_c, K_c; A_c^*, K_c^*) as discussed in the next section.

III. COMPLEX POLES IN FIELD THEORIES IN GENERAL AND BPW IN PARTICULAR

It has long been realized that relativistic, local field theories are plagued with apparent inconsistencies which can be identified with discrete, nonnormalizable states of indefinite metric.[9] This was evidenced in the celebrated soluble model of Lee[10] as well as in various approximate solutions of more physical field theories. In QED, for example, the regularization procedure effected by subtracting the contribution of a massive lepton corresponds to introducing states of negative norm.

Lee and Wick[11] have discussed the role of complex poles extensively, and provided an interpretation of such states in terms of an indefinite metric. The BPW nucleon Green's function can be afforded the Lee-Wick interpretation. Details are given in Appendix A, the results of which are summarized here.

Following Lee and Wick, a Hermitian metric matrix, η, is introduced which has eigenvalues ± 1 and satisfies, therefore, $\eta^2 = 1$. The Hamiltonian is not Hermitian but pseudo-Hermitian:

$$\eta H^h \eta = H.$$

The superscript h is used for Hermitian conjugate. The Hamiltonian and field equations are constructed as in BPW, with equal-time anticommutation

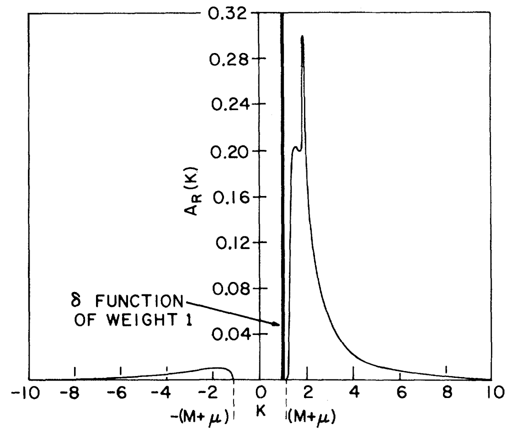

FIG. 4. The BPW spectral function.

relations given by

$$\{\psi^\dagger(\vec{r}, t), \psi(\vec{r}', t)\} = \delta^3(\vec{r} - \vec{r}').$$

$\psi^\dagger \equiv \bar{\psi}\gamma_0$, which appears in the anticommutation relations and in H, is not equal to ψ^h. To obtain the BPW results, however, the η matrix and Hermitian conjugate operators need not be constructed; only the anticommutation relations are required. (Similar statements may be made for the meson field operators.)

The Green's function which results from the consistent solution of the BPW factorization is of the form

$$G_R(p) = \int \frac{A_R(K)dK}{\gamma \cdot p + K} + \frac{A_c}{\gamma \cdot p + K_c} + \frac{A_c^*}{\gamma \cdot p + K_c^*}.$$

The first term on the right-hand side is the usual Lehmann-Källén representation, which is based on the assumption that the complete set of eigenstates of the system consists only of states which have real eigenvalues and are normalizable with a real metric. The appearance of remaining terms shows that the states with real eigenvalues are not complete, but must be supplemented by (at least) a pair of conjugate states of indefinite metric; a linear combination of the pair can be constructed to give one state of positive and one of negative metric. Although such states are not physically realizable, they must be included for completeness in the representation. (There *may* exist another representation in which negative-metric states can be "diagonalized away.") The location and residues of the complex poles are displayed in Fig. 5.

The points which should be emphasized here are that complex poles were not *introduced* by BPW, but emerged from the consistent solution of their factorized equations, and that there exists an interpretation of the corresponding states based on the work of Lee and Wick.

IV. THE ADLER-WEINBERG CONSISTENCY CONDITION

The Adler-Weinberg (AW) condition is a virtual or "soft" pion limit on the charge-symmetric invariant amplitude. The invariant amplitudes are defined from the T matrix,

$$T_{\beta\beta'}(p', q'; pq) = \frac{1}{16\pi^5}\bar{u}_{\lambda' p'}[-A_{\beta\beta'}(s,t,u) + \gamma\cdot q B_{\beta\beta'}(s,t,u)]u_{\lambda p}, \quad (9)$$

where the Mandelstam variables are used:

$$s \equiv -(p+q)^2,$$
$$t \equiv -(q'-q)^2,$$
$$u \equiv -(p'-q)^2,$$

with

$$s+t+u = 2M^2 + 2\mu^2.$$

The invariant amplitudes are decomposed into charge-symmetric and -antisymmetric terms:

$$A_{\beta\beta'} = \delta_{\beta\beta'}A^{(+)} + \tfrac{1}{2}[\tau_\beta, \tau_{\beta'}]A^{(-)},$$
$$B_{\beta\beta'} = \delta_{\beta\beta'}B^{(+)} + \tfrac{1}{2}[\tau_\beta, \tau_{\beta'}]B^{(-)}. \quad (10)$$

ABA produces expressions for the invariant amplitudes defined in Eq. (10):

$$A^{(\pm)}(s,t,u) = g_\pi^2 \int (K-M)A(K)dK\left(\frac{1}{K^2-s} \pm \frac{1}{K^2-u}\right), \quad (11)$$

$$B^{(\pm)}(s,t,u) = g_\pi^2 \int A(K)dK\left(\frac{1}{K^2-s} \pm \frac{-1}{K^2-u}\right). \quad (12)$$

The AW conditions are given by

$$A^{(+)}(M^2, t, M^2) = -g_\pi^2/M,$$
$$A^{(-)}(M^2, t, M^2) = 0, \quad (13)$$

in the approximation that the effect of the form factor on the first condition, which is of order μ^2/M^2, is ignored. The second condition is a result of crossing symmetry and is trivially satisfied. The Born approximation cannot satisfy the first condition since $A^{(+)}$ vanishes identically for that approximation. ABA produces

$$A^{(+)}(M^2, t, M^2) \simeq -0.8 g_\pi^2/M$$

due almost exclusively to the complex poles. [The continuum part of $A(k)$ gives about $0.16\, g_\pi^2/M$ and these poles give approximately $-0.96 g_\pi^2/M$.]

Inclusion of complex poles in the nucleon Green's function provides an approximate fulfillment of the Adler-Weinberg consistency condition. Attendant with this approximate fulfillment is the suppression of the πN S-wave amplitudes and the total πN cross section at threshold, as well as alteration of the S-wave $\pi\pi$ helicity amplitudes over the whole range. These effects are described in the next two sections.

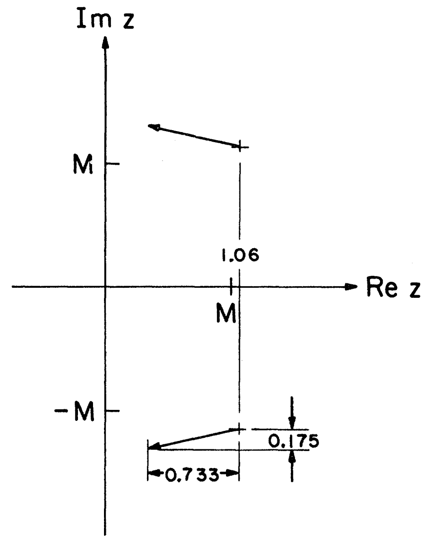

FIG. 5. The location of the complex poles. The arrow represents the residue of the pole.

V. PION-NUCLEON SCATTERING

The total cross section for πN scattering may be decomposed into the isospin $\frac{3}{2}$ and $\frac{1}{2}$ channels. The isospin $\frac{3}{2}$ and $\frac{1}{2}$ amplitudes are related to the symmetric and antisymmetric amplitudes as follows:

$$F^{(3/2)} = F^{(+)} - F^{(-)},$$
$$F^{(1/2)} = F^{(+)} + 2F^{(-)}. \quad (14)$$

The cross sections are given by

$$\sigma^n(W) = \frac{1}{16\pi E(q)W}\int_{-1}^{1} dx[\Gamma^n(x)q^2 x + H^n(x)], \quad (15)$$

where W is the total energy, $E(q) = (M^2 + \vec{q}^2)^{1/2}$, x is the angle between $\vec{p}$ and $\vec{p}'$ (the incoming and outgoing nucleons) in the center-of-momentum frame of the pion-nucleon system, and

$$\Gamma^n(x) = |B^n(x)|^2(W^2 - M^2) - |A^n(x)|^2 - 2M\,\mathrm{Re}[A^n(x)B^n(x)^*],$$

$$H^n(x) = |B^n(x)|^2\{E(q)w(q)(W^2 - M^2) + \mu^2[M^2 - E(q)W]\} + |A^n(x)|^2(2M^2 + q^2) - 2M\,\mathrm{Re}[A^n(x)B^n(x)^*] \times (W^2 - M^2 - \mu^2 - q^2),$$

with $\vec{q}$ being the momentum of the incoming pion and $w(q) = (\mu^2 + \vec{q}^2)^{1/2}$.

Figures 6 and 7 display the Born approximation, the "augmented Born approximation," and the experimental results.[12] The threshold behavior is significantly improved in both channels and, with the exception of the $(3, 3^+)$ resonance, most of the features of the cross sections are reproduced in a qualitative way.

The peak in the $\frac{1}{2}$ channel could be adjusted to produce a cross section much closer to the experimental value by varying the ω coupling constant when calculating the Green's function. The effect on the πN cross section in the threshold region is quite small for even a 50% increase in the coupling constant. The magnitude of the peak would be about right for a 10% increase in the coupling constant. The peak would remain shifted too far to the right because the peaks in the experimental cross section are due mostly to lower-mass, higher–angular momentum resonances which are not included in ABA.

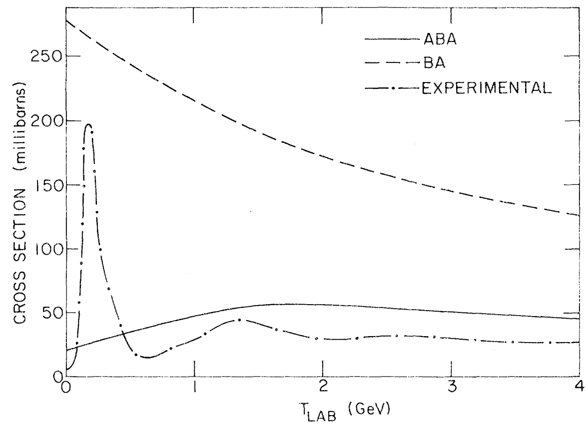

FIG. 6. Pion-nucleon elastic cross sections in isospin-$\frac{3}{2}$ channel.

The T matrix can be decomposed into good isospin and total angular momentum channels to reveal the existence of any resonances. An angular momentum decomposition of T results in the following expression[13]:

$$f_{lj}^{(\pm)} = -\frac{g_\pi^2}{16\pi W}\int dK\, A(K)\left\{(E+M)\left[\frac{I_l}{K+W} \pm (K+W-2M)J_l\right] - (E-M)\left[\frac{I_j}{K-W} \pm (K-W-2M)J_j\right]\right\}, \tag{16}$$

where

$$I_l \equiv \int_{-1}^{1} dx\, P_l(x) = 2\delta_{l,0},$$

$$J \equiv \frac{1}{2q^2}\int_{-1}^{1} dx\, \frac{P_l(x)}{H+x},$$

and

$$H \equiv (K^2 + W^2 - 2M^2 - 2\mu^2)/2q^2 - 1 .$$

The subscripts l and j are orbital and total angular momentum, respectively. The superscript $(\pm)$ indicates that the amplitude is charge-symmetric or -antisymmetric.

The $(\frac{1}{2}, \frac{1}{2}^+)$ amplitude is given by $f_{1,1/2}^{(+)} + 2f_{1,1/2}^{(-)}$, the $(\frac{1}{2}, \frac{1}{2}^-)$ amplitude by $f_{0,1/2}^{(+)} + 2f_{0,1/2}^{(-)}$. An imaginary part may arise only in the $I=\frac{1}{2}$ channel due to the form of $f_{lj}^{(\pm)}$. [See Eq. (14).] Thus this approximation cannot represent any of the $I=\frac{3}{2}$ resonances.

Figure 8 displays the amplitudes for the $(\frac{1}{2}, \frac{1}{2}^-)$ and the $(\frac{3}{2}, \frac{1}{2}^-)$ channels. The magnitudes of both of these S-wave ABA amplitudes are somewhat smaller than the corresponding BA amplitudes. This is one manifestation of "pair suppression" and produces better agreement with experimental amplitudes. No resonances are present in the isospin-$\frac{1}{2}$ channel since the imaginary part does not peak strongly and the real part has no ripples.

The $(\frac{1}{2}, \frac{1}{2}^+)$ channel displays a very distinct resonance at the correct mass to represent the $N^*(1780)$. Figure 9 shows this resonant behavior and in addition shows the close agreement of the real parts of BA and ABA for low energy.

VI. $\pi\pi$ HELICITY AMPLITUDES

A program for calculating the two-nucleon potential via dispersion relations has been created independently by Amati, Leader, and Vitale[14] and by Cottingham and Vinh Mau.[15] The inputs for the calculation are the S- and P-wave helicity amplitudes for the reaction $N\bar{N} \to \pi\pi$. The t channel of

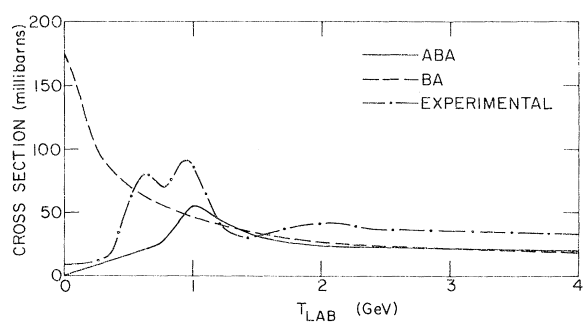

FIG. 7. Pion-nucleon elastic cross sections in isospin-$\frac{1}{2}$ channel.

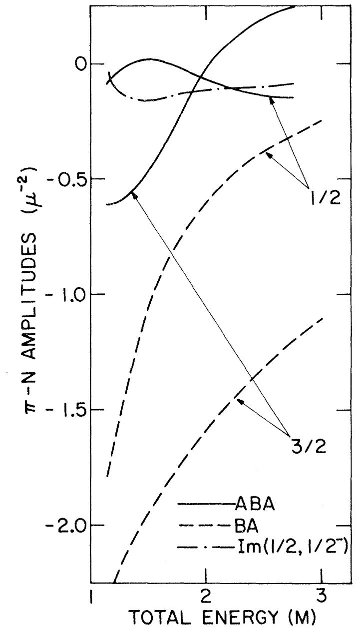

FIG. 8. S-wave πN amplitudes.

Figs. 1 and 3 gives precisely this reaction. Because the same diagrams are employed, the $\pi\pi$ helicity amplitudes are readily derived from the πN amplitude.[16] The helicity amplitudes are

$$\phi_+^0 = \frac{1}{8\pi}[-p^2 A_0^{(+)} + MpqB_1^{(-)}], \tag{17}$$

$$\phi_+^1 = \frac{1}{8\pi}\left(-\frac{p}{q}A_1^{(-)} + \frac{M}{3}[2B_2^{(-)} + B_0^{(+)}]\right), \tag{18}$$

$$\phi_-^1 = \frac{1}{8\pi}\frac{\sqrt{2}}{3}[B_0^{(-)} - B_2^{(-)}], \tag{19}$$

where

$$(A_J^{(\pm)}, B_J^{(\pm)}) \equiv \int_{-1}^{1} dx\, P_J(x)(A^{(\pm)}, B^{(\pm)}),$$

and x is the πN scattering angle. In the t channel,

$$p^2 = \tfrac{1}{4}t - M^2,$$

$$q^2 = \tfrac{1}{4}t - \mu^2.$$

Substituting Eqs. (11) and (12) into (17), (18), and (19) gives the expression for the helicity amplitudes:

$$\phi_+^0 = -\frac{g_\pi^2}{16\pi^2}\int dK\, A(K)\left[\frac{ip}{q}(K-M)\arctan\frac{1}{h} + M\left(1 - h\arctan\frac{1}{h}\right)\right], \tag{20}$$

$$\phi_-^1 = -\frac{g_\pi^2}{16\pi^2}\frac{i}{\sqrt{2}\,q}\int dK\, A(K)(h^2 - h + 1)\arctan\frac{1}{h}, \tag{21}$$

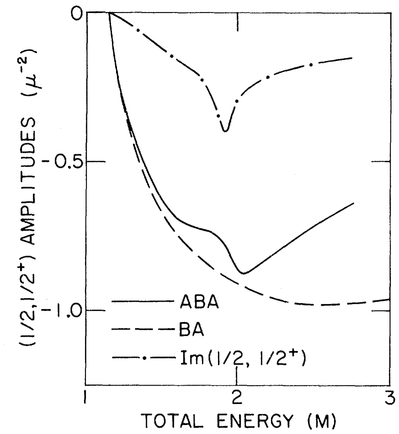

FIG. 9. Pion-nucleon $(\frac{1}{2}, \frac{1}{2}^+)$ channel amplitudes.

$$\phi_+^1 = -\frac{g_\pi^2}{16\pi^2}\frac{1}{q}\int dK\, A(K)\left(1 - h\arctan\frac{1}{h}\right)\times\left(\frac{K-M}{q} + \frac{iMh}{p}\right), \tag{22}$$

where $h \equiv i(K^2 + \frac{1}{2}t - M^2 - \mu^2)/2pq$.

All of the amplitudes are real and cannot produce any resonances. The P waves are not plotted, as the difference between the two approximations is negligible. Figure 10 displays the resulting S-wave amplitudes along with the S wave calculated by Nielsen, Petersen, and Pietarinen (NPP)[17] by ana-

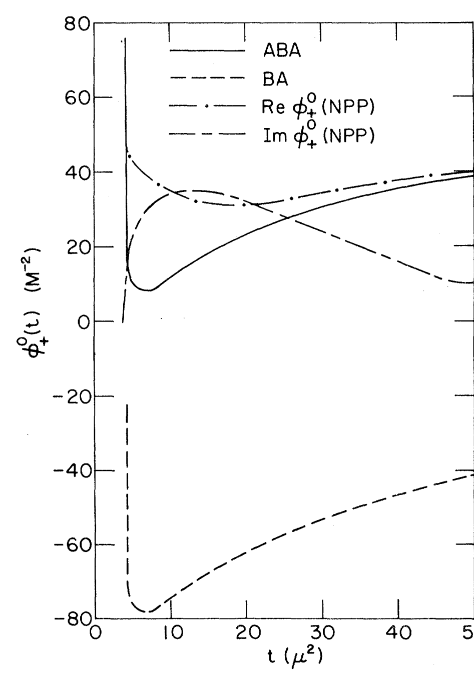

FIG. 10. S-wave helicity amplitude for $NN \to \pi\pi$ reaction.

lytic continuation from πN scattering. ABA is significantly closer than BA to the real part of NPP.

VII. SUMMARY

The inclusion of the complex poles in the augmented Born approximation results in a near satisfaction of the Adler-Weinberg consistency condition for πN scattering. The real parts of the S-wave πN and $\pi\pi$ amplitudes, as well as the πN total cross section, are significantly improved with respect to experimental results. The πN scattering also reflects the $N^*(1780)$ resonance of the BPW Green's function.

The very large values of the S-wave amplitudes in the Born approximation for πN scattering have been attributed to the intermediate nucleon-antinucleon pair which arises from a different time ordering of the graphs in Fig. 1.[18] Thus the very small S waves in πN scattering produced by the augmented Born approximation are interpreted here as pair suppression.

The improvement over the Born approximation comes about with little cost in computation, and, although the $(3, 3^+)$ resonance is not included in this approximation, the results are qualitatively reasonable elsewhere. The S matrices resulting from ABA, as in the usual BA, do not possess unitarity. The two-pion contribution to the nucleon-nucleon potential is dominated by the t-channel S- and P-wave amplitudes, which enter as the square of their absolute values.[19] Thus the existence of a small imaginary part in the low-t channel is not very important. In calculating the two-pion contribution to the nucleon-nucleon potential, the "augmented Born approximation" is more realistic than the Born approximation.

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APPENDIX: SUMMARY OF BPW AND COMPLEX POLES

A recapitulation of BPW and a discussion of the complex poles which result from their approximation of the two-body Green's function are presented here.

Consider a system of nucleons and, for example, π mesons described by the Hamiltonian

$$H = H_N + H_\pi + H' , \tag{A1}$$

where

$$H_N = \tfrac{1}{2}\int d^3r[\bar{\psi}_\zeta(x)(\vec{\gamma}\cdot\vec{p} + M_0)_{\zeta\zeta'}\psi_{\zeta'}(x) - (\vec{\gamma}\cdot\vec{p} + M_0)_{\zeta\zeta'}\psi_{\zeta'}(x)\bar{\psi}_\zeta(x)] , \tag{A2}$$

$$H_\pi = \tfrac{1}{2}\int d^3r(\pi^j\pi^j + \vec{\nabla}\phi^j\cdot\vec{\nabla}\phi^j + \mu_0^2\phi^j\phi^j) , \tag{A3}$$

and

$$H' = \tfrac{1}{2}g_{0\pi}\int d^3r[\bar{\psi}_\zeta(x)(\tau_j\gamma_5)_{\zeta\zeta'}\psi_{\zeta'}(x)\phi^j(x) - \psi_{\zeta'}(x)(\tau_j\gamma_5)_{\zeta\zeta'}\bar{\psi}_\zeta(x)\phi^j(x)] . \tag{A4}$$

The indices ζ and ζ' on the nucleon field operators, ψ, denote both spin (γ) and isospin (τ). The j index on the pion field operators, ϕ^j, refers to the Hermitian components of the pion field. π^j is the canonical conjugate to ϕ^j.

The fields satisfy the usual equal-time commutation relations

$$\{\psi_\zeta(\vec{r}, t), \bar{\psi}_{\zeta'}(\vec{r}', t)\} = (\gamma_0)_{\zeta\zeta'}\delta^3(\vec{r} - \vec{r}') , \tag{A5}$$

$$[\pi^j(\vec{r}, t), \phi^{j'}(\vec{r}', t)] = -i\delta_{jj'}\delta^3(\vec{r} - \vec{r}') . \tag{A6}$$

The field equations follow from these commutation relations and the operator relation $i\dot{O} = [O, H]$. They may be written for the general case of nonderivative coupling as

$$(\gamma\cdot p + M_0)_{\zeta\zeta'}\psi_{\zeta'}(x) = -\Omega^j_{\zeta\zeta'}\psi_{\zeta'}(x)\phi^j(x) , \tag{A7}$$

$$(p^2 + \mu_0^2)\phi^j(x) = -\tfrac{1}{2}[\psi_{\zeta'}(x), \Omega^j_{\zeta\zeta'}\psi_{\zeta'}(x)] , \tag{A8}$$

where the coupling matrix for π mesons is

$$\Omega^j_{\zeta\zeta'} = g_{0\pi}(\tau_j\gamma_5)_{\zeta\zeta'} . \tag{A9}$$

The n-particle Green's function is

$$G_n(1\cdots n; 1'\cdots n') = i^n\langle T(\psi(1)\cdots\psi(n)\bar{\psi}(n')\cdots\bar{\psi}(1'))\rangle , \tag{A10}$$

where T is the Wick time-ordering operator, which contains a factor of $(-1)^p$, where p is the number of permutations of nucleon field operators. The meson field is defined in a similar fashion, but the factor $(-1)^p$ is not included since bosons commute.

Formally eliminating the meson degrees of freedom yields

$$G_1^0(1\,1'')G_n(1''\,2\cdots n; 1'\cdots n') = \sum_{l=1}^{n}(-1)^{l+1}\delta(1l')G_{n-1}(2\cdots n; 1'\cdots \text{omit } l'\cdots n') + i\langle 1, n+1|v|n+2, n+3\rangle G_{n+1}[2\,3\cdots n, n+2, n+3; (n+1)^{\pm}1'\cdots n'] , \tag{A11}$$

where

$$\langle 1\,2\,|v|\,3\,4\rangle = -\sum_j \Omega^j_{\zeta_1\zeta_3}\Omega^j_{\zeta_2\zeta_4}\delta(x_1-x_3)\mathcal{G}^j_0(x_1-x_2)\delta(x_2-x_4), \tag{A12}$$

with G^0_1 denoting the noninteracting nucleon Green's function and $\mathcal{G}^j_0$ denoting the noninteracting meson Green's function.

The BPW approximation to this equation for G_1 is written as

$$G_2(1\,2;1'\,2') = G_1(1\,1')G_1(2\,2') - G_1(1\,2')G_1(2\,1'). \tag{A13}$$

The equation for G_1 is given by substituting (A13) into (A11) for the case $n=1$ and employing the subsidiary condition, $G_0 = 1$. In momentum space

$$G_1^{-1}(p)_{\zeta_1\zeta_1'} = G_1^{0-1}(p)_{\zeta_1\zeta_1'} + i\sum_j \Omega^j_{\zeta_1\zeta_3}\Omega^j_{\zeta_2\zeta_1'}\int_{q_4}\mathcal{G}^j_0(p-q)G_1(p)_{\zeta_3\zeta_2}\cos(q_0 0^+), \tag{A14}$$

where

$$\int_{q_4} \equiv \int \frac{d^4q}{(2\pi)^4}.$$

Closing of the contour in q_0 is governed by the factor $\cos(q_0 0^+)$, which requires equal contributions from upper and lower closings (this is of no consequence below). The Dirac algebra is eliminated by the introduction of the projection operators

$$P_\pm(p) = -\tfrac{1}{2}\left(\pm\frac{\gamma\cdot p}{\omega_p} - 1\right), \tag{A15}$$

where $\omega_p = (-p^2)^{1/2}$. The Green's function is written as

$$G_1(p) = P_+(p)\overline{G}_+(\omega_p) + P_-(p)\overline{G}_-(\omega_p), \tag{A16}$$

where the fact that $\overline{G}_+$ and $\overline{G}_-$ must be Lorentz invariant requires that they depend only on ω_p. The inverse Green's function may then be written as

$$G_1^{-1}(p) = P_+(p)\overline{G}_+^{-1}(\omega_p) + P_-(p)\overline{G}_-^{-1}(\omega_p). \tag{A17}$$

Equation (A14) is written as two coupled equations for $\overline{G}_+$ and $\overline{G}_-$ which after a very simple manipulation reduce to one integral equation and a subsidiary condition. These are

$$\overline{G}_+^{-1}(\omega_p) = M_0 - \omega_p - \tfrac{3}{2}\int_{q_4}\frac{\cos((q^0-p^0)0^+)}{q^2+\mu^2-i\epsilon}\left[\left(1+\frac{p(p-q)}{\omega_p\omega_{p-q}}\right)\overline{G}_+(\omega_{p-q}) + \left(1-\frac{p(p-q)}{\omega_p\omega_{p-q}}\right)\overline{G}_+(-\omega_{p-q})\right](ig^2_{0\pi}) \tag{A18}$$

and

$$\overline{G}_+(\omega_p) = \overline{G}_-(-\omega_p).$$

If the imaginary part of $\overline{G}_+^{-1}(x)$ is written as $-\pi T(x)$ and the imaginary part of $\overline{G}_+(x)$ as $\pi A(x)$, then a relationship between these two functions may be written as

$$T(K) = 3\left(\frac{g_{0\pi}}{4\pi}\right)^2\int dK' K_e(K,K')A(K'), \tag{A19}$$

where the kernel is

$$K_e(k,k') = \frac{1}{2|k|}[k^4 - k^2(k'^2+\mu^2) + (k'^2-\mu^2)^2]^{1/2} \times\Theta(|k| - |k'+\mu|)[(k-k')^2-\mu^2]. \tag{A20}$$

The behavior of this kernel at large K determines the number of subtractions necessary to write $\overline{G}_+^{-1}$ in the form of a dispersion relation. The self-energy is written as a Hilbert transform of $-\pi T$:

$$\overline{G}_+^{-1}(z) = M_0 - z - \int\frac{T(K)dK}{K-z}. \tag{A21}$$

The requirement that G_1 have a pole at the physical mass of the nucleon gives one subtraction. The subtraction results in

$$\overline{G}_+^{-1}(z) = (M-z)\left(1 + \int\frac{T(K)dK}{(K-M)(K-z)}\right). \tag{A22}$$

The effects of the behavior of the kernel in (A20) can be summarized by

$$T(K)\xrightarrow[K\to\infty]{} C|K|, \tag{A23}$$

which means the integral in (A22) is logarithmically divergent. At this point a regularization is performed. In form it appears to be a typical renormalization, but a closer examination reveals that it does not correspond to a renormalization; however, it does remove the divergent behavior of the representation for $\overline{G}_+^{-1}$ by introducing a pair of complex zeros off the real axis. These act to remove the divergence for large K and lead to a finite, well-behaved solution.

Define a "renormalization constant," Z_2, which will produce a unit strength pole at the nucleon mass. This constant is

$$Z_2=\left(1+\int\frac{T(K)dK}{(K-M)^2}\right)^{-1}. \tag{A24}$$

When $\overline{G}_+^{-1}$ is multiplied by this constant, the resulting integral is finite and allows the following integral representation of $\overline{G}_R^{-1}$:

$$\overline{G}_R^{-1}(z)=(M-z)\left(1-(M-z)\int\frac{T_R(K)dK}{(K-M)^2(K-z)}\right), \tag{A25}$$

where the subscript, R, stands for "renormalized" (i.e., $T_R=T\times Z_2$). The Green's function is renormalized by dividing by Z_2, which means (A19) is invariant if $g_{0\pi}Z_2=g_\pi$. Thus Z_2 vanishes from the equations. Equation (A24) can be rewritten as

$$Z_2=1-\int\frac{T_R(K)dK}{(K-M)^2}. \tag{A26}$$

Since $T_R(K)\rightarrow C|K|$, this integral is logarithmically divergent. The operation called, in this case, renormalization is a form of regularizing subtraction. An examination of the large-z behavior of (A25) indicates that

$$\overline{G}_R^{-1}(z)\xrightarrow[z\to\infty]{} Cz\ln|z|-i\pi T_R(z). \tag{A27}$$

The function S, which is $\overline{G}_R^{-1}(x)/(M-z)$, thus has two zeros for its real part. The lines of constant value of the real and imaginary parts of S are orthogonal. For both large imaginary argument and large real argument, the real part of S approaches Z_2. This indicates that the contours of constant value for the real part form closed loops. For large x ($z=x+iy$) with y nonzero, the imaginary part is even and of the same sign as for large y. Since the imaginary part changes sign between $M+\mu$ and $-M-\mu$, there has to be at least one pair of complex poles at some point K_c and K_c^* with residues which are complex conjugates of one another. The complex poles represent states of indefinite metric which act as the regularizing agents.

The Green's function may now be written in terms of the imaginary part of G and the complex poles as in Sec. III.

The existence of these indefinite-metric states implies that the Hamiltonian defined by Eqs. (A1)–(A4) is not Hermitian, but pseudo-Hermitian.

Although the form of the Hamiltonian appears to be manifestly Hermitian, the assumption that $\psi^\dagger=\psi^h$ has not been made. While this is usually the case, no constraint has been placed on $\psi^\dagger$ which forces this relationship. Thus, there is really no argument to support the Hermiticity of the Hamiltonian. This admits complex poles in conjugate pairs. It also requires that the indefinite-metric states be included in any complete sum over states. For this reason, the usual Lehmann-Källén representation is not complete and it is necessary to include the contribution of complex poles. This alteration does not affect the formal solution posed by BPW but requires that the complex poles be explicitly included.

The inclusion of these indefinite-metric states is necessary for the agreement of G_R^{-1} and $1/G_R$.

The location and residues of the BPW complex poles are shown in Fig. 5.

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