Letter

Clockwork axion footprint on nanohertz stochastic gravitational wave background

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The recent pulsar timing arrays (PTAs) nano-Hz gravitational wave (GW) background signal can be naturally induced by the annihilation of domain walls (DWs) formed at a symmetry-breaking scale $f \simeq 200$ TeV in the clockwork axion framework. Based on our first successful and precise prediction, we, for the first time, suggest that the recent PTA observations strongly support the novel mechanism of the QCD instanton-induced DW annihilation in the clockwork axion framework. We also, for the first time, discover a novel correlation between dark matter relic abundance and nano-Hz GW background, which in turn indicates a natural connection between the axion decay constant and the symmetry-breaking scale in the clockwork framework. We find that the GW signal has a peak $h^2\Omega_{\rm GW} \simeq 10^{-6.6} - 10^{-6.1}$ at about 50 nHz, which is definite and testable for future PTA data at frequencies $\gtrsim$25 nHz and CMB-S4 experiment. We also propose various phenomena that may appear in PTAs and future GW interferometers.

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Introduction. It is well known that domain walls (DWs) produced with the spontaneous breaking of a discrete Z_N symmetry could dominate the Universe if they do not completely annihilate at a later time, and thus bury the predictions of standard cosmology [1,2]. The DWs formed above the grand unified theory (GUT) scales $M_{\rm G} \sim 10^{16}$ GeV might be adequately diluted by the inflation. Below $M_{\rm G}$, to break the degenerate vacua, one commonly introduces a bias potential with an energy magnitude $(v/M_{\rm pl})^2 v^4 \lesssim V_{\rm bias} \lesssim (v/M_{\rm pl}) v^4$ (v is the spontaneous symmetry-breaking scale, and $M_{\rm pl}$ is the Planck scale), where the lower bound ensures the DWs annihilate before dominating the Universe, and the upper bound indicates the DWs annihilate immediately after their formation. The symmetric part of the potential has an energy $V_{\rm sym} \sim v^4$. A hierarchy arises between the bias and the symmetric parts of the potential if the symmetry breaks much below the GUT scale. Since many models do not explicitly break the discrete symmetry, the bias term is often introduced as an extrinsic and free parameter while ignoring the hierarchy. In fact, the small explicit symmetry-breaking can be dynamically induced by some nonperturbative instanton effects if the global symmetry is anomalous in the underlying theory [2–6]. Two well-known examples are quantum gravity (QG) [7–9] and QCD instanton [2,10,11]. It is believed that all global symmetries are not respected by the QG effect [12–17], which can lead to the decays of dark matter (DM) and DW [18–20]. The symmetry breaking by the QG effects is described by some higher-dimensional operators. However, one can not make a predictive announcement since the size of the symmetry-breaking by the QG effect is not well specified.

In Ref. [21], two of us were the first to notice that the QCD instanton effect could induce a bias potential for the annihilation of DWs formed at a symmetry-breaking scale $f \simeq 200$ TeV in the clockwork axion framework, leading to a loud gravitational wave (GW) signal for NANOGrav 12.5-year (NG12) observation. However, this signal behaves as $\Omega_{\rm GW}(\nu) \propto \nu^\gamma$ with $\gamma = 3$, and thus was not supported by the NG12 data [22], which can be fitted by a flat spectrum with an amplitude $\Omega_{\rm GW}(5.5~\text{nHz}) \in (3\times 10^{-10}, 2\times 10^{-9})$ and an exponent $\gamma \in (-1.5, 0.5)$ [21]. During that time, the cosmic string [23–25] and primordial black hole (PBH) [26,27] were commonly considered as they generated a flat GW spectrum that aligned with the NG12 data. By fitting the NANOGrav 15-year (NG15) data [28] and IPTA-DR2 data [29], we find

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that the recent pulsar timing array (PTA) nano-Hz GW observations show a complete consistency with our prediction in Ref. [21], both in the amplitude and in the exponent of the GW spectrum (see Fig. 10 in Ref. [21] and Fig. S3 in the Supplemental Material [30]).

It is worth mentioning that NANOGrav [28,31], together with EPTA [32], Parkes pulsar timing array [33], and Chinese pulsar timing array [34] have recently presented the first convincing evidence for the Hellings-Downs angular correlation, which strongly supports the existence of nano-Hz stochastic GW background (GWB). Although the astrophysical source from a population of inspiraling supermassive black hole binaries (SMBHBs) does not fit better than new physics to the NG15 data, an understanding of the SMBHBs still remains to be improved [31]. A series of works on nano-Hz GWs also appeared recently, including DWs [19,35–39], cosmic string [40–42], scalar-induced GW [43–47], PBH [48–50], first-order phase transition [51–56], as well as model comparisons [57–60].

In this Letter, we, for the first time, systematically present the hierarchy in potential and suggest the mechanism of QCD instanton-induced DW annihilation for PTAs, whose novelty rests in three aspects. First, compared with other interpretations for PTAs, our scenario is predictive since the QCD phase transition definitely takes place in the early Universe. Second, compared with the QG, the size of the bias potential induced by the QCD instanton effect is quantitatively determined by the QCD confinement scale $\Lambda_{\rm QCD}$. Interestingly, the DWs annihilated at the QCD scale can naturally generate nano-Hz GWs for PTAs. Finally, the Peccei-Quinn (PQ) scale is restricted to be much higher than the electroweak scale by the astroparticle physics experiments, which therefore leads to a hierarchy in the potential. Notice that since the QCD instanton explicitly breaks the $U(1)$ symmetry down to the discrete shift symmetry, the QCD instanton will lead to the formation, rather than the annihilation of the DWs in the classical QCD axion model [2].

Although the existence of DM has been confirmed by various cosmological and astrophysical observations [61], the nature of DM still remains unknown. Since the bias potential is hierarchically suppressed, the global symmetry should be exact enough to provide a DM candidate. This motivates us to further explore the DM phenomenon in the clockwork axion framework. The axion produced at the symmetry-breaking scale f attains a mass at the QCD scale. For the first time, we find that one can obtain the correct order of the axion decay constant f_a for the axion DM relic abundance from axion oscillation with the estimate $f_a \sim f^2/\Lambda_{\rm QCD} \sim 10^{11}$ GeV, once we adopt $f \simeq 200$ TeV from the PTA observation. We expect that the DM relic abundance is closely related to the nano-Hz GWB, and the clockwork axion framework provides a natural realization.

GWs from clockwork DW annihilation. The clockwork axion framework [62–72], which was first proposed to break the canonical relation between the symmetry-breaking scale and axion decay constant [64], has recently been extended to other fields in its continuum limit [68,73,74], thus providing a solution to the Higgs naturalness problem. The large $N+1$ global $U(1)$ symmetries can appear as an accidental consequence of gauge invariance and five-dimensional locality in an extradimensional model [64,75–77]. The framework introduces $N+1$ copies of complex scalars $\Phi_j(x)$ with $j = 0, 1, \ldots, N$ and the following potential:

$$V(\Phi) = \sum_{j=0}^{N} (-m^2|\Phi_j|^2 + \lambda|\Phi_j|^4) - \varepsilon \sum_{j=0}^{N-1} (\Phi_j^\dagger \Phi_{j+1}^3 + \text{H.c.}), \tag{1}$$

where m^2, λ, and ε have been assumed to be real and universal. The first term respects a global $U(1)^{N+1}$ symmetry and is explicitly broken by the ε-dependent terms down to the N shift symmetries and a global $U(1)$ symmetry, which is identified as the PQ symmetry. For $N \gtrsim 3$, the stable string-wall network forms when the radial components acquire a vacuum expectation value $\langle \Phi_j \rangle = f/\sqrt{2}$, resulting in N massive axions A_i and one massless axion a (see the Supplemental Material [30]).

Because the discrete symmetries are anomalous under the QCD gauge symmetry, the QCD instanton effects generate a bias potential $V_{\rm bias} \sim \Lambda_{\rm QCD}^4$ (with $\Lambda_{\rm QCD} = (332 \pm 17)$ MeV [78]) during the QCD phase transition, which lifts the N degenerate vacua and breaks the residual $U(1)$ symmetry. The annihilation of DWs is significant when the surface energy becomes comparable to the bias energy, i.e., $\sigma H \simeq V_{\rm bias}$, where $\sigma \simeq 8 m_A f^2$ is the surface tension of the wall, and $m_A \simeq \varepsilon^{1/2} f$ is the mass of the massive axion. The DWs quickly annihilate one after another at the temperature

$$T_{\rm ann} \simeq 7.15 \times 10^{-2}\ \text{GeV} \varepsilon^{-1/4} \left(\frac{g_*(T_{\rm ann})}{10}\right)^{-1/4} \times \left(\frac{f}{100\ \text{TeV}}\right)^{-3/2} \left(\frac{\Lambda_{\rm QCD}}{100\ \text{MeV}}\right)^2. \tag{2}$$

The GW spectrum from the violent DW annihilation is characterized by a peak frequency determined by $\nu_{\rm peak}(t_{\rm ann}) \simeq H(t_{\rm ann})$, where $t_{\rm ann}$ is the cosmic time of DW annihilation. Then the red-shifted peak frequency today is found to be

$$\nu_{\rm peak}(t_0) \simeq 1.1 \times 10^{-8}\ \text{Hz} \left(\frac{g_*(T_{\rm ann})}{10}\right)^{1/2} \times \left(\frac{g_{*s}(T_{\rm ann})}{10}\right)^{-1/3} \left(\frac{T_{\rm ann}}{0.1\ \text{GeV}}\right), \tag{3}$$

where g_* and g_{*s} are the effective relativistic degrees of freedom (DOF) associated with energy and entropy, respectively. Equation (3) shows that the DW annihilation at the QCD scale naturally induces a GWB at nano-Hz frequencies.

The peak GW amplitude produced at the annihilation time is determined by $\Omega_{\rm GW}(\nu_{\rm peak}(t_{\rm ann})) \simeq (8\pi\tilde{\epsilon}_{\rm gw}G^2A^2\sigma^2)/(3H^2(t_{\rm ann}))$, with $\tilde{\epsilon}_{\rm gw} \simeq 0.7 \pm 0.4$ [79] and $A \simeq N$ [67,80,81] from simulations. The peak GW amplitude today is diluted by the cosmic expansion as

$$h^2\Omega_{\rm GW}^{\rm peak}(t_0) = 6.45\times10^{-6}\varepsilon\left(\frac{\tilde{\epsilon}_{\rm gw}}{0.7}\right)\left(\frac{A}{10}\right)^2\left(\frac{g_{*s}(T_{\rm ann})}{10}\right)^{-4/3} \times\left(\frac{f}{100\ \text{TeV}}\right)^6\left(\frac{T_{\rm ann}}{0.1\ \text{GeV}}\right)^{-4}. \quad (4)$$

We see that the DW annihilation in the clockwork framework can generate a GW amplitude of $\sim10^{-6}$, which falls in the sensitivity of current PTA experiments. Below the peak frequency, causality demands that the GW spectrum scale as $\propto(\nu/\nu_{\rm peak})^3$ [82]. Above the peak frequency, on the other hand, numerical simulations [79] indicate a $\propto(\nu/\nu_{\rm peak})^{-1}$ scaling behavior instead.

PTA data analysis and predictions. We carry out the standard Bayesian statistical analysis for the IPTA-DR2 dataset [83] and the recent NG15 dataset [28] (see the Supplemental Material [30] for more details). The Bayes estimators for the three input parameters $f/100$ TeV, ε, and N are 1.81 ± 0.21, 0.50 ± 0.26, and 12.21 ± 4.35 (1.76 ± 0.22, 0.48 ± 0.26, and 12.28 ± 4.35) by fitting to the NG15 (IPTA-DR2) dataset. These parameter values are very natural in the clockwork axion framework and are also in good agreement with the prediction in our previous work [21] for the NG12 data. In addition to considering merely the DWs, in the Supplemental Material [30] we also take into account potential astrophysical sources by including a power-law spectrum in our fit. We find that our interpretation is still supported by the NG15 data.

Note that although only the first 14 frequencies ($10^{-9.3}\lesssim\nu/\text{Hz}\lesssim10^{-7.6}$) of the NG15 dataset are adopted in the fit to avoid the possible high frequencies pulsar-intrinsic excess noise, we predict that the amplitude will continue to grow and have a peak $h^2\Omega_{\rm GW}\simeq10^{-6.6}-10^{-6.1}$ at $\nu\simeq10^{-7.3}$ Hz (see Fig. 1 above and Fig. 10 of [21]). This is because the peak frequency is entirely determined by the QCD instanton effect. GWs can contribute to the radiation energy density and affect the expansion of the Universe. This gives a constraint, $h^2\Omega_{\rm GW}(t_0)\lesssim5.6\times10^{-6}\Delta N_{\rm eff}$ [84]. The current upper bound on the number of extra neutrino species at 95% confidence level (CL) from the Planck observation is $\Delta N_{\rm eff}\leq0.29$ [85] (the grey region in Fig. 1), which can be further tightened to 0.11 and 0.06 by the upcoming Simons Observatory [86] and CMB-S4 experiment [87]. We find that the predicted peak amplitude can generate detectable effects in the CMB-S4 experiment. We also observe in Fig. 2 that the predicted GW signals are detectable for future GW experiments in a wide frequency range.

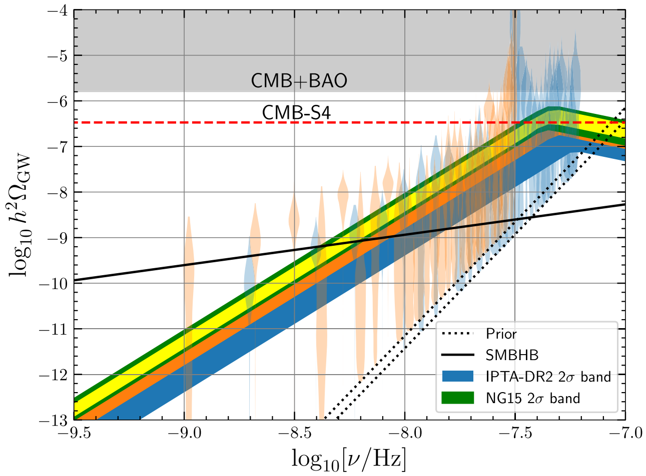

FIG. 1. The black line denotes the SMBHB with the power-law spectrum $(A_{\rm BHB},\gamma_{\rm BHB})=(-14.7,13/3)$. The posteriors of the free spectrum for IPTA-DR2 [29] and NG15 [28] are reproduced by the light-orange and light-blue violins, with the prior choices of lower limits shown by the dotted lines. The yellow (green) region and orange (blue) region represent the 1σ (2σ) uncertainty band by fitting to the NG15 and IPTA-DR2 datasets, respectively.

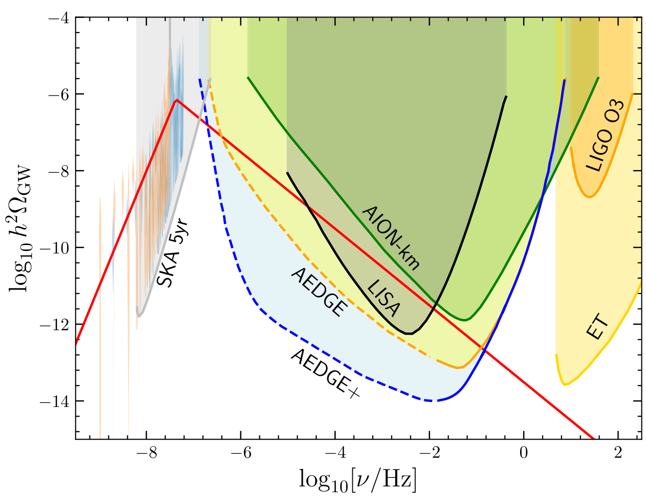

FIG. 2. The stochastic gravitational wave background sensitivity curves of SKA 5 yr (silver) [88], atom interferometer observatory and network (AION)-km (light-green) [89], atomic experiment for dark matter and gravity exploration (AEDGE) (yellow) [90], AEDGE+ (light-blue), Laser Interferometer Space Antenna (LISA) (grey) [91], Laser Interferometer Gravitational Wave Observatory (LIGO) O3 (orange) [92], *Einstein* telescope (ET, gold) [93]. The red line corresponds to a GW signal with $(f,\varepsilon,N)=(180\ \text{TeV},0.5,12)$. Note that the AEDGE sensitivities presented in [94] do not consider many possible sources of instrumental noise.

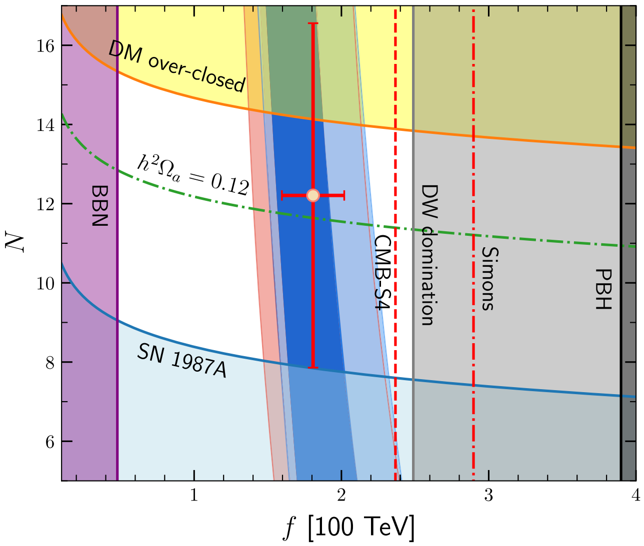

FIG. 3. The blue and red vertical bands with $f \simeq 200$ TeV represent the 1 and 2σ parameter bands favored by NG15 [28] and IPTA-DR2 [29] datasets, respectively. The yellow region with $f_a \gtrsim 10^{12}$ GeV is excluded since the Universe is over-closed by the axion DM.

Constraints. We summarize various constraints in Fig. 3. Axions can be produced via the nucleon-nucleon axion bremsstrahlung in the core of a supernova and accelerate the stellar cooling. The observation of neutrinos from supernova 1987A excludes the light-blue region with $f_a \lesssim 10^9$ GeV [95–97]. The emission of axions from the string-wall network is dominated by the late time decay at $t \sim t_{\rm QCD}$ [98]. The axion energy density from the decay of the scaling string is estimated as $\rho_{a,\rm str} \sim \pi f^2 H^2(t_{\rm QCD}) \ln(t_{\rm QCD}/t_s)$, where t_s is the string formation time. The QCD axion obtains a mass m_a and gets mixed with the massive axions A_i with a mixing angle $\vartheta_i \sim q^N m_a^2/m_A^2$ at the QCD scale. Then the QCD axion energy density from the collapse of the DW network is found to be $\rho_{a,\rm wall} \sim \sum_i \vartheta_i^2 \rho_w(t_{\rm QCD}) \sim 8N m_a^4 f_a^2 H(t_{\rm QCD})/m_A^3$. We find that the contributions from the scaling strings and collapsing walls to the cold axion relic abundance [with energy $\omega(t_{\rm QCD}) \sim H(t_{\rm QCD})$] and $\Delta N_{\rm eff}$ are both negligibly small. The most significant contribution to ρ_a from the topological defects is found to be the oscillations of DWs with a frequency $\omega(t_{\rm QCD}) \gg H(t_{\rm QCD})$ at the QCD scale, which optimistically gives $\rho_{a,\rm osc} \sim 0.1 v_w^2 \rho_w(t_{\rm QCD})$ [71]. This radiation population contributes to cosmic expansion by $\Delta N_{\rm eff} \simeq 6.4 \times 10^{-3} v_w^2 \epsilon^{1/2} (f/100\,\text{TeV})^3$, where we adopt the wall velocity $v_w \sim 1$ for a conservative estimate. The bound from the current Planck observation falls much behind the constraint from DW domination (light grey region). The constraints from the upcoming Simons Observatory and CMB-S4 experiment are represented by the red dot-dashed and red dashed lines.

Since most of the energy from DW annihilation is poured into the Standard Model (SM) plasma via the prompt decays of massive axion to the SM particles, we therefore require the annihilation to take place before the big bang nucleosynthesis (BBN), i.e., $T_{\rm ann} > T_{\rm BBN} \simeq 5$ MeV, so that the successful BBN processes are not altered by the DW collapse (purple region). At the QCD scale, the closed DWs may collapse into massive PBHs [99], whose lifetime can be longer than the Universe. We find in the Supplemental Material [30] that if $M_{\rm pl}^2 \gtrsim 5\pi\sigma^2/V_{\rm bias}$, then the PBH could be formed after the DW annihilation (black region). We also confirm that a negligible fraction $f_{\rm PBH} \lesssim 10^{-4}$ of DM consists of PBHs formed in the collapse of DWs. Finally, we note that the constraints from searches for axions in the laboratory are currently not competitive with the astrophysical bounds (see [100] for a review).

Phenomenology. Axions start oscillations at $H(t) \sim m_a$ and generate a DM relic abundance $\Omega_a h^2 \simeq 0.2(f_a/10^{11}\,\text{GeV})^{7/6} \langle\theta_a^2\rangle/(3\pi^2)$ [101–105], where $\theta_a \equiv a/f_a$. Since the decay constant $f_a \simeq q^N f$ (with $q = 3$) in the clockwork framework, the large number $N = 12.21 \pm 4.35$ (point with error bars in Fig. 3) from the Bayes estimator naturally improves the axion decay constant to generate the correct axion DM relic abundance (dot-dashed green curve in Fig. 3). Note that we have not taken into account the DM relic abundance observation in our fit. This is the first report on the novel correlation between the nano-Hz GWB and the DM relic abundance. Such a correlation can be estimated by having $f_a \sim f^2/\Lambda_{\rm QCD} \sim 10^{11}$ GeV in the clockwork axion framework. Equivalently, the DM relic abundance requires $f \simeq 200$ TeV, leading to the DW decays just before it would dominate the Universe. Future telescope observations on stellar cooling [106,107] and axion experiments may reveal the existence of axion [100].

The freeze-out of the massive particles can lead to changes in the relativistic DOF, and therefore, in the expansion rate $H(t)$ of the Universe. A rise in $H(t)$ after the annihilation of massive particles will dilute the subhorizon modes of the primordial GW spectrum, while the superhorizon modes are frozen and remain unaffected [108–114]. Thus, the freeze-out of N massive axions will leave an imprint on a primordial GW spectrum from inflation [115–117], reheating/preheating [118–121], or cosmic strings [23–25], and are detectable for future space-based interferometers, like LISA [91], Taiji [122], and TianQin [123]. The spectrum of superhorizon modes can be affected by the equation of state (EOS) when these modes enter the Hubble horizon during the QCD crossover, and therefore, leads to a slight departure from the ν^3 behavior in the causality tail of the spectrum [53,124–126]. Equation (3) indicates that the superhorizon mode begins around 10^{-8} Hz. Therefore PTA observations can be used to test the EOS at the QCD scale if the DW annihilation in our scenario is indeed the GW source for the PTAs. Furthermore, the spontaneous breaking of the approximate $U(1)$ symmetries at 200 TeV may lead to a cosmological first-order phase transition [21], whose accompanying GW emissions with a characteristic spectrum peaked at even higher frequencies can be tested by the LIGO O3 run [92] and ET [93].

Conclusions and outlook. For the first time, by fitting to the NG15 and IPTA-DR2 datasets, we found that our predictions of the nano-Hz GW signal in Ref. [21] with a set of very natural parameters in the clockwork axion framework have been successfully and precisely confirmed by the recent PTA observations. Such success has not been observed in other works for the PTA observations, and thus strongly supports the novel mechanism of the QCD instanton-induced DW annihilation. Furthermore, we found that the PTA data analysis naturally results in a correct DM relic abundance, which therefore indicates a strong correlation between the nano-Hz GWB and DM relic abundance. Such a correlation stems from the hierarchy between the bias and the symmetric parts of the potential. We, for the first time, proposed a novel relation $f_a \sim f^2/\Lambda_{\rm QCD} \sim 10^{11}$ GeV in the clockwork axion framework to estimate the correlation. We showed that the GW signal has a peak of $h^2\Omega_{\rm GW} \simeq 10^{-6.6} - 10^{-6.1}$ at about 50 nHz and can be tested by PTAs and CMB-S4 experiment. Moreover, we expected a slight departure from the ν^3 scaling in the PTA spectrum at frequencies below about 10^{-8} Hz. Our analysis may hint at the violation of Lorentz invariance in the extra dimension at 200 TeV. Future GW observations will shed more light on model construction.

We acknowledge the publicly available packages `enterprise` [127], `enterprise_extensions` [128], `ceffyl` [129], and `PTArcade` [130], where we have modeled the DW signal and implemented the Monte Carlo sampling with `PTMCMC` [131].

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