Letter

Deriving the Gibbons-Maldacena-Nunez no-go theorem from the Raychaudhuri equation

Mir Mehedi Faruk
Department of Physics, McGill University, Montreal, Quebec H3A 2T8, Canada

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In this article, we point out that to solve the null Raychaudhuri equation for higher-dimensional spacetime with an accelerating FRW solution in external directions and static compact internal directions, it is necessary to violate the strong-energy condition in higher dimensions. This constraint is well-known in obtaining accelerating cosmological solutions in string compactification, first described by Gibbons-Maldacena-Nunez. In deriving this constraint, we do not make any assumptions regarding the matter content.

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Introduction. We have compelling evidence that the present Universe is dominated by dark energy and going through a period of accelerated expansion. It is, therefore, natural to ask ourselves what quantum gravity theories, such as string/M-theory can tell about accelerating cosmological solutions [1]. However, the construction of accelerating cosmological solutions in string theory is still an open problem even after almost twenty years of the subject. The research direction took an interesting turn due to the obstacles such as the no-go theorems [2,3] and the proposed swampland conjectures [4–6]. The most interesting of them is the well-known no-go first proposed by Gibbons [2,7]; later on, a more refined version came from Maldacena and Nunez [3] which took into account the supergravity fluxes and the (anti-)D-branes. The most significant point made in this no-go theorem is that to obtain a d-dimensional accelerating cosmological solution by compactifying a $D > d$ dimensional theory, it is necessary to violate the D-dimensional strong-energy condition (SEC). Afterward, many other no-go theorems were constructed in string cosmology paradigm from different viewpoints, including metric-based constraints [8–11], world sheet symmetry [12–14], energy conditions [6,8,15–21], supersymmetry [22–24] string/M theory [20,25–45], spacetime entropy, and quantum gravity in de Sitter (dS) space [46–50]. We have made considerable progress in understanding the properties of gravitational thermodynamics [51–55]. In these works as well as in the Hawking-Penrose singularity theorems [56,57] Raychaudhuri equation provides a fundamental contribution.

The Raychaudhuri equation, a well-known geometric identity, is used extensively to enhance our understanding of various disciplines involving gravity, from astrophysics [58] to holography [59] and quantum gravity [60,61]. Recently, in an interesting article [62] this geometric identity was used to further derive new no-go theorems in string compactification. In the no-go theorem, the article concluded that accelerating backgrounds in string theory can only solve the Raychaudhuri equation when the null energy condition (NEC) is violated and/or the internal directions have a positive curvature. Besides, the well-studied flux compactification schemes in de Sitter (dS), such as the Kachru-Kallosh-Linde-Trivedi scheme, are revisited in light of this new no-go theorem in Ref. [62]. The authors point out that the matter sources or geometries that can potentially evade many of the previous no-go theorems and are considered to be essential ingredients in building putative dS solutions are unfortunately ruled out by the NEC violation constraint [63]. This is bad news for dS compactifications as we know that the four-dimensional dS maintains NEC [64]. The NEC is also well-known to be satisfied by a large set of matter content, which will further restrict the models with extra dimensions. In this article, we carefully study the conditions to satisfy the Raychaudhuri equation for a D-dimensional spacetime solution where we have a d-dimensional Friedmann-Robertson-Walker (FRW) solution in the external direction and a compact internal manifold of dimension n [65]. There are several arguments [1] whether such a cosmological background with accelerating cosmology can be obtained in string compactification. The pursuit of constructing four-dimensional accelerating solutions in string theory has led to claims ranging from multiple solutions [25,66,67] to none at all [6,68], but here we reexamine the necessary constraints for such backgrounds to satisfy the Raychaudhuri equation. In particular, we consider if any violation of NEC is really necessary to satisfy the Raychaudhuri equation. It is well-known that NEC also plays a significant role in establishing the existence of the big bang singularity, as well as proving the second law of thermodynamics for black holes [69]. There are also some hints that accelerating cosmology should satisfy the NEC to have a UV completion in string theory [15]. For example, the Virasoro constraint in string theory coming from world sheet theory precisely gives rise to the NEC in the geometry [13]. Because of NEC violation, it is quite difficult to obtain wormhole solutions, the creation of laboratory universes, and the building of time machines [13]. We demonstrate here that to solve the Raychaudhuri equation with four-dimensional accelerating cosmology in an external direction successfully, we must violate the SEC in higher dimensions. It is quite remarkable as this is the precise statement of the Gibbons-Maldacena-Nunez (GMN) no-go theorem. We briefly discuss the energy conditions, the null Raychaudhuri equation, and the GMN no-go theorem. Then, in Sec. IV, we work out the details to look for the constraints to solve the Raychaudhuri equation for the background metric in (12).

A brief review on energy conditions. In this section we briefly discuss the null and strong-energy conditions as well as the constraints it provides on the "scale factor" of the FRW metric [64]. The NEC simply states that for all the null-like vectors l^M we have the following constraint on Ricci tensor:

$$R_{MN}(x)l^M l^N \geq 0, \qquad g_{MN}(x)l^M l^N = 0. \tag{1}$$

Similarly, the SEC implies that for any timelike vector t^M,

$$R_{MN}t^M t^N \geq 0, \qquad t^2 < 0. \tag{2}$$

Considering the FRW metric in physical time coordinates gives,

$$\tilde{g}_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j. \tag{3}$$

As we know, $H = \frac{\dot{a}}{a}$ is the Hubble scale. Accelerating solutions are identified with

$$\ddot{a}/a = \dot{H} + H^2 > 0. \tag{4}$$

For a power-law scale factor, $a(t) \propto t^\gamma$, we have

$$\text{SEC} \Leftrightarrow 0 < \gamma \leq 1, \tag{5}$$

$$\text{NEC} \Leftrightarrow \gamma \geq 0, \tag{6}$$

If the NEC violation is necessary to maintain the Raychaudhuri equation, then it would rule out many the four-dimensional cosmology such as dS. It is much more difficult to violate the NEC than it is to violate the SEC. Violating the NEC, on the other hand, is very difficult, and no known classical energy-momentum sources or fields are known do so [70].

The null Raychaudhuri equation. In order to establish a singularity theorem, it is necessary to have an effective means of anticipating the appearance of focal points along the geodesics. Raychaudhuri's equation offers such a methodology [71]. Raychaudhuri's equation reveals that the occurrence of focal points is quite common as gravity has a propensity to focus nearby geodesics. The Raychaudhuri equation for null geodesic congruences is the following:

$$\frac{d\theta}{d\lambda} = -\frac{1}{D-2}\theta^2 - \sigma^2 - R_{MN}l^M l^N, \tag{7}$$

where l^N are the null vectors. Now, the expansion parameter and the shear tensor are identified as

$$\theta = \frac{1}{\sqrt{-g_D}}\partial_M(\sqrt{-g_D}l^M), \tag{8}$$

$$\sigma_{MN} = \frac{1}{2}(\nabla_M l_N + \nabla_N l_M) - \frac{1}{D-2}\hat{h}_{MN}\theta. \tag{9}$$

And, $\hat{h}_{MN}$ is the transverse metric, i.e.,

$$\hat{h}_{MN}l^M = 0, \tag{10}$$

which are, of course, transverse to null rays.

Gibbons-Maldacena-Nunez no-go theorem. We quickly review the GMN no-go theorem [2,3,7] in this section. We will consider the following warped product $D = d + n$ dimensional manifold as a solution of higher-dimensional quantum gravity theory such as string theory ($D = 10$) following [65]. In the external direction we take the four-dimensional FRW metric, and the internal direction is given by a (time-independent) compact n-dimensional manifold,

$$ds^2 = e^{-2A(y^m)}g_{\mu\nu}dx^\mu dx^\nu + e^{2A(y^m)}g_{mn}(y^m)dy^m dy^n. \tag{11}$$

The above metric can also be written in the following way using conformal rescaling of the internal metric [62]:

$$ds^2 = \Omega^2(y^m)[\tilde{g}_{\mu\nu}dx^\mu dx^\nu + \tilde{h}_{mn}(y^m)dy^m dy^n]. \tag{12}$$

We will refer to the Ω as the warp factor. We take the compact manifold has no boundary, and the warp factor is nonsingular. Calculating the Ricci tensor for the D-dimensional metric in the external directions following (B2) gives,

$$R^{(D)}_{\mu\nu} = R^{(d)}_{\mu\nu}(\tilde{g}) - \tilde{g}_{\mu\nu}[\nabla^2(\ln\Omega) + (D-2)(\nabla\ln\Omega)^2]. \tag{13}$$

The covariant derivative of Eq. (13) is simply along the compact directions. Using the Einstein equation for the full metric (12), we find that,

$$R^{(D)}_{\mu\nu} = T_{\mu\nu} - \frac{\Omega^2}{D-2}\tilde{g}_{\mu\nu}T^M_M. \tag{14}$$

Comparing (13) and (14) and taking the trace over $\tilde{g}$ we get,

$$\frac{1}{(D-2)}\frac{\nabla^2\Omega^{D-2}}{\Omega^{D-2}} = R^{(d)} + \Omega^2\left(-T^\mu_\mu + \frac{d}{D-2}T^M_M\right). \quad (15)$$

As a result [3], when we want to have positive curvature spacetime, i.e., $R^{(d)} > 0$ this implies,

$$\boxed{(D-d-2)T^\mu_\mu > dT^m_m}. \quad (16)$$

Alternatively, a similar statement is (when we want to have accelerated expansion $\frac{\ddot{a}}{a} \geq 0$ in warped compactification) that the D-dimensional Einstein field equations allow a time-independent compactification to an accelerating solution of dimension $d < D$ if the D-dimensional stress tensor violates the SEC. This can be viewed easily in physical time coordinates of the FRW metric. From Eq. (11) we find that

$$R^{(D)}_{00} = -(d-1)(\dot{H} + H^2) + \frac{\Omega^{-(D-2)}}{D-2}\nabla^2\Omega^{(D-2)}. \quad (17)$$

Multiplying both sides with Ω^{D-2} and integrating over compact space leads to

$$(d-1)\frac{G_D}{G_d}(\dot{H} + H^2) = -\int d^n\tilde{y}\,\sqrt{\tilde{h}}\,\Omega^{(D-2)}R^{(D)}_{00}, \quad (18)$$

where G_D and G_d are Newton's constant in D and d dimensions, respectively. We can clearly see to obtain the accelerating solution we need to violate (integrated version of) the D-dimensional SEC, i.e.,

$$\int d^n\tilde{y}\,\sqrt{\tilde{h}}\,\Omega^{D-2}R^{(D)}_{00} < 0. \quad (19)$$

If the higher-dimensional SEC was satisfied we would always have $\frac{\ddot{a}}{a} \leq 0$, i.e., a nonaccelerating cosmology in the four-dimensional external directions. We should also note when we have an accelerating FRW solution it automatically indicates

$$R^{(4)} = 3\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right) > 0, \quad (20)$$

which is a positive curvature [73] as the second term is clearly non-negative by nature. We will further prove in the next section that the accelerating backgrounds in (12) should maintain the condition (19) to satisfy the Raychaudhuri equation.

The Raychaudhuri equation and GMN no go theorem. We start by examining the background (12) to see if this can satisfy the Raychaudhuri equation following [62]. In other words, we would like to understand what the constraints for background (12) are in order to maintain the Raychaudhuri equation. We first take the affine null vectors such as

$$N^M = \frac{1}{\Omega^2}(1, 0, 0, 0, \tilde{n}^m), \quad (21)$$

where $\tilde{n}_m$ is an affine unit n-dimensional spacelike vector with respect to metric $\tilde{h}_{mn}$, which means [62]

$$\tilde{h}_{mn}\tilde{n}^m\tilde{n}^n = 1, \qquad \tilde{n}^m\tilde{\nabla}_m\tilde{n}^n = 0. \quad (22)$$

The expansion parameter associated with N_A is

$$\theta = N^m\partial_m[\ln(\Omega^{D-2}\sqrt{\tilde{h}})] + \frac{3H}{\Omega^2}, \quad (23)$$

where $\sqrt{\tilde{h}} = [\det(\tilde{h}_{mn})]^{\frac{1}{2}}$. The shear tensors have the following nonzero components:

$$\sigma_{tt} = -\tilde{h}^{mn}\partial_n(\ln\Omega)N_m, \quad (24)$$

$$\sigma_{tm} = \partial_m(\ln\Omega), \quad (25)$$

$$\sigma_{mn} = -\Gamma^p_{mn}N_p - \frac{\theta}{D-2}\hat{h}_{mn}, \quad (26)$$

$$\sigma_{ij} = \Gamma^M_{ij}N_M - \frac{\theta}{D-2}\hat{h}_{ij}, \quad (27)$$

where, Γ^A_{BC} are the Christoffel coefficients of (12). (They are listed in the Appendix of Ref. [19].) Combining Eqs. (7), (23), and (24)–(27), we find out (exactly as Ref. [62]),

$$3(H^2 + \dot{H}^2) = \tilde{R}^{(n)}_{mn}\tilde{n}^m\tilde{n}^n + A_{mn}(\Omega)\tilde{n}^m\tilde{n}^n - \Omega^4 R^{(D)}_{MN}N^M N^N, \quad (28)$$

where $A_{mn}(\Omega)$ is defined as

$$A_{mn}(\Omega) = (D-2)[\partial_m(\ln\Omega)\partial_n(\ln\Omega) - \nabla_m\partial_n(\ln\Omega)].$$

Rewriting (28) as

$$3(H^2 + \dot{H}) = \tilde{R}^{(n)}_{mn}\tilde{n}^m\tilde{n}^n + A_{mn}(\Omega)\tilde{n}^m\tilde{n}^n - R^{(D)}_{00} - R^{(D)}_{mn}\tilde{n}^m\tilde{n}^n, \quad (29)$$

we can see $R^{(D)}_{mn}$ is related to $\tilde{R}^{(n)}_{mn}$ by a conformal symmetry similar to (13) which we can use to further simplify our calculation [using (B2)],

$$R^{(D)}_{mn} = \tilde{R}^{(n)}_{mn}(\tilde{h}) - \tilde{h}_{mn}((D-2)\partial_p\ln\Omega\partial^p\ln\Omega + \Box\ln\Omega) + (D-2)(\partial_m(\ln\Omega)\partial_n(\ln\Omega) - \nabla_m\partial_n(\ln\Omega)). \quad (30)$$

We can clearly see the last term in Eq. (30) exactly cancels $A_{mn}(\Omega)$ in Eq. (29). So we rewrite Eq. (29) as

$$3(H^2+\dot{H}) = -R^{(D)}_{00} + ((D-2)\partial_p \ln\Omega \partial^p \ln\Omega + \Box \ln\Omega)\tilde{h}_{mn}\tilde{n}^m\tilde{n}^n. \quad (31)$$

Simplifying Eq. (31) further using Eq. (22) [74] and multiplying both sides by Ω^{D-2} we get

$$3(D-2)(H^2+\dot{H})\Omega^{D-2} = -(D-2)R^{(D)}_{00}\Omega^{D-2} + \Box\Omega^{D-2}. \quad (32)$$

Let us now perform an integral over the compact internal space which gives,

$$3(H^2+\dot{H})\frac{G_D}{G_d} = -\int d^n\tilde{y}\,\sqrt{\tilde{h}}\,\Omega^{D-2}R^{(D)}_{00}. \quad (33)$$

Throughout, we have used the fact that the integral of the Laplacian of the warp factor over the compact manifold is zero as the warp factor is nonsingular and the compact manifold has no boundary. Let usfocus on

(A) The dS solution, where we know that $\dot{H}=0$. As a result, the left-hand side is a positive definite quantity.

(B) In the case of other accelerating FRW solutions,

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} > 0,$$

and finally, the left-hand side of (33) is also a positive definite quantity as well. In both cases, we find out that the left-hand side is positive. The only way the right-hand side of (33) is a positive quantity is if

$$\int d^n\tilde{y}\,\sqrt{\tilde{h}}\,\Omega^{D-2}R^{(D)}_{00} < 0. \quad (34)$$

This looks like an (integrated) constraint which indicates towards (averaged) SEC violation in higher dimensions but this is the main essence of the GMN no-go theorem (19), i.e., one needs to violate the SEC in the higher dimensions to obtain a four-dimensional accelerating FRW solution. As we point out here, the Raychaudhuri equation results an integrated (averaged) SEC violation constraint. This follows entirely following a geometric identity, i.e., the Raychaudhuri equation, not having any presumption on the matter content of the theory. In Ref. [62], the authors argued that the constraints one obtains to solve this geometric identity leads to much stronger condition than the existing no-go theorems in that the NEC must be violated at every point. Therefore, the NEC violation constraint puts considerable restrictions on constructing any accelerating cosmology. The NEC is the weakest of the energy conditions in the sense that a violation of the NEC implies a violation of the other energy conditions, such as weak, dominant, and strong-energy conditions. The minimal coupling of NEC violating matter to Einstein gravity is likely inconsistent with string theory and black hole thermodynamics [75]. Also, NEC violating theories often display unsettling characteristics, namely superluminal propagation [76] and unbounded negative Hamiltonians [77]. Therefore, if NEC violation is essential for the background (12) to satisfy the Raychaudhuri equation, it could be problematic as NEC violation would cause the aforementioned problems.

Investigating the problem carefully, we realize [from Eq. (33)] that it is important to violate the D-dimensional SEC [or at least the integrated version (34)] to satisfy the Raychaudhuri equation when backgrounds have accelerating FRW solutions in the external directions. The geometric identity (7) does not impose any condition on the curvature of compact internal space and/or on the NEC for the background under consideration in (12). As a consequence violating the NEC is not an essential condition for accelerating geometries. Satisfying this geometric identity for the background (12) leads to a constraint which indicates the same conclusion as the GMN no-go theorem.

This can be understood from the seminal work of Jacobson [51,52]; the main result of the GMN no-go theorem is dependent upon satisfying the Einstein equation. We basically take the trace over the higher-dimensional Einstein equation in the external direction (see Sec. III). We do not explicitly perform such a step to find the relevant constraints to satisfy the Raychaudhuri equation but as pointed out by works of Jacobson [51,52] making the use of null Raychaudhuri equation along with the fundamental heat flow equation $\delta Q = TdS$, one can obtain the Einstein equation as the equation of state. As a result, although we have not explicitly evoked the Einstein equation in the derivation in Sec. IV, it is already implied due to Jacobson [51,54]. One of the main reasons we reach a different conclusion compared to [62] is because when the authors reach (29) they do not relate $\tilde{R}^{(n)}_{mn}$ with $R^{(D)}_{mn}$. However, as we know these quantities are related by simple conformal transformation (30). When we use it to derive the constraint, we find out that not only does the contribution from spacelike parts completely disappear but also the warp factor contribution can be nicely removed when we perform an integral over compact space. Consequently, we are left with an integrated constraint involving only timelike directions of the Ricci tensor, i.e., R_{00}. This allows us to conclude that we need to violate the SEC to satisfy the Raychaudhuri equation and in the process we reinvent the main essence of GMN no-go theorem. We have not discussed the status of the apparent horizon and antitrapped surfaces in this article [62]. In the future we would like to revisit these issues on time-dependent background because, as pointed out by Townsend [29,41] and Steinhardt [11], to evade all the no-go theorems for dS compactification we might need to go beyond the time-independent setup. In particular we would like to understand how such backgrounds can be realized as a coherent state [78,79] to bypass the Swampland conjectures [6] and trans-Planckian problems [80].

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Appendix A: Christoffel coefficients. The list Christoffel coefficients associated with (12) are

$$\Gamma^{\rho}_{\alpha m} = \partial_m(\ln\Omega)\delta^{\rho}_{\alpha},$$
$$\Gamma^{\rho}_{mn} = 0,$$
$$\Gamma^{m}_{\mu r} = 0,$$
$$\Gamma^{m}_{\mu\nu} = \tilde{g}_{\mu\nu}\tilde{h}^{mn}\partial_n(\ln\Omega),$$
$$\Gamma^{m}_{nr} = \tilde{\Gamma}^{m}_{nr}(\tilde{h}) + \frac{1}{\Omega}(\delta^m_r\partial_n\Omega + \delta^m_n\partial_r\Omega - \tilde{h}_{nr}\partial^m\Omega),$$
$$\Gamma^{\rho}_{\mu\nu} = \tilde{\Gamma}^{\rho}_{\mu\nu}(\tilde{g}).$$

Appendix B: Ricci tensor. In any warped geometry represented by the metric

$$ds^2 = \tilde{h}(z)d\tilde{s}^2 = \tilde{h}(z)\tilde{G}_{MN}dz^M dz^N, \tag{B1}$$

the Ricci tensor for the full metric $R(G_{MN})$ is related to Ricci tensor of the metric $R(\tilde{G}_{MN})$ by

$$R(G_{MN}) = R(\tilde{G}_{MN}) - (D-2)\left(\nabla_M\nabla_N\left(\frac{\ln\tilde{h}}{2}\right) - \nabla_M\left(\frac{\ln\tilde{h}}{2}\right)\nabla_N\left(\frac{\ln\tilde{h}}{2}\right)\right) - \tilde{h}_{MN}\left(\nabla_P\nabla^P\left(\frac{\ln\tilde{h}}{2}\right) + (D-2)\nabla_P\left(\frac{\ln\tilde{h}}{2}\right)\nabla^P\left(\frac{\ln\tilde{h}}{2}\right)\right). \tag{B2}$$

Block letter indices such as M can be thought of as the directions $0\ldots, D-1$ for the metric (12).

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