Letter

Long-range correlations of the stress tensor near the Cauchy horizon

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We show that the stress tensor of a real scalar quantum field on Reissner-Nordström-de Sitter spacetime exhibits correlations over macroscopic distances near the Cauchy horizon. These diverge as the Cauchy horizon is approached and are universal, i.e., state independent. This signals a breakdown of the semiclassical approximation near the Cauchy horizon. We also investigate the effect of turning on a charge of the scalar field and consider the correlation of the stress tensor between the two poles of the Cauchy horizon of Kerr-de Sitter spacetime.

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Introduction. On Minkowski space and in the vacuum state, the correlations $\langle \hat{T}_{\alpha\beta}(x)\hat{T}_{\gamma\delta}(x')\rangle$ of stress tensor components of a massless field at spacelike separated points x, x' fall off as d^{-8} with the distance d (exponentially for massive fields). Hence, correlations of the stress tensor over macroscopic distances are negligible. This is one of the (not always outspoken) assumptions underlying the semi-classical Einstein equation (here Λ is the cosmological constant, $T^{\mathrm{class}}_{\mu\nu}$ the stress tensor of classical matter and Ψ the state of the quantum matter),

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi\left(T^{\mathrm{class}}_{\mu\nu} + \langle \hat{T}_{\mu\nu}\rangle_{\Psi}\right). \tag{1}$$

An example of a situation in which correlations of the stress tensor over macroscopic distances are not negligible would be a quantum superposition of two macroscopically different spatial distributions of quantum matter, also called a *gravitational cat* state. Such superpositions played a crucial role in Feynman's famous Gedanken experiment [1] which was used to argue that in a consistent theory comprising gravity and quantum matter also the gravitational field must be quantized. The analysis of this and similar Gedanken experiments has been a major activity in quantum gravity research in recent years [2–6] and the actual realization and study of gravitational cat states is a key goal of experimental studies of quantum gravity [7,8].

In quantum field theory on curved spacetimes (QFTCS), correlations of quantum fields over macroscopic distances are crucial in inflationary cosmology [9], where they provide the seeds of cosmological structure formation.

In the following, we show that also near the Cauchy horizon inside black holes, there are correlations of the stress tensor over macroscopic distances. These are generically of the same order as the square of the expectation value of the stress tensor, and in particular they diverge as the Cauchy horizon is approached. Moreover, this behavior is universal, i.e., the leading divergence of the correlations is independent of the quantum state. Hence, the occurrence of gravitational cat states (defined as states with non-negligible correlations of the stress tensor over macroscopic distances) on the Cauchy horizon of a black hole is a robust prediction of QFTCS.

All stationary black hole solutions (with the exception of nonrotating, uncharged black holes) possess a Cauchy horizon in their interior. While the metric can be smoothly extended beyond it, the extension is nonunique, as is the extension of any other field subject to hyperbolic field equations. Hence, the occurrence of a Cauchy horizon signals the breakdown of predictivity. It was conjectured by Penrose [10] that this breakdown is not generic, i.e., under generic perturbations of the gravitational and/or matter fields, the Cauchy horizon should become singular. This strong cosmic censorship (sCC) conjecture motivates the study of classical [11–25] and quantum [26–37] fields near a Cauchy horizon.

For our study, we mostly consider a scalar (charged or uncharged) field on Reissner-Nordström-de Sitter (RNdS) spacetime, describing a static charged black hole in a spacetime with positive cosmological constant. This choice is motivated both by physical and practical considerations: From a practical point of view, RNdS has the advantage of spherical symmetry as well as having a further (cosmological) horizon at a finite radius, which simplifies the computation of the required scattering coefficients.

But RNdS is also interesting from a conceptual point of view, as (the Christodoulou formulation [38] of) sCC can

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be violated in that case [21,24,25], i.e., the classical stress tensor diverges weaker than V^{-1} at the Cauchy horizon. Here V is a (Kruskal) coordinate with which one can smoothly extend the metric beyond the Cauchy horizon, situated at $V = 0$. In contrast, for quantum fields, one finds the leading divergence,

$$\langle \hat{T}_{VV} \rangle \sim C V^{-2}, \tag{2}$$

for the expectation value of the renormalized stress-tensor component $\hat{T}_{VV}$ near the Cauchy horizon [30,31], with a universal (state-independent) coefficient C [31,39].

In this work, we will consider the correlations,

$$\Delta\hat{T}_{VV}(\delta\theta) \coloneqq \langle \hat{T}_{VV}(\theta)\hat{T}_{VV}(\theta+\delta\theta)\rangle_{\mathrm{U}} - \langle \hat{T}_{VV}\rangle_{\mathrm{U}}^2, \tag{3}$$

of $\hat{T}_{VV}$ in the Unruh state at angular separation $\delta\theta$ near the Cauchy horizon (we used spherical symmetry to simplify the last term on the rhs). We find that for the uncharged scalar field on RNdS,

$$\Delta\hat{T}_{VV}(\delta\theta) \sim D(\delta\theta) V^{-4}, \tag{4}$$

with a non-negative coefficient $D(\delta\theta)$ which is (i) universal (state independent), (ii) related to the coefficient C of (2) as $\lim_{\delta\theta\to 0} D(\delta\theta) = 2C^2$, and (for near-extremal RNdS) (iii) essentially flat (independent of $\delta\theta$) except near spacetime parameters where C vanishes.

This implies that the correlations of $\hat{T}_{VV}$ are of the same order as (the square of) its expectation value and that these strong correlations exist over macroscopic distances (the whole Cauchy horizon), putting the applicability of the semiclassical Einstein equation into question. Of course, also the divergence (2) indicates a breakdown of the semiclassical Einstein equation for $V \to 0$. However, for the present argument we do not need to consider this limit. In fact, our argument would also apply in a regime where $|\langle \hat{T}_{VV}\rangle|$ is still small, but $\Delta\hat{T}_{VV}$ is of the same order of magnitude as $\langle \hat{T}_{VV}\rangle^2$ over macroscopic distances.

For the charged scalar field, one still finds (4), but for increasing charge q of the field $D(\delta\theta)$ is more and more localized near $\delta\theta = 0$, i.e., correlations over macroscopic distances are suppressed. This calls into question the genericity of the result obtained for the uncharged scalar field. As realistic black holes are (essentially) uncharged but rotating, we finally consider the case of Kerr-de Sitter (KdS) spacetime. Since all fields "couple" to the angular momentum in a fashion quite analogous to the "coupling" of a charged field to the charge of the black hole, it is conceivable by analogy to the charged case that correlations over macroscopic distances are suppressed in that case. We calculate the correlation between the North and the South Pole and again find correlations which are of the same order as the (square of the) expectation value. Hence, also in this case, there are correlations over macroscopic distances near the Cauchy horizon, so that these can be considered as a generic feature of (essentially massless) quantum fields in black hole spacetimes.

To the best of our knowledge, the explicit calculation of correlations of the stress tensor (or more generally quantum fields) on black hole spacetimes has up to now been limited to timelike separated points [40], or to toy models which neglect scattering [41–43]. Hence, we also present the first calculation of quantum correlations at spacelike separation in black hole spacetimes.

Reissner-Nordström-de Sitter spacetime. The RNdS spacetime is characterized by the metric

$$g = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2 \tag{5}$$

$$f(r) = -\frac{\Lambda}{3}r^2 + 1 - \frac{2M}{r} + \frac{Q^2}{r^2}. \tag{6}$$

Here, $d\Omega^2$ is the area element of the unit 2-sphere, Λ the cosmological constant, M the mass, and Q the charge of the black hole. These are chosen such that $f(r)$ has three distinct positive roots $r_- < r_+ < r_c$, which determine the location of the Cauchy ($\mathcal{CH}$), event ($\mathcal{H}$) and cosmological horizon ($\mathcal{H}_c$). The black hole exterior consists of region $\mathrm{III} = \mathbb{R}_t \times (r_c, \infty) \times \mathbb{S}^2$ beyond the cosmological horizon and region $\mathrm{I} = \mathbb{R}_t \times (r_+, r_c) \times \mathbb{S}^2$ causally connected to the black hole, as well as the horizon $\mathcal{H}_c^L$ connecting them. The black hole interior up to the Cauchy horizon, region $\mathrm{II} = \mathbb{R}_t \times (r_-, r_+) \times \mathbb{S}^2$, is connected to region I along the event horizon $\mathcal{H}^R$. Region II is connected to region IV containing the singularity by the Cauchy horizon $\mathcal{CH}^R$. A Penrose diagram of the spacetime is shown in Fig. 1.

We introduce a double null coordinate system. First, we define the tortoise coordinate r_* by $f dr_* = dr$. Then, one defines $u = t - r_*$ and $v = t + r_*$. These coordinates run over $\mathbb{R}$ in each of the regions I, II, and III separately. To cover the horizons, one can introduce Kruskal-type coordinates. The null coordinate that is regular across the Cauchy horizon $\mathcal{CH}^R$ is defined in region II by

$$V = -e^{-\kappa_- v}, \qquad \kappa_i = \frac{1}{2}|\partial_r f(r_i)|, \tag{7}$$

so $V = 0$ on $\mathcal{CH}^R$. These coordinates are indicated in Fig. 1.

The other Kruskal coordinates relevant to this work are defined in region I by $U = -e^{-\kappa_+ u}$ and $V_c = -e^{-\kappa_c v}$.

Correlations of the energy flux. We will mostly consider a real, minimally coupled massive scalar field of mass $\mu^2 = 2\Lambda/3$, i.e., satisfying the Klein-Gordon equation,

$$(\nabla^\nu \nabla_\nu - \mu^2)\phi = 0. \tag{8}$$

This is the same equation as for a massless, conformally coupled scalar field on RNdS (or KdS), so that we say that

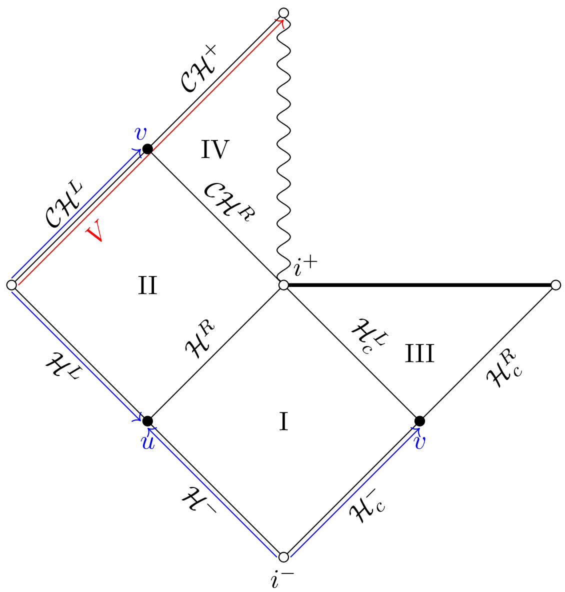

FIG. 1. The Penrose diagram of RNdS. Regions I and III comprise the black hole exterior. The black hole interior consists of II and IV. The blue arrows point towards increasing u and v, the red arrow towards increasing V.

the field has "conformal mass". For this choice the corresponding mode equation can be brought into Heun form [44] (as for massless fields of higher spin, so that ϕ can be seen as a proxy for the electromagnetic field), which simplifies numerical calculations. For such a scalar field, the classical stress tensor is given by

$$T_{\nu\rho} = \partial_\nu\phi\partial_\rho\phi - \frac{1}{2}g_{\nu\rho}(\partial_\alpha\phi\partial^\alpha\phi - \mu^2\phi^2). \tag{9}$$

In the quantum theory, this needs to be renormalized, since it is quadratic and local in the quantum field. In QFTCS, this must be done in a local and covariant way [45], i.e., by Hadamard point split renormalization (up to possible finite renormalizations, which are irrelevant for the divergent behavior near the Cauchy horizon). In this scheme, the renormalized expectation value of a Wick square with any number of derivatives is given by

$$\langle\partial^\alpha\phi(x)\partial^\beta\phi(x)\rangle_\Psi^{\mathrm{ren}} = \lim_{x'\to x}\partial_x^\alpha\partial_{x'}^\beta\big(\langle\phi(x)\phi(x')\rangle_\Psi - H(x,x')\big), \tag{10}$$

with α, β multi-indices. Here, $H(x,x')$ is the Hadamard parametrix for the Klein-Gordon operator $\nabla^\nu\nabla_\nu - \mu^2$. If the state Ψ is Hadamard, i.e., with a two-point function whose singularities for $x' \to x$ agree with those of $H(x,x')$, then the resulting expectation value is finite. Thus, one may write the operator-valued distribution for the vv-component of the renormalized stress tensor (henceforth referred to as the energy flux) on RNdS (somewhat formally) as

$$\hat{T}_{vv}(x) = \lim_{x'\to x}(\partial_v\phi(x)\partial_v\phi(x') - \partial_v\partial_v' H(x,x')\mathbf{1}). \tag{11}$$

We are interested in the correlations of this operator, defined by

$$\Delta\hat{T}_{vv}(x,y)_\Psi = \langle\hat{T}_{vv}(x)\hat{T}_{vv}(y)\rangle_\Psi - \langle\hat{T}_{vv}(x)\rangle_\Psi\langle\hat{T}_{vv}(y)\rangle_\Psi. \tag{12}$$

If the state Ψ is quasifree (Gaussian), as is the case for the Unruh state considered below, then one finds, using Wick's formula,

$$\Delta\hat{T}_{vv}(x,y)_\Psi = 2(\langle\partial_v\phi(x)\partial_v\phi(y)\rangle_\Psi)^2. \tag{13}$$

We would like to compute this correlation for the Unruh state on the Cauchy horizon $\mathcal{CH}^R$ of RNdS.

The Unruh state for the real scalar field on RNdS is Hadamard in I ∪ II ∪ III, i.e., up to the Cauchy horizon [31]. It can be defined by expanding the quantum scalar field in terms of mode solutions to the Klein-Gordon equation (8),

$$\phi(x) = \sum_{\lambda,\ell,m}\int_0^\infty \phi^\lambda_{\omega\ell m}(x)a^\lambda_{\omega\ell m} + \bar{\phi}^\lambda_{\omega\ell m}(x)a^{\lambda\dagger}_{\omega\ell m}\mathrm{d}\omega, \tag{14}$$

where the $a^\lambda_{\omega\ell m}$ and $a^{\lambda\dagger}_{\omega\ell m}$ are creation and annihilation operators and $\phi^\lambda_{\omega\ell m}$ form a complete set of symplectically normalized mode solutions to (8). The index λ runs over two families of such modes, called "in" and "up" modes, while ℓ and m are the usual angular quantum numbers.

For the Unruh state, the "up" modes vanish near $\mathcal{H}_c^- \cup \mathcal{H}_c^R$, and are of positive frequency with respect to U near $\mathcal{H}^L \cup \mathcal{H}^-$, while the "in" modes vanish near $\mathcal{H}^L \cup \mathcal{H}^-$ and are of positive frequency with respect to V_c near $\mathcal{H}_c^- \cup \mathcal{H}_c^R$.

For the evaluation of the correlations at the Cauchy horizon, we will pick x as any point on $\mathcal{CH}^R$, and $y = x + \delta\theta$ separated from x in the θ-direction. Due to the spherical symmetry of the Unruh state, the correlations will only depend on $\delta\theta$.

As the Unruh state is stationary, we can also compute the correlations on $\mathcal{CH}^L$, which is advantageous. To explain this, we recall the calculation of the expectation value $\langle\hat{T}_{vv}\rangle_{\mathrm{U}}$ on $\mathcal{CH}^R$ as performed in [31]. There, a further stationary "comparison" state $\langle\cdot\rangle_{\mathrm{C}}$ is introduced, which is defined by final data on $\mathcal{CH}^L \cup \mathcal{CH}^+$ (see Fig. 1), and which is Hadamard in II ∪ IV, i.e., across $\mathcal{CH}^R$. Due to the latter property, the renormalized expectation value of $\hat{T}_{vv}$ in this state must vanish at $\mathcal{CH}^R$, so that instead of $\langle\hat{T}_{vv}\rangle_{\mathrm{U}}$ one can consider $\langle\hat{T}_{vv}\rangle_{\mathrm{U}} - \langle\hat{T}_{vv}\rangle_{\mathrm{C}}$. The advantage is that the Hadamard parametrix drops out in this difference and the result can be computed (on $\mathcal{CH}^L$) in terms of a mode integral as

$$\langle \hat{T}_{vv} \rangle_{\mathrm{U}} = \sum_{\ell=0}^{\infty} T_{vv}^{(\ell)}, \qquad T_{vv}^{(\ell)} = \frac{2\ell+1}{16\pi^2 r_-^2} \int_0^\infty \mathrm{d}\omega\, \omega n_\ell(\omega), \tag{15}$$

with $n_\ell(\omega)$, explicitly given in [31, Eq. (123)], a function of certain scattering coefficients on RNdS.

Returning to the correlations (13), we note that in principle these do not require any renormalization for spacelike separated x, y (as is the case for $\delta\theta \neq 0$ on $\mathcal{CH}^R$). However, to improve the weak convergence of the mode integral, we again use stationarity to evaluate the expression on $\mathcal{CH}^L$ and subtract the correlation of $\hat{T}_{vv}$ in the comparison state. This does not alter the result for $\delta\theta \neq 0$, as, on $\mathcal{CH}^L$, $\langle \partial_v \phi(x) \partial_v \phi(x + \delta\theta) \rangle_{\mathrm{C}} = 0$ for $\delta\theta \neq 0$ (a "blind spot" in the terminology of [46]). Thus, one finds

$$\Delta \hat{T}_{vv}(\delta\theta)_{\mathrm{U}} = 2 \left(\sum_{\ell=0}^{\infty} P_\ell(\cos\delta\theta) T_{vv}^{(\ell)} \right)^2, \tag{16}$$

with $P_\ell(x)$ the Legendre polynomials. In particular, we see that the angular dependence comes from the terms with $\ell > 0$, and that $\lim_{\delta\theta \to 0} \Delta \hat{T}_{vv}(\delta\theta)_{\mathrm{U}} = 2 \langle \hat{T}_{vv} \rangle_{\mathrm{U}}^2$.

From the result for $\Delta \hat{T}_{vv}(\delta\theta)_{\mathrm{U}}$, one straightforwardly obtains (with the tensor transformation law) the divergent correlation (4), with $D(\delta\theta) = \kappa_-^{-4} \Delta \hat{T}_{vv}(\delta\theta)_{\mathrm{U}}$. Furthermore, using the arguments of [31,39], one sees that this result is universal, i.e., the coefficient $D(\delta\theta)$ of the leading divergence is the same for all states which are Hadamard in $\mathrm{I} \cup \mathrm{II} \cup \mathrm{III}$.

Numerical results. We now present numerical results for $\Delta \hat{T}_{vv}(\delta\theta)_{\mathrm{U}}$. A method for the numerical computation of the integrand $n_\ell(\omega)$ with *Mathematica* has been developed in [32]. We will focus on the regime of large Q, since this is where sCC is violated classically.

When fixing M and Λ and studying $T_{vv}^{(\ell)}$ as a function of Q, one finds that generically contributions with $\ell > 0$ are suppressed with respect to the $\ell = 0$ term. The only exception is a parameter region around Q_0, where $T_{vv}^{(0)}$ vanishes and changes sign. This parameter region also contains the value Q_* at which $\langle \hat{T}_{vv} \rangle_{\mathrm{U}}$ vanishes due to a cancellation of $T_{vv}^{(0)}$ and the higher ℓ-modes, mostly $T_{vv}^{(1)}$. In Fig. 2, we focus on a neighborhood of Q_* (indicated by a red line) in parameter space. We see that away from Q_* the correlations are essentially independent of $\delta\theta$, as expected due to the dominance of the $\ell = 0$ term. Furthermore, they coincide approximately with $2\langle T_{vv} \rangle_{\mathrm{U}}^2$ (indicated by the purple line). It follows that the correlations of $\hat{T}_{VV}$ in the Unruh state diverge as V^{-4} near the Cauchy horizon, with a coefficient of the same order as that of the leading divergence of $\langle \hat{T}_{VV} \rangle_{\mathrm{U}}^2$. We thus see strong fluctuations of the stress tensor which are correlated over macroscopic distances.

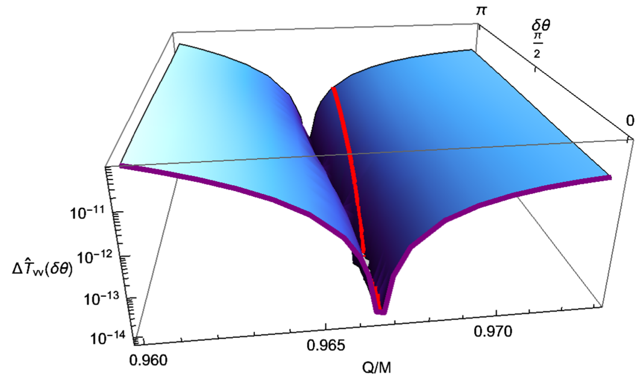

FIG. 2. Correlations of the energy flux at the Cauchy horizon at angular separation $\delta\theta$ for different values of Q/M and $\Lambda M^2 = 0.02$. The red line marks Q_* at which the energy flux vanishes. The purple line represents twice the square of the flux as a function of Q/M.

At Q_*, the correlations of $\hat{T}_{vv}$ at nonzero angular separations are nonzero and of a size similar to $\langle \hat{T}_{vv} \rangle_{\mathrm{U}}^2$ at other (nearby) values of Q. This means that even at Q_*, the typical realization of $\hat{T}_{vv}$ in a single measurement must be of the same order of magnitude as for other nearby values of Q. The positive and negative measurement results only cancel out on average. Thus, even if the leading divergence of the expectation value of the energy flux vanishes for this particular choice of parameters, one would expect that in a typical realization quantum effects will still lead to a quadratic divergence of the stress tensor and thereby restore sCC.

We note that at Q_*, the correlations at large angular separation are larger than at small angular separation. This is counterintuitive, as one would usually expect the correlations to decay with the separation.

To see how generic our finding of strong correlations of the energy flux over macroscopic distances near the Cauchy horizon is, we study two variations of the above. The first one is to turn on a charge q of the scalar field. This indeed alters the picture substantially. First, if q is sufficiently large, the sign change of $T_{vv}^{(0)}$ as a function of Q/M that we observed for the uncharged field is absent, and therefore also the sign change in $\langle \hat{T}_{vv} \rangle_{\mathrm{U}}$. Second, as q is increased, the relative size of the higher-ℓ modes, $T_{vv}^{(\ell)}$ with $\ell \geq 1$, compared to $T_{vv}^{(0)}$, increases. This leads to a stronger dependence of the correlations on $\delta\theta$. This can be observed in Fig. 3, where $\Delta \hat{T}_{vv}(\delta\theta)_{\mathrm{U}}$ (normalized with $\langle \hat{T}_{vv} \rangle_{\mathrm{U}}^2$) is shown as a function of $\delta\theta$ for different values of the field charge q (for a fixed choice of spacetime parameters Λ and Q). We see that as q increases, the correlations start to localize stronger around $\delta\theta = 0$.

Finally, we consider the case of a real scalar field (of conformal mass) on KdS, which describes a rotating black

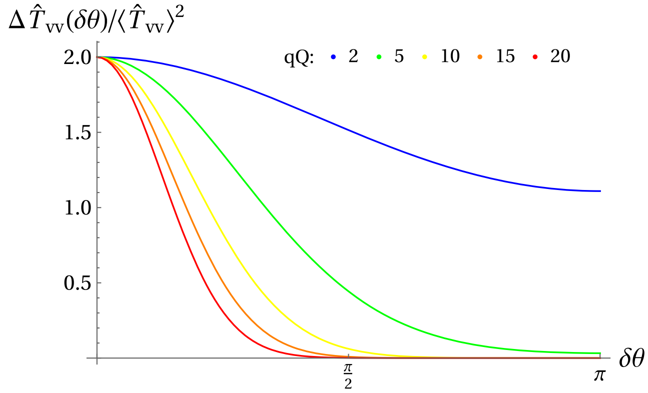

FIG. 3. Correlation of the energy flux, normalized to the square of the expectation value, as a function of $\delta\theta$ for different values of the scalar field charge q at $Q/M = 1$ and $\Lambda M^2 = 0.14$. For larger qQ, the range of the correlations shrinks.

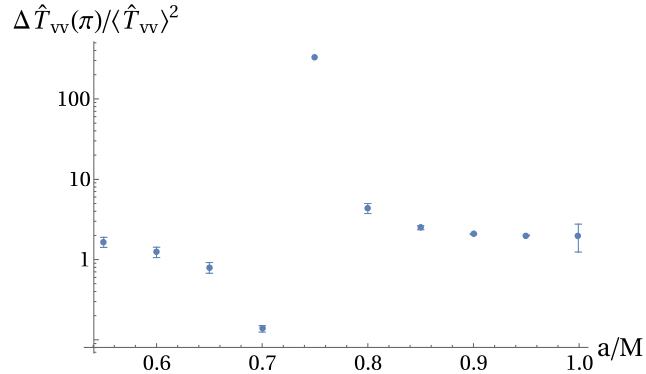

FIG. 4. Correlation of the energy flux between the poles of KdS as a function of the black hole angular momentum a/M for $\Lambda M^2 = 1/270$. The correlations are of the same order as the square of the expectation value, except near $a/M \simeq 0.75$, where $\langle\hat{T}_{vv}\rangle(\theta = 0)$ changes its sign (for $a/M = 0.75$, the error bar is omitted, due to the small denominator).

hole in the presence of a positive cosmological constant. Due to the lack of spherical symmetry, this is considerably more involved than RNdS. Results for the expectation values of $\hat{T}_{vv}$ (and $\hat{T}_{v\varphi}$) on Kerr (KdS) were recently obtained in [35] ([47]). For simplicity, we restrict to the correlation between the two poles. Using the results of [47], one obtains the correlations shown in Fig. 4. These are of the same order as $\langle\hat{T}_{vv}\rangle^2_{\mathrm{U}}$ at the pole, except around the parameter value at which $\langle\hat{T}_{vv}\rangle^2_{\mathrm{U}}$ vanishes. Hence, there are strong correlations over macroscopic distances, also near the Cauchy horizon of KdS.

An intuitive understanding for this finding can easily be given: At the poles, only the $m = 0$ modes are relevant, which do not couple to the angular momentum of the black hole, analogously to the real scalar field not coupling to the black hole charge; hence, the similarity to the behavior of the real scalar on RNdS. However, we also verified the existence of strong correlations (of the order of the square of the expectation value) away from the poles (for pairs of points related by reflection at the equatorial plane). In this case, the long-range correlations can be attributed to the dominance of the modes with $m = -\ell$, which are symmetric under reflection at the equatorial plane, and thus do not contribute to a suppression of the correlations for the pairs of points under consideration.

Quite generally, we expect strong correlations of the energy flux over macroscopic distances near the Cauchy horizon whenever only a small number of ℓ- and m modes contribute to the expectation value of the energy flux. This is expected to no longer hold for fields of a sizeable mass, in appropriate units; the effective potential governing the one-dimensional scattering problem for the modes of the real scalar (in the spherically symmetric case) is $V_{\mathrm{eff}} = f(\ell(\ell+1)r^{-2} + \mu^2 + r^{-1}f')$, so that if the Compton wavelength corresponding to μ is much smaller than r_-, then the first (ℓ dependent) term is negligible with respect to the second one for a large range of ℓ values. Hence, in this case (which applies to astrophysical black holes and the known massive elementary particles, possibly with the exception of neutrinos), the scattering coefficients, and thus $n_\ell(\omega)$ should be essentially independent of ℓ for a large range of ℓ values, leading to a strong localization of the correlations. However, as there is at least one massless particle, the photon, the strong correlations over macroscopic distances near the Cauchy horizon persist.

Conclusion. We have seen that there are strong (divergent as V^{-4}) correlations of the component $\hat{T}_{VV}$ of the stress tensor near the Cauchy horizon which, in some cases, extend over macroscopic distances and are of the same order as the square of the expectation value. We expect this to be the generic behavior for fields of vanishing or small mass.

If the correlations of $\hat{T}_{VV}$ are of the same order as the square of its expectation value over macroscopic distances, then correlations, i.e., fluctuations, can no longer be neglected, calling into question the applicability of the semiclassical Einstein equation, even if the expectation value of $\hat{T}_{VV}$ is still sufficiently small. We refer to [48,49] for approaches to take correlations of the stress tensor into account in the description of backreaction. Considering the results obtained in this work, an ansatz incorporating also the fluctuations of the stress tensor will be necessary to unravel the effect of quantum fields on the formation of singularities at the Cauchy horizon.

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