Letter

Consistent first-order action functional for gauge theories

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A novel first-order action principle has been proposed as the possible foundation for a more fundamental theory of general relativity and the Standard Model. It is shown in this article that the proposal consistently incorporates gravity and matter fields, and guides one to a new and robust path toward unification of fundamental interactions.

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Introduction. Lorentz symmetry is a cornerstone of modern physics. The Standard Model is formulated as a quantum field theory based on the global Lorentz symmetry of special relativity, the fields being classified according to the representations of the (complexified) Lorentz group [1]. While gravity has been understood to arise from the "gauging" of the Poincaré group of the inhomogeneous Lorentz transformations in the Einstein-Cartan-Sciama-Kibble theory[1] and its generalizations [3,4], this has not yet lead to a reconciliation of general relativity and quantum mechanics.

A new take on the gauge theory of spacetime and gravity is based on precisely the homogeneous (complexified) Lorentz group[2] [10]. In general, gravitational models with polynomial actions can accommodate the zero ground state of the metric [14–17], which we refer to as the "pregeometric" property [18–21]. The natural idea that spacetime arises via a spontaneous symmetry breaking that selects a preferred direction of time [22–24] is often implemented by additional fields on top of the geometry, but in-built to the Lorentz gauge theory wherein the symmetry breaking is necessary to emerge from the pregeometric state. The subtle elaboration of the mechanism entails an apparently drastically different description of gravity and spacetime, where even the Minkowski space has dynamical curvature and torsion [25]. A recent Hamiltonian analysis established the consistency of the Lorentz gauge theory [13], and the possibility of a new cosmological paradigm was speculated [26].

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[1]This procedure consists of a combined gauging of the global "internal" Lorentz symmetry of fermionic actions and promotion of the symmetry of Standard Model actions under diffeomorphisms generated by the Killing vectors of Minkowski space (whose commutators satisfy the Lie algebra of the Poincaré group) to a full diffeomorphism symmetry. It could be argued that the latter part of the procedure is superfluous both mathematically (as manifest in the by-construction diffeomorphism-invariant language of differential forms) and physically (the introduction of the corresponding gauge force is not supported by the interpretation of gravity according to the equivalence principle) [2].

[2]Possible formulations of a Lorentz gauge theory of gravity had been considered earlier [5–9], but a key point of the new theory [10] is the realization that chiral asymmetry [11,12] is required for the existence of a general-relativistic limit to the solutions [13].

In view of the SO(10) grand unification of the Standard Model gauge interactions [27], the new $SO_{\mathbb{C}}(1,3) \cong SO_{\mathbb{C}}(4)$ gravitational gauge theory would naturally seem to fit into a yet grander SO(N) unification along the lines of the grand unified theory including gravity proposals [28–33]. However, the coupling of the Standard Model to the Lorentz gauge theory calls for a pregeometrization of also the internal gauge field sector [34]. Whereas the standard spinor actions are polynomial in the fields, first-order in the derivatives, and possess the pregeometric property, a more fundamental action principle was required for the Yang-Mills gauge bosons. The suggested theory [34] can differ already classically from previous first-order formulations [35–38].

The characteristic feature of the new first-order gravity is the appearance of an effective dark matter source term. Interestingly, it was recently pointed out by Kaplan *et al.* [39,40], that since unitary evolution in quantum mechanics is described by the Schrödinger equation which is first-order in time derivative, the classical limit of gauge theories, including gravity, could be generalized by the addition of *shadow charges*, whose presence reflects the fact that quantum fluctuations need not satisfy the constraints imposed by the standard, second-order formulation of gauge interactions. This motivates us to consider also a modified version of the first-order Yang Mills theory, wherein shadow charges could arise as integration constants in the solutions to the equations of motion, analogously to the theory of gravity [34,41].

We shall focus on the conserved charges in the framework of Lorentz gauge theory from the perspective of Noether's theorems, taking advantage of some recent developments in covariant phase space formalism [42–49]. This article reports the results of our derivations. In the following sections, we present the action, cover the currents in the gravitational sector, and cover the rest. All the charges are unambiguous and have a clear physical interpretation. We conclude that the consistency of the first-action formulation provides a valuable guiding principle in the quest for the final theory.

The action principle. We consider an action $I = \int \boldsymbol{L}$ with the four-form

$$\boldsymbol{L} = \boldsymbol{L}_G + \boldsymbol{L}_M, \tag{1}$$

where $\boldsymbol{L}_G$ is the gravitational Lagrangian four-form polynomial in the gravitational fields which are taken to be a connection for the (complexified) Lorentz group $\boldsymbol{\omega}^{ab}$ and a scalar field ϕ^a valued in the group's fundamental representation (which we term the *khronon* due to its potential to introduce a standard of time into gravitation). We choose

$$\boldsymbol{L}_G = \boldsymbol{B}^{ab} \wedge {}^+\boldsymbol{R}_{ab} \tag{2}$$

where we have introduced the shorthand for the (proto) area-element $\boldsymbol{B}^{ab}$,

$$\boldsymbol{B}^{ab} = \frac{i}{2}(\mathbf{D}\phi^a \wedge \mathbf{D}\phi^b), \tag{3}$$

and $\boldsymbol{R}^{ab} = \mathbf{d}\boldsymbol{\omega}^{ab} + \boldsymbol{\omega}^a_c \wedge \boldsymbol{\omega}^{cb}$ is the curvature two-form for $\boldsymbol{\omega}^{ab}$. The ${}^{\pm}X = (1 \mp i\star)X$ are the projectors to the self-dual (left-handed) or anti-self-dual (right-handed) sectors, $\star\, {}^{\pm}X = \pm i^{\pm}X$. It was demonstrated in [10] that (2) realizes an extension to general relativity, when the metric tensor g is identified as $g = \mathbf{D}\phi^a \otimes \mathbf{D}\phi_a$.

In (1) we take into account minimally coupled matter fields ψ which may be some p-forms. $\boldsymbol{L}_M = \boldsymbol{L}_M(\mathbf{D}\phi^a, \psi, {}^+\mathbf{D}\psi)$ is the Lagrangian four-form for ψ which includes the gravitational fields, but we have excluded nonminimal couplings of ${}^-\boldsymbol{\omega}$ to ψ. We parametrize the material energy current $\boldsymbol{t}_a$ and the spin current $\boldsymbol{O}_{ab}$, respectively, as

$$\boldsymbol{t}_a = -\frac{\partial \boldsymbol{L}_M}{\partial \mathbf{D}\phi^a}, \qquad \boldsymbol{O}_{ab} = -\mathrm{rep}_{ab}\psi \wedge \frac{\partial \boldsymbol{L}_M}{\partial \mathbf{D}\psi}, \tag{4}$$

where rep_{ab} represents the Lorentz generator for ψ. Detailed examples are considered in the *Sources* section.

The variation of the total action,

$$\delta \boldsymbol{L} = \delta\phi^a \boldsymbol{E}_a + \delta\boldsymbol{\omega}^{ab} \wedge \boldsymbol{E}_{ab} + \delta\psi \wedge \boldsymbol{E}_\psi + \mathbf{d}\boldsymbol{\Theta}, \tag{5}$$

then yields the equations of motion (EoM) for the khronon, the gauge potential, and the matter fields, respectively,

$$\boldsymbol{E}_a = -\mathbf{D}(i\mathbf{D}^+\Box\phi_a - \boldsymbol{t}_a), \tag{6a}$$

$$\boldsymbol{E}_{ab} = {}^+\mathbf{D}\boldsymbol{B}_{ab} - i\phi_{[a}\mathbf{D}^+\Box\phi_{b]} + \phi_{[a}\boldsymbol{t}_{b]} - \boldsymbol{O}_{ab}, \tag{6b}$$

$$\boldsymbol{E}_\psi = \frac{\partial \boldsymbol{L}_M}{\partial \psi} + (-1)^{p+1}\mathbf{D}\frac{\partial \boldsymbol{L}_M}{\partial \mathbf{D}\psi}, \tag{6c}$$

and the symplectic potential

$$\boldsymbol{\Theta} = \delta\phi^a(i\mathbf{D}^+\Box\phi_a - \boldsymbol{t}_a) + {}^+\delta\boldsymbol{\omega}^{ab} \wedge \boldsymbol{B}_{ab} + \delta\psi \wedge \frac{\partial \boldsymbol{L}_M}{\partial \mathbf{D}\psi}, \tag{7}$$

where $\Box = \mathbf{DD}$ is the curvature two-form operator and ${}^+\Box = {}^+\mathbf{D}^+\mathbf{D}$. This shows that the action is stationary on-shell given Dirichlet boundary conditions for the variations of the gravitational and matter fields [43]. There are no boundary conditions[3] for the anti-self-dual potential ${}^-\delta\boldsymbol{\omega}^{ab}$. The EoM $\boldsymbol{E}_a$ and $\boldsymbol{E}_{ab}$ imply that on-shell $\approx$

$$i\mathbf{D}^+\Box\phi^a \approx \boldsymbol{t}^a + \boldsymbol{M}^a, \tag{8a}$$

$${}^+\mathbf{D}\boldsymbol{B}^{ab} \approx \phi^{[a}\boldsymbol{M}^{b]} + \boldsymbol{O}^{ab}, \tag{8b}$$

where $\boldsymbol{M}^a$ is a three-form that satisfies $\mathbf{D}\boldsymbol{M}^a = 0$.

[3]This depends crucially on the precise form of the action functional, and would not hold e.g. for the choice $\boldsymbol{L} = (i/2)\Box\phi_a \wedge {}^+\Box\phi^a + \boldsymbol{L}_M$ which is equivalent to (1) up to a total derivative.

Symmetries. We consider transformations δ that act on the dynamical fields. The transformation is a symmetry of $\boldsymbol{L}$ if $\delta \boldsymbol{L} = \mathbf{d}\boldsymbol{\ell}$, and exact if $\boldsymbol{\ell} = 0$. Besides the Lorentz and diffeomorphism symmetry, the action (1) has a peculiar shift symmetry. Below we report the currents $\boldsymbol{J}$ corresponding to the three classes of symmetry transformations. Each current is manifestly conserved on-shell, $\mathbf{d}\boldsymbol{J} \approx 0$. For a gauge symmetry, the current is on-shell an exact form, $\boldsymbol{J} \approx \mathbf{d}\boldsymbol{j}$, where $\boldsymbol{j}$ is called the Noether-Wald charge [42,43]. The charges are given as the integrated $\oint \boldsymbol{j}$ of the Noether-Wald charge over a closed surface.

Lorentz transformation: Consider a Lorentz transformation of the fields with infinitesimal parameters $\lambda^a{}_b$,

$$\delta_\lambda \phi^a = \lambda^a{}_b \phi^b, \tag{9a}$$

$$\delta_\lambda \boldsymbol{\omega}^a{}_b = -\mathbf{D}\lambda^a{}_b, \tag{9b}$$

$$\delta_\lambda \psi = \lambda^{ab} \mathrm{rep}_{ab} \psi. \tag{9c}$$

The Lorentz symmetry is exact $\delta_\lambda \boldsymbol{L} = 0$, and we take this to be the case also independently for the matter four-form $\delta_\lambda \boldsymbol{L}_M = 0$. Then we obtain Noether identities independently for the gravitational and matter sector. These are derived from (5) by considering parameters λ^{ab} which vanish at the boundary such that we can neglect all the total derivatives in the variations. We obtain the two identities,

$$^{+}\Box \boldsymbol{B}_{ab} = i\mathbf{D}\phi_{[a} \wedge \mathbf{D}^{+}\Box \phi_{b]}, \tag{10a}$$

$$\mathbf{D}\boldsymbol{O}_{ab} = \mathbf{D}\phi_{[a} \wedge \boldsymbol{t}_{b]} + \mathrm{rep}_{ab}\psi \wedge \boldsymbol{E}_\psi. \tag{10b}$$

The Noether current,

$$\boldsymbol{J}_\lambda = \lambda^{ab}\boldsymbol{E}_{ab} - \mathbf{d}({}^{+}\lambda^{ab}\boldsymbol{B}_{ab}), \tag{11}$$

is an exact form on-shell $\boldsymbol{J}_\lambda \approx \mathbf{d}\boldsymbol{j}_\lambda$, where the Noether charge two-form is now $\boldsymbol{j}_\lambda = {}^{+}\lambda^{ab}\boldsymbol{B}_{ab}$. Only the self-dual Lorentz transformations are associated with nontrivial charges.

Shift symmetry: The action $\int \boldsymbol{L}$ enjoys a shift symmetry, the invariance under constant translations of the khronon,[4]

$$\delta_\chi \phi^a = \chi^a \quad \text{where } \mathbf{D}\chi^a = 0, \tag{12a}$$

$$\delta_\chi \boldsymbol{\omega}^a{}_b = 0, \qquad \delta_\chi \psi = 0. \tag{12b}$$

The Noether identity is trivial for this transformation. The charge that we obtain using (7) and then (8a),

$$\boldsymbol{J}_\chi = \chi^a (i\mathbf{D}^{+}\Box \phi_a - \boldsymbol{t}_a) \approx \chi^a \boldsymbol{M}_a, \tag{13}$$

describes the energy-momentum carried by the effective matter three-form $\boldsymbol{M}_a$. This can be contrasted with Poincaré gauge theory, where the local translation is called a trivial gauge symmetry since it has zero charge. (One has to break covariance in order to extract a nonzero charge. We will return to this point later.)

Diffeomorphism: In the Lorentz gauge theory, spacetime geometry (coframe and curvature) is generated by Lie-dragging the fundamental fields (khronon and gauge potential) *covariantly*[5] along a vector ξ:

$$\delta_\xi \phi^a = \xi \lrcorner \mathbf{D}\phi^a, \tag{14a}$$

$$\delta_\xi \boldsymbol{\omega}^a{}_b = \xi \lrcorner \boldsymbol{R}^a{}_b, \tag{14b}$$

$$\delta_\xi \psi = \{\xi \lrcorner, \mathbf{D}\}\psi, \tag{14c}$$

where $\lrcorner$ is the interior product on differential forms, and here and in what follows, the $\mathbf{D}$ is always the total covariant derivative, thus involving also internal gauge fields in the case that the fields ψ have internal gauge charge. This gauge symmetry is not exact in the sense of $\boldsymbol{L}$ being invariant under the transformation, but $\delta_\xi \boldsymbol{L} = \mathbf{d}(\xi \lrcorner \boldsymbol{L})$. We obtain the Noether identity for gravity,

$$i(\xi \lrcorner \mathbf{D}\phi^a)\Box^{+}\Box \phi_a = \xi \lrcorner \boldsymbol{R}^{ab} \wedge ({}^{+}\mathbf{D}\boldsymbol{B}_{ab} - i\phi_{[a}\mathbf{D}^{+}\Box \phi_{b]}),$$

and for the invariance of $\int \boldsymbol{L}_M$ we get

$$(\xi \lrcorner \mathbf{D}\phi^a)\mathbf{D}\boldsymbol{t}_a + \xi \lrcorner \boldsymbol{R}^{ab} \wedge (\phi_{[a}\boldsymbol{t}_{b]} - \boldsymbol{O}_{ab}) = -\delta_\xi \psi \wedge \boldsymbol{E}_\psi.$$

In a nondegenerate spacetime wherein $\mathbf{e}^a \equiv \mathbf{D}\phi^a$ has an inverse $\mathbf{e}_a$, these can be rewritten as

$$i\Box^{+}\Box \phi_a = \mathbf{e}_a \lrcorner \boldsymbol{R}^{bc} \wedge {}^{+}\mathbf{D}\boldsymbol{B}_{ab} + i\boldsymbol{T}^a \wedge \mathbf{D}^{+}\Box \phi_a, \tag{15a}$$

$$-\delta_{\mathbf{e}_a}\psi \wedge \boldsymbol{E}_\psi = \mathbf{D}\boldsymbol{t}_a - \mathbf{e}_a \lrcorner \boldsymbol{T}^b \wedge \boldsymbol{t}_b - \mathbf{e}_a \lrcorner \boldsymbol{R}^{bc} \wedge \boldsymbol{O}_{bc}, \tag{15b}$$

where $\boldsymbol{T}^a = \mathbf{D}\mathbf{e}^a = \Box \phi^a$. The Noether current vanishes identically $\boldsymbol{J}_\xi = \xi \cdot \boldsymbol{\Theta} - \xi \lrcorner \boldsymbol{L} = 0$, and thus implies that a change of coordinates is a trivial gauge transformation. The matter sources have to be formulated consistently such that

$$(\xi \lrcorner \mathbf{D}\phi^a)\boldsymbol{t}_a = \delta_\xi \psi \wedge \frac{\partial \boldsymbol{L}_M}{\partial \mathbf{D}\psi} - \xi \lrcorner \boldsymbol{L}_M, \tag{16}$$

which means that the Hilbert (i.e. the metrical) and the Noether (i.e. the canonical) energy-momenta are equivalent.

[4]Perchance this could be understood due to ϕ^a representing the symmetry of not the group but the torsor; see https://math.ucr.edu/home/baez/torsors.html.

[5]The transformation can be considered as the minimal coupling of the frame-dependent definition discussed below.

On frame-dependent charges: One can combine transformations from the above three classes of symmetry transformations. An example is the coordinate diffeomorphism,

$$\mathcal{L}_\xi \phi^a = \xi \lrcorner \mathbf{d}\phi^a, \tag{17a}$$

$$\mathcal{L}_\xi \boldsymbol{\omega}^{ab} = \mathbf{D}(\xi \lrcorner \boldsymbol{\omega}^{ab}) + \xi \lrcorner \boldsymbol{R}^{ab}, \tag{17b}$$

$$\mathcal{L}_\xi \psi = \{\xi \lrcorner, \mathbf{d}\}\psi, \tag{17c}$$

which is the combination of a Lorentz transformation and a proper diffeomorphism, $\mathcal{L}_\xi = \delta_\xi + \delta_{\lambda=\xi\lrcorner\boldsymbol{\omega}}$. The possible physical relevance of this transformation is subject to case-dependent subtleties. The way that the fields are dragged along a vector ξ has no Lorentz-covariant meaning. The corresponding charge has no Lorentz-invariant interpretation. With some manipulations, using e.g. (4) and assuming (16), one can verify that the Noether current from (17) is given, as expected, precisely by (11) with the Lorentz transformation parameter $\lambda^{ab} = \xi \lrcorner \boldsymbol{\omega}^{ab}$. So, the charge is frame dependent because the parameter is noncovariant.

Nevertheless, it is very well known that the currents generated by $\mathcal{L}_\xi$ correctly describe the physical energy and momenta in many relevant special cases. This is so because energy and momentum can only be defined with respect to a reference frame, and thus it is expected that these charges are frame dependent.[6] The basic example is the standard result in Minkowski space that the symmetry of matter actions in the fixed background under diffeomorphisms corresponding to the Killing vectors of Minkowski space can—with "improvements"—lead to the conservation of the stress-energy-momentum tensor and the six conservations associated with the boost and rotation Killing vectors. This can be generalized to a maximally symmetric space, available perhaps globally, locally, asymptotically, or say, as an extra-dimensional embedding. These considerations apply as such in the geometric phase of Lorentz gauge theory.

Sources. Next we consider fermions, gauge bosons, and scalars. A unimodular version of the theory is also briefly checked.

Fermion matter: Dirac's theory of the electron and Weyl's theory of the neutrino pass the pregeometric standards and need no modifications. Let ψ in here denote the Dirac spinor. The γ_a in the spin-1/2 $\text{rep}_{ab} = -\gamma_{[a}\gamma_{b]}/2$ are matrices which obey $\gamma_{(a}\gamma_{b)} = -\eta_{ab}$. The Dirac spinor ψ has the conjugate $\bar{\psi} = \psi^\dagger\gamma^0$. In this representation, $\star = i\gamma^5 = -\gamma^0\gamma^1\gamma^2\gamma^3$, and we can project the two Weyl spinors ${}^\pm\psi = (1 \mp \gamma^5)\psi/2$. Define also $\star\, \mathbf{D}\phi^a = \epsilon^a{}_{bcd}\mathbf{D}\phi^b \wedge \mathbf{D}\phi^c \wedge \mathbf{D}\phi^d/3!$ and $\star\, 1 = \epsilon_{abcd}\mathbf{D}\phi^a \wedge \mathbf{D}\phi^b \wedge \mathbf{D}\phi^c \wedge \mathbf{D}\phi^d/4!$. Then, adopting the prescription of Ref. [51],

$$\boldsymbol{L}_M = \frac{i}{2}(\star\, \mathbf{D}\phi^a) \wedge (\bar{\psi}\gamma_a \mathbf{D}^+\psi - \mathbf{D}\bar{\psi}\gamma_a{}^-\psi) - \bar{\psi}\psi \star m. \tag{18}$$

From the variation

$$\delta \boldsymbol{L}_M = -\delta(\mathbf{D}\phi^a) \wedge \boldsymbol{t}_a + \delta\boldsymbol{\omega}_{ab} \wedge \delta\boldsymbol{O}^{ab} + \delta\bar{\psi}\boldsymbol{E}_{\bar{\psi}} + \boldsymbol{E}_\psi \delta\psi + \mathbf{d}\boldsymbol{\Theta}, \tag{19}$$

we obtain the currents

$$\boldsymbol{t}_a = (\star\, \boldsymbol{B}_{ab}) \wedge (\bar{\psi}\gamma^b \mathbf{D}^+\psi - \mathbf{D}\bar{\psi}\gamma^{b-}\psi) + m\bar{\psi}\psi \star \mathbf{D}\phi_a, \tag{20a}$$

$$\boldsymbol{O}^{ab} = \frac{i}{8}(\star\, \mathbf{D}\phi_c)\bar{\psi}(\gamma^c\gamma^{[a}\gamma^{b]+}\psi + \gamma^{[a}\gamma^{b]}\gamma^{c-}\psi) = \frac{i}{2}\bar{\psi}^+(\star\, \mathbf{D}\phi^{[a}\gamma^{b]})\gamma^5\psi, \tag{20b}$$

the EoM,

$$\boldsymbol{E}_{\bar{\psi}} = \frac{i}{2}\gamma_a(\star\, \mathbf{D}\phi^a) \wedge \mathbf{D}\psi - \gamma^a(\star\, \boldsymbol{B}_{ab}) \wedge \boldsymbol{T}^{b-}\psi - \psi \star m, \tag{21a}$$

$$\boldsymbol{E}_\psi = -\frac{i}{2}(\star\, \mathbf{D}\phi^a) \wedge \mathbf{D}\bar{\psi}\gamma_a + (\star\, \boldsymbol{B}_{ab}) \wedge \boldsymbol{T}^{b+}\bar{\psi}\gamma^a - \star\, m\bar{\psi}, \tag{21b}$$

and the symplectic potential,

$$\boldsymbol{\Theta} = \frac{i}{2}(\star\, \mathbf{D}\phi^a)(\delta\bar{\psi}\gamma_a{}^-\psi - \bar{\psi}\gamma_a\delta^+\psi). \tag{22}$$

In a real frame, $\boldsymbol{E}_{\bar{\psi}} = \bar{\boldsymbol{E}}_\psi$. The identity (16) is consistent with the energy current (20a).

Yang-Mills fields: The first-order pregeometric Yang-Mills theory [34] is formulated in terms of the interface (proto) area element

$$\tilde{\boldsymbol{B}}^{ab} = \boldsymbol{h}^a \wedge \mathbf{D}\phi^b, \tag{23}$$

with the "one foot outside" and the other $\boldsymbol{h}^a$, valued in the adjoint representation of the Yang-Mills gauge group,

[6]According to a recent proposal, the frame dependence is the consequence of the equivalence principle, and the physical criterion that uniquely fixes the reference frame is the vanishing of its local energy-momentum current [47]. However, it is outside this article's scope to implement this so-called $\mathrm{G}_\parallel$R principle [50] in the Lorentz gauge theory.

a "vierbein" spanning an internal hyperspace.[7] We recall that $\mathbf{D}$ is the total covariant derivative, thus involving also the Yang-Mills gauge field $\boldsymbol{A}$ whose field strength is denoted by $\boldsymbol{F}$. Now the field excitation $*\boldsymbol{F}$ (where $*$ is the Hodge dual) is not postulated *a priori*, but the gist of this new approach to gauge interactions is that the field excitation $*\tilde{\boldsymbol{B}} = \eta^{ab} * \tilde{\boldsymbol{B}}_{ab} \approx *\boldsymbol{F}$ emerges from the variational principle. An action density which achieves this is

$$\boldsymbol{L}_M = \langle \tilde{\boldsymbol{B}}^{ab} \wedge (\star\, \tilde{\boldsymbol{B}}_{ab} - \eta_{ab}\boldsymbol{F})\rangle - \langle \boldsymbol{A} \wedge \tilde{\boldsymbol{J}}\rangle, \tag{24}$$

where $\boldsymbol{A}$ is the Yang-Mills gauge field, $\tilde{\boldsymbol{J}}$ is its material source, and $\langle\cdot\rangle$ is the trace over the Lie algebra.

Standard theory. The variation

$$\begin{aligned}\delta \boldsymbol{L}_M = &-\delta(\mathbf{D}\phi^a) \wedge \boldsymbol{t}_a + \delta\boldsymbol{\omega}^{ab} \wedge \boldsymbol{O}_{ab} \\ &+\langle \delta \boldsymbol{h}^a \wedge \tilde{\boldsymbol{E}}_a\rangle + \langle \delta\boldsymbol{A} \wedge \tilde{\boldsymbol{E}}\rangle + \mathbf{d}\boldsymbol{\Theta},\end{aligned} \tag{25}$$

yields us the EoM,

$$\tilde{\boldsymbol{E}}_a = -2 \star \tilde{\boldsymbol{B}}_{ab} \wedge \mathbf{D}\phi^b + \boldsymbol{F} \wedge \mathbf{D}\phi_a, \tag{26a}$$

$$\tilde{\boldsymbol{E}} = \mathbf{D}\tilde{\boldsymbol{B}} - \tilde{\boldsymbol{J}}, \tag{26b}$$

and the symplectic potential,

$$\boldsymbol{\Theta} = -\langle \delta\boldsymbol{A} \wedge \tilde{\boldsymbol{B}}\rangle. \tag{27}$$

The gravitational source currents are

$$\boldsymbol{t}_a = 2\langle \star\, \tilde{\boldsymbol{B}}_{ab} \wedge \boldsymbol{h}^b\rangle - \langle \boldsymbol{F} \wedge \boldsymbol{h}_a\rangle, \tag{28a}$$

$$\boldsymbol{O}_{ab} = 0. \tag{28b}$$

It is not difficult to see that the internal symmetry transformation,

$$\delta_g \boldsymbol{h}^a = [g, \boldsymbol{h}^a], \qquad \delta_g \boldsymbol{A} = -\mathbf{D}g, \tag{29}$$

results in the expected current $\boldsymbol{J}_g \approx \tilde{\boldsymbol{J}}$. It has to be concluded that this prescription is the mere reformulation of the standard Yang-Mills theory. In particular, the symplectic current (27) assumes its expected form, and the energy current (28a) fails the consistency requirement (16).

A slightly more economic reformulation considers instead the six degrees of freedom (d.o.f.) of the excitation carried in the fundamental variational d.o.f. α^{ab} valued in the adjoints of both the Lorentz and the Yang-Mills gauge groups, such that $\boldsymbol{h}^a = \alpha^a{}_b \mathbf{e}^b$. However, this would not change the conclusions.

Modified theory. A more radical alternative is to encode the variational d.o.f. into the isokhronon α^a living in the fundamental representation of the Lorentz group and giving rise to the internal hyperspacetime $\boldsymbol{h}^a = \mathbf{D}\alpha^a$ in an analogy to the khronon ϕ^a in the external spacetime. Then an analogy of dark matter may also arise in the form of nontrivial vacua. This describes the situation in quantum mechanics wherein the field force lines need not be strictly attached to the material source points. The case $*\tilde{\boldsymbol{B}} \approx *\boldsymbol{F}$ is just one of the solutions, and therefore the solution space can be constrained by phenomenological data.[8]

The variation (25) should then be reconsidered,

$$\begin{aligned}\delta \boldsymbol{L}_M = &-\delta(\mathbf{D}\phi^a) \wedge \boldsymbol{t}_a + \delta\boldsymbol{\omega}^{ab} \wedge \boldsymbol{O}_{ab} \\ &+\langle \delta\alpha^a \mathbf{D}\tilde{\boldsymbol{E}}_a\rangle + \langle \delta\boldsymbol{A} \wedge \tilde{\boldsymbol{E}}\rangle + \mathbf{d}\boldsymbol{\Theta},\end{aligned} \tag{30}$$

since now the three-form $\tilde{\boldsymbol{E}}_a$ in (26a) is closed but may not vanish on-shell. Nontrivial modifications now enter into the expression for the symplectic potential,

$$\boldsymbol{\Theta} = \langle \delta\alpha^a (2 \star \tilde{\boldsymbol{B}}_{ab} - \eta_{ab}\boldsymbol{F})\rangle \wedge \mathbf{D}\phi^b - \langle \delta\boldsymbol{A} \wedge \tilde{\boldsymbol{B}}\rangle, \tag{31}$$

as well as the gravitational source currents,

$$\boldsymbol{t}_a = 2\langle \star\, \tilde{\boldsymbol{B}}_{ab} \wedge \mathbf{D}\alpha^b\rangle - \langle \boldsymbol{F} \wedge \mathbf{D}\alpha_a\rangle, \tag{32a}$$

$$\boldsymbol{O}_{ab} = \langle \alpha_{[a}\mathbf{D}\phi_{b]} \wedge \boldsymbol{F}\rangle - 2\langle \alpha_{[a} \star \tilde{\boldsymbol{B}}_{b]c}\rangle \wedge \mathbf{D}\phi^c. \tag{32b}$$

Remarkably, the energy current (32a) identically satisfies (16). So, the results for the three classes of gravitational charges in the *Symmetries* section remain intact in the presence of the modified Yang-Mills interactions.

It can be verified that the internal symmetry transformation $\delta_g \alpha^a = [g, \alpha^a]$, $\delta_g \boldsymbol{A} = -\mathbf{D}g$ is associated with the current

$$\begin{aligned}\boldsymbol{J}_g = &-\langle g\tilde{\boldsymbol{E}}\rangle + \mathbf{d}\langle g\tilde{\boldsymbol{B}}\rangle + \langle [g, \alpha^a](2 \star \tilde{\boldsymbol{B}}_{ab} - \eta_{ab}\boldsymbol{F})\rangle \\ &\wedge \mathbf{D}\phi^b \approx \tilde{\boldsymbol{J}},\end{aligned} \tag{33}$$

where in the last step we used the EoM (26) (see Sec. III C of [34]). The possible contribution to the divergence of $\tilde{\boldsymbol{B}}$ due to a vacuum polarization or magnetization [see Eq. (50) of [34]] is cancelled by the second term in (33), and we recover the canonical gauge current. A novel property of isokhronon theory is the shift symmetry,

$$\delta_{\tilde{\chi}} \alpha^a = \tilde{\chi}^a, \quad \text{where } \mathbf{D}\tilde{\chi}^a = 0. \tag{34}$$

The conserved current,

$$\boldsymbol{J}_{\tilde{\chi}} = \tilde{\chi}^a (2 \star \tilde{\boldsymbol{B}}_{ab} - \eta_{ab}\boldsymbol{F}) \wedge \mathbf{D}\phi^b \approx \tilde{\chi}^a \boldsymbol{X}_a, \tag{35}$$

[7] On frames constructed from material fields in condensed matter physics, see Ref. [52].

[8] In cosmology [26] it remains to be seen whether $\boldsymbol{M}_a$ could be related to dark matter and the $\boldsymbol{X}_a$ in the result (35) to magnetic fields.

is the integration form $\boldsymbol{X}_a$ responsible for the possible vacuum excitation [34]. It is the analogy of the integration form $\boldsymbol{M}_a$ in the gravity sector.[9]

An important caveat is that one is now not free to choose both integration forms independently for arbitrary solutions. Therefore this theory is probably not a viable modification of the Standard Model gauge interactions. Let us briefly speculate on a possible refinement of the unified theory, first restricting to the case of an Abelian gauge field $\boldsymbol{A}$. Now, if we consider, instead of ϕ^a, a field in $((\frac{1}{2}\otimes\bar{0})_-\otimes(\frac{1}{2}\otimes\bar{\frac{1}{2}})_+)$ of the complex Lorentz group, and instead of the α^a, a field in $((0\otimes\bar{\frac{1}{2}})_-\otimes(\frac{1}{2}\otimes\bar{\frac{1}{2}})_+)$, then both of these fields are coupled to an independent SU(2) connection. Consequently, there always exist solutions with $\boldsymbol{X}_a = 0$, apparently restoring the viable limit to standard gauge theory. However, this prescription is not without other repercussions as then the $\tilde{\boldsymbol{B}}$ is not a scalar but carries the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ charges from the anti-self-dual sector of the Lorentz group. Optimistically, this hints to the structure of the gravielectroweak theory and to the geometrization of the Higgs mechanism operated by the isokhronon in the hyperspacetime.

Scalar fields: Putting the above speculation aside, since the Standard Model features a Higgs scalar field, for completeness we take into account a scalar field ζ. In the first-order formulation, it is accompanied by a Lorentz vector z_a, and a possible action is

$$\boldsymbol{L}_M = z_a \star \mathbf{D}\phi^a \wedge \mathbf{D}\zeta + \left(\frac{1}{4} z_a z^a + U(\zeta)\right) \star 1, \quad (36)$$

leaving open the possibility of a nontrivial potential $U(\zeta)$. We obtain the EoM,

$$\boldsymbol{E}_\zeta = \mathbf{D}(z_a \star \mathbf{D}\phi^a) + U'(\zeta) \star 1, \quad (37a)$$

$$\boldsymbol{E}^a = \star\, \mathbf{D}\phi^a \wedge \mathbf{D}\zeta + \frac{1}{2} z^a \star 1, \quad (37b)$$

the symplectic contribution,

$$\boldsymbol{\Theta} = -z_a \star \mathbf{D}\phi^a \delta\zeta, \quad (38)$$

and the source current,

$$\boldsymbol{t}_a = i\epsilon_{abcd} z^b \boldsymbol{B}^{cd} \wedge \mathbf{D}\zeta - \left(\frac{1}{4} z_b z^b + U(\zeta)\right) \star \mathbf{D}\phi_a, \quad (39a)$$

while for scalar fields $\boldsymbol{O}_{ab} = 0$.

Cosmological constant: Perhaps the simplest energy source is a cosmological constant. The contribution to the matter action is given by a Lagrangian with two new fields, a scalar Λ and a three-form $\boldsymbol{\kappa}$,

$$\boldsymbol{L}_M = \frac{1}{2}\Lambda(\mathbf{d}\boldsymbol{\kappa} - \star 1). \quad (40)$$

The source contributions (4) is

$$\boldsymbol{t}^a = -\frac{1}{2}\Lambda \star \mathbf{D}\phi^a, \qquad \boldsymbol{O}^{ab} = 0. \quad (41)$$

The EoM for the twi fields dictate that $\mathbf{d}\boldsymbol{\kappa} \approx \star 1$ and $\mathbf{d}\Lambda \approx 0$. Thus $\boldsymbol{L}_M \approx 0$. In the derivation of the diffeomorphism Noether current, we have to take into account that now (16) does not hold. We obtain $\boldsymbol{J}_\xi = \Lambda * \xi/2$, so it would seem that the Λ does contribute. The nontrivial charge reflects the effective breaking of the longitudinal diffeomorphisms.

The three-form gauge symmetry $\boldsymbol{\kappa} \to \boldsymbol{\kappa} + \boldsymbol{k}$, where $\boldsymbol{k}$ is an arbitrary two-form, has a nontrivial charge that is given as the integral of $\boldsymbol{j}_{\boldsymbol{\kappa}} = \Lambda \boldsymbol{k}/2$ over a 2-surface.

Conclusion. Conserved charges lie at the heart of gauge theories. They characterize the observables of the theory and their algebra governs the structure of the theory. Charges are of paramount importance in holography and play a central role in (most approaches to) quantum gravity. In fact, the putative quantum theory might be entirely deduced from the charge algebra, according to the corner proposal and related current developments [44,45].

In this article we presented the physical charges in the new Lorentz gauge theory of spacetime and gravitation. The charges associated with the Lorentz symmetry and diffeomorphism symmetry are the direct extrapolation ($\mathbf{e}^a \to \mathbf{D}\phi^a$) of the results in Poincaré gauge theory. A novel feature is the "dark shadow matter" current $\boldsymbol{M}_a$ associated with the shift symmetry of the action (1).

The theory was coupled to the pregeometrized Standard Model of particle physics, and it was shown that its matter fields generate consistently both the energy-momenta and the angular momenta source currents. However, the most straightforward implementation of the Standard Model gauge fields inherits the issue in their usual, second-order geometric formulation, which does not consistently describe the gravitational sources by the canonical Noether currents. It has often puzzled theoreticians that the canonical energy-momentum currents have the wrong expression, unless modified by some of the proposed "improvements" [1,47,54–58]. We considered a possible modification of the pregeometric first-order theory, which would provide a solution to the issue, and features the newly suggested shadow charges,

[9] Indeed, we recover the gravity action with a cosmological constant when we set $\alpha^{a,bc} = \epsilon^{abcd}\phi_d$ and identify $\boldsymbol{A}$ with ${}^+\boldsymbol{\omega}$. In this sense, the actions for Yang-Mills fields and gravitation have a similar character. A perturbative hint of this similarity is already well known from the context of amplitudes, as the so called double copy structure [53], manifest in (3) vs (23).

associated with the shift symmetry of the first-order fields in the internal sector.

The modified theory is not yet a phenomenologically viable replacement of the Standard Model interactions (though it might describe hypothetical new interactions e.g. in cosmology), but it calls for the elaboration toward a more final theory. We conclude that the first-order action principle provides a new robust framework to negotiate the unification of internal and spacetime gauge interactions and the reconciliation of gravity and quantum mechanics.

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