Missing mass as an alternative to rapidity gap in the experimental study of diffraction at high energy*

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The study of Regge and multi-Regge processes can be carried out either in terms of the rapidity or the missing-mass variables. In theoretical studies it is more advantageous to use the rapidity variable, while in practice missing masses are easier to measure. The asymptotic relations between the two variables, known in theory, are not reliable at presently available energies. We derive here a formula that provides a statistical relation between the two variables, reliable even at present energies, and demonstrate its validity by applying it to a sample of events in which both the missing-mass and the rapidity variables are known.

Regge and multi-Regge processes may be studied in terms of either the rapidity-gap variables or the missing-mass variables. Consider for example the process depicted in Fig. 1(a). When the gap Δ is large, we may either analyze the process in terms of Δ—using it as the asymptotic variable in the Regge expansion, studying the differential cross section $d\sigma/d\Delta$ as a function of Δ, and so on—or we may equally well carry out the analysis in terms of the variable s/M_B^2, where s is the overall c.m. energy squared and M_B is the "missing" mass recoiling against particle a in the final state. Similar considerations apply to multi-Regge reactions. For example, the double-Regge reaction of Fig. 1(c) may be described either in terms of Δ_1 and Δ_2 or in terms of s/m_B^2 and s/m_A^2, where M_B and M_A are the missing masses recoiling against a and b respectively in the final state.

The coexistence of the two sets of variables is due to the fact that each has certain merits the other does not have. For example, the final-particle phase space is easy to imagine [Figs. 1(a), 1(b), 1(c)] in terms of the rapidity variable which exploits the dynamical feature that most of the interesting physics is in the longitudinal direction. The rapidity-gap variables have the further advantage that they are local—for example, in the double-Regge process of Fig. 1(c), each Reggeon is described by the corresponding Δ, in contrast to the overlapping missing-mass variables s/M_A^2 and s/M_B^2 which obscure the factorization property of the multi-Regge pole expansion. Next, the rapidity gaps Δ_1 and Δ_2 of Fig. 1(c) (the simplest example of a multi-Regge process) satisfy an additive kinematical constraint which is easy to visualize and implement,[1] whereas the equivalent constraints in the variables M_A^2 and M_B^2 are complicated.[2] On the other hand, if we switch our attention from the contemplation of these processes to the measurement of their cross sections, the situation is reversed—the rapidity gaps are impossible to measure except for fitted events, owing to undetectable neutrals in the final state, whereas the missing masses can be determined free of this ambiguity.

We have presented the above facts to motivate the need for a formula that connects the two sets of variables so that using it we can pass freely from one to the other, therefore exploiting the merits of both. We know in theory that the two sets of variables are equivalent in the *asymptotic* Regge limit—for example, in the case of Fig. 1(a) we know that

$$\Delta \underset{\Delta\to\infty}{\sim} \ln(s/M_B^2) + C_\perp \tag{1}$$

where $C_\perp$ is a function of certain transverse momenta. In the Regge limit, as $\Delta\to\infty$, and the transverse momenta are frozen, one can ignore $C_\perp$. In practice, when the overall rapidity space is at most 8 units long the omission of $C_\perp$ could be unwarranted. In what follows we will derive a formula for the average value of $C_\perp$ based on a certain assumed average behavior in the transverse direction. We will then subject this formula to a quantitative test.

Let us begin with the formula for the c.m. energy squared of two particles with masses M_1 and M_2:

$$\begin{aligned} s_{12} &= M_1^2 + M_2^2 + 2\mu_1\mu_2\cosh(y_1 - y_2) - 2\vec{p}_{\perp 1}\cdot\vec{p}_{\perp 2} \\ &\underset{s_{12}\text{ large}}{\simeq} \mu_1\mu_2 e^{|y_1-y_2|}\,, \end{aligned} \tag{2}$$

where $\vec{p}_{\perp i}$ is the transverse momentum, y_i is the rapidity, and $\mu_i = (M_i^2 + |p_{\perp i}|^2)^{1/2}$, the transverse mass of particle i. Turning to Fig. 1(a) we see that if we treat the cluster B as a single object of mass M_B and call y_B the location of its center of mass, the final c.m. energy squared is

$$s \simeq \mu_{a'}\, \mu_B\, e^{y_B - y_{a'}} \simeq \mu_{a'} M_B\, e^{y_B - y_1 + \Delta} \tag{3}$$

if $\mu_B \simeq M_B$ for large M_B. To calculate y_B we use the definition of center of mass, measuring all momenta in the rest frame of particle N;

$$\begin{aligned} M_B \sinh(y_N - y_B) &\simeq \tfrac{1}{2} M_B\, e^{y_N - y_B} \\ &= \sum_{i=1}^{N-1} p_{\parallel i} \\ &= \sum_{i=1}^{N-1} \mu_i \sinh(y_N - y_i)\ . \end{aligned} \tag{4}$$

If particle 1 is well separated from particle 2 in rapidity, so that $e^{y_N - y_1} \gg e^{y_N - y_2}$, we can approximate[3] the sum by the leading term $p_{\parallel 1}$ to get

$$M_B\, e^{y_N - y_B} \simeq \mu_1\, e^{y_N - y_1}\ . \tag{5}$$

Inserting this into Eq. (3) gives

$$s \simeq \frac{\mu_{a'}}{\mu_1} M_B^2\, e^{\Delta}\ . \tag{6}$$

Assuming that on the average $\langle p_\perp \rangle \simeq 350$ MeV/c for all particles, and that particle 1 is a pion (on the basis of the preponderance of pions among produced particles), we get

$$\Delta = \ln\left(\frac{s}{M_B^2}\right) + \ln\left(\frac{\langle \mu_\pi \rangle}{\langle \mu_{a'} \rangle}\right)\ , \tag{7}$$

where $\langle \mu_i \rangle = (M_i^2 + \langle p_\perp \rangle^2)^{1/2}$. If particle a is a pion the second term (the average value of $C_\perp$) vanishes, while if a is a proton $\ln(\langle \mu_\pi \rangle / \langle \mu_p \rangle) \simeq -1$. Clearly the latter constant is not ignorable compared to the rapidity gaps currently being studied.

The gap in Fig. 1(b) may be described by Eq. (7), after we perform the substitutions $a' \leftrightarrow b'$ and $A \leftrightarrow B$. The double-Regge reaction of Fig. 1(c) is described by the repeated application of Eq. (7), once for each gap.

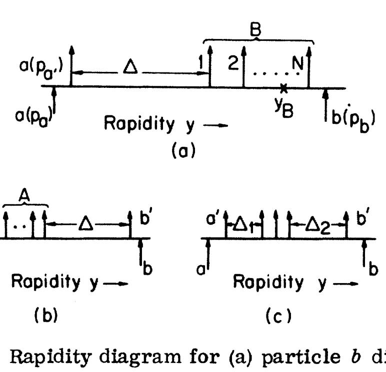

FIG. 1. Rapidity diagram for (a) particle b dissociation, (b) particle a dissociation, and (c) double-Pomeron exchange (DPE).

To examine the reliability of Eq. (7), we analyzed a sample of events of the reaction $\pi^- p \rightarrow \pi^-\pi^+\pi^- p$ at 205 GeV/c.[4] The final-particle momenta were determined by a 4-constraint fit, and the particles were labeled α, β, γ, and δ in the order of increasing rapidity. The events satisfy α = proton and $\delta = \pi^-$.

Plotted in Fig. 2(a) is the measured rapidity gap $R_{\alpha\beta}$ between α and β, versus $\ln(s/M_{3\pi}^2)$ for each event. The straight line of unit slope and intercept $\simeq -1$ represents our expectation based on Eq. (7). The scatter of points about this line reflects the statistical fluctuations around the assumed mean behavior, and reminds us that Eq. (7) is to be used not event by event but over an ensemble.

While it is apparent from Fig. 2(a) that Eq. (7)

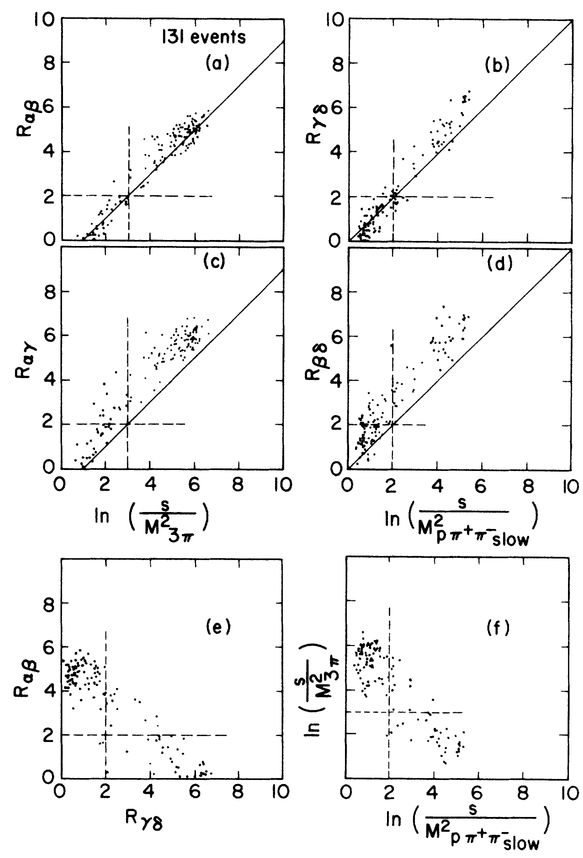

FIG. 2. (a) Measured rapidity gap $R_{\alpha\beta}$ between the proton and the nearest pion vs $\ln(s/M_{3\pi}^2)$. (b) Measured rapidity gap $R_{\gamma\delta}$ between the fast π^- and the nearest π vs $\ln(s/M_{p\,\pi^+\pi^-_{\rm slow}}^2)$. The lines with the unit slope are our expectations from formula (7). The broken lines represent the cuts corresponding to $\Delta_0 = 2$. (c) "Measured" rapidity gap $R_{\alpha\gamma}$ vs $\ln(s/M_{3\pi}^2)$ (β is the fake π^0). (d) "Measured" rapidity gap $R_{\beta\delta}$ vs $\ln(s/M_{p\,\pi^+\pi^-_{\rm slow}}^2)$ (γ is the fake π^0). (e) $R_{\alpha\beta}$ vs $R_{\gamma\delta}$. (f) $\ln(s/M_{3\pi}^2)$ vs $\ln(s/M_{p\,\pi^+\pi_{\rm slow}}^2)$.

TABLE I. Comparison of Eq. (7) with experimental data of Ref. 4.

Process \ Selection	Missing-mass formula for rapidity gap $\Delta_0=2$	Missing-mass formula for rapidity gap $\Delta_0=3$	Measured rapidity gap ($R_{\alpha\beta}$ and $R_{\gamma\delta}$) $\Delta_0=2$	Measured rapidity gap ($R_{\alpha\beta}$ and $R_{\gamma\delta}$) $\Delta_0=3$	"Measured" rapidity gap for faked π^0 ($R_{\alpha\gamma}$ and $R_{\beta\delta}$) $\Delta_0=2$	"Measured" rapidity gap for faked π^0 ($R_{\alpha\gamma}$ and $R_{\beta\delta}$) $\Delta_0=3$
π^- dissociation	94	85	95	91	107	98
p dissociation	48	33	46	37	79	51
double-Pomeron exchange (DPE)	12	0	12	2	···	···

is in qualitative agreement with experiment, we subject it to a quantitative test in the following manner. We select all events in which the rapidity gap next to the proton exceeds some Δ_0, first on the basis of the directly measured gaps $R_{\alpha\beta}$ and then on the basis of missing mass, selecting those events which, according to Eq. (7), have a gap exceeding Δ_0. In Fig. 1(a), the two modes of selection correspond to picking events above the line $R_{\alpha\beta}=\Delta_0$ and to the right of the line $\ln(s/M_{3\pi}^2)\simeq\Delta_0+1$, respectively. The former selection yields 95 events and the latter yields 94 events, for $\Delta_0=2$. These results and the corresponding ones for $\Delta_0=3$ are summarized in Table I under "π^- dissociation."

The results of a similar analysis of the gap next to the fast π^- [Fig. 2(b)] are contained in Table I under "proton dissociation."

We now consider the gaps next to the slow proton (α) and the fast $\pi^-(\delta)$ simultaneously. In Fig. 2(e) are plotted the measured gaps $R_{\alpha\beta}$ and $R_{\gamma\delta}$, and in Fig. 2(f) are shown the values of $\ln(s/M_{3\pi}^2)$ and $\ln(s/M_{p\pi^+\pi^-_{\rm slow}}^2)$ for each event. We selected events in which both gaps exceeded Δ_0—first from Fig. 2(e), and then from Fig. 2(f), using Eq. (7). The results for $\Delta_0=2$ and 3 are summarized in Table I under "double-Pomeron exchange (DPE)".

We wish to emphasize two points here. Firstly, we use the terms π^- dissociation, proton dissociation, and DPE only to label the various selections and do not imply that for some dynamical reason the conditions $\Delta>\Delta_0$ mark the onset of such processes. Secondly, the comparisons we have performed so far only confirm the integrated form of Eq. (7)—to test it differentially one needs to bin the events in the rapidity-gap variable, comparing numbers within each bin. Unfortunately the present statistics do not permit a meaningful comparison of the latter type.

In a sample such as ours, where all the rapidities are determined by the 4-constraint fit, Eq. (7) provides the connection between an analysis based on rapidity gaps and one based on "missing" masses. While such a connection is interesting in itself, the real utility of Eq. (7) lies in unfitted reactions, where it allows the (indirect) measurement of the two gaps closest to the extreme particles—if one assumes that the momenta of the latter are measurable.

To illustrate this fact, we pretended that the pion (β) next to the proton is neutral. If we select events with a gap next to the proton larger than Δ_0 directly on the basis of just the "observable" gap $R_{\alpha\gamma}$ we overestimate, whereas the indirect determination of the gap on the basis of the missing mass recoiling across the proton was seen to be in fair agreement with the analysis based on $R_{\alpha\beta}$ [see Fig. 2(c)]. A similar consideration applies to the proton dissociation events [Fig. 2(d)] if we pretend that the pion (γ) next to the fast $\pi^-(\delta)$ is unobservable. The results of the selection based on the "observable" gaps ($R_{\alpha\gamma}$ and $R_{\beta\delta}$) are summarized in Table I and we find that substantial overestimation can result if one tries to measure rapidity gaps directly.

In conclusion, we have presented here a formula that provides a statistical relation between the rapidity-gap and missing-mass variables and have demonstrated its validity (in the integrated sense). We have shown that for an important class of (unfitted) events the indirect determination of rapidity gaps from the missing masses using this formula is more reliable than any direct gap measurement that ignores possible neutrals.

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[1]D. M. Chew *et al.*, LBL Report No. LBL-2106, 1973 (unpublished) (talk presented at the 1973 Berkeley Meeting of the Division of Particles and Fields of the APS); see also D. M. Chew *et al.*, LBL Reports Nos. LBL-2464 and LBL-2465, 1974 (unpublished).

[2]D. M. Chew, Nucl. Phys. B82, 422 (1974).

[3]This approximation, which is clearly an oversimplification, is made in the interest of simplicity. Its justification comes *a posteriori*, when the formulas derived on this basis are found to be quite reliable.

[4]For a detailed study of this experiment see D. Bogert *et al.*, Phys. Rev. Lett. 31, 1271 (1973); F. C. Winkelmann *et al.*, in *Particles and Fields–1973*, proceedings of the 1973 Meeting of the Division of Particles and Fields of the American Physical Society, Berkeley, Calif., edited by H. H. Bingham *et al.* (A.I.P., New York, 1973), p. 359; see H. Bingham *et al.*, Phys. Lett. 51B, 397 (1974) for an estimate of background (< 15%).