Letter

Complete determination of $SU(3)_F$ amplitudes and strong phase in $\Lambda_c^+ \to \Xi^0 K^+$

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The BESIII collaboration has recently reported the first time measurement of the decay asymmetry $\alpha(\Lambda_c^+ \to \Xi^0 K^+) = 0.01 \pm 0.16(\mathrm{stat}) \pm 0.03(\mathrm{syst})$ and also a sizable phase shift of $\delta_P - \delta_S = -1.55 \pm 0.25$ or 1.59 ± 0.25 between S- and P-wave amplitudes. This implies significant strong phase shifts in the decay amplitudes. The strong phases indicate the existence of rescattering or loop effects, which are challenging to calculate due to nonperturbative effects. By employing the flavor $SU(3)_F$ symmetry and applying the Körner-Pati-Woo theorem to reduce the number of parameters, we find that the current data already allow us to obtain, for the first time, model-independent decay amplitudes and their strong phases. The establishment of the existence of sizable strong phases opens a window for future investigations into CP violation. In our fit, a notable discrepancy emerges in the branching ratio of $\Xi_c^0 \to \Xi^- \pi^+$. The direct relationship between $\Gamma(\Lambda_c^+ \to \Lambda e^+ \nu_e)$ and $\Gamma(\Xi_c^0 \to \Xi^- e^+ \nu_e)$, along with newly discovered $SU(3)_F$ relations, collectively suggests an underestimation of $\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+)$ in experimental findings.

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Recent results the BESIII collaboration have reported $\alpha(\Lambda_c^+ \to \Xi^0 K^+) = 0.01 \pm 0.16(\mathrm{stat}) \pm 0.03(\mathrm{syst})$ [1]. This supplements the previously established $\mathcal{B}(\Lambda_c^+ \to \Xi^0 K^+) = (0.55 \pm 0.07)\%$ [2], highlighting the importance of this channel in deepening our understanding of baryon decays. Moreover, BESIII data also indicates a nonzero $\beta(\Lambda_c^+ \to \Xi^0 K^+)$ to be negative, implying a strong phase shift between the S- and P-waves of $\delta_P - \delta_S = -1.55 \pm 0.25$ or 1.59 ± 0.25 [1,3]. These strong phases can be induced by rescattering processes and loop effects, where the intermediate particles are on-shell. This is an important feature originating from quantum theory and an essential ingredient for observing CP violation in particle and antiparticle decays. In two-body baryonic decays, the strong phase shifts manifest their effects in the Lee-Yang parameters (α, β) [4], which have played significant roles in understanding weak interaction.

Theoretically, first principle calculations for these decay amplitudes are extremely difficult due the low energy scale involved where nonperturbative QCD effects become important. Determinations for such decays need to wait for a full lattice calculation. In the meantime, analyses of low energy physics have proven to be useful [5] with the help of the flavor $SU(3)_F$ symmetry to a good approximation [6,7]. This flavor symmetry reduces the number of amplitudes by relating some of them together. When enough data are accumulated, it is possible to determine the decay amplitudes in a model-independent way and make testable predictions.

Efforts have been made in this direction recently. Previous studies based on the flavor $SU(3)_F$ symmetry predicted a large value close to one for $\alpha(\Lambda_c^+ \to \Xi^0 K^+)$ assuming real decay amplitudes [8–12] which also lead to zero strong phase shifts. The reasons for assuming real decay amplitudes were that there were no hints of strong phase shifts and also not enough data points to obtain useful information. The new data from BESIII now show the needs of having nonzero strong phase shifts, calling for a new theoretical understanding. In this work, we show that the decay amplitudes and their strong phases for two body weak decays of antitriplet charmed baryons can be completely determined from available data by applying the Körner-Pati-Woo (KPW) theorem to further reduce the

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number of parameters [13,14] and therefore explain the measured nonzero strong phase. We achieved for the first time a model-independent determination of two body decays of charmed antitriplet baryon.

$\Lambda_c^+ \to \Xi^0 K^+$ is one of the weak decays of a charmed antitriplet baryon ($T_{c\bar{3}}$) to an octet charmless baryon ($\mathbf{B}$) and a nonet pseudoscalar (P). Their $SU(3)_F$ representations are given by

$$T_{c\bar{3}} = (\Xi_c^0, -\Xi_c^+, \Lambda_c^+), \tag{1}$$

and

$$\mathbf{B} = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}, \quad P = \begin{pmatrix} \frac{1}{\sqrt{2}}(\pi^0 + c_\phi \eta + s_\phi \eta') & \pi^+ & K^+ \\ \pi^- & \frac{1}{\sqrt{2}}(-\pi^0 + c_\phi \eta + s_\phi \eta') & K^0 \\ K^- & \bar{K}^0 & -s_\phi \eta + c_\phi \eta' \end{pmatrix}, \tag{2}$$

where $(c_\phi, s_\phi) = (\cos\phi, \sin\phi)$ with $\phi = 39.3°$ [15] the mixing angle.

The decay amplitude for an initial baryon $\mathbf{B}_i$ to a final baryon $\mathbf{B}_f$ and a meson P, can be written as:

$$\mathcal{M} = \langle \mathbf{B}_f P | \mathcal{H}_{\rm eff} | \mathbf{B}_i \rangle = \bar{u}_f (F - G\gamma_5) u_i, \tag{3}$$

where $u_{i(f)}$ denotes the Dirac spinor for the initial (final) baryon and F (G) indicates a generic amplitude which violates (conserves) parity, associated with the S (P) partial wave. The values for F (G) depends on processes. The decay width, Γ, and the other decay observables are given by:

$$\begin{aligned} \Gamma &= \frac{p_f}{8\pi} \frac{(M_i + M_f)^2 - M_P^2}{M_i^2} (|F|^2 + \kappa^2 |G|^2), \\ \alpha &= \frac{2\kappa {\rm Re}(F^* G)}{|F|^2 + \kappa^2 |G|^2}, \qquad \beta = \frac{2\kappa {\rm Im}(F^* G)}{|F|^2 + \kappa^2 |G|^2}, \\ \gamma &= \frac{|F|^2 - \kappa^2 |G|^2}{|F|^2 + \kappa^2 |G|^2}, \qquad F^* G = |F^* G| e^{i(\delta_P - \delta_S)} \end{aligned} \tag{4}$$

where $M_{i,f}$ and M_P are the respective masses of $\mathbf{B}_{i,f}$ and P, $\kappa = p_f/(E_f + M_f)$, and $p_f(E_f)$ is the 3-momentum (energy) of $\mathbf{B}_f$ in the rest frame of $\mathbf{B}_i$.

The effective Hamiltonian inducing a charmed antitriplet baryon weak decay is given by

$$\begin{aligned} \mathcal{H}_{\rm eff} &= \frac{G_F}{\sqrt{2}} (c_+ \mathcal{H}(\mathbf{15})_k^{ij} + c_- \mathcal{H}(\bar{\mathbf{6}})_{lk} \epsilon^{lij}) \\ &\quad \times (\bar{q}_i q^k)_{V-A} (\bar{q}_j c)_{V-A}, \end{aligned} \tag{5}$$

where $c_\pm$ are the Wilson coefficients. In this work, we use $i, j, k, l \in \{1, 2, 3\}$ as flavor indices with $(q_1, q_2, q_3) = (u, d, s)$ which forms the fundamental representation of $SU(3)_F$. The $\mathcal{H}(\bar{\mathbf{6}})$ and $\mathcal{H}(\mathbf{15})$ are tensors of $SU(3)_F$ whose nonzero entries are given by

$$\begin{aligned} &\mathcal{H}(\mathbf{15})_2^{13} = \mathcal{H}(\bar{\mathbf{6}})_{22} = V_{ud} V_{cs}^*, \\ &\mathcal{H}(\mathbf{15})_2^{12} = -\mathcal{H}(\mathbf{15})_3^{13} = \mathcal{H}(\bar{\mathbf{6}})_{23} = V_{ud} V_{cd}^*, \\ &\mathcal{H}(\mathbf{15})_3^{12} = \mathcal{H}(\bar{\mathbf{6}})_{33} = V_{us} V_{cd}^*, \end{aligned} \tag{6}$$

while the other nonvanishing elements are obtained by using $\mathcal{H}(\mathbf{15})_k^{ij} = \mathcal{H}(\mathbf{15})_k^{ji}$ and $\mathcal{H}(\bar{\mathbf{6}})_{ij} = \mathcal{H}(\bar{\mathbf{6}})_{ji}$. The symmetric structures indicate $\bar{q}_i$ and $\bar{q}_j$ in Eq. (5) are color-symmetric for the term originated from $\mathcal{H}(\mathbf{15})$ whereas antisymmetric from $\mathcal{H}(\bar{\mathbf{6}})$. The same also applies to q^k and c. Here we have omitted $\mathcal{H}(\mathbf{3}) = (V_{ub} V_{cb}^*, 0, 0)$, which has a minimal impact on CP-even quantities.

Since the decay amplitudes must remain invariant under the $SU(3)_F$ transformation in the symmetric limit, they should be $SU(3)_F$ singlets. Accordingly, the flavor indices are fully contracted. For different ways of contraction, we assign an undetermined parameter, respectively. Then the decay amplitudes are decomposed into several invariant amplitudes [9,12], which can be extracted by the following parametrization for the Lagrangian

$$\begin{aligned} \mathcal{L}_{\mathbf{B}_c \to \mathbf{B}P} &= (P^\dagger)_n^l \overline{\mathbf{B}_m^k} (F_{ijkl}^{mn} - G_{ijkl}^{mn} \gamma_5) T_c^{ij}, \\ F_{ijkl}^{mn} &= \tilde{f}^a \mathcal{H}(\bar{\mathbf{6}})_{ik} \delta_j^m (\delta^\dagger)_l^n + \tilde{f}^b \mathcal{H}(\bar{\mathbf{6}})_{il} \delta_k^n \delta_j^m \\ &\quad + \tilde{f}^c \mathcal{H}(\bar{\mathbf{6}})_{ik} \delta_j^n (\delta^\dagger)_l^m + \tilde{f}^d \mathcal{H}(\bar{\mathbf{6}})_{kl} \delta_j^n (\delta^\dagger)_i^m \\ &\quad + \tilde{f}^e \mathcal{H}(\mathbf{15})_l^{mn} \epsilon_{ijk}/2, \end{aligned} \tag{7}$$

where $T_c^{ij} = \epsilon^{ijk} (T_{c\bar{3}})_k$ and $x \in \{a, b, c, d, e\}$. Replacing $\tilde{f}^x$ in F_{ijkl}^{mn} by $\tilde{g}^x$, one obtains expression for G_{ijkl}^{mn}. These amplitudes will be expressed as $\tilde{f}^x = f^x \exp(i\delta_f^x)$ and $\tilde{g}^x = g^x \exp(i\delta_g^x)$, where f^x and g^x are strictly positive.

Considering only the flavor structure, there are five different ways of contracting the $SU(3)_F$ indices for $\mathcal{H}(\mathbf{15})$: $(T_{c\bar{3}})_i \mathcal{H}(\mathbf{15})_j^{\{ik\}} (\mathbf{B}^\dagger)_k^j P_l^l$, $(T_{c\bar{3}})_i \mathcal{H}(\mathbf{15})_j^{\{ik\}} (\mathbf{B}^\dagger)_k^l P_l^j$, $(T_{c\bar{3}})_i \mathcal{H}(\mathbf{15})_j^{\{ik\}} (\mathbf{B}^\dagger)_l^j P_k^l$, $(T_{c\bar{3}})_i \mathcal{H}(\mathbf{15})_l^{\{jk\}} (\mathbf{B}^\dagger)_j^l P_k^i$ and $(T_{c\bar{3}})_i \mathcal{H}(\mathbf{15})_l^{\{jk\}} (\mathbf{B}^\dagger)_j^i P_k^l$. However, after taking into account of that the color indices of the quarks originated from $\mathcal{H}(\mathbf{15})$ (baryons) must be (anti)symmetric, these five terms can be reduced into one proportional to $(\mathbf{B}^\dagger)^j_i \mathcal{H}(\mathbf{15})^{\{ik\}}_l (P^\dagger)^l_k (T_{c\bar{3}})_j$. This is a remarkable result of the KPW theorem [13,14] for hyperon decays when applied to charmed baryon decays. This reduction of the number of decay amplitudes enable us to use available data to completely determine the flavor $SU(3)_F$ decay amplitudes.

In the following, we elucidate the configuration of the $SU(3)_F$ global fit. Given that both F and G encompass five complex amplitudes each, by omitting one unphysical overall phase shift, say δ^b_f, we are left with a total of 19 parameters. If one does not consider decays involving η and η', one can further neglect $\tilde{f}^a$ and $\tilde{g}^a$. In that case there are only 15 parameters to work with. On the other hand, there are in total 29 (23 without η and η' data points) experimental data points [16,17]. The $SU(3)_F$ invariant amplitudes can therefore be completely determined from a global fit. The experimental data are listed in Table I. Note that had one kept the sub-leading terms in $\mathcal{H}(\mathbf{15})$, a global analysis becomes impossible at present.

We determine the best fit values for the decay amplitudes $\tilde{f}^x$ and $\tilde{g}^x$ by minimizing the χ^2 function defined as

$$\chi^2(\tilde{f}^x, \tilde{g}^x) = \sum_{\text{exp}} \left(\frac{O_{\text{th}}(\tilde{f}^x, \tilde{g}^x) - O_{\text{exp}}}{\sigma_{\text{exp}}} \right)^2, \tag{8}$$

where O_{th} is the theoretical value of an observable, and O_{exp} is the experimental value with the standard deviation of σ_{exp}. In conducting the global fit, we incorporate all of the experimental branching ratios and asymmetry parameters, α_i, available to date. For the decays of Λ_c^+ and Ξ_c^+, and $\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+) = (1.43 \pm 0.32)\%$, we rely on the absolute branching ratios documented in the Particle Data Group [16]. While for the others, the reported ratios of $\mathcal{R}_X \coloneqq \mathcal{B}(\Xi_c^0 \to X)/\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+)$ from Belle are utilized [18,19]. Employing $\mathcal{R}_X$ as opposed to $\mathcal{B}(\Xi_c^0 \to X)$ is crucial, as the former is what has been actually measured. Although measurements exist for β_i, their associated uncertainties are substantial [20], making them insignificant to χ^2. Consequently, they will not be incorporated into the fit.

The resultant best fit values of the decay amplitude parameters and the error bars are given as follows:

TABLE I. Predictions from the $SU(3)_F$ global fit for the observed decays. The experimental uncertainties are combined quadratically, and the numbers in the parentheses are the uncertainties counting backward in digits, for example, $1.59(8) = 1.59 \pm 0.08$. The empty cells in the table indicate either α_{exp} are missing or the theory imposes no constraint on the quantities. Asterisks denote the numbers of standard deviations against the theory.

Channels	$\mathcal{B}_{\text{exp}}(\%)$	α_{exp}	$\mathcal{B}(\%)$	α	β	γ
$\Lambda_c^+ \to pK_S$	1.59(8)	*0.18(45)	1.55(7)	−0.40(49)	0.32(29)	−0.86(19)
$\Lambda_c^+ \to \Lambda^0\pi^+$	1.30(6)	−0.755(6)	1.29(5)	−0.75(1)	−0.13(19)	−0.64(4)
$\Lambda_c^+ \to \Sigma^0\pi^+$	1.27(6)	−0.466(18)	1.27(5)	−0.47(2)	0.88(2)	−0.05(27)
$\Lambda_c^+ \to \Sigma^+\pi^0$	1.25(10)	−0.48(3)	1.27(5)	−0.47(2)	0.88(2)	−0.05(27)
$\Lambda_c^+ \to \Xi^0 K^+$	**0.55(7)	0.01(16)	0.40(3)	−0.15(14)	−0.29(22)	0.94(7)
$\Lambda_c^+ \to \Lambda^0 K^+$	0.064(3)	−0.585(52)	0.063(3)	−0.56(5)	0.82(5)	0.10(27)
$\Lambda_c^+ \to \Sigma^0 K^+$	0.0382(25)	−0.54(20)	0.0365(21)	−0.52(10)	0.48(24)	−0.71(17)
$\Lambda_c^+ \to n\pi^+$	0.066(13)		0.067(8)	−0.78(12)	−0.63(15)	−0.04(20)
$\Lambda_c^+ \to \Sigma^+ K_S$	0.048(14)		0.036(2)	−0.52(10)	0.48(24)	−0.71(17)
$\Lambda_c^+ \to p\pi^0$	<0.008		0.02(1)		−0.82(32)	0.57(48)
$\Lambda_c^+ \to \Sigma^+\eta$	0.32(4)	−0.99(6)	0.32(4)	−0.93(4)	−0.32(16)	−0.16(23)
$\Lambda_c^+ \to p\eta$	0.142(12)		0.145(26)	−0.42(61)	0.64(40)	−0.65(20)
$\Lambda_c^+ \to \Sigma^+\eta'$	0.437(84)	−0.46(7)	0.420(70)	−0.44(25)	0.86(6)	0.25(35)
$\Lambda_c^+ \to p\eta'$	0.0484(91)		0.0520(114)	−0.59(9)	0.76(14)	−0.26(33)
$\Xi_c^+ \to \Xi^0\pi^+$	1.6(8)		0.90(16)	−0.94(6)	0.32(21)	−0.07(20)
$\Xi_c^0 \to \Xi^-\pi^+$	****1.43(32)	*−0.64(5)	2.72(9)	−0.71(3)	0.36(20)	−0.60(12)
Channels	$\mathcal{R}_X^{\text{exp}}$	α_{exp}	$\mathcal{R}_X$	α	β	γ
$\Xi_c^0 \to \Lambda^0 K_S$	0.225(13)		0.233(9)	−0.47(29)	0.66(20)	−0.58(21)
$\Xi_c^0 \to \Xi^- K^+$	**0.0275(57)		0.0410(4)	−0.75(4)	0.38(20)	−0.55(13)
$\Xi_c^0 \to \Sigma^0 K_S$	0.038(7)		0.038(7)	−0.07(117)	−0.83(28)	0.55(41)
$\Xi_c^0 \to \Sigma^+ K^-$	0.123(12)		0.132(11)	−0.21(18)	−0.39(29)	0.90(13)

$$
\begin{aligned}
f^x &= 3.60(70), 3.64(1.20), 3.84(0.18), 1.25(1.24),\\
&\quad 2.19(2.52),\\
g^x &= 12.21(3.34), 28.05(1.18), 2.76(1.72), 5.23(1.55),\\
&\quad 6.49(5.35),\\
\delta_f^x &= 1.66(31), 0, -2.20(39), -0.57(31), -0.58(50),\\
\delta_g^x &= -1.77(34), 2.60(0.37), 2.03(0.43), 2.39(0.74),\\
&\quad 1.98(1.03),
\end{aligned}
\tag{9}
$$

in the order of $x = a, b, c, d, e$, respectively, and both f^x and g^x are in units of $10^{-2} G_F$ GeV^2. We note that without measurement results for β_i, χ^2 suffers from a Z_2 ambiguity of $(\delta_f^x, \delta_g^x) \to (-\delta_f^x, -\delta_g^x)$. In addition, at the $SU(3)_F$ limit, κ are the same for all channels resulting in an additional ambiguity of $(f^x, \delta_f^x) \leftrightarrow (\kappa g^x, -\delta_g^x)$ without measured γ_i as input. To break the degeneracies, we fix it by using $\beta(\Lambda_c^+ \to \Xi^0 K^+) < 0$ and $\gamma(\Lambda_c^+ \to \Lambda^0 \pi^+) < 0$ from the experiment [1,20].

The fitting values for $\mathcal{B}, \alpha, \beta$ and γ are collected in Table I for the observed decay modes. The presence of empty cells signifies that either the corresponding $\alpha_{\rm exp}$ values are absent, or the theoretical framework does not impose any constraints on those particular quantities. Asterisks are used to denote the number of standard deviations by which the observed values deviate from the theoretical central values. Predictions for the unobserved decays with 148 decay observables are collected in Table II for the future experiment verification.

Note that the phases of $\delta^c_{f,g}$ are sizable which give phase shift to the decay amplitude of $F(\Lambda_c^+ \to \Xi^0 K^+) = -\tilde{f}^c$ and $G(\Lambda_c^+ \to \Xi^0 K^+) = -\tilde{g}^c$. In particular, we have $\delta_P - \delta_S = -2.06 \pm 0.50$, which is consistent with the experimental finding of -1.55 ± 0.25. As $\alpha \propto \cos(\delta_P - \delta_S)$, this is crucial in obtaining a small value of $\alpha(\Lambda_c^+ \to \Xi^0 K^+) = -0.15 \pm 0.14$ to be consistent with the BESIII measurement. If we remove $\alpha(\Lambda_c^+ \to \Xi^0 K^+) = 0.01 \pm 0.16$ as an input in the $SU(3)_F$ fit, then $\delta_P - \delta_S = -2.82 \pm 0.51$. The new BESIII data pulled the strong phase shift away from $-\pi$ which corresponds to no strong phase shift. Therefore, the data for $\alpha(\Lambda_c^+ \to \Xi^0 K^+)$ is crucial for reflecting the phase shift in $\Lambda_c^+ \to \Xi^0 K^+$. In addition, in contrast to the previous $SU(3)_F$ literature with $\alpha(\Lambda_c^+ \to \Sigma^0 K^+) \approx -1$ [9–12], we find that $\alpha(\Lambda_c^+ \to \Sigma^0 K^+) = -0.52 \pm 0.10$ which is consistent with the current experimental data.

The establishment of sizable strong phases makes the study of CP violations possible [21,22] when combined with theoretical calculation for $\mathcal{H}(\mathbf{3})$ contributions. The direct CP asymmetries are expected to be at the size of 10^{-3} with the details given elsewhere.

There are several direct relations appear when the color symmetry is considered. In particular, $\Gamma^{\Lambda_c^+}_{\Sigma^+ K_S} = \Gamma^{\Lambda_c^+}_{\Sigma^0 K^+}$ is well satisfied by the experimental data [16], which partly justifies our approach in this work. An important new relation is

TABLE II. Legend as in Table I but for unobserved decays.

Channels	$\mathcal{B}(10^{-3})$	α	β	γ	Channels	$\mathcal{B}(10^{-4})$	α	β	γ
$\Lambda_c^+ \to pK_L$	15.20(67)	−0.40(44)	0.33(27)	−0.86(17)	$\Lambda_c^+ \to nK^+$	0.13(2)	−0.90(6)	0.31(20)	−0.32(2)
$\Xi_c^+ \to \Sigma^+ K_S$	0.59(49)				$\Xi_c^+ \to \Lambda^0 \pi^+$	3.24(90)	0.29(29)	−0.47(30)	−0.83(13)
$\Xi_c^+ \to pK_{S/L}$	1.90(15)	−0.37(7)	0.35(18)	−0.86(8)	$\Xi_c^+ \to n\pi^+$	0.34(4)	−0.27(23)	−0.51(35)	0.81(26)
$\Xi_c^+ \to \Sigma^+ \pi^0$	2.12(14)	−0.49(49)	0.83(26)	−0.26(27)	$\Xi_c^+ \to \Sigma^0 K^+$	1.17(4)	−0.68(3)	0.35(19)	−0.65(26)
$\Xi_c^+ \to \Sigma^+ \eta$	0.70(33)	−0.80(69)	−0.41(77)	−0.43(104)	$\Xi_c^+ \to p\pi^0$	0.17(2)	−0.27(23)	−0.51(35)	0.81(26)
$\Xi_c^+ \to \Sigma^+ \eta'$	1.13(24)	−0.44(30)	0.88(23)	−0.19(42)	$\Xi_c^+ \to p\eta$	1.72(37)	−0.41(7)	0.67(15)	−0.62(26)
$\Xi_c^+ \to \Sigma^0 \pi^+$	3.04(10)	−0.59(3)	0.75(7)	−0.29(22)	$\Xi_c^+ \to p\eta'$	0.94(18)	−0.53(5)	0.73(18)	−0.43(26)
$\Xi_c^+ \to \Xi^0 K^+$	1.04(14)	−0.73(12)	−0.59(14)	0.35(17)	$\Xi_c^+ \to \Lambda^0 K^+$	0.37(4)	−0.44(12)	0.63(21)	0.65(26)
$\Xi_c^0 \to \Sigma^0 K_L$	0.97(17)		−0.53(39)	0.84(28)	$\Xi_c^0 \to pK^-$	1.96(19)	−0.26(22)	−0.50(34)	0.83(20)
$\Xi_c^0 \to \Xi^0 \pi^0$	7.10(41)	−0.49(9)	0.46(23)	−0.74(15)	$\Xi_c^0 \to nK_{S/L}$	7.10(62)	−0.44(3)	0.83(8)	−0.36(23)
$\Xi_c^0 \to \Xi^0 \eta$	2.94(97)	0.04(22)	0.83(13)	0.55(21)	$\Xi_c^0 \to \Lambda^0 \pi^0$	0.89(17)	−0.32(50)	−0.40(31)	−0.86(24)
$\Xi_c^0 \to \Xi^0 \eta'$	5.66(93)	−0.58(15)	0.74(6)	0.34(25)	$\Xi_c^0 \to n\pi^0$	0.06(1)	−0.27(23)	−0.51(35)	0.81(26)
$\Xi_c^0 \to \Lambda^0 K_L$	7.07(24)	−0.47(24)	0.71(17)	−0.53(21)	$\Xi_c^0 \to \Lambda^0 \eta$	4.31(1.10)	−0.02(52)	0.12(30)	−0.99(2)
$\Xi_c^0 \to \Sigma^+ \pi^-$	0.21(2)	−0.22(19)	−0.41(30)	0.88(14)	$\Xi_c^0 \to \Lambda^0 \eta'$	6.83(1.32)	−0.67(6)	0.74(8)	−0.09(26)
$\Xi_c^0 \to \Sigma^0 \pi^0$	0.34(3)	−0.33(48)	−0.38(27)	−0.87(23)	$\Xi_c^0 \to \Sigma^- K^+$	0.78(3)	−0.68(3)	0.35(19)	−0.65(26)
$\Xi_c^0 \to \Sigma^0 \eta$	0.12(5)	−0.80(69)	−0.41(77)	−0.43(104)	$\Xi_c^0 \to p\pi^-$	0.11(1)	−0.27(23)	−0.51(35)	0.81(26)
$\Xi_c^0 \to \Sigma^0 \eta'$	0.19(4)	−0.44(30)	0.88(23)	−0.19(42)	$\Xi_c^0 \to n\eta'$	0.31(6)	−0.53(5)	0.73(18)	−0.43(26)
$\Xi_c^0 \to \Sigma^- \pi^+$	1.83(6)	−0.65(3)	0.33(18)	−0.69(9)	$\Xi_c^0 \to n\eta$	0.57(12)	−0.41(7)	0.67(15)	−0.62(26)
$\Xi_c^0 \to \Xi^0 K_{S/L}$	0.43(2)	−0.47(2)	0.88(1)	0.06(26)					

$$\frac{\tau_{\Lambda_c^+}}{\tau_{\Xi_c^0}}\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+) = \mathcal{B}(\Lambda_c^+\to\Sigma^0\pi^+) + 3\mathcal{B}(\Lambda_c^+\to\Lambda\pi^+) - \left|\frac{V_{ud}}{V_{cd}}\right|^2\mathcal{B}(\Lambda_c^+\to n\pi^+). \quad (10)$$

By plugging the measured data at BESIII for Λ_c^+ decays on the right, we obtain $\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+) = (2.96\pm0.31)\%$, which differs significantly to $(1.43\pm0.32)\%$ from PDG [16]. On the other hand, with the relation of $\Gamma^{\Lambda_c^+}_{\Xi^0K^+} = \Gamma^{\Xi_c^0}_{\Sigma^+K^-}$, the experimental values of $\mathcal{B}(\Lambda_c^+\to\Xi^0K^+)$ and $\mathcal{R}^{\rm exp}_{\Sigma^+K^-} = 0.123\pm0.012$ [19] collectively lead to $\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+) = (3.37\pm0.52)\%$, echoing with the discussion above. Intriguingly, the current algebra approach, exemplary in the Λ_c^+ sector, predicts $\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+) = 6.47\%$ [23].

The study of semileptonic decays might offer further insights on $\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+)$. Using the experimental ratio $\mathcal{R}^{\rm exp}_{\Xi^-e^+\nu_e} = 0.730\pm0.044$ [24] alongside $\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+) = (2.72\pm0.09)\%$ from Table I, we derive $\mathcal{B}(\Xi_c^0\to\Xi^-e^+\nu_e) = (1.98\pm0.12)\%$. This aligns with $\mathcal{B}(\Xi_c^0\to\Xi^-e^+\nu_e) = (2.38\pm0.44)\%$ from lattice QCD [25] and $\mathcal{B}(\Xi_c^0\to\Xi^-e^+\nu_e) = (2.17\pm0.20)\%$ under the exact $SU(3)_F$ symmetry [26]. Conversely, $\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+) = (1.43\pm0.32)\%$ implies $\mathcal{B}(\Xi_c^0\to\Xi^-e^+\nu_e) = (1.04\pm0.24)\%$, a deviation from the lattice QCD by 2.6σ. We note that the latest preliminary result of lattice QCD indicates also a larger $\mathcal{B}(\Xi_c^0\to\Xi^-e^+\nu_e)$ [27].

The above analysis indicates that the current experimental value for $\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+)$ might have underestimated its true value which needs to be scrutinized more carefully further. The global fit here yields a χ^2 per degree of freedom (χ^2/d.o.f.) value of 3.7. This cannot be considered to be a good fit. The largest contributions to χ^2 come from the experimental ratio of $\mathcal{R}^{\rm exp}_{\Xi^-K^+} = 0.275\pm0.057$ and $\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+) = (1.43\pm0.32)\%$. If one removes these two data points, χ^2/d.o.f. is reduced to 1.5 which is a much better fit. If we use the original data of $\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+) = (1.80\pm0.52)\%$ [28] from Belle, the overall χ^2/d.o.f. reduces to 1.9 indicating a better overall fit. Meanwhile, in these cases the predictions for other quantities do not change much. We therefore would like to emphasize that our analysis hints that $\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+) = (1.43\pm0.32)\%$ is inconsistent with the direct relation of the nonleptonic decays and the indirect relation from the semileptonic decays.

Should forthcoming experimental results confirm the diminished magnitude of $\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+)$, the presence of a substantial gluon component within the Ξ_c^0 should be considered. In such a scenario, a rigorous examination of the subleading terms from $\mathcal{H}(\mathbf{15})$ would become imperative in theoretical discussions.

In this analysis, we have assumed $SU(3)_F$ symmetry to be exact and the applicability of the KPW theorem to the processes under consideration. Possible corrections from $SU(3)_F$ symmetry breaking effects due to quark mass differences and, also, KPW theorem breaking effects due to baryon states containing gluon Fock states, have been neglected. When the experimental data become more comprehensive, it is advisable to consider these effects to achieve greater theoretical precision. At the current stage, treating the $SU(3)_F$ symmetry and the KPW theorem as exact offers valuable guidance. Future experimental data will provide further insights.

Finally we would also like to point out that the establishment of strong phases in the decay amplitudes have far reaching implications for opportunities of finding CP violations in charmed baryon decays. One expects to have nonzero CP violating rate asymmetries for charmed baryon decays. Experimental searches should be carried out. We will present related theoretical studies elsewhere.

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