Letter

Anomalous interactions between mesons with nonzero spin and glueballs

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Topologically nontrivial fluctuations control the anomalous interactions for the η and η' pseudoscalar mesons. We consider the anomalous interactions for mesons with higher spin, the heterochiral nonets with $J^{PC} = 1^{+-}$ and 2^{-+}. Under the approximation of a dilute gas of instantons, the mixing angle between nonstrange and strange mesons decreases strongly as J increases, and oscillates in sign. Anomalous interactions also open up new, rare decay channels. For glueballs, anomalous interactions indicate that the $X(2600)$ state is primarily gluonic.

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Introduction. Quantum Chromodynamics (QCD) is close to the chiral limit, where the up, down and strange quarks (u, d, and s) are very light. Consequently, when the global chiral symmetry is spontaneously broken in the vacuum, from $SU(3)_L \times SU(3)_R \times U_A(1) \to SU(3)_V$, nine pseudo-Goldstone bosons should appear. Instead, there are only eight: the usual octet of pions, kaons, and the η meson, while the η' is much heavier than expected.

This occurs because the axial $U_A(1)$ symmetry of the classical theory is broken by quantum effects, through the anomaly of Adler, Bell, and Jackiw [1,2]. This splits the singlet η' meson from the octet mesons, and gives it a mass through fluctuations which are topologically nontrivial [3–7]. The most familiar example are instantons: classical solutions of the gluon field equations in Euclidean spacetime [3], whose effects can be computed semiclassically [4,5]. While instantons dominate at high temperature, in vacuum truly quantum fluctuations also contribute [7].

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While anomalous interactions are especially dramatic for the pseudoscalar multiplet, it is natural to ask how the axial anomaly affects other mesons, such as conventional mesons with higher spin, or unconventional ones, such as glueballs. As both mesons with nonzero spin and glueballs are massive, the effects of the axial anomaly are more subtle, affecting the mass splittings, mixing, and decays of some fields in these multiplets.

In Ref. [8], mesons are divided into "heterochiral" and "homochiral." In the chirally symmetric phase, heterochiral mesons are a mixture of a left-handed anti-quark and a right-handed quark (or vice versa), as for the pseudo-Goldstone bosons. Homochiral mesons are formed just from a left (or right) handed anti-quark and a quark. These begin with the vector mesons, $J^{PC} = 1^{--}$: the $\rho_\mu(770)$, $\omega_\mu(782)$, and $\phi_\mu(1020)$ mesons.

The anomalous interactions between heterochiral and homochiral mesons are very different. Heterochiral mesons have anomalous interactions with no derivatives, which directly affect their mass spectrum, and with few derivatives, which affect their decays. In contrast, homochiral mesons only have anomalous interactions with many derivatives, through the Wess-Zumino-Witten term [9].

In this paper we construct the anomalous interactions for the underlying quark operators, and their counterparts for heterochiral mesons and for the pseudoscalar glueball, in a dilute gas of instantons (DGI). After reviewing the well known case of $J = 0$, the extension to heterochiral mesons with spin $J = 1$ and $J = 2$, and then with a glueball, is direct. Because of the axial anomaly, massless quarks have exact zero modes, so that instanton contributions to anomalous operators can be computed by saturating these operators with these zero modes [4,5]. The only change with nonzero spin is that the vertices which tie the zero modes differs.

At the outset we acknowledge that the topological structure of the vacuum is surely more complicated than a dilute gas of instantons [7]. Nevertheless, the anomalous operators which we compute in this work are novel, and we expect a DGI to give a first estimate of their magnitude. Indeed, a recent analysis of the chiral phase transition near the chiral limit suggests that a DGI may well *under*estimate the effects of topologically nontrivial fluctuations [10].

The present analysis is meant to motivate further study from numerical simulations on the lattice, and especially from experiment. Thus we concentrate on phenomenology, notably the splitting and mixing between mesons in a given multiplet, and on new decay channels which open up for mesons and glueballs.

Heterochiral multiplets. Mesons are classified according to their quantum numbers under spin, parity, and charge conjugation, J^{PC}. The total spin $J = L + S$ is the sum of angular momentum L and the spin S, with $P = (-1)^{L+1}$ and $C = (-1)^{L+S}$. With massless quarks, classically left and right handed quarks are invariant the symmetry group of $\mathcal{G}_{\rm cl} = SU_L(3) \times SU_R(3) \times U_A(1)$:

$$q_{L,R} \longrightarrow e^{\mp i\alpha/2} U_{L,R} q_{L,R}. \tag{1}$$

Here $q_{L,R} = \mathbb{P}_{L,R} q$, where $\mathbb{P}_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$. U_L and U_R are flavor rotations in $SU_L(3)$ and $SU_R(3)$, respectively, while $\exp(\mp i\alpha/2)$ is a rotation for axial $U_A(1)$. This transformation relates nonets with the same spin and opposite parity.

A heterochiral meson with spin zero is proportional to the quark operator $\bar{q}_L q_R$; those of higher spin are given just by inserting powers of the covariant color derivative, $\overleftrightarrow{D}_\mu$, between the quark fields. For $J = 0$, 1, and 2, these are Φ, Φ_μ and $\Phi_{\mu\nu}$, as shown in Table I. Because $\overleftrightarrow{D}_\mu$ only acts upon color and not flavor, these mesons all transform identically under chiral rotations [8].

Typically bosonic fields in an effective Lagrangian have dimensions of mass. To ensure this it is necessary to introduce the dimensionful constants M_0, M_1, and M_2 for $J = 0$, 1, and 2 in Table I. Since the spin is increased by inserting more powers of $\overleftrightarrow{D}_\mu$, the power of M increases with J, $\sim 1/M_J^{J+2}$. A major concern in the phenomenological analysis below is the relative magnitude of these mass scales.

TABLE I. Heterochiral fields for multiplets with spin zero, one, and two.

Chiral nonet	$\mathcal{G}_{\rm cl}$
$\Phi = \bar{q}_L q_R / M_0^2$	$e^{i\alpha} U_L^\dagger \Phi U_R$
$\Phi_\mu = i\bar{q}_L \overleftrightarrow{D}_\mu q_R / M_1^3$	$e^{i\alpha} U_L^\dagger \Phi_\mu U_R$
$\Phi_{\mu\nu} = \bar{q}_L (g_{\mu\nu} \overleftrightarrow{D}^2/4 - \overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu) q_R / M_2^4$	$e^{i\alpha} U_L^\dagger \Phi_{\mu\nu} U_R$

The unbroken symmetry group of the quantum theory is not $\mathcal{G}_{\rm cl}$, but $\mathcal{G}_{\rm qu} = SU_L(3) \times SU_R(3)$ [8]. Each $SU(3)$ contains the element $U = \exp(2\pi i/3)$, which generates an abelian $Z(3)$ subgroup. Anomalous interactions violate $U_A(1)$, but are invariant under this $Z(3)$. For spin zero, this begins with the cubic invariant, $\sim \det(\Phi)$, in Eq. (6). The anomalous interactions for fields with higher spin, Eqs. (9), (13), (16), and (17), generalize this term.

We begin by reviewing the experimental evidence for heterochiral multiplets.

(i) *Heterochiral mesons with* $J = 0$: Besides the usual pions and kaons, there are the flavor eigenstates, $\eta_N \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d)$ and $\eta_S \equiv \bar{s}s$. Because of the axial anomaly, Eq. (6), these mix to form the physical η and η' states:

$$\begin{pmatrix} \eta(547) \\ \eta'(958) \end{pmatrix} = \begin{pmatrix} \cos\beta_0 & \sin\beta_0 \\ -\sin\beta_0 & \cos\beta_0 \end{pmatrix} \begin{pmatrix} \eta_N \\ \eta_S \end{pmatrix}, \tag{2}$$

The mixing angle, $\beta_0 = -43.4°$ [11], is large and negative. This demonstrates that the axial anomaly ensures that the physical states are closer to the octet and singlet configurations, respectively [12,13]. In all they form a pseudoscalar nonet, $P_{ij} = \frac{1}{2}\bar{q}_j i\gamma^5 q_i$. The assignment for the scalar mesons, with $J^{PC} = 0^{++}$, is still under debate [14–21] [22]. In all, $\Phi = S + iP$, Table I.

(ii) *Heterochiral mesons with* $J = 1$: The pseudovector mesons with $J^{PC} = 1^{+-}$ corresponds to $P_\mu = \{b_1(1235), \bar{K}_{1B} \equiv K_1(1270)/K_1(1400)$ [23], $h_1(1170), h_1(1415)\}$ [26]. The mixing angle between $h_1(1170)$ and $h_1(1415)$ takes the same expression as in Eq. (2), β_1. The value of β_1 is not yet known, and is discussed below. Their chiral partners with $J^{PC} = 1^{--}$ are the orbitally excited vector mesons $S_\mu = \{\rho(1700), K^*(1680), \omega(1650), \phi(2170)\}$. The full multiplet is $\Phi_\mu = S_\mu + iP_\mu$, Table I.

(iii) *Heterochiral mesons with* $J = 2$: The pseudotensor mesons $J^{PC} = 2^{-+}$ listed in the PDG [26], denoted as $P_{\mu\nu} = \{\pi_2(1670), K_{2P} \equiv K_2(1770)/K_2(1820)$ [28], $\eta_2(1645), \eta_2(1875)\}$, are members of the heterochiral nonet with spin 2. The isoscalar mixing analogous to Eq. (2) via the angle β_2, is under debate, but according to the phenomenological studies of Refs. [30,31], it might be large. The chiral partners of the pseudotensor mesons are expected to be the orbitally excited tensor mesons $S_{\mu\nu}$ with $J^{PC} = 2^{++}$ [32]. The full multiplet is $\Phi_{\mu\nu} = S_{\mu\nu} + iP_{\mu\nu}$, Table I.

Instanton induced interactions. It is well known that instantons generate the interaction [4,5,13]

$$\mathcal{L}_{\rm eff}^{J=0} = -\frac{k_0}{3!}\left(\det(\bar{q}_L q_R) + \det(\bar{q}_R q_L)\right). \tag{3}$$

Anticipating later results, we introduce the J-dependent coupling

$$k_J = (8\pi^2)^3 \int_0^{\Lambda_{\overline{\rm MS}}^{-1}} d\rho n(\rho)\rho^{9+2J}. \tag{4}$$

This is a weighted average over the instanton density $n(\rho)$, which for three massless quarks and three colors is given by [34–36]:

$$n(\rho) = \exp\left(-\frac{8\pi^2}{g^2(\rho\Lambda_{\overline{\rm MS}})} - 7.07534\right)\frac{1}{\pi^2\rho^5}\left(\frac{16\pi^2}{g^2(\rho\Lambda_{\overline{\rm MS}})}\right)^6. \tag{5}$$

The expression for the running coupling constant $g(\rho\Lambda_{\overline{\rm MS}})$ is given in the Supplemental Material [37] to two loop order, while the instanton density $n(\rho\Lambda_{\overline{\rm MS}})$ is illustrated in Fig. 1. See the Supplemental Material (SM) [37] for further details. Taking the renormalization mass scale $\Lambda_{\overline{\rm MS}} = 300$ MeV [26], for $J = 0$ we obtain $k_0 \approx 2.57 \times 10^6\ {\rm GeV}^{-5}$ [38].

Assuming that the effective bosonic field Φ is proportional to the quark bilinear [13,39,40],

$$\mathcal{L}_{\rm eff}^{J=0} = -a_0(\det\Phi + \det\Phi^\dagger). \tag{6}$$

The bosonic coupling a_0 depends on k_0 and the constant M_0 in Table I, $a_0 = k_0 M_0^6/48 > 0$.

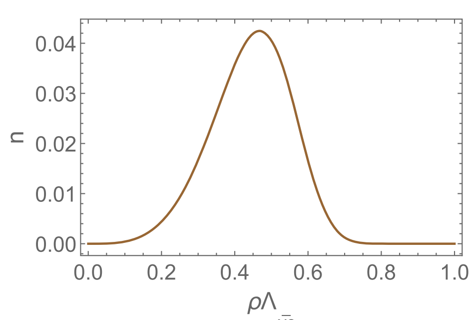

FIG. 1. The density of instantons for $N_c = N_f = 3$.

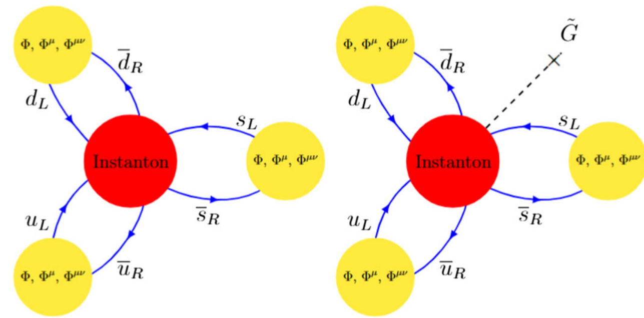

FIG. 2. Anomalous processes induced by instantons: to left, cubic couplings between the heterochiral-type, Φ's, and to the right, their coupling to a glueball field, $\tilde{G}$.

The mixing angle of Eq. (2) is then [41]

$$\beta_0 = \frac{1}{2}\tan^{-1}\left(\frac{-2.6\sqrt{2}a_0\phi_N}{(m_{\eta'}^2 - m_\eta^2)\cos 2\beta_0}\right) < 0, \tag{7}$$

where the chiral condensate of nonstrange quarks ϕ_N can be expressed in terms of the pion decay constant $\phi_N = \langle 0|\eta_N|0\rangle \simeq 1.7 f_\pi \approx 160$ MeV. A dilute gas of instantons gives negative β_0, in agreement with phenomenology. Imposing the phenomenological value $\beta_0 = -43.6°$ and using the parameters of Refs. [42,43],

$$a_0 = 1.3\ {\rm GeV}; \qquad M_0 = 170\ {\rm MeV}, \tag{8}$$

so that the value of M_0 is close to that for ϕ_N.

The generic anomalous interaction for three flavors is illustrated in the left part of Fig. 2. The only change with higher spin is that as J increases, powers of $\overleftrightarrow{D}_\mu$ are inserted between the zero modes. This is responsible for the factor of ρ^{2J} in the anomalous interactions, k_J in Eq. (4).

For spin one, the simplest anomalous interaction is quadratic in Φ_μ and linear in Φ:

$$\begin{aligned}\mathcal{L}_{\rm eff}^{J=1} &= -\frac{k_1}{3!}\left(\epsilon\left[(\bar{q}_L q_R)(\bar{q}_L\overleftrightarrow{D}_\mu q_R)^2\right] + R \leftrightarrow L\right)\\ &= a_1(\epsilon[\Phi\Phi_\mu\Phi^\mu] + {\rm c.c.}),\end{aligned} \tag{9}$$

where we introduce the symbol [44]

$$\epsilon[ABC] = \epsilon^{ijk}\epsilon^{i'j'k'}A_{ii'}B_{jj'}C_{kk'}/3!, \tag{10}$$

with i, j, k and i', j', k' are $SU_L(3)$ and $SU_R(3)$ indices. Since $\epsilon[AAA] = \det A$, $\epsilon[ABC]$ represents a type of generalized determinant between dissimilar matrices [45]. Given the transformation properties of Φ and Φ_μ in Table I, Eqs. (6) and (9) are manifestly invariant under $SU_L(3) \times SU_R(3)$. Similarly, as the product of three heterochiral fields, these terms are not invariant under $U_A(1)$, but $Z(3)$. These anomalous interactions were first obtained in Ref. [8]

entirely from considerations of symmetry. In this paper we now compute their magnitude, as well as anomalous glueball interactions, in a DGI.

To relate the k_J to physical processes, we need the values for the constants of proportionality M_J between quark and mesonic operators. For spin one, we find $k_1 = 9.91 \times 10^6\ \mathrm{GeV}^{-7}$, which for $M_1 = M_0$ gives

$$a_1 = -\frac{k_1 M_1^6 M_0^2}{48} \approx -0.14\ \mathrm{GeV} < 0. \tag{11}$$

The corresponding mixing angle is approximately:

$$\beta_1 \simeq \frac{1}{2}\tan^{-1}\left(\frac{-\sqrt{2}a_1\phi_N/3}{2(m_{K_{1B}}^2 - m_{b_1}^2) - \sqrt{2}a_1\phi_S/6}\right) > 0. \tag{12}$$

For a DGI this mixing angle is positive. Using the value for a strange quark condensate $\phi_S = \langle 0|\eta_S|0\rangle \approx 130$ MeV, and assuming $M_1 = M_0 = 170$ MeV, we obtain a small value of $\beta_1 \simeq 0.75°$; for a larger value of $M_1 = 270$ MeV, the mixing angle increases to $\beta_1 \simeq 10°$. As illustrated in Fig. 3 [51], experimental results favor a positive value [54–57], as do numerical simulations on the lattice [58].

The anomalous interactions in Eq. (9) also open up new decay modes. For example, $\Gamma(\rho(1700) \to h_1(1415)\pi) = 0.027(M_1/M_0)^6$ MeV, which if $M_1 = M_0$ is rather small. Other anomalous decays are discussed in the Supplemental Material [37]. Measuring such processes can be used to fix the value of M_1.

An interaction term with one $J = 0$ meson and two $J = 2$ heterochiral mesons is

$$\mathcal{L}_{\mathrm{eff}}^{J=2} = -\frac{k_2}{3!}\left(\epsilon\left[(\bar{q}_L q_R)\left(\bar{q}_L\left(\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu - g_{\mu\nu}\overleftrightarrow{D}^2/4\right)q_R\right)^2\right] + R \leftrightarrow L\right) = -a_2(\epsilon[\Phi\Phi_{\mu\nu}\Phi^{\mu\nu}] + \mathrm{c.c.}). \tag{13}$$

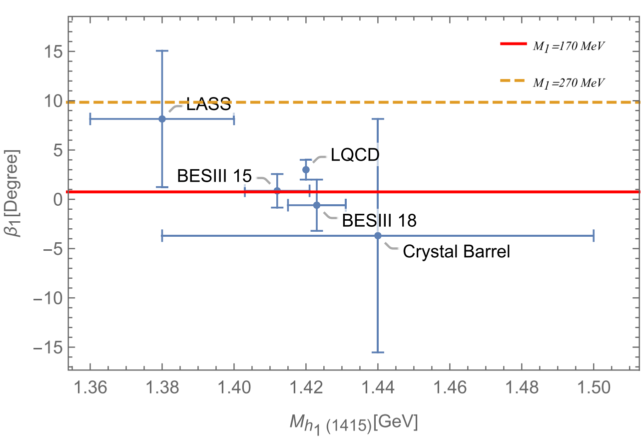

FIG. 3. β_1 in a DGI compared to the experiment [54–57] and the lattice (LQCD) [58], for $M_1 = M_0 = 170$ MeV and $M_1 = 270$ MeV.

We find $k_2 = 4.05 \times 10^7\ \mathrm{GeV}^{-9}$, so when $M_2 = M_0$,

$$a_2 = \frac{k_2 M_2^8 M_0^2}{48} \approx 0.017\ \mathrm{GeV} > 0. \tag{14}$$

The mixing angle for the pseudotensor multiplet is negative [59],

$$\beta_2 \simeq \frac{1}{2}\tan^{-1}\left(\frac{-\sqrt{2}a_2\phi_N/3}{2(m_{K_{2P}}^2 - m_{\pi_2}^2) - \sqrt{2}a_2\phi_S/6}\right) < 0. \tag{15}$$

Assuming that $M_2 = M_0$, the DGI gives a small mixing angle, $\beta_2 \approx -0.05°$. This agrees with lattice QCD [61], but not with the large value of $\beta_2 \simeq -42°$ extracted in Ref. [30] from the decay rates. To fit such a large mixing angle requires $M_2 = 2.4M_0$.

We see that anomalous terms generate mixings between the octet and singlet for *all* (pseudo-)heterochiral mesons. These mixing angles do decrease strongly with J, for two reasons. First, comparing the values in Eqs. (8), (11), and (14), each a_J decreases by about $\approx 1/10$ as J increases by one (*assuming* that $M_0 = M_1 = M_2$). This is because anomalous coupling k_J in Eq. (4) involves ρ^{2J}, and a DGI peaks at small $\rho\Lambda_{\overline{\mathrm{MS}}} \sim 0.5$, Fig. 1. Second, $\tan\beta_J$ is proportional to the inverse of the mass squared of the mesons, Eqs. (7), (12) and (15). For $J = 0$, the η and η' are pseudo-Goldstone bosons, and so *much* lighter than ordinary mesons, with $J = 1$ and 2. The former *may* be an artifact of a dilute gas of instantons; the latter is not. Further, that the *sign* of β_J flips as J changes is dynamical, and does not follow just from the chiral symmetry. This is a nontrivial test of our model, and appears to agree with experiment.

Besides mixing terms, there are also anomalous terms which involve derivatives of the spin zero field Φ, and so exclusively affect decays. For example, a term which couples heterochiral mesons with $J = 0$, 1, and 2 is

$$\mathcal{L}_{b_2} = -b_2\left(\epsilon\left[(\partial_\mu\Phi)\Phi_\nu\Phi^{\mu\nu}\right] + \mathrm{c.c.}\right). \tag{16}$$

In a DGI $|b_2| = k_2 M_0^2 M_1^3 M_2^4/48$; with $M_0 = M_1 = M_2$, $|b_2| \approx 0.099$.

An anomalous interaction coupling two heterochiral $J = 0$ mesons to a $J = 2$ meson is

$$\mathcal{L}_{c_2} = -c_2\left(\epsilon\left[(\partial_\mu\Phi)(\partial_\nu\Phi)\Phi^{\mu\nu}\right] + \mathrm{c.c.}\right). \tag{17}$$

For a DGI, $|c_2| = k_2 M_0^2 M_2^4/48$, with $|c_2| = 0.474\ \mathrm{GeV}^{-1}$ when $M_2 = M_0$.

Again, numerous anomalous decay channels open up. For example, $\Gamma(\eta_2(1870) \to \rho(1700)\pi) = 1.5 \times 10^{-6} M_1^3 M_2^4/M_0^7$ MeV. Measuring such processes will *significantly* constrain the values of M_1 and M_2, and test the consistency of our approach.

Besides anomalous mesonic interactions, those involving glueballs follow immediately, and are illustrated in the right part of Fig. 2. An anomalous interaction between a pseudoscalar glueball and heterochiral mesons is given by the term

$$\mathcal{L}_{c_g} = -\mathrm{i}c_g\tilde{G}_0(\det\Phi - \det\Phi^\dagger). \tag{18}$$

In a DGI $c_g \approx 11$. Then, by using Ref. [62], we obtain $\Gamma(\tilde{G}_0 \to K\bar{K}\pi) \approx 0.24$ GeV and $\Gamma(\tilde{G}_0 \to \pi\pi\eta') \approx 0.05$ GeV. In contrast to the anomalous decays between heterochiral mesons, these are *large* values. Notably, the BESIII collaboration has recently seen a pseudoscalar resonance, denoted as $X(2600)$, in the $\pi\pi\eta'$ channel [63]. Our results support the interpretation of this resonance as mostly gluonic, with a decay enhanced by the chiral anomaly [64].

Further anomalous decays involving heterochiral mesons with higher spin follow directly, and include interactions such as $\tilde{G}_0(\epsilon[\Phi\Phi_\mu\Phi^\mu] - \text{c.c.})$.

We conclude by noting that there are *many* other anomalous interactions which can be computed with our techniques. These include baryon decays [66], tetraquarks [67], glueballs and hybrid states [68,69], and the H dibaryon [70,71]. In summary, the effects of the axial anomaly merely begin with the η and the η' mesons, but most certainly do not end there.

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