Letter

Entropy of flat space cosmologies from celestial dual

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We construct a one-dimensional dual theory that effectively describes the sector of the $(2+1)$D flat gravity phase space near a flat space cosmology saddle labeled by definite mass and angular momentum. This Schwarzian-type action describes the dynamics of the (pseudo-)Goldstone bosons of BMS_3 algebra on a circle as the symmetry is spontaneously and anomalously broken. This 1D theory, living on the celestial circle, provides the first explicit construction of a celestial dual theory in $(2+1)$D. We use it to calculate the semiclassical entropy of flat space cosmologies and find perfect agreement with existing literature.

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Introduction. Understanding the holographic principle in flat spacetime is an intriguing problem; $(3+1)$D flat spacetime has been shown to have a 2D *celestial* conformal field theory (CFT) description in several scenarios: via holographic reduction [1]; recasting 4D scattering amplitudes in terms of CFT_2 correlators [2,3] from an asymptotic symmetry analysis [4–6]; interpreting bulk soft theorems as CFT_2 Ward identities [7,8] etc. This connection, which can be generalized as a correspondence between $(d+1)$-dimensional gravitational theories in flat space and $(d-1)$-dimensional CFTs, is in stark contrast with the more familiar $\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$ holographic principle. The complete description of a celestial dual theory for 4D flat space is however not understood yet. We pose the question for gravity in $(2+1)$D flat spacetime, which is a much simpler setup due to the absence of local gravitons. However, the gravity phase space is spanned by the asymptotic symmetry modes. The asymptotic symmetries form the BMS_3 group [9] consisting of 3D versions of *supertranslations* and *superrotations*, which is an infinite extension of Poincaré symmetry. There are also solutions resembling asymptotically flat black holes, called flat space cosmological (FSC) solutions.

One of the standard approaches towards flat space holography in 3D is to take a large radius limit of $\mathrm{AdS}_3/\mathrm{CFT}_2$, dubbed as the $\mathrm{BMS}_3/\mathrm{GCFT}_2$ correspondence [10,11]. Using an analog of the Cardy regime, the entropy of FSCs was calculated from this dual 2D description [12,13]. Another approach is to recast the 3D gravity theory as a 3D Chern-Simons theory, where the dual theory turns out to be a 2D Liouville-like theory [14–16]. These results suggest that the holographic dual must be a 2D theory lying on the null boundary of 3D flat spacetime, which is in apparent contradiction with the celestial holography program [17]. Our major goal would be to show that indeed $(2+1)$D asymptotically flat spacetimes can be endowed with a 1D celestial dual structure and understand the FSC thermodynamics from a 1D perspective. Thus in this regime, the effective 1D theory encapsulates all the details of the BMS_2 field theory.

In our earlier work [20], we have shown the emergence of an effective 1D Schwarzian theory dual to the subspace of $(2+1)$D asymptotically flat spacetimes, closed under superrotations. The computations rely on foliating these "superrotated" spacetimes into asymptotically $\mathrm{(A)dS}_2$ leaves and use Wedge holography [21] to obtain the dual description. Foliation of supertranslated spacetimes is however much more involved [22]. The superrotation symmetries in 3D form a Virasoro subgroup which is softly broken by the Schwarzian theory. This pattern can be seen in [23,24] where the holomorphic part of the CFT_2 partition function was realized as the path integral of a Schwarzian theory of pseudo-Goldstone modes corresponding to softly broken Virasoro symmetry.

In this work, we take a group theoretic approach [25,26] to incorporate supertranslations into the picture to describe the full phase space of the theory via a one-dimensional dual. This leads us to obtain an effective action for the (pseudo-)Goldstone modes of BMS_3 group. As we will see, the superrotation part of the dual theory is again a

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Schwarzian, agreeing with the results of [20]. The 1D theory is the anticipated celestial dual which effectively describes the dynamics of $(2+1)$D gravity near an FSC solution. To understand the validity of this proposal, in this work, we also compute the semiclassical entropy of 3D flat cosmological solutions from the 1D theory perspective and find agreement with [12]. This also implies that our 1D theory is able to capture the Cardy regime of a GCFT_2 [13].

Gravity in $(2+1)$D *and* BMS_3 *symmetry*. The general form of $(2+1)$D asymptotically flat spacetimes in leading order is [9]

$$ds^2 = \mathcal{M}(\theta)du^2 - 2dudr + (u\mathcal{M}'(\theta) + 2\mathcal{J}(\theta))dud\theta + r^2 d\theta^2. \tag{1}$$

The vector fields that leave the form of the above metric invariant up to leading order are called asymptotic Killing vectors and they form the infinite-dimensional BMS_3 algebra. These vectors are parametrized by arbitrary functions $T(\theta)$ and $Y(\theta)$ generating supertranslations and superrotations, respectively. Starting from the flat space solution $\mathcal{M} = -1/8G$ and $\mathcal{J} = 0$, a generic superrotation would generate "superrotated spacetimes" with nontrivial angle dependent $\mathcal{M}(\theta) = \{Y(\theta), \theta\}$ and $\mathcal{J} = 0$ [20]. Here, $\{Y(\theta), \theta\}$ denotes the Schwarzian derivative of $Y(\theta)$. The corresponding conserved charge is given by [27]

$$Q[\mathcal{M}, \mathcal{J}] = \frac{1}{16\pi G}\int d\theta(\mathcal{M}T + \mathcal{J}Y). \tag{2}$$

G is the 3D Newton constant. These charges, when endowed with a proper Dirac bracket, satisfy the 3D BMS (Bondi–van der Burg–Metzner–Sachs) algebra or $\mathfrak{bms}_3$. Written in terms of the modes of $\mathcal{M}$ and $\mathcal{J}$, the algebra takes the following form:

$$\begin{aligned}
[J_m, J_n] &= (m-n)J_{m+n} + \frac{c_1}{12}(m^3 - m)\delta_{m+n,0},\\
[J_m, M_n] &= (m-n)M_{m+n} + \frac{c_2}{12}(m^3 - m)\delta_{m+n,0},\\
[M_m, M_n] &= 0.
\end{aligned} \tag{3}$$

For pure gravity, $c_1 = 0$ and $c_2 = 3/G$. Nonzero c_1 arises if we add a Lorentz Chern-Simons term along with pure gravity.

Since pure gravity in $(2+1)$ dimensions does not contain local degrees of freedom, the only nontriviality comes from the boundary. Hence the functions $\mathcal{M}(\theta)$ and $\mathcal{J}(\theta)$ span the phase space of the theory. On the other hand, the vector field labeled by $T(\theta)$ and $Y(\theta)$ transforms the elements of this phase space into one another. These two sets of functions can then be thought of as elements of dual spaces where the inner product is given by the surface charge (2).

Superrotated spacetimes and their dual theory: In an earlier work [20], we have considered the part of the phase space consisting of superrotated spacetimes. Choosing an origin in the bulk, we may slice up these spacetimes into foliations of fixed proper time (distance). The region inside the light cone looks like warped product of $(\mathrm{A})\mathrm{AdS}_2 \ltimes \mathbb{R}$,

$$ds^2 = -d\tau^2 + \tau^2\left[d\tilde{\rho}^2 + \left(\frac{1}{4}e^{2\tilde{\rho}} - \frac{\mathcal{M}}{2} + \frac{\mathcal{M}^2}{4}e^{-2\tilde{\rho}}\right)d\theta^2\right], \tag{4}$$

where $(\mathrm{A})\mathrm{AdS}_2$ stands for asymptotically anti–de Sitter spacetimes in 2D in the sense of [28]. Similarly, the spacetime outside of the light cone has an asymptotically $\mathrm{dS}_2 \ltimes \mathbb{R}$ geometry. Next, we consider a wedge region in the future light cone bounded by two $(\mathrm{A})\mathrm{AdS}_2$ surfaces at τ_1 and τ_2 and perform a dimensional reduction of the pure gravity theory inside the wedge onto these boundaries via wedge holography [20,21,29]. The idea is to eventually cover the full spacetime by taking $\tau_1 \to 0$, $\tau_2 \equiv \tau_\infty \to \infty$ and also by doing the same procedure for dS slices. We find that the massless modes of the dimensionally reduced theory give us the nondynamical pure gravity action on the boundary $(\mathrm{A})\mathrm{AdS}_2$ slices, proportional to the Euler character of the manifold. This led us to the insight that if superrotation symmetry is unbroken, then there is no dynamics in this smaller subspace of phase space.

To get nontrivial dynamics, we break the superrotation symmetry by fluctuating the $(\mathrm{A})\mathrm{AdS}_2$ slices along the transverse direction, sourced by a scalar mode ϕ. This gives an effective Jackiw-Teitelboim (JT) theory on the $(\mathrm{A})\mathrm{AdS}_2$ slices, where ϕ plays the role of dilaton. Integrating the dilaton, we get a 1D Schwarzian theory on the celestial circle [20],

$$S_{1D} = \frac{\phi_r}{8\pi G_2}\int d\theta\mathcal{M}(\theta), \qquad G_2 \equiv \frac{G}{\tau_\infty}, \tag{5}$$

here ϕ_r is the boundary value of dilaton. The insight we get from this is that the bulk superrotation symmetry has a nonlinear realization in terms of Goldstone modes on the celestial circle. Thus to generalize this result for the full phase space of the 3D gravity theory, we need to understand the effective field theory of Goldstone modes corresponding to the full asymptotic symmetries. Around a classical solution of Einstein's equations, this effective theory would govern the semiclassical dynamics and would describe an explicit celestial holographic dual description. Below we describe the bulk solutions in brief detail along with their thermodynamic properties. We will see that these solutions and their properties emerge from the explicit 1D dual that we will construct later.

Flat space cosmologies: Among the asymptotically flat metrics, we will be particularly interested in FSCs, which are analogs of black holes in 3D flat space. They are parametrized by mass $M > 0$ and angular momentum J and their form is described below [12,30],

$$ds^2 = 8GMdu^2 - 2dudr + 8GJdud\theta + r^2 d\theta^2. \quad (6)$$

In the phase space of 3D gravity, the flat space is disconnected from the FSC solutions [30], much like the spectrum of Bañados-Teitelboim-Zanelli (BTZ) black hole and global AdS_3. The FSC solution has a cosmological horizon generated by the Killing vector $K = \partial_u + \Omega\partial_\theta$ and it is located at

$$r = r_C = \sqrt{\frac{2GJ^2}{M}}. \quad (7)$$

The FSC can be associated with angular velocity and inverse temperature given by

$$\Omega = -\frac{2M}{J}, \qquad T = \beta^{-1} = \frac{2}{\pi}\sqrt{\frac{2GM^3}{J^2}}. \quad (8)$$

The thermodynamics [31] of these solutions are well studied in the literature [12]. The Bekenstein-Hawking entropy is given by

$$S_{\mathrm{FSC}} = \frac{2\pi r_C}{4G} = \frac{\pi|J|}{\sqrt{2GM}}. \quad (9)$$

In [12], it was shown that the thermodynamics of these solutions can have a holographic interpretation similar to BTZ black holes in AdS_3. Here, the entropy can be derived from a suitable limit of the Cardy formula in CFT. This was also computed from a 2D perspective in [13] along the line of $BMS_3/GCFT_2$ correspondence. Logarithmic correction to the semiclassical entropy has also been calculated in [32]. To make this connection, the FSC solution is analytically continued to the Euclidean signature. The demand of a smooth geometry fixes the periodicity of the Euclidean time direction to β. Whereas, the periodicity of the angular direction gets fixed to $i\beta\Omega = i\Phi$, typical for rotating black holes [12].

The main aim of this paper is to compute the semiclassical entropy from an independent 1D celestial dual theory.

Pseudo-Goldstone bosons of BMS_3. To construct the 1D dual theory of pseudo-Goldstone modes of BMS_3, we would extensively use the alternate representation of $\mathfrak{bms}_3$ algebra in terms of vector fields on a circle [25,26,33]. The superrotations of bulk act on the circle as diffeomorphisms, which form the Virasoro subgroup of the centrally extended BMS_3 group. Infinitesimal diffeomorphisms are generated by the vector fields on the circle, which span the adjoint space of Virasoro group. The BMS_3 group is the centrally extended *exceptional* semidirect product of the group of superrotations and the Abelian group of vector fields corresponding to supertranslations, which are acted on by the Virasoro group according to the adjoint action. The elements of the BMS_3 group are labeled as $(f, \lambda; \alpha, \mu)$ where (f, λ) constitutes the superrotation subgroup and (α, μ) constitutes the supertranslation subgroup. λ and $\mu \in \mathbb{R}$ are the central extensions. The infinitesimal transformations are generated by the elements of the corresponding Lie algebra $\mathfrak{bms}_3$. The dual space of the Lie algebra, on which the BMS_3 group has a coadjoint action, is the space of *quadratic densities*, whose conformal transformation properties resemble that of a weight-2 primary operator in CFT. The coadjoint elements are labeled by $(j, c_1; p, c_2)$, where (j, c_1) corresponds to the "stress tensor" dual to superrotation and (p, c_2) is a "weight-2 current" dual to supertranslation.

We begin by defining the states in the Hilbert space of the dual theory. Since $(j(\theta), p(\theta))$ are the stress tensor equivalents for BMS_3, we should label the states on the Hilbert space via the mutually commuting zero modes of these two fields. This precisely follows [11] where a similar description comes from considerations of Galilean conformal algebra, $\mathfrak{gca}_2$. Thus the states are labeled as $|h_j, h_p\rangle$ such that

$$\langle h_j, h_p|j(\theta)|h_j, h_p\rangle = h_j, \qquad \langle h_j, h_p|p(\theta)|h_j, h_p\rangle = h_p. \quad (10)$$

The modes of the fields $(j(\theta), p(\theta))$ satisfy the $\mathfrak{bms}_3$ algebra (3) when the zero modes are shifted accordingly. Thus in the holographic setup, these zero modes are related to the mass M and angular momentum J of the corresponding bulk FSC as $J = h_j + \frac{c_1}{24}, M = h_p + \frac{c_1}{24}$.

Now suppose we have a state labeled by (h_j, h_p) on the dual theory. For large charges $h_j, h_p \gg c_1, c_2$, i.e. in the "Cardy regime," the state has a large degeneracy. Hence, this can be thought of as an ensemble labeled by a finite temperature and a chemical potential, which arise due to the stress tensor and the weight-2 current respectively. This finite energy density of these states breaks the BMS_3 symmetry spontaneously into the global Poincaré subgroup, giving rise to an infinite number of massless Goldstone modes. However, BMS symmetry is also anomalous due to the presence of the central charges. Thus we expect that the symmetry is nonlinearly described in this part of the Hilbert space via the pseudo-Goldstone Bosons, where the Goldstone modes acquire small mass sourced by the central charges. This idea imitates [23] where the authors describe the holomorphic sector of a CFT near a finite energy state in terms of pseudo-Goldstone modes.

To identify the pseudo-Goldstone modes, we start from our chosen state $|h_j, h_p\rangle$ and do a small BMS_3 transformation labeled by $(f(\theta), \lambda; \alpha(\theta), \mu)$ such that the transformed state still belongs to the same region of the Hilbert space in appropriate sense. Then the expectation values of the charges change according to the coadjoint action of the BMS_3 group [25,26,33],

$$\langle p(\theta)\rangle = h_p[f'(\theta)]^2 - \frac{c_2}{12}S[f](\theta), \tag{11}$$

$$\langle j(\theta)\rangle = h_j[f'(\theta)]^2 - \frac{c_1}{12}S[f](\theta) - \alpha(f(\theta))\left(2f''(\theta)h_p - \frac{c_2}{12}\frac{S[f]'(\theta)}{f'(\theta)}\right). \tag{12}$$

Here, $S[f](\theta) = -\frac{1}{2}(\frac{f''}{f'})^2 + (\frac{f''}{f'})'$ is the Schwarzian derivative of f. Let us look at the transformation properties closely. The central terms of the BMS transformation do not affect the transformations. The supertranslation parameter α does not appear at all in the transformation (11). Under pure superrotation ($\alpha = 0$), both the charges transform like holomorphic weight-2 operators of CFT. If we restrict ourselves to the superrotation subsector of the asymptotic symmetries, we only get nontrivial transformation for the supertranslation charge in pure Einstein gravity ($c_1 = 0$). This feature will essentially connect the 1D dual theory obtained here with the results of [20].

The main idea is to promote these transformation parameters $f(\theta)$ and $\alpha(\theta)$ to local quantum fields. In fact, these are the Goldstone bosons of BMS_3. Then if we enlarge the part of the Hilbert space near the state $|h_j, h_p\rangle$, we can promote the expectation values (11) and (12) to operator identities. From the knowledge of the conserved charges, we can write down an effective action on the celestial circle,

$$I = -\frac{\beta}{2\pi}\int_0^{2\pi} d\theta\left([f'(\theta)]^2 h_p - \frac{c_2}{12}S[f](\theta)\right) - \frac{\Phi}{2\pi}\int_0^{2\pi} d\theta\left[(f'(\theta))^2 h_j - \frac{c_1}{12}S[f](\theta) - \gamma(\theta)\left(2f''(\theta)h_p - \frac{c_2}{12}\frac{S[f]'(\theta)}{f'(\theta)}\right)\right], \tag{13}$$

where $\gamma \equiv \alpha \circ f$ acts as a Lagrange multiplier. The coupling constants β and Φ are still arbitrary but they will soon be related to bulk quantities as suggested by the notations.

We propose that the above action captures the gravitational dynamics near a flat space cosmological solution. As a consistency check, let us return to the case of purely superrotated spacetimes in Einstein gravity. For these spacetimes $h_j = 0, c_1 = 0$ and also γ is absent. We rescale the field $f(\theta) \longrightarrow \sqrt{\frac{24h_p}{c_2}}f(\theta)$ and then define $Y(\theta) = \tanh(f(\theta)/2)$ such that the action becomes

$$I_s = \frac{\beta}{8\pi G}\int_0^{2\pi} d\theta\{Y(\theta), \theta\}. \tag{14}$$

We exactly get the form of (5) back given that the parameter β and the dilaton boundary value ϕ_r are related appropriately. In the next section we use the action (13) to calculate the bulk thermodynamic properties providing further evidence of its validity.

Properties of the effective action. Our claim is that the path integral corresponding to the celestial dual theory is equivalent to the 3D gravity path integral around an FSC saddle. To check the validity of this statement, our preliminary goal would be to understand whether we can map 3D gravity solutions in this 1D theory language. Then we would like to obtain the Bekenstein-Hawking entropy of FSCs, which are labeled by mass M and angular momentum J. These saddle points belong to different coadjoint orbits of the BMS_3 group.

1D representative of FSC saddle: Let us start with the state $|h_j = 0, h_p = 0\rangle$, which corresponds to the trivial flat space cosmology with zero temperature and zero angular velocity. Then we would try to evaluate the representative state at finite eigenvalues. To do so, locally we will think of BMS_3 as the following set of infinitesimal transformations,

$$\theta \to \theta + \epsilon(\theta), \quad x \to x + g(\theta); \quad g(\theta) \equiv \frac{\alpha(\theta)}{\epsilon(\theta)}, \tag{15}$$

where θ is the coordinate along the circle on which diffeomorphism acts. The interesting point to note is that the transformation of the parameter x depends only on the coordinate of the circle; i.e. we have an angle-dependent translation along the x direction. To understand the implications of such an auxiliary direction, we can start from the usual 2D notion of BMS_3 group where it transforms the coordinate along null infinity $u \to u + u\epsilon'(\theta) + \alpha(\theta)$ and then define x via $u \equiv \epsilon(\theta)x$. Note that we have already discussed that both $\alpha(\theta)$ and $\epsilon(\theta)$ are generated by vector fields on S^1. Hence, the transformation properties of $g(\theta)$ dictate that it is a function or a density of weight zero. The effective 1D description comes because the transformation parameters depend only on the coordinate on the circle and the u dependence is trivial. Hence, all the data available at a particular constant u-slice can be trivially evolved to any other slice. Thus from a Kaluza-Klein perspective, we can *exactly* interpret the 2D coordinate transformations as coordinate and gauge transformations in 1D. The transformations (15) exactly do this job. Now we can think of the direction x to be trivially parametrizing the fiber on the base manifold S^1 and null infinity can essentially be thought of as the principal bundle.

Then we can consider a finite version of the above transformations by exponentiating the infinitesimal ones,

$$\theta \to f(\theta), \qquad x \to x + G(\theta). \tag{16}$$

To go to the finite parameter saddle from the $h_j = h_p = 0$ vacuum, we would like to put certain periodicity conditions on the circle and the fiber [34,35]. Thus we have the following BMS_3 transformations:

$$f(\theta) = e^{\frac{2\pi\theta}{\Phi}}, \quad G(\theta) = -\frac{\beta}{\Phi}\theta, \quad \Rightarrow \alpha \circ f(\theta) = -\frac{2\pi\beta}{\Phi^2}\theta f(\theta), \tag{17}$$

where we get the identification $(\theta, x) \sim (\theta + i\Phi, x - i\beta)$ and the conditions $f(\theta + i\Phi) = f(\theta) + i\Phi$ and $G(\theta + i\Phi) = G(\theta)$. We can put these forms back in (11) and (12) to get the following relations:

$$h_p = \frac{c_2 \pi^2}{6\Phi^2}, \qquad h_j = \frac{c_1 \pi^2}{6\Phi^2} - \frac{c_2 \pi^2}{3}\frac{\beta}{\Phi^3}. \tag{18}$$

For a cosmological solution with $M, J \gg c_2$, and for pure gravity $c_1 = 0$. So, from above equation, it follows that for pure gravity,

$$M = \frac{c_2 \pi^2}{6\Phi^2}, \qquad J = -\frac{c_2 \pi^2}{3}\frac{\beta}{\Phi^3}. \tag{19}$$

These match with [12] and also justify our notation of the parameters β and Φ as inverse temperature and angular velocity. Having correctly identified the 3D saddles from a 1D perspective, we will now show the computation of the thermodynamic variables using the 1D theory (13), when we identify the couplings with these β and Φ.

Entropy of flat space cosmologies: With the information of the coadjoint orbits at hand, we may now write down the path integral of the 1D theory formally as

$$Z = \int \mathcal{D}f\mathcal{D}\alpha e^{-I}, \tag{20}$$

with I given by (13) with the couplings identified with the "inverse temperature" and "angular velocity" parameters obtained in the earlier part of this section. To evaluate this partition function completely would require us to understand the measure in the phase space. We leave this complete analysis for a later work and instead focus here on a semiclassical computation.

In the large charge limit, the path integral can be approximated by saddle point contributions only. For this, we would consider the solutions to the equations of motion of the action (13). The Lagrange multiplier field $\gamma = \alpha \circ f$ puts a constraint on f given as

$$2f''(\theta)h_p - \frac{c_2}{12}\frac{S[f]'(\theta)}{f'(\theta)} = 0. \tag{21}$$

The solutions to this equation are of the form $f = A\theta$. The solution for γ comes from the equation of motion of the field f. γ being a Lagrange multiplier, its explicit form does not appear in the construction. Under saddle point approximation, the partition function is given by the on shell action for $f = A\theta$ such that

$$\log Z = -I_{\text{on shell}} = -A^2 \frac{c_2 \pi^2}{6}\frac{\beta}{\Phi^2} + A^2 \frac{c_1 \pi^2}{6\Phi}. \tag{22}$$

From the definition of charges, i.e. $-\partial_\beta \log Z = h_p$ and $-\partial_\Phi \log Z = h_j$, we fix the constant $A = 1$. Therefore, we have identified a solution in the 1D theory that is the representative of the FSC solution in 3D gravity.

Now we consider the entropy that is given by the Laplace transform of partition function [36],

$$e^{-S(h_j, h_p)} = \int d\beta d\Phi e^{\beta h_p + \Phi h_j} Z(\beta, \Phi). \tag{23}$$

Under saddle point approximation, we have $-S(h_j, h_p) = \beta h_p + \Phi h_j + \log Z$, where β, Φ take the thermodynamic values. Using (18) and $h_p \approx M, h_j \approx J$, we get the semiclassical entropy,

$$S = \frac{\pi}{\sqrt{6}}\left(\sqrt{\frac{c_2}{M}}J + c_1\sqrt{\frac{M}{c_2}}\right). \tag{24}$$

Our result is in perfect agreement with the galilean conformal field theory (GCFT) computation in [13,32]. For pure gravity, we have $c_1 = 0$ and $c_2 = 3/G$, such that

$$S = \pi \frac{|J|}{\sqrt{2GM}}. \tag{25}$$

This agrees with the semiclassical entropy of FSC [12].

Discussions. In this work, we have found a 1D theory (13) of pseudo-Goldstone modes corresponding to the spontaneously and anomalously broken BMS_3 symmetry. This theory lies on the celestial circle of flat space. To arrive at this theory, we have considered the transformation properties of supertranslation and superrotation charges under the action of finite BMS_3 transformations. We have identified a solution in this 1D theory that represents the 3D FSC solution with definite mass and angular momentum. Using saddle point approximation to compute the path integral corresponding to the 1D theory, we have obtained the semiclassical entropy for this 1D solution. This perfectly agrees with the Bekenstein-Hawking entropy of the 3D FSC [12] in Einstein gravity. The 1D theory also captures the Cardy regime of a $GCFT_2$ [13]. This generalizes the relation between Carollian and celestial approaches to flat space holography [18,19] in lower dimensions.

In an earlier work [20], we obtained a Schwarzian theory on the celestial circle describing the dynamics of superrotation modes. We arrived at this theory via holographic reduction of 3D Einstein theory to JT gravity on a cutoff (A)dS surface and eventually using JT-Schwarzian duality. We observed that the boundary theory is an integration of the supertranslation current, which is a Schwarzian derivative of the reparametrization mode of the boundary. This theory also depends on two arbitrary parameters τ_∞ and ϕ_r. Here τ_∞ is the choice of cutoff surface and ϕ_r governs the boundary behavior of the fluctuation in the location of this cutoff surface. As pointed out earlier, from (11) we understand how supermomentum is blind to the supertranslation transformations. To compare with pure Einstein theory we consider $c_1 = 0$ and $c_2 = 3/G$. Considering the action (13) of our current work, we identify that the first part of the action coming from $p(\theta)$ is what we have obtained via dimensional reduction. This matching can be obtained by fixing the parameters τ_∞ and ϕ_r to appropriate values, which were arbitrary in the context of our earlier work.

Therefore, we have explicitly given an example of "celestial holography" for 3D gravity with zero cosmological constant. Part of this theory was obtained via holographic reduction as well. Our theory correctly reproduces the semiclassical entropy of 3D FSC, which further validates the claim. We would like to compute the logarithmic corrections to FSC entropy [32] from the 1D theory in a future work.

We are interested in computing the complete one-loop path integral of this 1D theory, which can potentially capture the 3D gravity path integral around an FSC saddle. The 1D theory is a Schwarzian coupled to a Lagrange multiplier field, thus it would be intriguing to analyze whether there is a connection between this 1D effective theory and BMS_3 invariant matrix models [37] along the line of Schwarzian and Hermitian matrix model duality [38]. It would be interesting to properly compare the 2D holographic notion and our 1D approach with other observables. In this context, the geometric action on BMS_3 coadjoint orbits and their connection to Liouville-like theories could be helpful [39,40].

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