Letter

From the Horndeski action to the Callan-Giddings-Harvey-Strominger model and beyond

Susobhan Mandal,* Tausif Parvez,† and S. Shankaranarayanan‡

Department of Physics, Indian Institute of Technology Bombay, Mumbai 400076, India

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The knowledge of what entered a black hole (BH) is completely lost as it evaporates. This contradicts the unitarity principle of quantum mechanics and is referred to as the information loss paradox. Understanding the end stages of BH evaporation is key to resolving this paradox. As a first step, we need to have exact models that can mimic 4D BHs in general relativity in classical limit and have a systematic way to include high-energy corrections. While there are various models in the literature, there is no systematic procedure by which one can study high-energy corrections. In this work, for the first time, we obtain Callan, Giddings, Harvey, and Strominger (CGHS)—a $(1+1)$-D—model from 4D Horndeski action—the most general scalar-tensor theory that does not lead to Ostrogradsky ghosts. We then show that 4D Horndeski action can systematically provide a route to include higher-derivative terms relevant at the end stages of black hole evaporation. We derive the leading order Hawking flux while discussing some intriguing characteristics of the corrected CGHS models. We compare our results with other works and discuss the implications for primordial BHs.

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Introduction. The observation of the merging black holes (BHs) by LIGO-VIRGO served as an extraordinary validation of the tenets of general relativity (GR). However, it remains unclear whether BHs adhere to the same principles of quantum mechanics (QM) as all other known objects [1,2]. Classically, BHs are perfect absorbers but cannot emit anything; their physical temperature is absolute zero, and their entropy is infinite [3–7]. Nevertheless, once QM is applied, the BH evaporates by emitting Hawking radiation that carries *no information* about their microstates [8]. This conundrum known as *BH information paradox* has remained unresolved for four decades [9–20].

Most BHs detected by LIGO-VIRGO are heavier than the previously known population of stellar-mass BHs that were indirectly inferred from EM observations [21,22]. The anomalously heavy BHs detected by LIGO have renewed interest in primordial black holes (PBHs) [23–25]. Current constraints suggest the PBHs in mass windows 10^{17}–10^{23} gm are potential dark matter candidates [26–29]. Interestingly, Hawking radiation is significant for these masses, and Hawking temperature [8,30,31]

$$T_{\mathrm{H}} = \left(\frac{\hbar c^3}{G k_B}\right)\frac{1}{8\pi M} \sim 10^{-7}\left(\frac{M_{\odot}}{M}\right) K \tag{1}$$

is large. Note $k_B(G)$ is Boltzmann (4D Newton's) constant, and M is the BH mass. Higher-order curvature terms must be considered since smaller masses experience substantial tidal forces. For Schwarzschild BH, this is evident from Kretschmann scalar:

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \propto (GM)^{-4}. \tag{2}$$

From the above two expressions, we see that as the BH mass decreases due to evaporation, the Hawking temperature and the strength of the Kretschmann scalar near the horizon increase rapidly. This clearly shows that the evaporation of small mass PBHs whose mass ranges from $10^{-15} - 10^{-5} M_{\odot}$ necessitates including higher-order curvature terms and hence requires one to go beyond GR [32]. The accelerated emission process for smaller mass BHs leads to a cataclysmic release of radiation, annihilating the BHs [8,30,31].

The evolution of radiating BH requires the theory of quantum gravity, which remains elusive. Due to the nonlinear nature of gravity, semiclassical gravity fails at the Planck length scale or involving singularities [33,34]. Hence, accurate characterization of the ultimate phase of BH evaporation within a semiclassical framework is unattainable. However, the situation is not entirely pessimistic; *a glimmer of hope exists*. Numerical results of 4D BHs show around 90% of Hawking radiation is in s waves [30,31]. Thus, restricting to s waves captures almost all the essential physics of BH radiation [35,36].

For 4D spherically symmetric space-times, the s wave corresponds to picking out the t-r plane and ignoring (or

*sm12ms085@gmail.com
†214120002@iitb.ac.in
‡shanki@iitb.ac.in

integrating) the two angular degrees of freedom—*effective 2D gravity*. However, 2D gravity must be distinct from GR since the Einstein tensor is topologically trivial. The simplest possible modification to obtain dynamics in 2D is via direct coupling of the Ricci scalar and scalar (dilaton) field [37–48]. Such a 2D gravity can be obtained by either treating the system as $D=4$ and imposing spherical symmetry in the equations of motion [49] or imposing spherical symmetry in the action [37,38,50]. Both approaches are classically equivalent.

Given this, we ask: Can we study the modifications to Hawking radiation in s waves due to the higher-order curvature corrections? If yes, what modifications must be included, and how can the comparison be made? In this work, we address these issues within the framework of the CGHS model [40,51] described by the action:

$$S_{\mathrm{CGHS}} = \int \frac{d^2x}{4\pi}\sqrt{-g}\, e^{-2\varphi}[\mathcal{R} + 4\nabla_\alpha\varphi\nabla^\alpha\varphi + 4\lambda^2], \quad (3)$$

where, φ is the dilaton field, λ^2 is *cosmological constant*, and $\mathcal{R}$ is the 2D Ricci scalar. The above action has an exact classical BH solution and hence, is a useful toy model for studying BH thermodynamics. The standard calculations reveal Hawking radiation with a temperature of $\lambda/2\pi$. It was shown that the effective action including the back-reaction of the Hawking radiation gives the evaporating BH analogy [42].

However, the CGHS action (3) cannot be derived from a 4D gravity action. This has been a key obstacle in generalizing the CGHS model by including higher-order curvature corrections and obtaining corrections to Hawking radiation at the late stages of evolution. *This work fills this void by obtaining the above CGHS action from 4D Horndeski gravity* [52–54]. We explicitly show that the spherical reduction of the lowest order terms in 4D Horndeski action identically leads to the above action. We then systematically obtain the higher curvature corrections to CGHS action and study the implications. In particular, we study two models to discuss the qualitative features of Hawking radiation and compare them with the earlier results [55,56].

4D gravity to CGHS. Let us consider the following Horndeski action [52–54]:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g^{(4)}}[G_2(X,\Phi) + G_4(\Phi)R^{(4)} - G_3(X,\Phi)\Box\Phi - G_5(\Phi)G^{(4)}_{\mu\nu}\nabla^\mu\Phi\nabla^\nu\Phi], \quad (4)$$

where $R^{(4)}$ is the 4D Ricci scalar corresponding to the metric $g^{(4)}_{\mu\nu}$, Φ is the mass-dimension zero scalar field, G_i's are functions of Φ,[1] and G_2 and G_3 are also functions of $X \equiv -\frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi$. In the above action, the last two terms contain higher-derivatives. First, we set

$$G_2 = \Omega(\Phi)X - V(\Phi); \qquad G_3 = G_5 = 0; \qquad G_4 = \Phi. \quad (5)$$

To do spherical reduction, we choose the metric ansatz:

$$ds^2 = g_{AB}dx^Adx^B + \rho^2(\{x^A\})(d\theta^2 + \sin^2\theta d\phi^2), \quad (6)$$

where (θ,ϕ) are the spherical polar coordinates, ρ acts as a dynamical radial coordinate that depends on x^A, and $A=0$, 1. For the above line element, we have

$$R^{(4)} = \mathcal{R} + 2(1 + \nabla_\mu\rho\nabla^\mu\rho - \Box\rho^2)/\rho^2, \quad (7)$$

where the covariant derivative is defined *with respect to* the 2D metric g_{AB}. Substituting Eq. (7) in action (4), we get

$$S = \frac{1}{4G}\int d^2x\sqrt{-g}[\rho^2\Phi\mathcal{R} + 2\Phi + 2\Phi\nabla_\mu\rho\nabla^\mu\rho - 2\Phi\Box\rho^2 + \Omega(\Phi)X - V(\Phi)]. \quad (8)$$

Since the above action has two scalar fields (Φ and ρ), we must choose a relation between the two scalars that will lead to CGHS action (3). Substituting

$$\Omega(\Phi) = -6/\Phi; \qquad V(\Phi) = A\Phi + B\Phi^3; \qquad \Phi = \Lambda/\rho, \quad (9)$$

in action (8), we have

$$S = \frac{\Lambda}{4G}\int d^2x\sqrt{-g}\left[\rho\mathcal{R} + \frac{2-B\Lambda^2}{\rho} + \rho[\nabla\log\rho]^2 - A\rho\right],$$

where Λ, A, and B are dimensionful constants. Redefining $\rho = \Lambda e^{-2\varphi}$ in the above action leads to

$$S = \frac{\Lambda^2}{4G}\int d^2x\sqrt{-g}\left[e^{-2\varphi}[\mathcal{R} + 4(\nabla\phi)^2 - A] + \left[\frac{2}{\Lambda^2} - B\right]e^{2\varphi}\right].$$

Setting

$$A = -4\lambda^2 \quad \text{and} \quad B = 2/\Lambda^2, \quad (10)$$

in the above action leads to the following:

$$S = (\Lambda^2\pi/G)\, S_{\mathrm{CGHS}}. \quad (11)$$

This is the first key result of this work, regarding which, we want to stress the following points: First, up to an overall

[1]The last term in Eq. (4) is different compared to Ref. [54] by a boundary term.

constant factor, the above reduced action is identical to the CGHS action (3). Since Λ is an arbitrary constant (not related to the cosmological constant), setting $\Lambda^2 = G/\pi$, we get CGHS action, including the factors. To our knowledge, this is the first time such a mapping has been established. Second, the reader might consider this approach to be contrived. As shown in Sec. I in the Supplemental Material [57], obtaining the CGHS action from a dimensional-reduced Einstein-Hilbert action in arbitrary dimensions is impossible. The result is also valid for conformally related 4D spherically symmetric spacetimes. The analysis can be extended to different topologies like $S^m \times S^n$, $(m, n \in \mathbb{Z}^+)$, and it can be shown that the Einstein-Hilbert action cannot lead to CGHS. This implies that a pure metric theory of gravity in arbitrary dimensions via dimensional reduction cannot lead to CGHS action. As shown above, the CGHS action *can only* be obtained from the dimensional reduction of scalar-tensor gravity theory [58–62], which can be interpreted as a time-varying gravitational "constant" represented by a scalar field φ. Third, an attentive reader might identify that the Horndeski action (4) in the limit $G_3 = G_5 = 0$ corresponds to Brans-Dicke theory [58–62]. It is interesting to note that CGHS action (3) is equivalent to 2D Brans-Dicke theory. To see this, redefining the dilaton field (φ) as $\Psi = e^{-2\varphi}$ in the CGHS action (3), we get

$$S_{\mathrm{CGHS}} = \frac{1}{4\pi}\int d^2x\sqrt{-g}\left[\Psi\mathcal{R} + \frac{1}{\Psi}\nabla_\mu\Psi\nabla^\mu\Psi + 4\lambda^2\Psi\right]. \tag{12}$$

The above action maps to 2D Brans-Dicke action exactly for $\Omega(\Psi) = -1/\Psi$ and $V(\Psi) = -4\lambda^2\Psi$.

Fourth, since CGHS action is obtained from 4D Brans-Dicke theory, the spherically symmetric solutions of Brans-Dicke theory are solutions of the CGHS action [63]. Also, as shown first by Hawking [64] and generalized by Faraoni and Sotiriou [65], a stationary spherically symmetric solution as the end state of collapse for a large class of scalar-tensor theories of gravity isolated is the same as that of general relativity. Hence, the static line element (6) is that of GR. Lastly, the above mapping provides a route to introduce higher-order curvature corrections relevant at the end stages of BH evaporation. It is well known that higher curvature terms introduce higher-derivative terms, leading to Ostrogradski instability [66–68]. However, the Horndeski action (4) is the most general scalar-tensor gravity action that leads to second-order equations of motion and hence does not possess Ostrogradski instability [52–54]. Thus, the spherical reduction of Horndeski action (4) will contain higher curvature corrections without leading to any instability and can provide crucial insights about the late stages of Hawking radiation. As G_3 and G_5 [in action (4)] are unknown, many ways exist to include higher-curvature corrections to CGHS. In the rest of this work, we consider two specific forms that indicate possible effects of higher curvature corrections at the Hawking flux.

Beyond classical CGHS model. To go beyond CGHS, we need to switch on the coefficients G_3 and G_5 in the action (4). Specifically, we set

$$G_3(X,\Phi) = Xf(\Phi); \qquad \Omega(\Phi) = -6/\Phi + h(\Phi)/\Lambda^2, \tag{13}$$

and G_5 is an arbitrary function of Φ. Using the metric ansatz (6), making the spherical reduction and substituting the relations, $\Phi = \Lambda/\rho$, $\rho = \Lambda e^{-2\varphi}$, the action (4) reduces to

$$\begin{aligned} S = {}& \frac{\Lambda^2\pi}{G}S_{\mathrm{CGHS}} - \frac{\Lambda^2}{4G}\int d^2x\sqrt{-g}\Bigg[\frac{4h(\Phi)}{\Lambda^2}X_\varphi \\ & - 8e^{2\varphi}f(\Phi)X_\varphi[\Box\varphi - 4X_\varphi] - G_5(\Phi)\Bigg[16\nabla_\mu\nabla_\nu\varphi\nabla^\mu\varphi\nabla^\nu\varphi \\ & + \frac{8}{\Lambda^2}e^{4\varphi}X_\varphi + 64X_\varphi^2 + 32X_\varphi\Box\varphi\Bigg]\Bigg], \end{aligned} \tag{14}$$

where $X_\varphi \equiv -\frac{1}{2}\nabla_\mu\varphi\nabla^\mu\varphi$. Section II in Ref. [57] contains the detailed derivation of the above action.

As can be seen, the corrections to CGHS contain three unknown coefficients—$f(\Phi)$, $h(\Phi)$ and $G_5(\Phi)$. We consider two simple forms of these three functions, which leads to two classes of CGHS corrected models:

$$\begin{aligned} &\textbf{Model 1}\ G_5 = \mathcal{G}; \qquad h(\Phi) = 2\Phi^2\mathcal{G}; \qquad f(\Phi) = -\frac{6\mathcal{G}}{\Phi} \\ &\textbf{Model 2}\ G_5 = \frac{\mathcal{G}}{\Phi}; \qquad h(\Phi) = 2\Phi\mathcal{G}; \qquad f(\Phi) = -\frac{6\mathcal{G}}{\Phi^2}, \end{aligned} \tag{15}$$

where $\mathcal{G}$ is a constant. Varying the above action with respect to the dilaton field (φ) and the 2D metric leads to

$$\begin{aligned} \partial_\pm^2\varphi &= 2\partial_\pm\omega\partial_\pm\varphi - 2\epsilon_i G_5[\Phi(\varphi)]e^{2(\varphi-\omega)}(\partial_\pm\varphi)^3\partial_\mp\varphi \\ \partial_+\partial_-\varphi &= \frac{\lambda^2 e^{2\omega}}{2} + 2\partial_+\varphi\partial_-\varphi + \epsilon_i G_5[\Phi(\varphi)]e^{2(\varphi-\omega)}(\partial_+\varphi\partial_-\varphi)^2 \\ \partial_+\partial_-\omega &= \frac{\lambda^2}{2}e^{2\omega} + 2\partial_+\varphi\partial_-\varphi[1 + \epsilon_i\lambda^2 G_5[\Phi(\varphi)]e^{2\varphi}] \\ &\quad + C_i\epsilon_i G_5[\Phi(\varphi)]e^{2(\varphi-\omega)}(\partial_+\varphi\partial_-\varphi)^2, \end{aligned} \tag{16}$$

where $i = 1, 2$ corresponding to the two models,

$$C_1 = 10, \quad C_2 = 7; \quad \epsilon_1 = 2048G\lambda^2/\mathcal{G}, \quad \epsilon_2 = 2560G\lambda^2/\mathcal{G}.$$

In the above equations, the 2D line element is set to be

$$ds^2 = -e^{2\omega(x_+,x_-)}dx_+dx_- = -e^{2\tilde{\omega}(\sigma_+,\sigma_-)}d\sigma_+d\sigma_-, \tag{17}$$

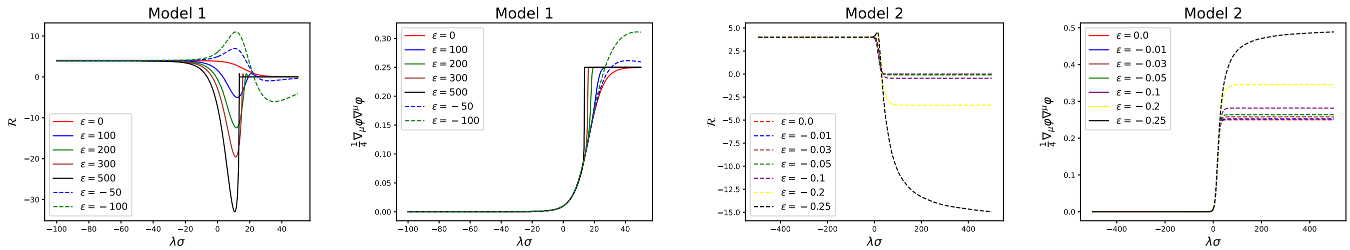

FIG. 1. Numerical solution of the 2D Ricci scalar and X_φ for different values of ϵ for Model 1 and Model 2.

where $x_\pm, \sigma_\pm$ are the null coordinates. Sections III and IV in Ref. [57] contain the details of the derivation.

For static configurations, the above expressions can be written in a compact form by rewriting in terms of X_φ (see Sec. V in Ref. [57]):

$$\frac{\partial_\sigma X_\varphi}{\partial_\sigma \varphi} = 1 + 2X_\varphi - \frac{\epsilon_i}{2}\frac{G_5(\Phi)\Phi}{\mathcal{G}}X_\varphi^2; \qquad (\partial_\sigma\varphi)^2 = -2e^{2\omega}X_\varphi$$
$$\partial_\sigma^2\omega = -4e^{2\omega}\left[\frac{1}{2} + X_\varphi\left[1 + \epsilon_i\frac{G_5(\Phi)\Phi}{\mathcal{G}}\right] + \frac{c_i\epsilon_i}{4}\left[\frac{G_5(\Phi)\Phi}{\mathcal{G}}\right]X_\varphi^2\right], \tag{18}$$

where $\sigma = (\sigma^+ - \sigma^-)/2$ and $\lambda x^\pm \to \pm e^{\pm\lambda\sigma^\pm}$ [40]. The above equations form a set of coupled differential equations of φ, X_φ, and ω. Before we proceed with the solution, we want to mention the following key points: First, equations of motion of the CGHS and the two models possess shift symmetry—$\varphi \to \varphi + c_0$ where c_0 is a constant. Note that the action (14) is not invariant under this symmetry. This is analogous to the scaling symmetry (of the coordinates) of a simple harmonic oscillator. This is a symmetry of the EOM of the harmonic oscillator but not of the Lagrangian. Second, the shift symmetry corresponds to $\rho \to e^{-2c_0}\rho$. Specifically, the solutions to the above equations for a given $\rho(x_-, x_+)$ will lead to an infinite number of identical solutions scaled by e^{2c_0}.

It is challenging, if not impossible, to obtain exact and analytical solutions for these coupled nonlinear differential equations. As we show below, the numerical solutions with high accuracy can provide crucial insights into the behavior of the classical solutions. In Fig. 1, we plotted the solutions of field equations as a function of σ for different values of ϵ for Models 1 and 2, respectively. Specifically, we have plotted 2D Ricci scalar ($\mathcal{R}$) and the kinetic term of the dilaton field ($-X_\varphi$).

From Fig. 1, we infer the following salient features for Model 1: First, the 2D Ricci scalar and X_φ saturate at spatial infinity for various positive values of ϵ as $\sigma \to \infty$. Second, it is interesting to note that the saturated values do not change even when changing the values of the ϵ parameter. Third, to compare with the analytical expressions for CGHS, we have plotted these quantities for CGHS model ($\epsilon = 0$). Our numerical results match with the analytical expression obtained in Ref. [69]:

$$\nabla_\mu\varphi\nabla^\mu\varphi = -\frac{\lambda^4 x^+ x^-}{M/\lambda - \lambda^2 x^+ x^-} = \frac{\lambda^2 e^{2\lambda\sigma}}{M/\lambda + e^{2\lambda\sigma}}$$
$$\mathcal{R} = \frac{4M\lambda^2}{M - \lambda^3 x^+ x^-} = \frac{4M\lambda^2}{M + \lambda e^{2\lambda\sigma}}. \tag{19}$$

Thus, our numerical results provide correct results for all values of $\lambda\sigma$ and show that the corrections to CGHS do not lead to any divergences. Lastly, the asymptotic values for Model 1 can be understood from the fact that for $\sigma \to \infty$, φ becomes negative, making the CGHS action dominant over the correction terms. On the other hand, in $\sigma \to -\infty$, $\nabla_\mu\varphi\nabla^\mu\varphi$ saturates toward the zero value, which also means the corrections to the CGHS action become subdominant.

From Fig. 1, we infer the following salient features for Model 2: First, the numerical results match the analytical expressions (19) for $\epsilon = 0$. Second, unlike Model 1, in this model, the saturated values of $\mathcal{R}$ and $\frac{1}{4}\nabla_\mu\varphi\nabla^\mu\varphi$ change significantly at $\sigma \to \infty$ depending on the values of ϵ. For the numerical computation, we have considered negative values of ϵ to make the field theory stable, which follows from the action of this model. Further, $\epsilon > -0.25$ to have a real-valued solution of X as a function of φ.

To understand the reason for the difference in the behavior of the various quantities in the two models, let us rewrite the equation of motion of the Horndeski action (4) in the following form [52–54]:

$$G_{\mu\nu} = 8\pi G_{\rm eff} T_{\mu\nu}^{\rm Corr},$$

where $T_{\mu\nu}^{\rm Corr}$ are the corrections to GR that are rewritten as matter corrections, and $G_{\rm eff}$ is the effective gravitational coupling. Due to the presence of a scalar field, $G_{\rm eff}$ may not be constant. Rewriting the Horndeski equations of motion in the above form leads to [70]

$$G_{\rm eff}^{-1} = 2(G_4 - 2XG_{4,X} + XG_{5,\Phi}). \tag{20}$$

For the above three cases, we have

$$\text{Brans-Dicke} \quad G_{\rm eff}^{-1} = \Phi, \tag{21}$$

$$\text{Corrected Model 1} \quad G_{\rm eff}^{-1} = (\Phi + X\mathcal{G}), \tag{22}$$

$$\text{Corrected Model 2} \quad G_{\rm eff}^{-1} = \left(\Phi + \frac{X\mathcal{G}}{\Phi}\right). \tag{23}$$

The two models have different dependencies on Φ, leading to different asymptotic values. Specifically, the corrected Model 2 has a "duality" under the transformation $\Phi \to \frac{X\mathcal{G}}{\Phi}$. This leads to similar dependence for large Φ and small Φ (assuming that X does not vanish).

Impact on Hawking radiation. So far, our analysis of corrections to the CGHS model is purely classical and without any extra matter field. For the CGHS model, it is known that the BH formation is an inevitable outcome, regardless of the nature of the matter field. For instance, Callan *et al.* [40] considered a massless scalar field f

$$S_m = -\frac{1}{4\pi}\int d^2x\sqrt{-g}\nabla_\mu f \nabla^\mu f, \tag{24}$$

and showed that a closed system of field equations can describe the evolution of dilaton gravity and scalar field. However, to our knowledge, a general analytical solution to the CGHS field equations with the scalar field is yet to be discovered. The numerical and approximate solutions have been obtained [71–74]. Given this, it may not be easy to obtain analytical solutions to the CGHS corrected models (14). However, it is possible to make certain concrete statements assuming that the black hole forms in these models in the presence of scalar field (24).

Keeping this in mind, we qualitatively analyze the effect of higher-derivative terms on the Hawking radiation for the 2D metric (17). The conformal invariance of scalar fields causes the trace of the classical stress tensor to vanish; however, the quantum expectation value of the trace does not vanish. As a result, new source terms of quantum origin enter the geometry's equations of motion, changing the geometry's evolution. The new source terms lead to modifications in the evolution of the geometry [33,34,75–78]. Specifically, for the above scalar field action (24), it has been shown that

$$\langle T^{(f)}\rangle = \frac{1}{24\pi}\mathcal{R}; \quad \langle T^{(f)}_{\pm\pm}\rangle = \frac{1}{12\pi}[\partial^2_\pm\omega - (\partial_\pm\omega)^2 + t_\pm], \tag{25}$$

where $t_\pm$ can be obtained from the boundary conditions. For the CGHS corrected Model 1, the trace of the stress tensor is

$$\langle T^{(f)}\rangle = -\frac{1}{6\pi}(\Box\varphi - \nabla_\mu\varphi\nabla^\mu\varphi + \lambda^2) + \frac{1}{3\pi}\Gamma G e^{2\varphi}[\Box\varphi\nabla_\mu\varphi\nabla^\mu\varphi + 2\nabla^\mu\varphi\nabla^\nu\varphi\nabla_\mu\nabla_\nu\varphi]. \tag{26}$$

As mentioned earlier, obtaining the exact solutions for the CGHS model with matter is hard. Since the corrections are nonlinear, obtaining the solutions is impossible, so the quantization is impractical. However, we can obtain the corrections to the CGHS by studying the linearized equations of motion (16), leading to

$$t_\pm - t_\pm^{\rm CGHS} = \epsilon[2\partial_\pm\bar{\omega}\partial_\pm\delta\omega - \partial^2_\pm\delta\omega] \simeq 2\epsilon\partial_\pm\bar{\omega}\partial_\pm\delta\omega, \tag{27}$$

where the initial conditions for $\bar{\omega}$ and $\delta\omega$ are set at $\sigma_+ \to -\infty$. (For details, see Sec. VI in Ref. [57]). Note that $\partial^2\delta\omega$ can be ignored in the linear regime. The corrections vanish if $\partial_\pm\delta\omega$ vanishes. In that case, the results confirm the earlier results of Jacobson and Unruh [55,56] that the trans-Planckian signatures do not modify the spectrum. However, as discussed earlier, such a thing requires highly fine-tuned parameters $\mathcal{G}$ and λ. As mentioned, evaluation of $\langle T^f_{\pm\pm}\rangle$ exactly requires the exact solutions of the modified field equations in terms of null coordinates $\sigma^\pm$, which will be discussed in future work.

Discussions. We have provided a systematic procedure to include a higher-derivative correction to the CGHS model and study the end stages of black hole evaporation. One of the advantages of this approach is that the higher-order corrections do not introduce Ostrogradsky instability (ghosts). Thus, the generic action (14) can be used to understand the higher-derivative corrections to the s wave of the 4D Hawking radiation. Our analysis provides a first step to investigate the effects of Hawking radiation from PBH in the mass range 10^{16}–10^{17} gm. Specifically, the positron annihilation rate implied by INTEGRAL's measurements of the Galactic 511 keV line has been used to place strong constraints on PBH in these mass ranges [79,80]. However, these results assume that the Hawking flux for these ranges is the same as that of larger BHs. However, as discussed above (27), there can be appreciable corrections to Hawking radiation for these mass ranges. Thus, our analysis can further constrain PBH as dark matter.

Our analysis can possibly provide a way to understand the effect of higher-derivative terms on the page curve [81–83]. For the BHs in GR, it is known that BH entropy increases monotonically until half its lifetime and decreases monotonically until the BH completely evaporates. If the higher-derivative corrections modify the Hawking flux for PBHs, there can likely be corrections to Page's analysis at the end stages of BH evaporation. This requires detailed numerical analysis along the lines of [73,74]. This is currently under investigation.

A mapping from 4D metric action (Einstein-Hilbert) to an effective two-dimensional model can be obtained through spherical reduction or via near horizon approximation [84–86]. However, the two approaches are quite distinct. In the case of spherical reduction, the Einstein-Hilbert action reduces to a dilaton gravity theory considering the areal radius as a field. This reduction is valid everywhere on the two-dimensional manifold, and one can mimic the radial collapse of matter leading to stationary black hole. However, the near horizon approximation (of a nondegenerate horizon) describes the two-dimensional metric in the t-r plane to be Rindler metric near the horizon. In this case, the areal radius do not play any role in determining the dynamics of the 2D theory and describes a stationary black hole solution. It will be interesting to explore the consequences of corrections on BMS symmetries [85,86].

For the quantum corrected 4D reduced Einstein-Hilbert action, Kazakov and Solodukhin showed that quantum corrections deform the line element of a Schwarzschild black hole leading to a nonsingular space-time [87]. It will be interesting to see whether the action (14) can lead to a nonsingular space-time. The above analysis can be extended to other 2D gravity models like Liouvelle gravity, JT gravity. These are currently under investigation.

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S. M. and T. P. contributed equally to this work.

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